

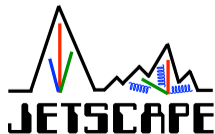
Far-off-equilibrium journeys through the QCD phase diagram

Ulrich Heinz



THE OHIO STATE UNIVERSITY

Symposium on collective flow in nuclear matter:
a celebration of Art Poskanzer's life and career
LBNL, December 9-10, 2022

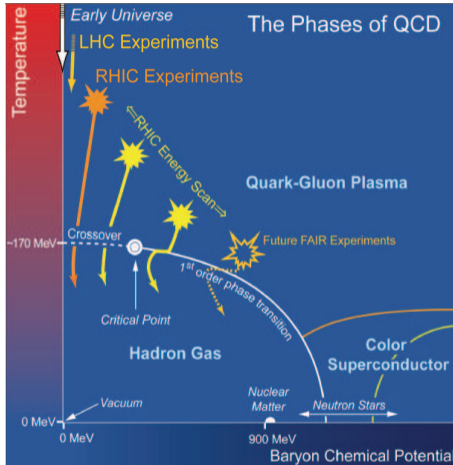




<https://www.flickr.com/photos/posk/229751790/in/album-72157594261031115/>



Exploring the QCD phase diagram: emergent phenomena in non-Abelian media



Phenomena:

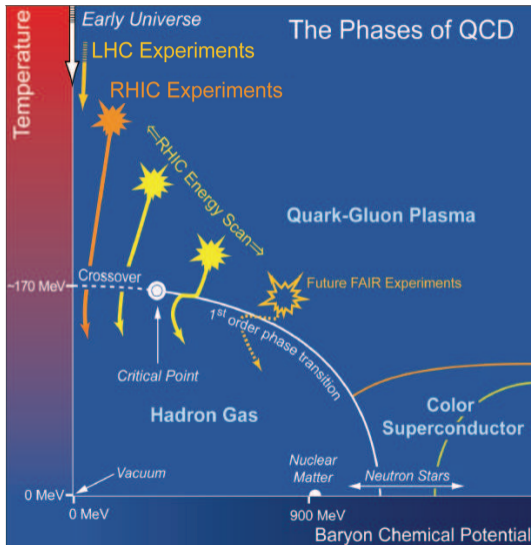
- (De-)confinement (clustered vs. homogeneous states)
- chiral symmetry restoration
- (almost) perfect fluidity
- order of the phase transition(s)
- critical end point?

(from the 2007 NSAC Nuclear Physics LRP)

What happened in the early universe about $10 \mu\text{s}$ after the Big Bang?

What changes when you **dope** the matter that filled the early universe with extra quarks/baryon number?

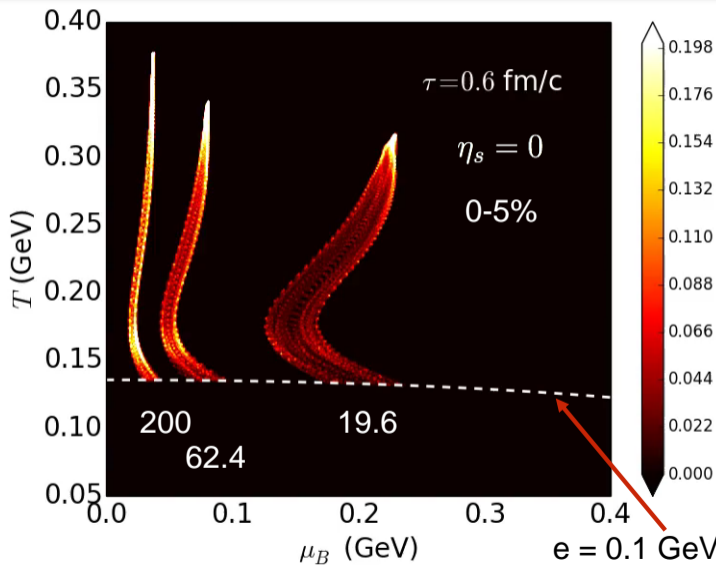
Exploring the QCD phase diagram: emergent phenomena in non-Abelian media



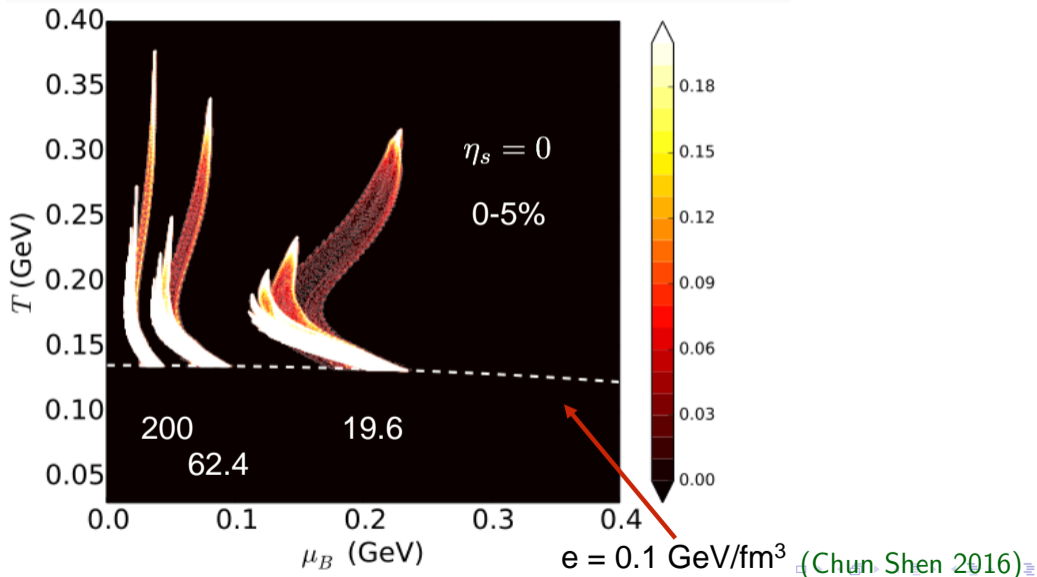
Probes:

- **Collective flow**
- Jet modification and quenching
- Thermal electromagnetic radiation
- Critical fluctuations
- . . .

