Two-photon effects in electromagnetic processes on nucleon

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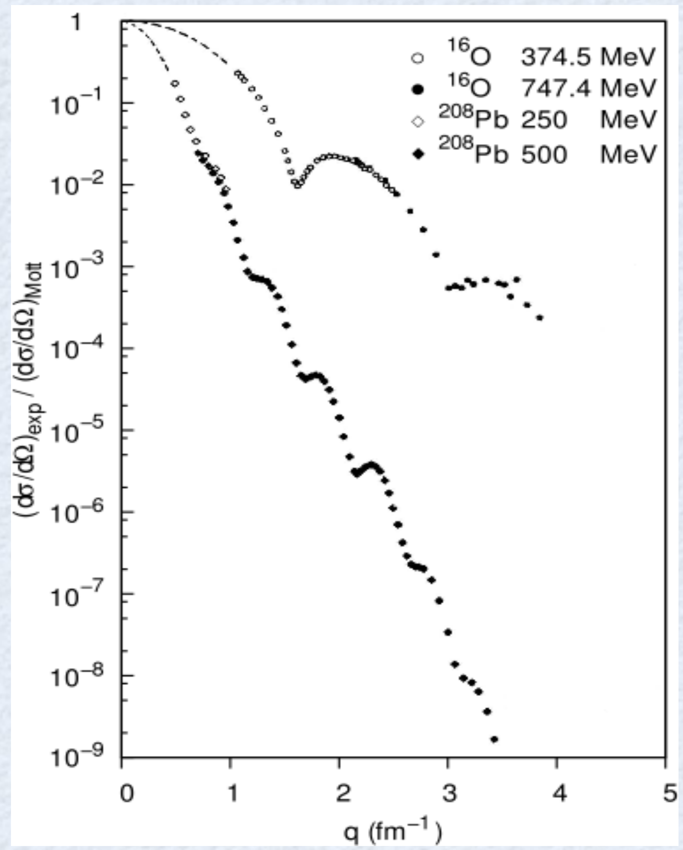
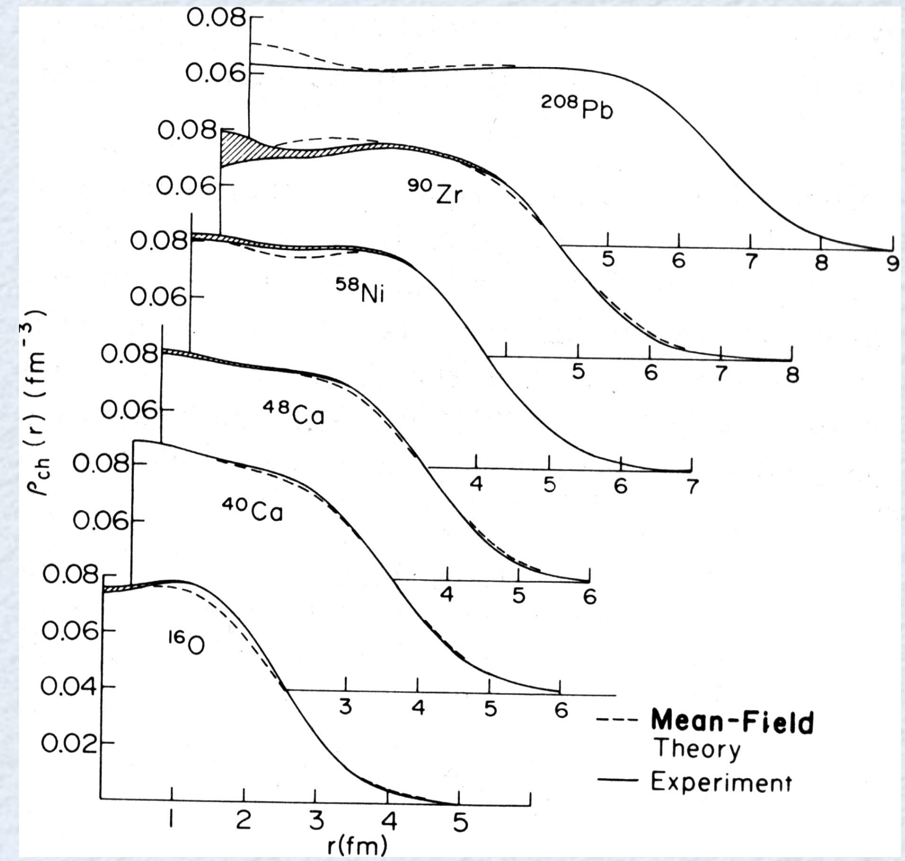
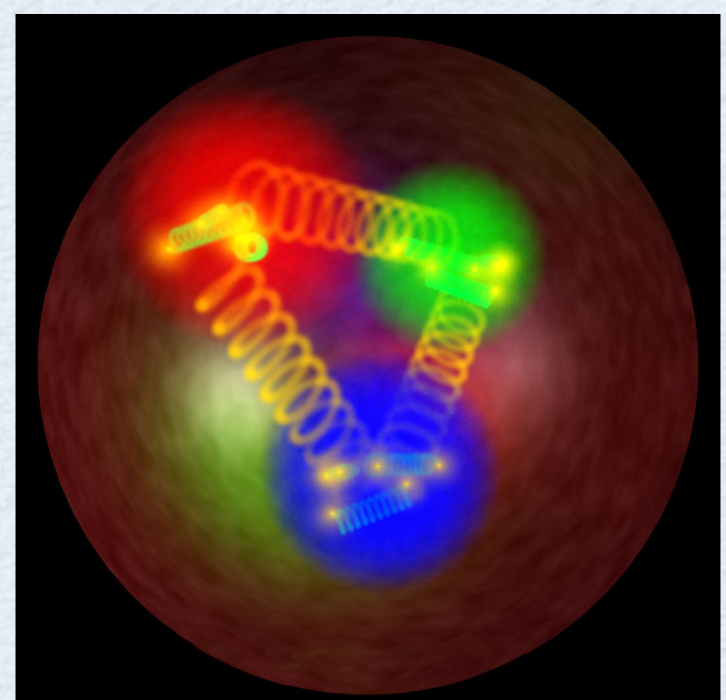
Electroweak probe: precision tool of hadron structure

time honored tool:
electroweak probe

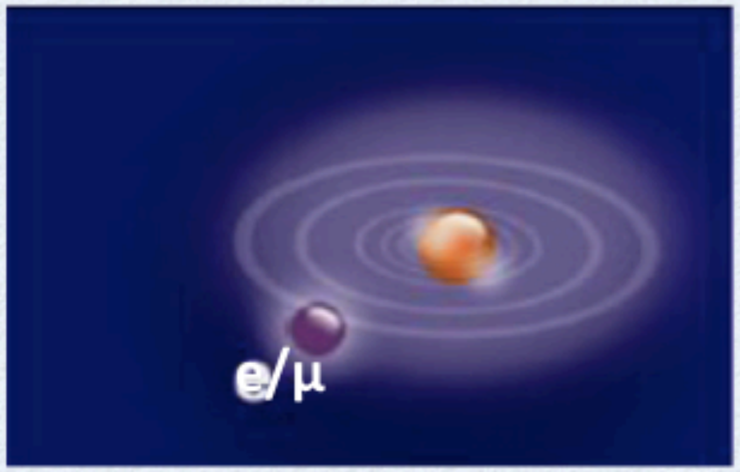


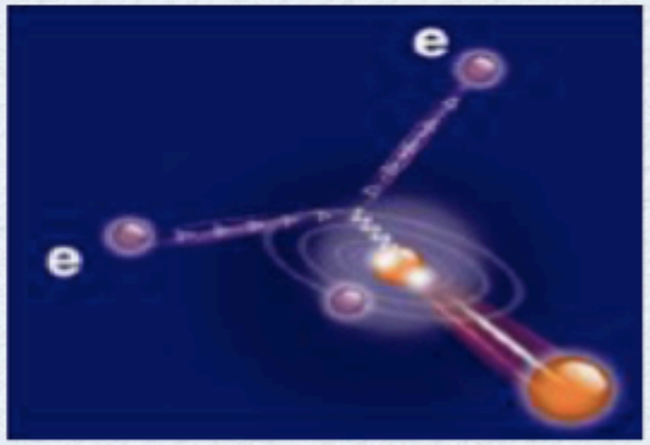


how accurate do we
know the proton size
and its spatial structure?

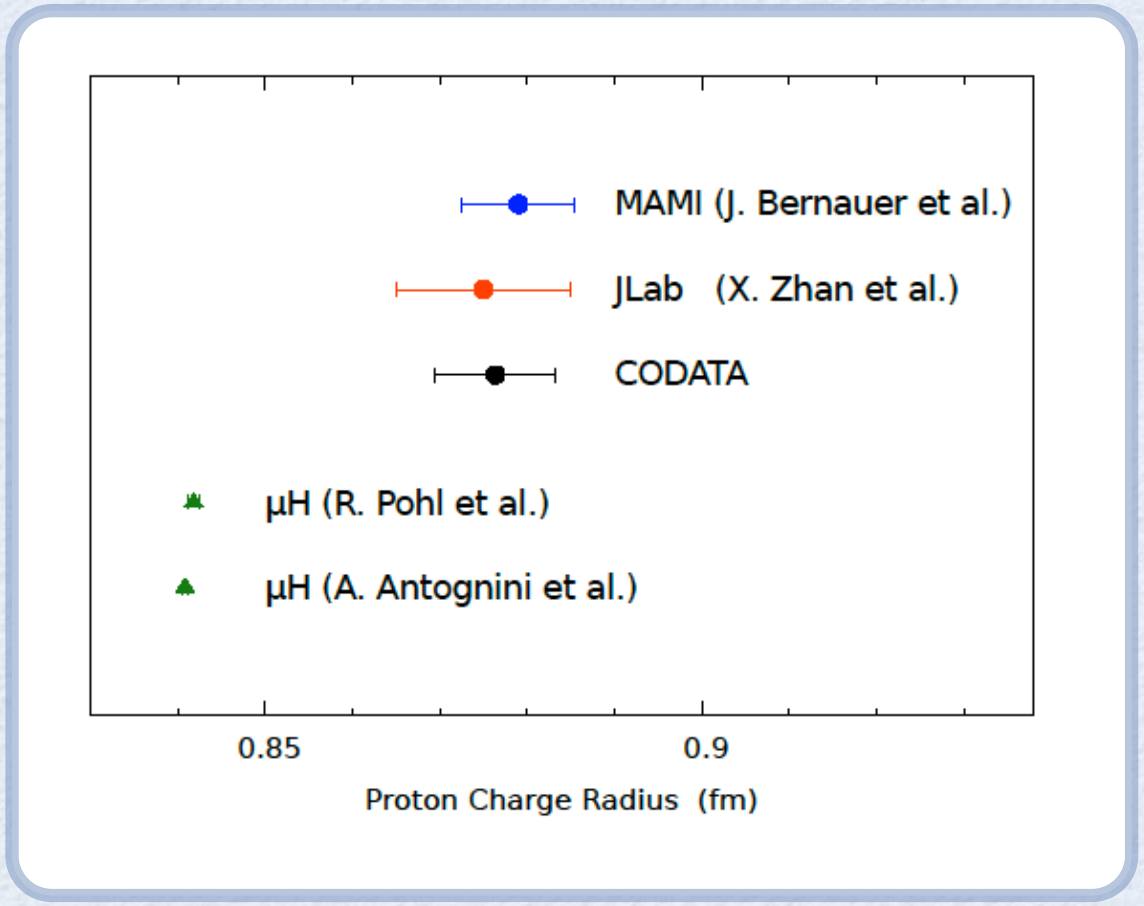


Two-photon exchange in Hydrogen spectroscopy





Proton radius puzzle



μH data:

Pohl et al. (2010)

Antognini et al. (2013)

$R_E = 0.8409 \pm 0.0004$ fm

5.6 σ difference !?