Compass for the QCD phase diagram



Compass for the QCD phase diagram



Isentropic evolution (ideal fluid dynamics)

conformal case ($m = 0$):

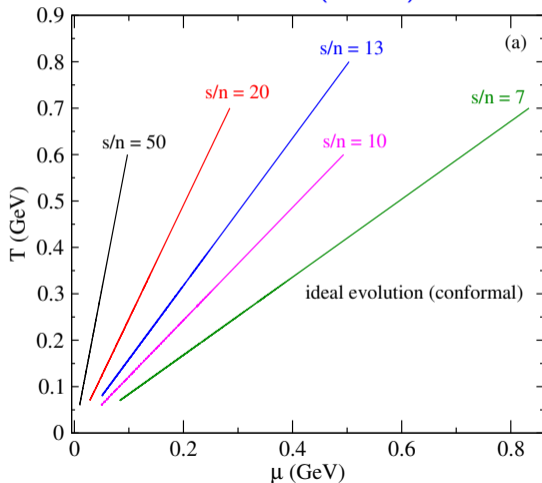
Consider a simple model:

noninteracting gas of gluons (bosons, $m_g = 0$) and quarks & antiquarks (fermions, $m_q = m$) with non-zero net baryon number in chemical equilibrium

More details:

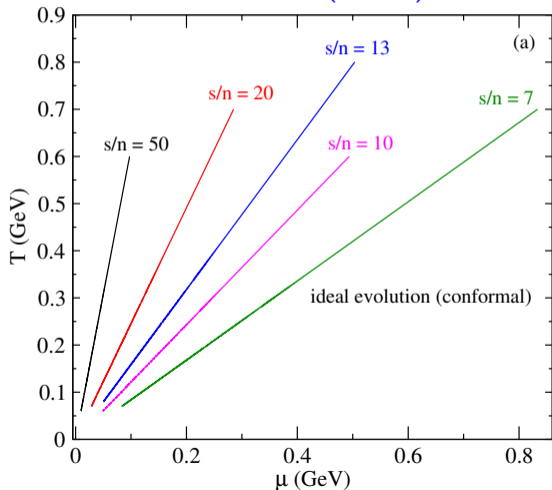
Chattopadhyay, UH, Schäfer, 2209.10483;

Florkowski, Maksymiuk, Ryblewski, 1710.07905

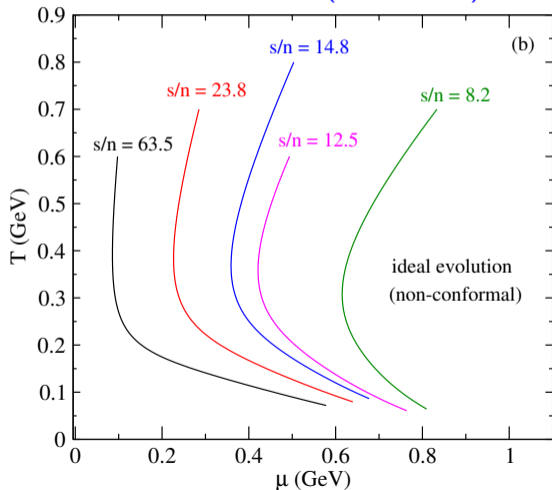


Isentropic evolution (ideal fluid dynamics) ($T, n \rightarrow 0 \Rightarrow \mu \rightarrow m$)

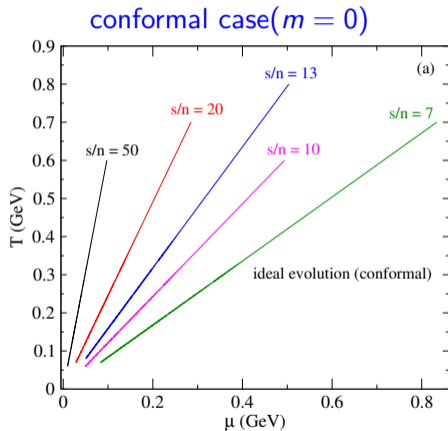
conformal case ($m = 0$)



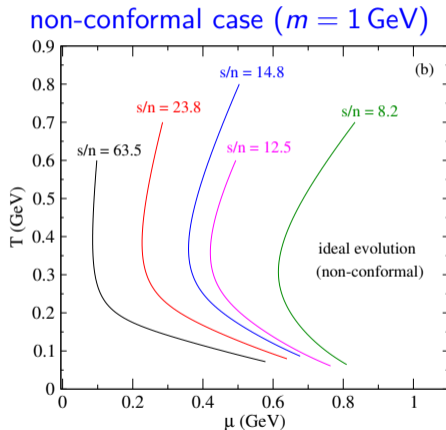
non-conformal case ($m = 1$ GeV)



What should we expect for dissipative expansion?



???