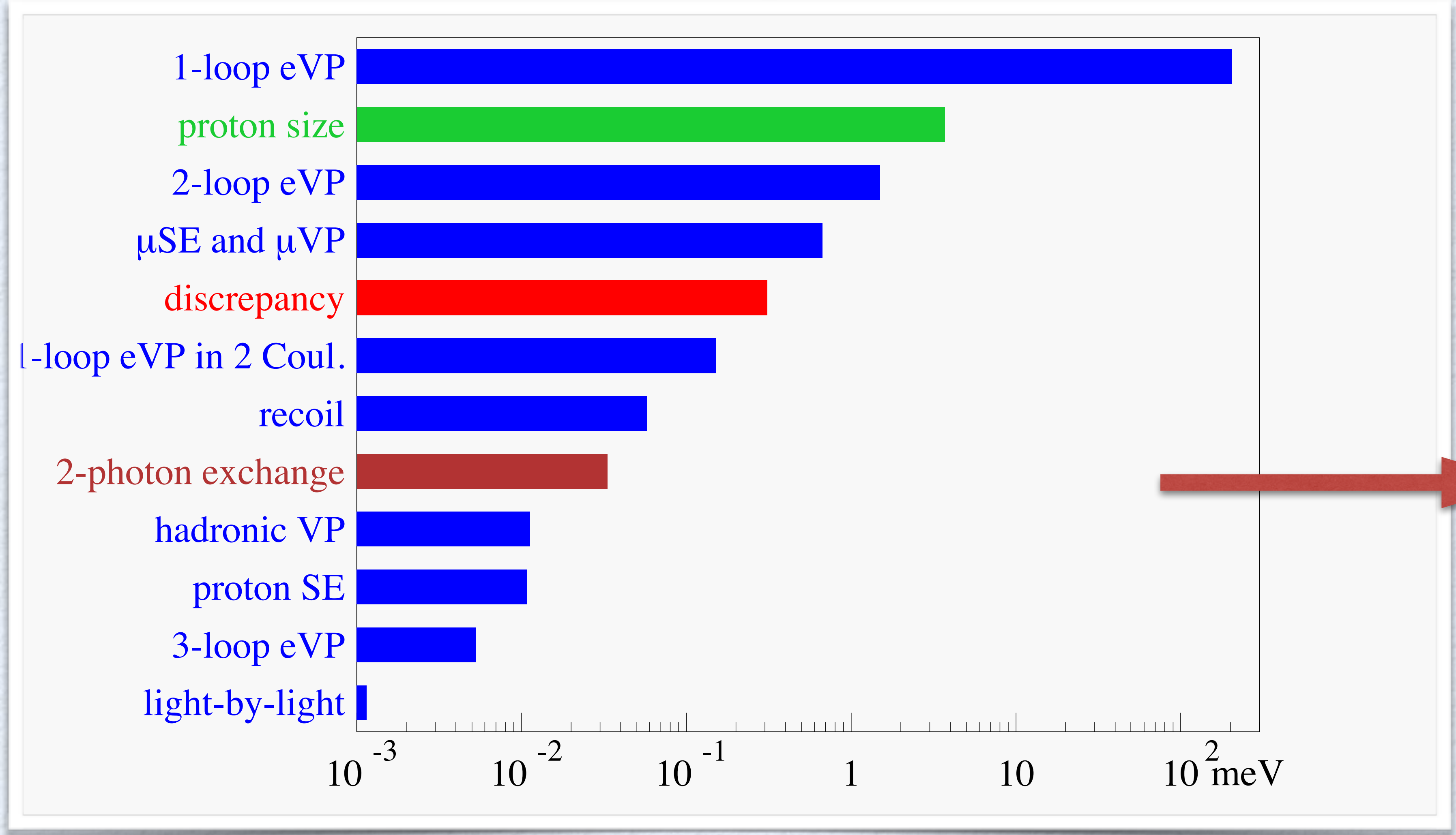
$R_E = 0.8775 \pm 0.0051$ fm

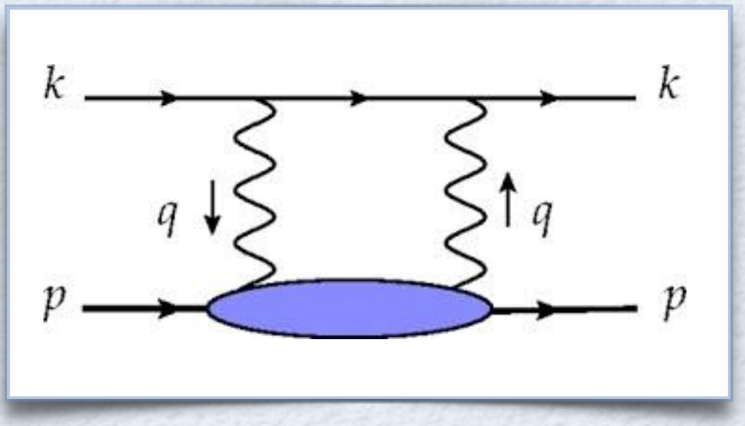
ep data:

CODATA (2012)

Lamb shift: status of known corrections

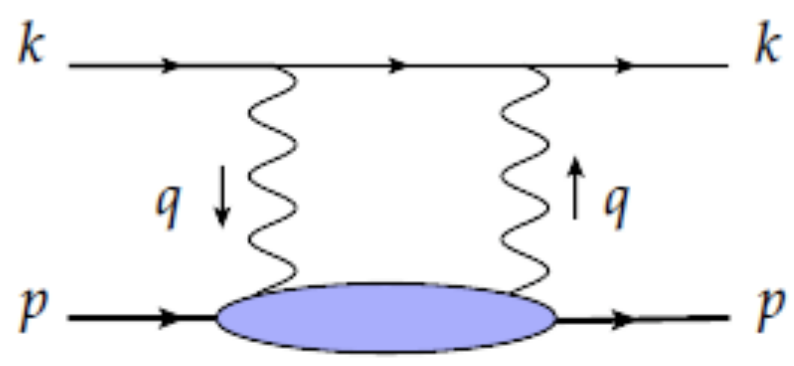
µH Lamb shift: summary of corrections





Two-photon exchange: largest theoretical uncertainty

Lamb shift: hadronic corrections



$$T^{\mu\nu}(p,q) = \frac{i}{8\pi M}\int d^4x\, e^{iqx}\langle p|Tj^\mu(x)j^\nu(0)|p\rangle$$
$$= \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) T_1(\nu, Q^2)$$
$$+ \frac{1}{M^2}\left(p^\mu - \frac{p\cdot q}{q^2}q^\mu\right)\left(p^\nu - \frac{p\cdot q}{q^2}q^\nu\right) T_2(\nu, Q^2)$$

- Lower blob contains both elastic (nucleon) and in-elastic states

  Information contained in **forward, double virtual Compton scattering**

  → **Hadron physics input required**

  - Described by two amplitudes T1 and T2: function of energy ν and virtuality Q^2

  - Imaginary parts of T1 , T2: unpolarized structure functions of proton

$$\mathrm{Im}\, T_1(\nu, Q^2) = \frac{1}{4M} F_1(\nu, Q^2)$$
$$\mathrm{Im}\, T_2(\nu, Q^2) = \frac{1}{4\nu} F_2(\nu, Q^2)$$

- ΔE evaluated through an integral over Q^2 and ν

$$\Delta E = \Delta E^{el} + \Delta E^{subtr} + \Delta E^{inel}$$

→ Elastic state: involves **nucleon form factors**

→ Subtraction: involves **nucleon polarizabilities**

→ Inelastic, dispersion integrals: involves **structure functions F1, F2**

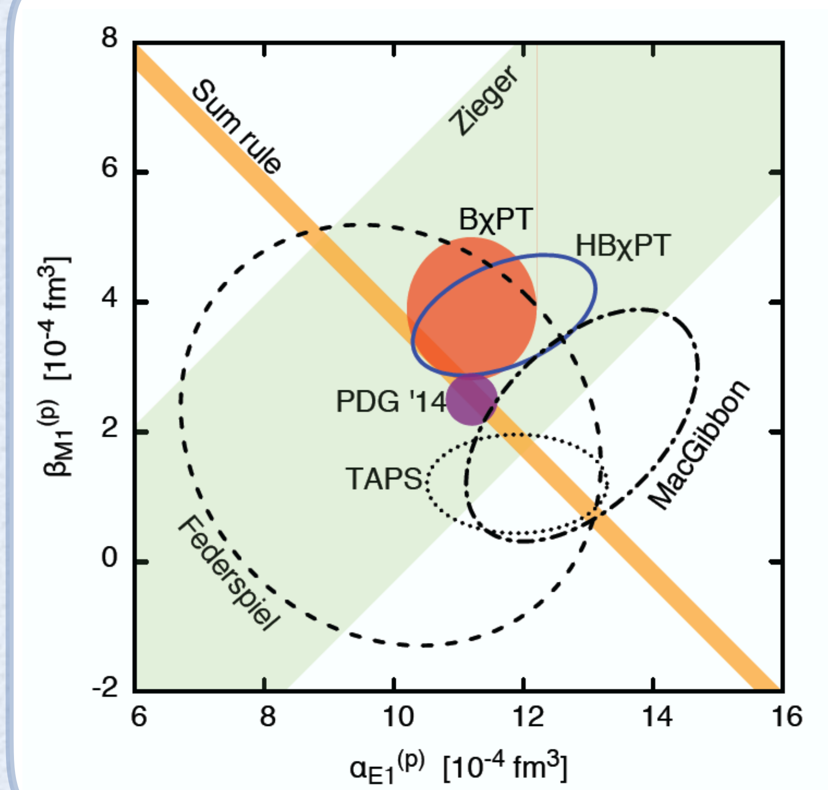
Lamb shift: subtraction function

- low-energy expansion of forward, doubly virtual Compton scattering contains a subtraction term $T_1(0,Q^2)$

effective Hamiltonian:

$$\mathcal{H} = -\frac{1}{2} 4\pi\alpha_E \vec{E}^{\,2} - \frac{1}{2} 4\pi\beta_M \vec{B}^{\,2}$$

electric / magnetic polarizabilities



Theory analyses:

BChPT
Lensky, Pascalutsa (2010)

HBChPT
Griesshammer, McGovern, Phillips (2013)

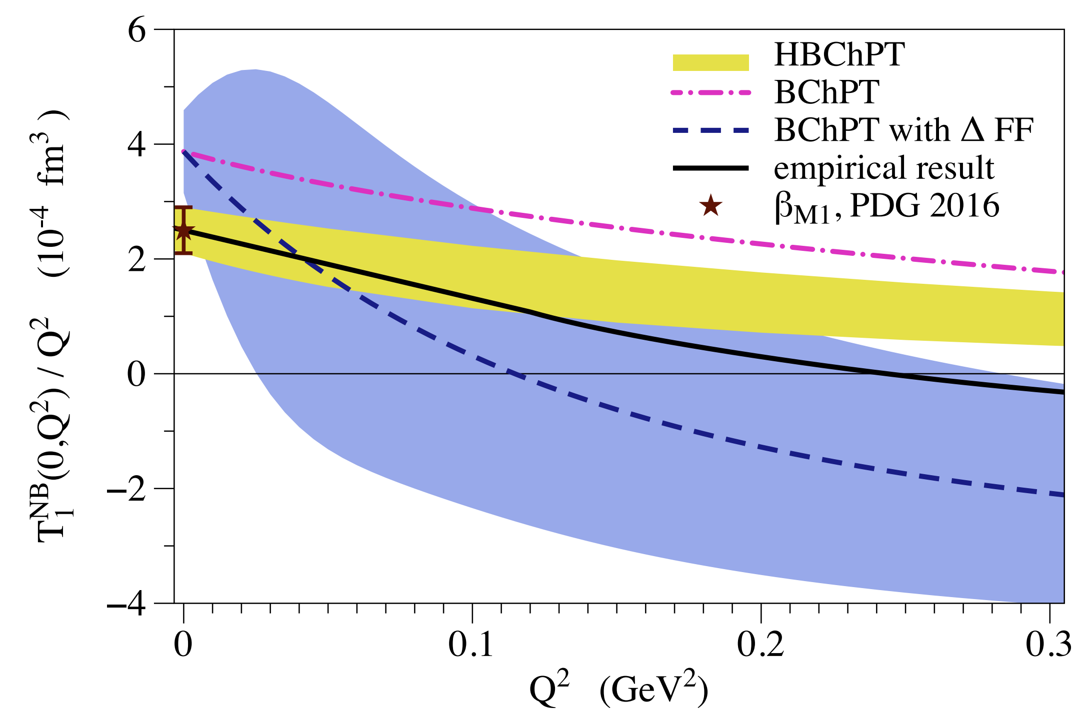
PDG '14 values:

$\alpha_E = (11.2 \pm 0.2) \times 10^{-4}\ \mathrm{fm}^3$

$\beta_M = (2.5 \pm 0.4) \times 10^{-4}\ \mathrm{fm}^3$

- subtraction term

$$T_1^{\mathrm{non-Born}}(0,Q^2) = Q^2 \beta_M + \mathcal{O}(Q^4)$$



HBChPT
Birse, McGovern (2012)

BChPT
Lensky, Hagelstein, Pascalutsa, Vdh (2018)

Empirical result
(based on HERA data)
Tomalak, Vdh (2016)

Lamb shift: hadronic corrections summary

polarizability correction on 2S level in μH in μeV

dispersive estimates

HBChPT

HBChPT + dispersive

BChPT

(μeV)	Pachucki [9]	Martynenko [10]	Nevado and Pineda [11]	Carlson and Vanderhaeghen [12]	Birse and McGovern [13]	Gorchtein et al. [14]	LO-BχPT [this work]
$\Delta E_{2S}^{(\mathrm{subt})}$	1.8	2.3	–	5.3 (1.9)	4.2 (1.0)	−2.3 (4.6)[a]	−3.0
$\Delta E_{2S}^{(\mathrm{inel})}$	−13.9	−13.8	–	−12.7 (5)	−12.7 (5)[b]	−13.0 (6)	−5.2
$\Delta E_{2S}^{(\mathrm{pol})}$	−12 (2)	−11.5	−18.5	−7.4 (2.4)	−8.5 (1.1)	−15.3 (5.6)	$-8.2(^{+1.2}_{-2.5})$

[a] Adjusted value; the original value of Ref. [14], +3.3, is based on a different decomposition into the 'elastic' and 'polarizability' contributions
[b] Taken from Ref. [12]

[9] K. Pachucki, Phys. Rev. A **60**, 3593 (1999).
[10] A. P. Martynenko, Phys. Atom. Nucl. **69**, 1309 (2006).
[11] D. Nevado and A. Pineda, Phys. Rev. C **77**, 035202 (2008).
[12] C. E. Carlson and M. Vanderhaeghen, Phys. Rev. A **84**, 020102 (2011).
[13] M. C. Birse and J. A. McGovern, Eur. Phys. J. A **48**, 120 (2012).
[14] M. Gorchtein, F. J. Llanes-Estrada and A. P. Szczepaniak, Phys. Rev. A 87, **052501** (2013).
[LO-BχPT] Alarcon, Lensky, Pascalutsa, EPJC (2014) 74:2852

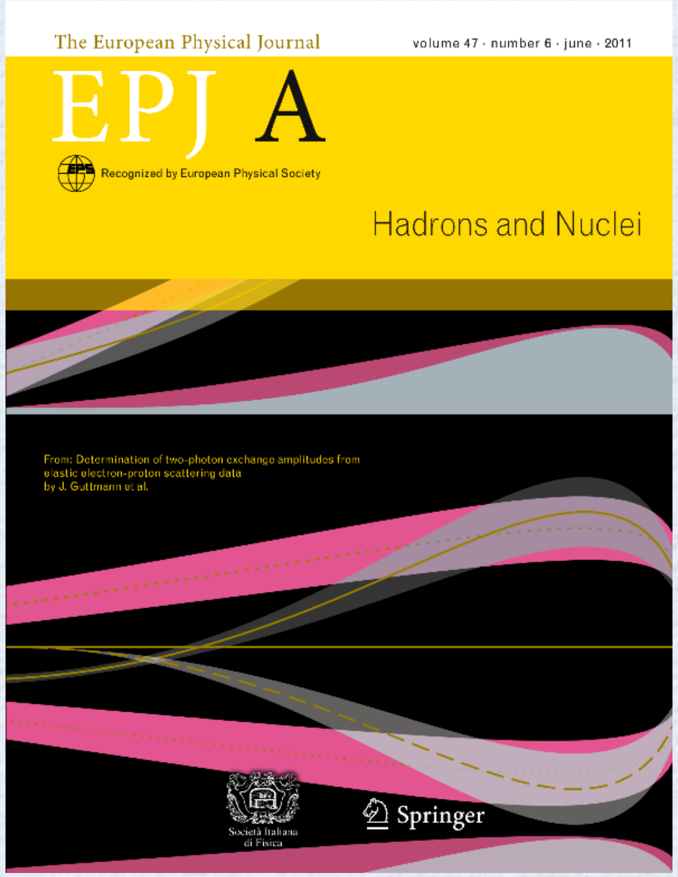
- elastic contribution on 2S level: ΔE_{2S} = -23 μeV
- inelastic contribution: Carlson, Vdh (2011) + Birse, McGovern (2012)