Viscous entropy production $\implies s/n$ increases

\implies (naively!) expect μ/T to decrease and expansion trajectories to shift to the left

Second-order Chapman Enskog hydrodynamics

Denicol *et al.*, 1407.4767; A. Jaiswal, Ryblewski, Strickland, 1407.7231

Simple flow model: 1-d Bjorken expansion:

$$\frac{de}{d\tau} = -\frac{1}{\tau} \left(e + P + \Pi - \pi \right),$$

$$\frac{dn}{d\tau} = \frac{n}{\tau},$$

$$\frac{d\Pi}{d\tau} + \frac{\Pi}{\tau_R} = -\frac{\beta_\Pi}{\tau} - \delta_{\Pi\Pi} \frac{\Pi}{\tau} + \lambda_{\Pi\pi} \frac{\pi}{\tau},$$

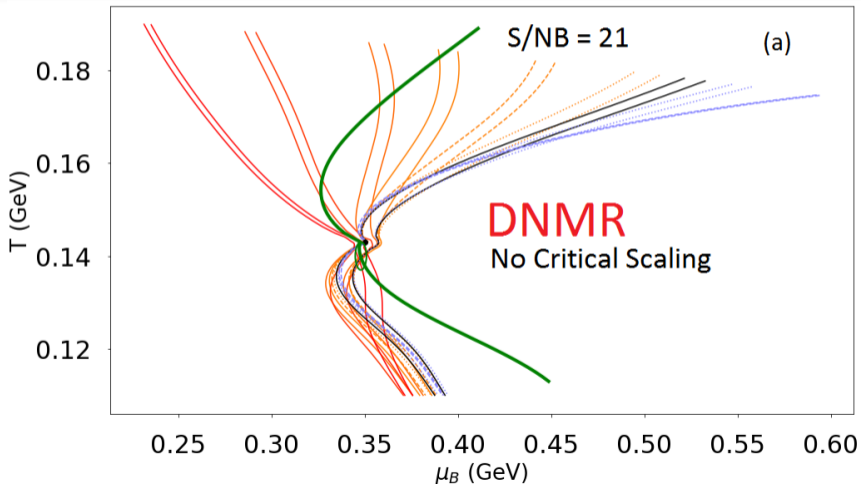
$$\frac{d\pi}{d\tau} + \frac{\pi}{\tau_R} = \frac{4}{3} \frac{\beta_\pi}{\tau} - \left(\frac{1}{3} \tau_{\pi\pi} + \delta_{\pi\pi} \right) \frac{\pi}{\tau} + \frac{2}{3} \lambda_{\pi\Pi} \frac{\Pi}{\tau}.$$

Transport coefficients from kinetic theory for a quark-gluon gas with nonzero quark mass

Chattopadhyay, UH, Schäfer, 2209.10483

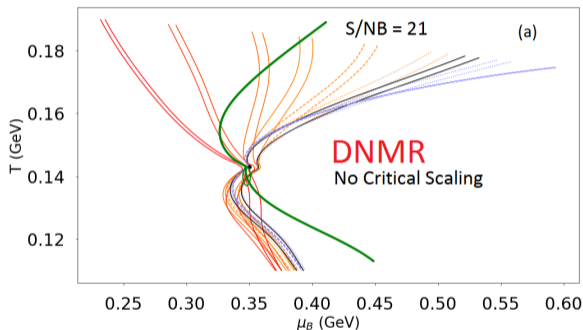
Dissipative fluid dynamic expansion trajectories

Dore, Noronha-Hostler, McLaughlin, 2007.15083



Dissipative fluid dynamic expansion trajectories

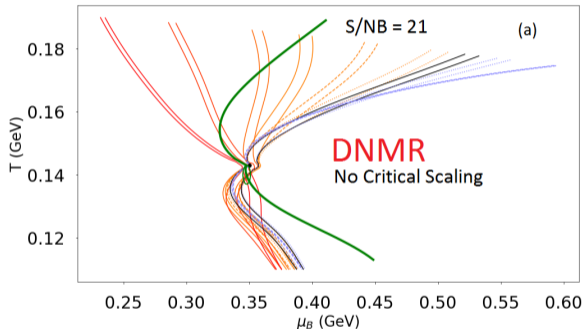
Dore, Noronha-Hostler, McLaughlin, 2007.15083



How can some of these trajectories evolve towards larger μ/T and therefore smaller s/n ?!

Dissipative fluid dynamic expansion trajectories

Dore, Noronha-Hostler, McLaughlin, 2007.15083

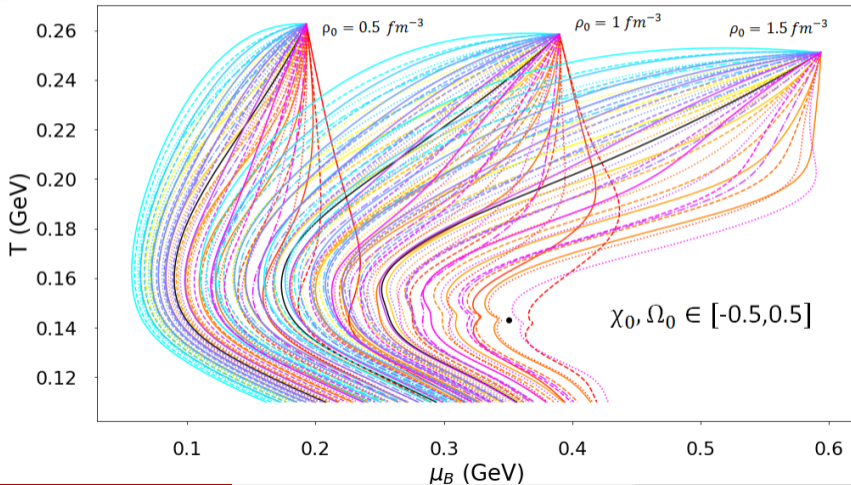


Correct question:

How can some of these trajectories evolve towards larger μ/T
and therefore smaller equilibrium s_{eq}/n ?