total hadronic correction on Lamb shift

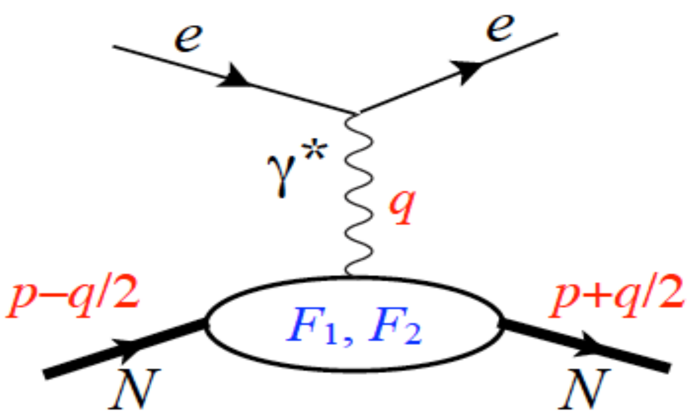
$\Delta E_{TPE}(2P - 2S) = (33 \pm 2)$ μeV

...or about 10% of needed correction

Two-photon exchange in lepton-nucleon scattering



e-p scattering: unpolarized cross sections



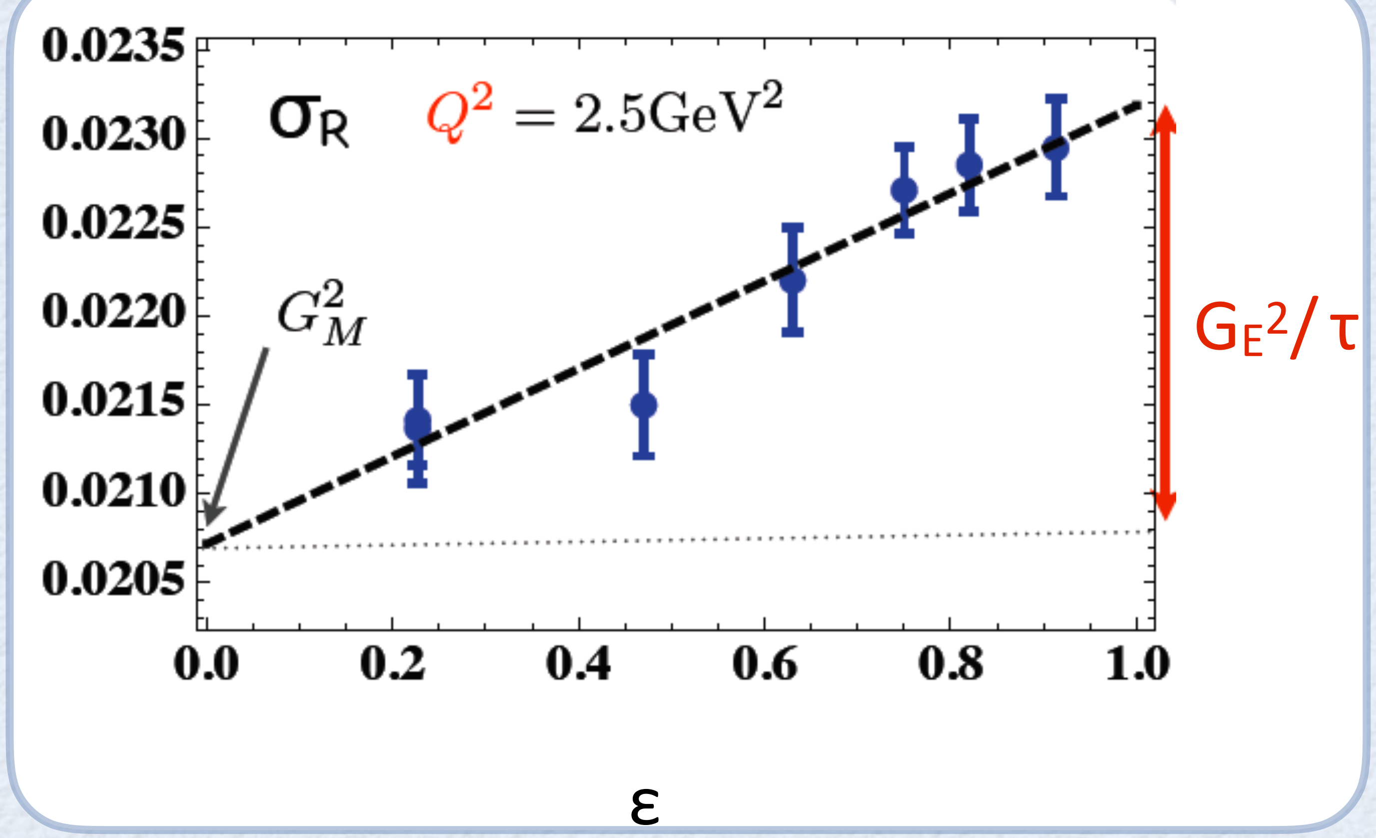
$G_M = F_1 + F_2$

$G_E = F_1 - \tau F_2$

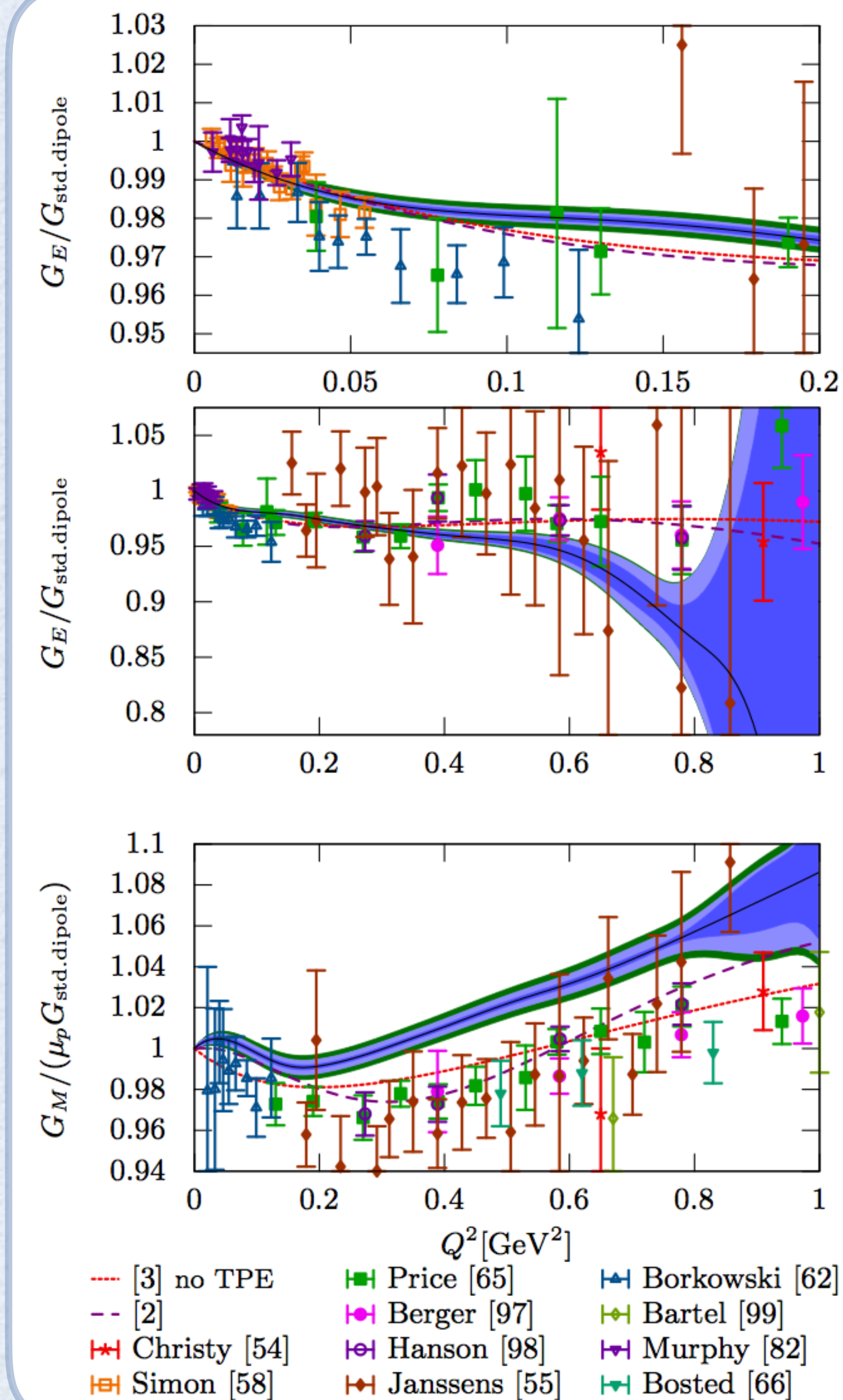
$$\tau \equiv \frac{Q^2}{4M^2} \qquad \frac{1}{\varepsilon} \equiv 1 + 2(1+\tau)\tan^2\frac{\theta}{2}$$

$$\sigma_{\rm R} = G_M^2 + \frac{\varepsilon}{\tau} G_E^2$$

Rosenbluth separation technique



Andivahis et al.(1994)

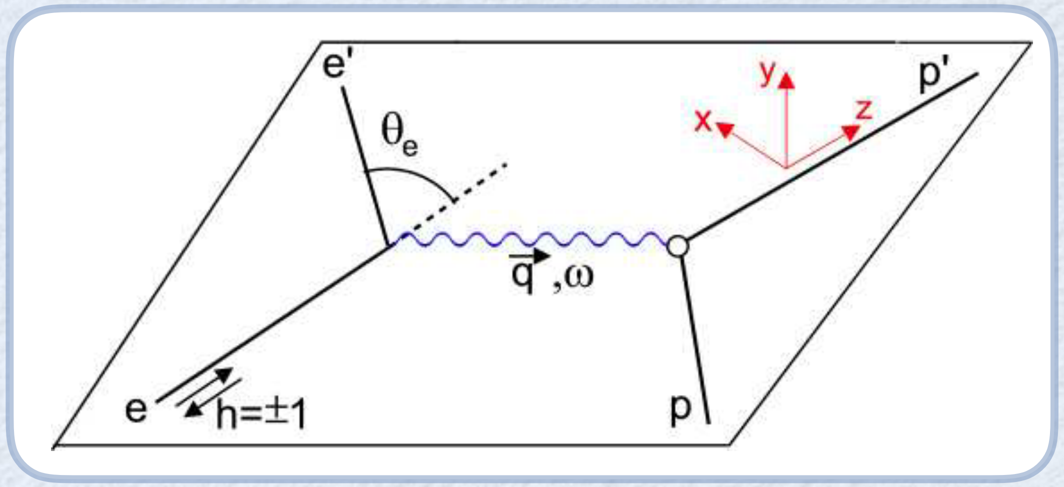


Bernauer et al.(2010, 2013)

e⁻p scattering: double polarization

$$\vec{e} + p \rightarrow e + \vec{p}$$

Akhiezer, Rekalo (1974)



$$d\sigma_{pol} = d\sigma_{unpol}(1 + h\,S_x\,P_t\; + \;h\,S_z\,P_l)$$

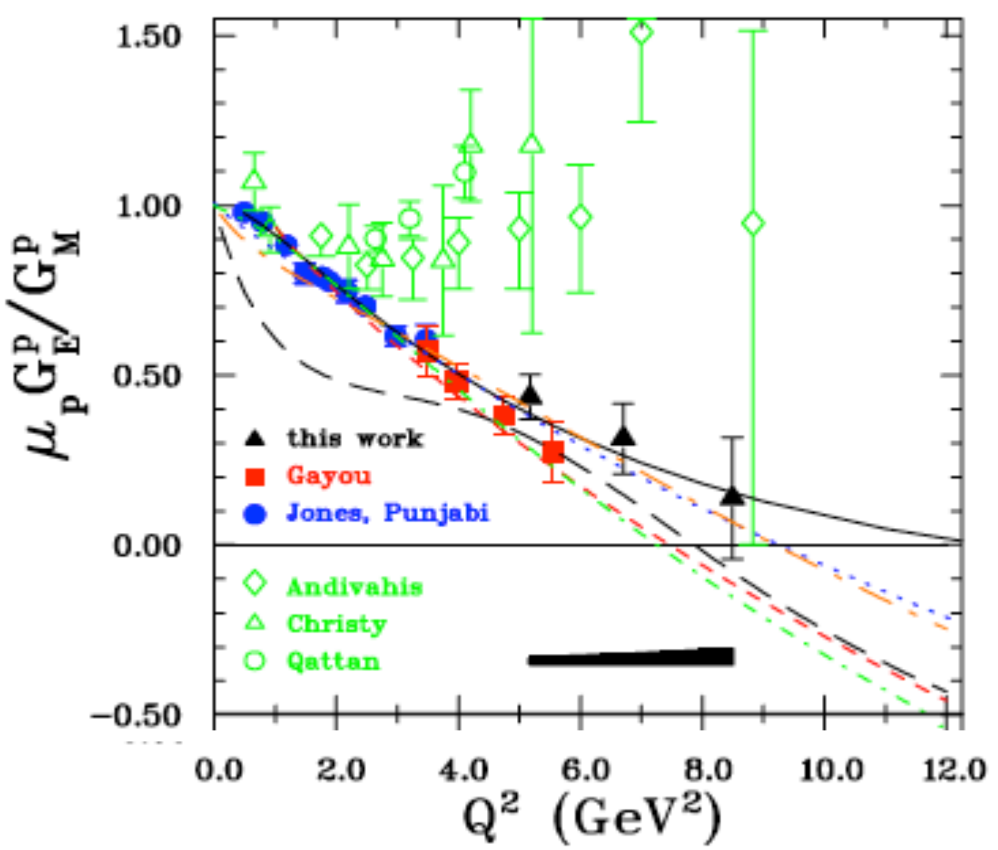
$$P_t = -\sqrt{\frac{2\varepsilon(1-\varepsilon)}{\tau}}\,\frac{G_E G_M}{\tau\,\sigma_R}$$

$$P_l = \sqrt{1-\varepsilon^2}\,\frac{G_M^2}{\tau\,\sigma_R}$$

→

$$\frac{P_t}{P_l} = -\sqrt{\frac{2\varepsilon}{\tau(1+\varepsilon)}}\,\frac{G_E}{G_M}$$