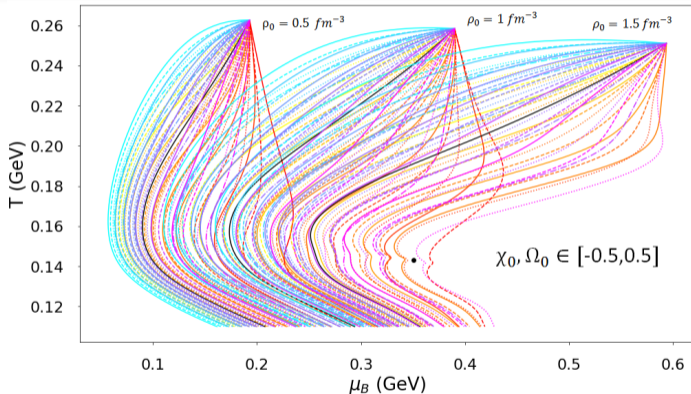
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How can some of these trajectories evolve towards larger μ/T
and therefore smaller equilibrium s_{eq}/n ?

Dissecting the problem Chattopadhyay, UH, Schäfer, 2209.10483

- **The equilibrium state maximizes the entropy**

⇒ for any given point (T, μ) in the phase diagram, $s/n \leq (s_{\text{eq}}/n)(T, \mu)$:

$$\frac{s}{n} = \frac{1}{n_{\text{eq}}(T, \mu)} \left(s_{\text{eq}}(T, \mu) - c_{\Pi}(T, \mu) \Pi^2 - c_{\pi}(T, \mu) \pi^{\mu\nu} \pi_{\mu\nu} + c_n(T, \mu) n^{\mu} n_{\mu} \right) + \dots$$

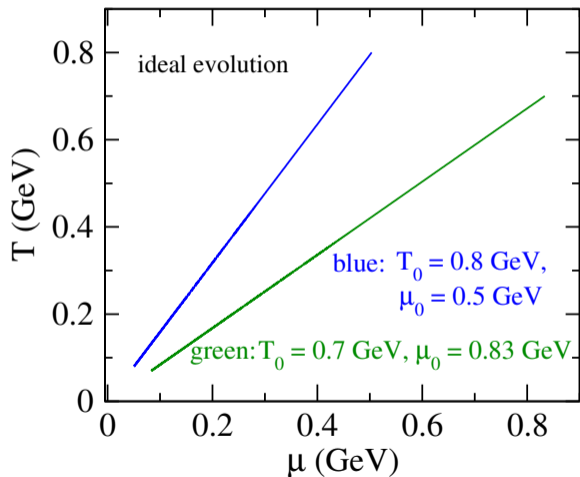
- **Dissipation creates entropy:** $T \partial_{\mu} S^{\mu} = \pi^{\mu\nu} \sigma_{\mu\nu} - \Pi \theta - T n^{\mu} \nabla_{\mu} \alpha + \dots \geq 0$,

where $\sigma_{\mu\nu}$ is the shear flow tensor, θ is the scalar expansion rate, and $\alpha \equiv \mu/T$.

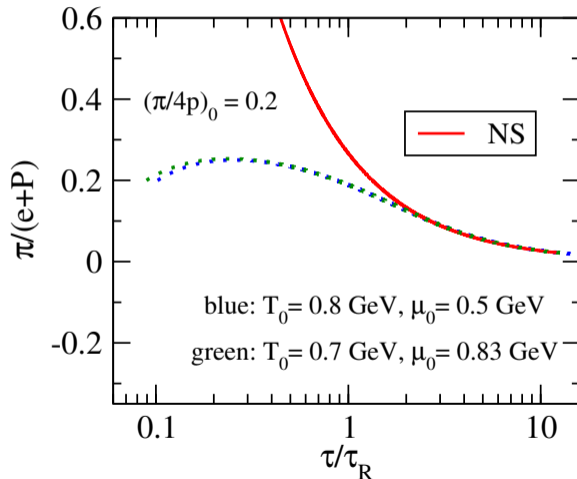
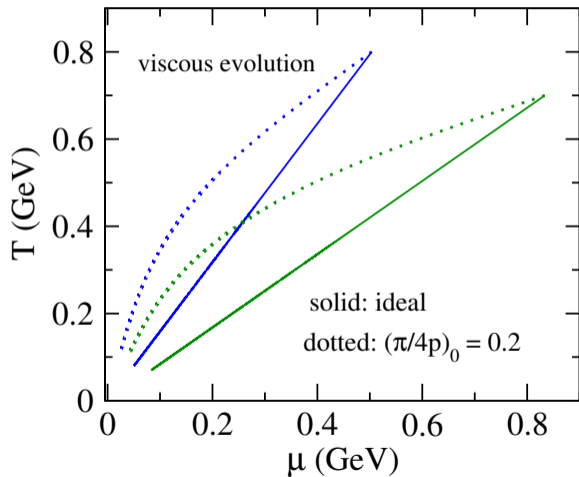
- **During evolution, entropy flows between the equilibrium and non-equilibrium sectors. For Bjorken flow**

$$\frac{d(s_{\text{eq}}\tau)}{d\tau} = \frac{\pi - \Pi}{T}.$$

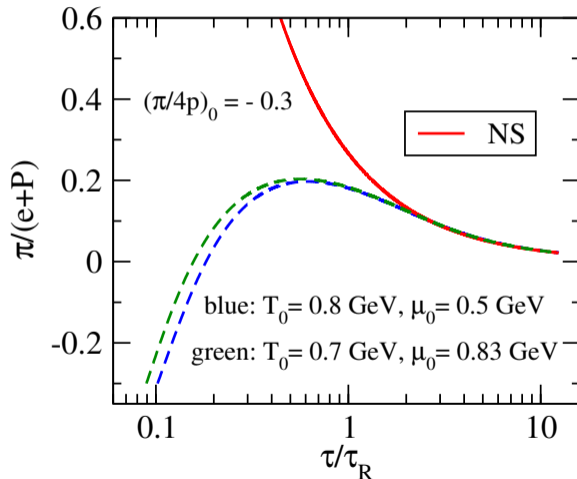
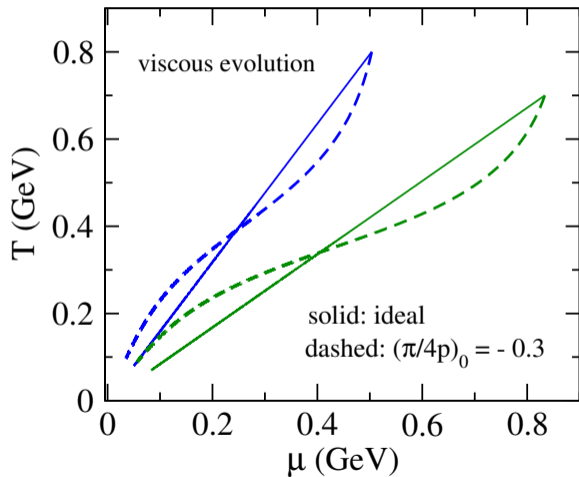
I. The conformal case $m = 0$



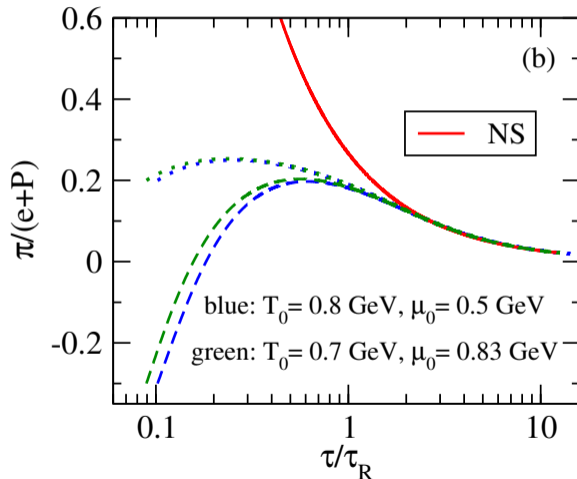
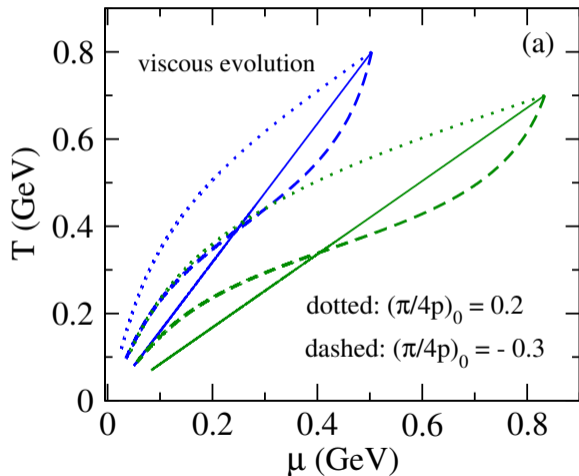
I. The conformal case $m = 0$; $\eta/s = 10/4\pi$