Rosenbluth vs polarization transfer measurements of G_E/G_M of proton



Rosenbluth data

SLAC, JLab (Hall A, C)

Polarization data

JLab (Hall A, C)

GEpI Jones et al.(2000)

Punjabi et al.(2005)

GEpII Gayou et al.(2002)

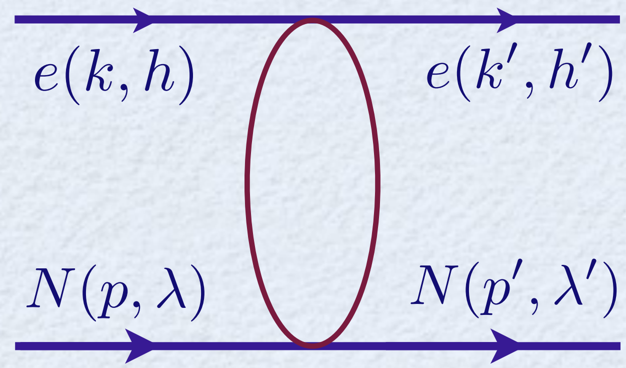
GEpIII Puckett et al.(2010)

Two methods: two different results

most likely: 2γ-exchange correction

2γ-exchange in e⁻ scattering: general

$$P = \frac{p+p'}{2} \qquad K = \frac{k+k'}{2}$$



$$t = (k-k')^2 \qquad u = (k-p')^2$$

$$s = (p+k)^2 \qquad \nu = \frac{s-u}{4}$$

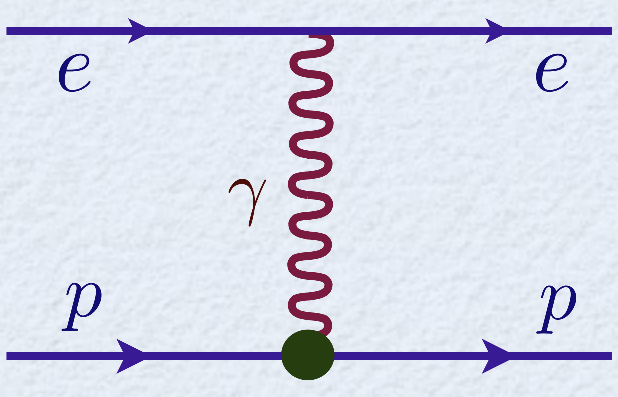
discrete symmetries + $m_e = 0$ ⟶ 3 structure amplitudes

$$T = \frac{e^2}{Q^2}\bar{e}(k',h')\gamma_\mu e(k,h).\bar{N}(p',\lambda')[\mathcal{G}_M(\nu,t)\gamma^\mu - \mathcal{F}_2(\nu,t)\frac{P^\mu}{M} + \mathcal{F}_3(\nu,t)\frac{\hat{K}P^\mu}{M^2}]N(p,\lambda)$$

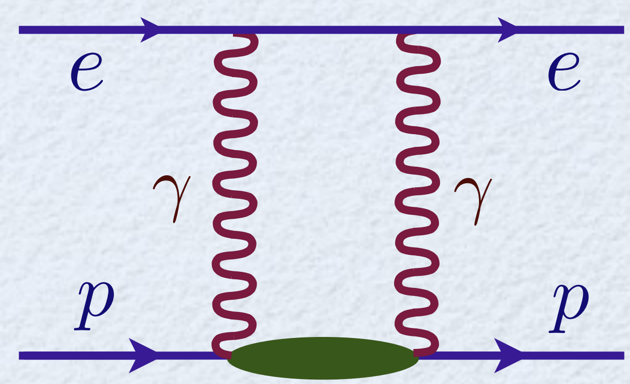
Guichon, Vdh (2003)

Leading contribution to cross section - interference term

1 photon diagram



2 photon exchange diagram



$$\delta_{TPE} \sim \Re\mathcal{G}_M, \Re\mathcal{F}_2, \Re\mathcal{F}_3$$

observables including 2γ-exchange

$$\begin{aligned}\tilde{G}_M(\nu,Q^2) &= G_M(Q^2)+\delta\tilde{G}_M\\ \tilde{F}_2(\nu,Q^2) &= F_2(Q^2)+\delta\tilde{F}_2\\ \tilde{F}_3(\nu,Q^2) &= 0+\delta\tilde{F}_3\end{aligned}$$

for real part:

3 independent observables

$$Y_{2\gamma}^M(\nu,Q^2)\equiv\mathcal{R}\left(\frac{\delta\tilde{G}_M}{G_M}\right)$$

$$Y_{2\gamma}^E(\nu,Q^2)\equiv\mathcal{R}\left(\frac{\delta\tilde{G}_E}{G_M}\right)$$

$$Y_{2\gamma}^3(\nu,Q^2)\equiv\frac{\nu}{M^2}\mathcal{R}\left(\frac{\tilde{F}_3}{G_M}\right)$$

$$\begin{aligned}&\tilde{G}_E\equiv\tilde{G}_M-(1+\tau)\,\tilde{F}_2\\ &\tilde{G}_E(\nu,Q^2)=G_E(Q^2)+\delta\tilde{G}_E\end{aligned}$$

$$\begin{aligned}\frac{\sigma_R}{G_M^2} &= 1+\frac{\varepsilon}{\tau}\frac{G_E^2}{G_M^2}\\ &+ 2\,Y_{2\gamma}^M+2\varepsilon\frac{G_E}{\tau G_M}\,Y_{2\gamma}^E+2\varepsilon\left(1+\frac{G_E}{\tau G_M}\right)Y_{2\gamma}^3\\ &+ \mathcal{O}(e^4)\end{aligned}$$