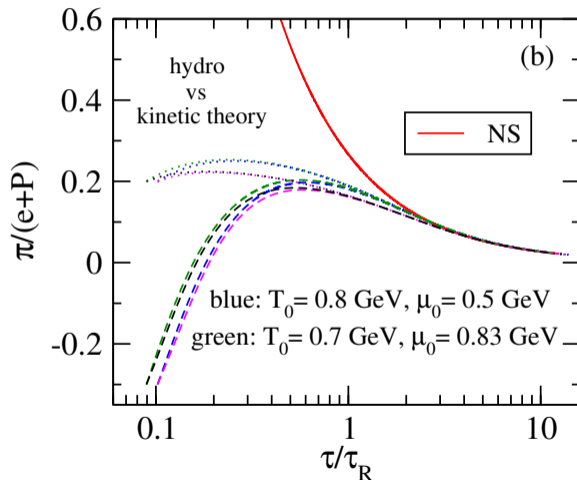
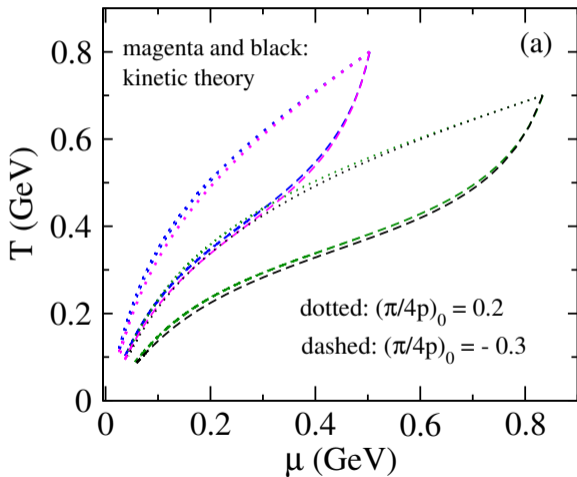
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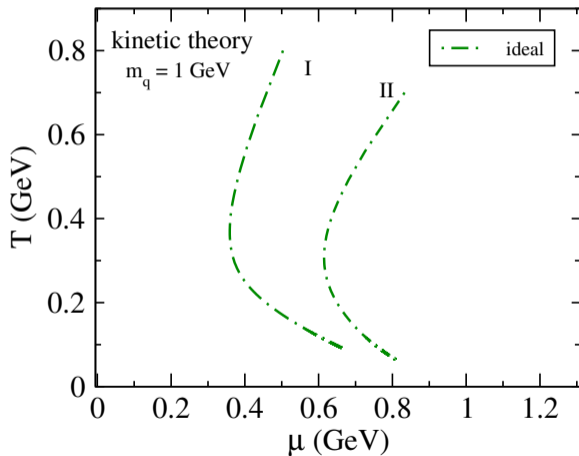
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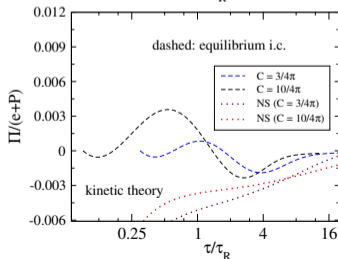
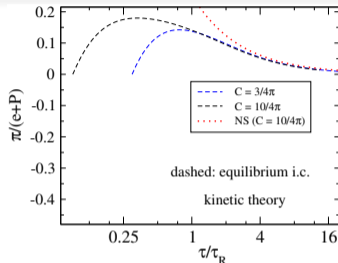
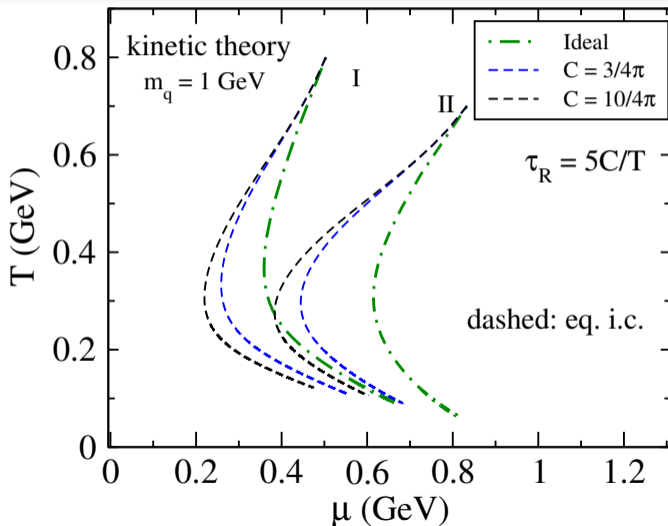
Is this an artifact of the hydrodynamic approximation? No!



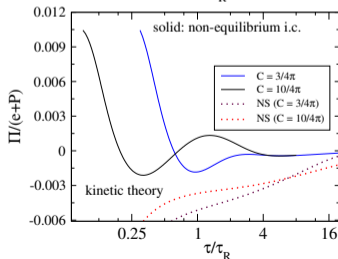
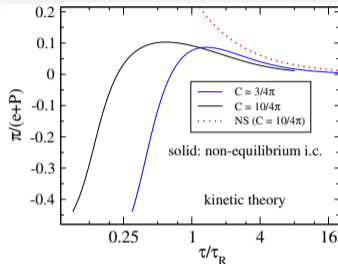
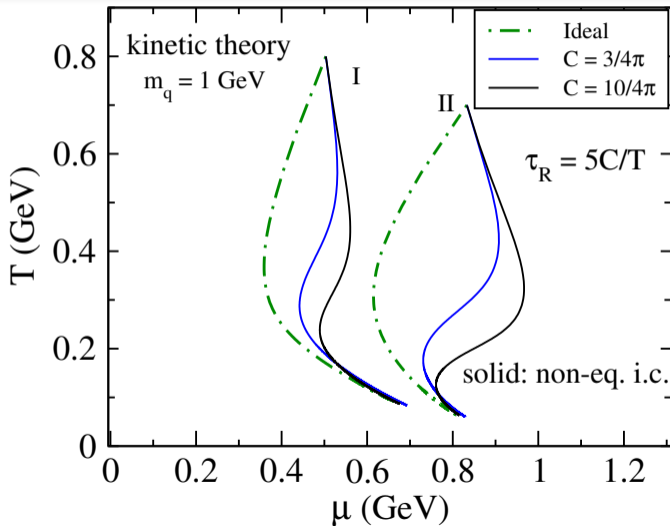
II. The non-conformal case for $m = 1$ GeV, ideal



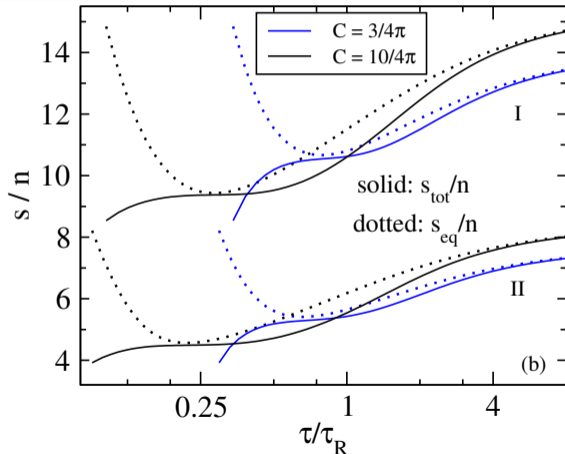
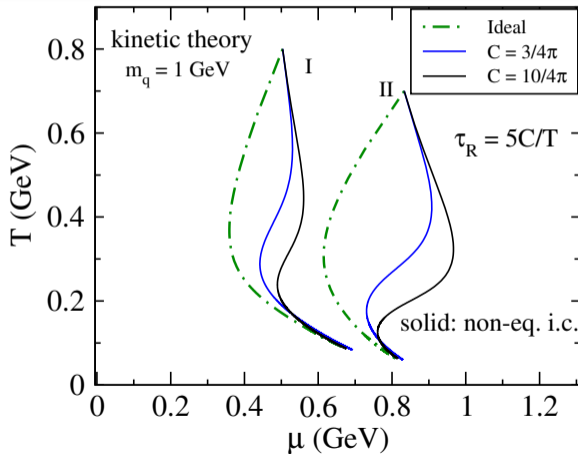
II. The non-conformal case for $m = 1 \text{ GeV}$, $\pi_0 = \Pi_0 = 0$



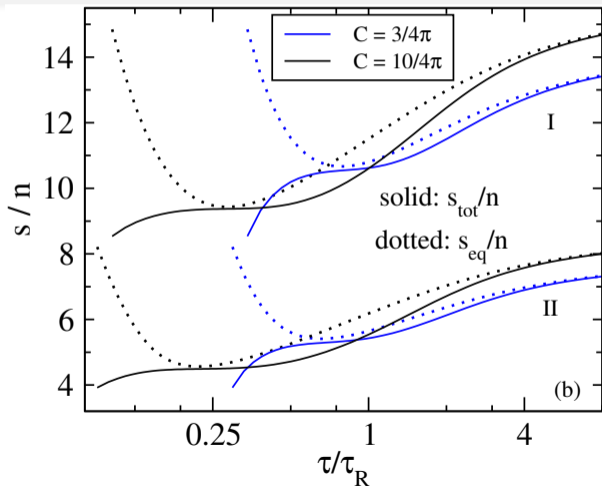
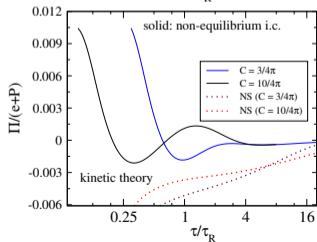
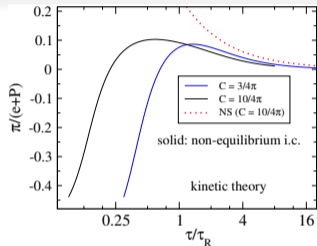
II. Non-conformal case: $m = 1 \text{ GeV}$, $[(\pi - \Pi)/(e + p)]_0 = -0.45$



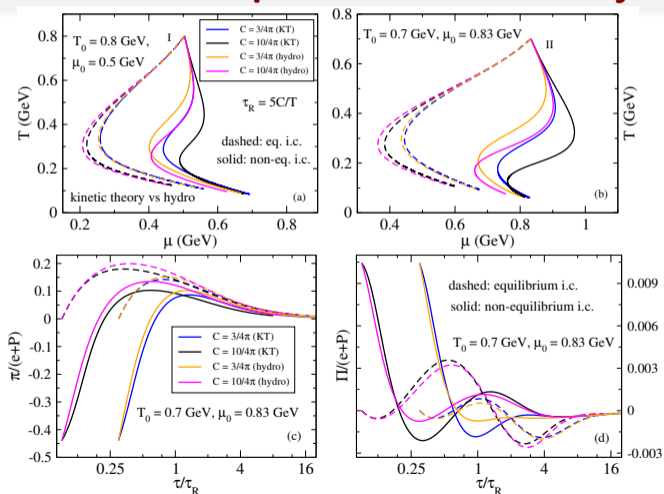
II. Non-conformal case: $m = 1 \text{ GeV}$, $[(\pi - \Pi)/(e + p)]_0 = -0.45$: Entropy production



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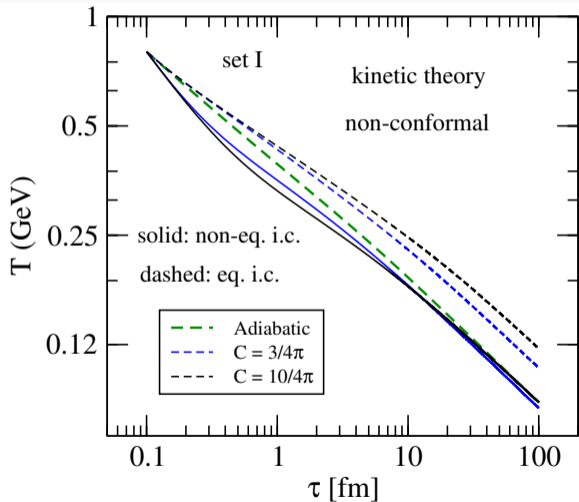


Kinetic theory vs. hydrodynamics: quantitative differences but qualitative consistency



“Viscous cooling”

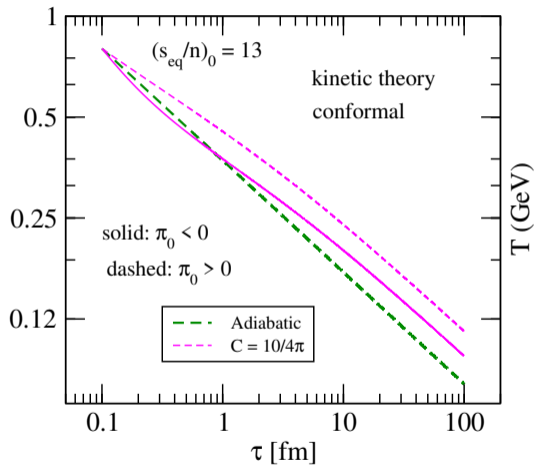
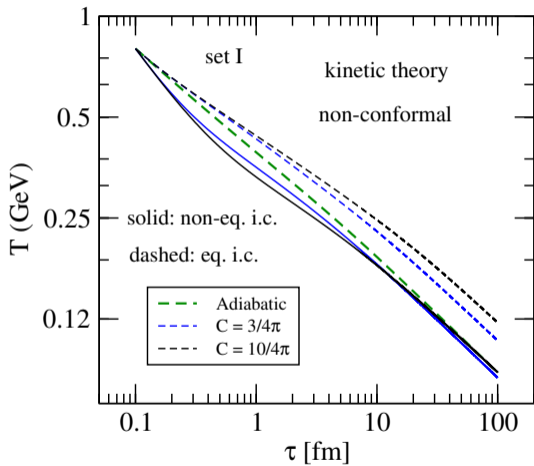
Chattopadhyay, UH, Schäfer, in preparation



- For equilibrium initial conditions system experiences **viscous heating** – cools more slowly than ideal fluid. Well-known effect.
- For non-equilibrium initial conditions with $[(\pi - \Pi)/(e + p)]_0 < 0$ system experiences **viscous cooling** – cools initially faster than ideal fluid.
- As the system approaches local equilibrium at late times (**Navier-Stokes stage**), **viscous cooling** turns into **viscous heating**.
- Both viscous heating and cooling phenomena last longer for larger relaxation times (i.e. for larger viscosities).

“Viscous cooling”

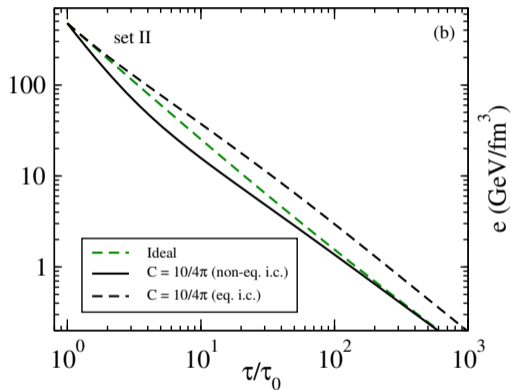
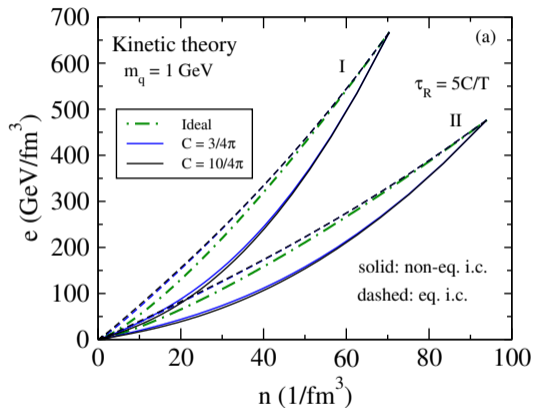
Chattopadhyay, UH, Schäfer, in preparation



Qualitatively similar but less pronounced viscous cooling effect seen in the conformal (massless) limit.

“Viscous cooling”

Chattopadhyay, UH, Schäfer, in preparation



Viscous cooling reduces the thermal energy density faster than for ideal expansion, both as a function of time and as a function of net baryon density.

So what?

- **Widespread folklore that must be un-learned:**
“Dissipation can be understood as internal friction that causes *viscous heating*.”
- For **far-off-equilibrium systems** there are situations where **negative entropy can flow from the non-equilibrium to the equilibrium sector**, causing the temperature and thermal energy density of the system to **decrease faster** than in an isentropically expanding ideal fluid \Rightarrow “**viscous cooling**”
- **Viscous cooling arises only for far-off-equilibrium conditions where the bulk and shear viscous pressures start out with the “wrong” signs** (i.e. opposite to their Navier-Stokes values towards which they evolve at late times).
- In non-conformal systems with sufficiently extreme initial conditions **viscous cooling** can cause the system to evolve initially **towards larger μ/T** values, seemingly violating deeply ingrained rules from our childhood.
- Unclear whether and under which conditions such far-off-equilibrium initial conditions can arise naturally in heavy-ion collisions.
- Interesting question: can *macroscopic* systems be initialized in analogous far-off-equilibrium initial states, in order to explore the dynamics of viscous cooling in the laboratory?

Thank you!

And many thanks to my collaborators and discussion partners on this project:

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Amaresh Jaiswal, Sunil Jaiswal, Dananjaya Liyanage, Mike McNelis,
Subrata Pal, Thomas Schäfer, and Mike Strickland**