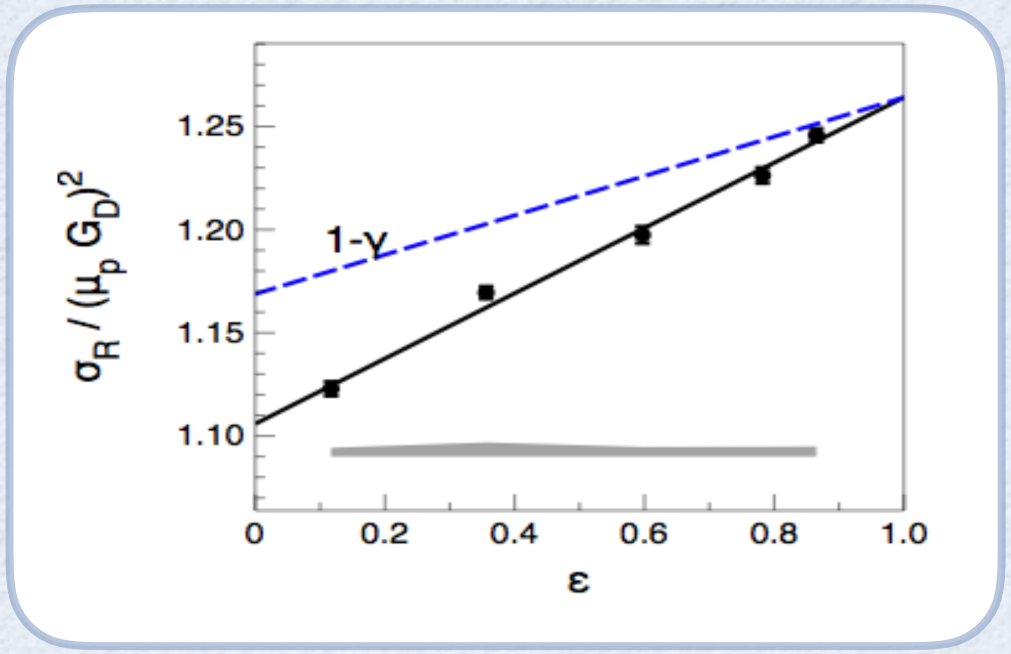
$$\begin{aligned}-\sqrt{\frac{\tau(1+\varepsilon)}{2\varepsilon}}\frac{P_t}{P_l} &= \frac{G_E}{G_M}\\ &+ Y_{2\gamma}^E-\frac{G_E}{G_M}\,Y_{2\gamma}^M+\left(1-\frac{2\varepsilon}{1+\varepsilon}\frac{G_E}{G_M}\right)Y_{2\gamma}^3\\ &+ \mathcal{O}(e^4)\end{aligned}$$

$$\begin{aligned}\frac{P_l}{P_l^{Born}} &= 1\\ &-2\varepsilon\left(1+\frac{\varepsilon}{\tau}\frac{G_E^2}{G_M^2}\right)^{-1}\left\{\left[\frac{\varepsilon}{1+\varepsilon}\left(1-\frac{G_E^2}{\tau G_M^2}\right)+\frac{G_E}{\tau G_M}\right]Y_{2\gamma}^3\right.\\ &\qquad\left.+\frac{G_E}{\tau G_M}\left[Y_{2\gamma}^E-\frac{G_E}{G_M}\,Y_{2\gamma}^M\right]\right\}\\ &+\mathcal{O}(e^4)\end{aligned}$$

extraction of 2γ-amplitudes: data

Rosenbluth data: JLab (Hall A)

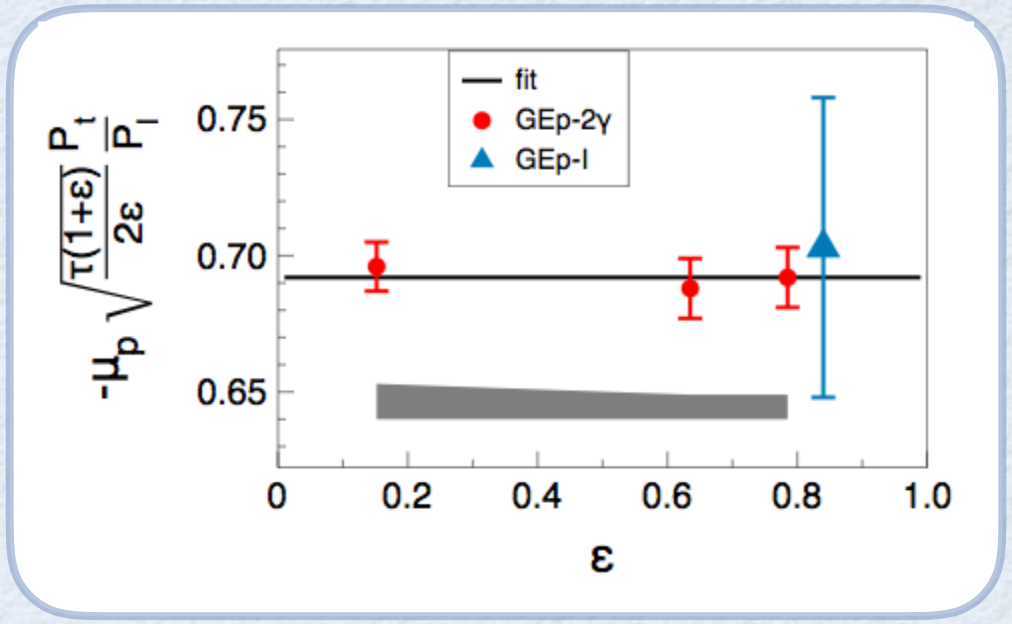
$Q^2 = 2.64\ GeV^2$

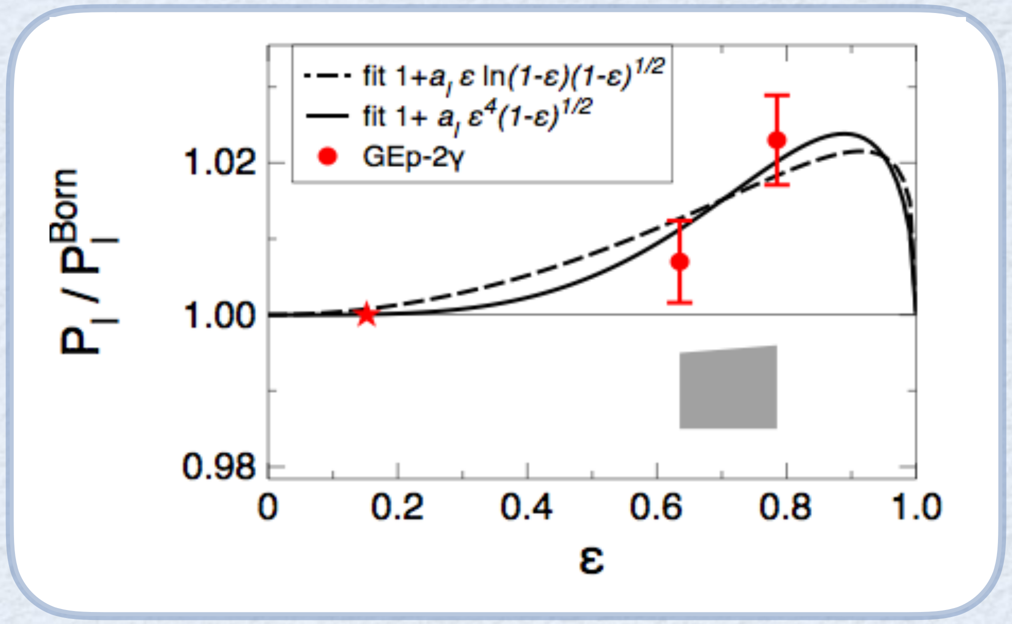


Qattan et al. (2005)

Polarization data: JLab (Hall C)

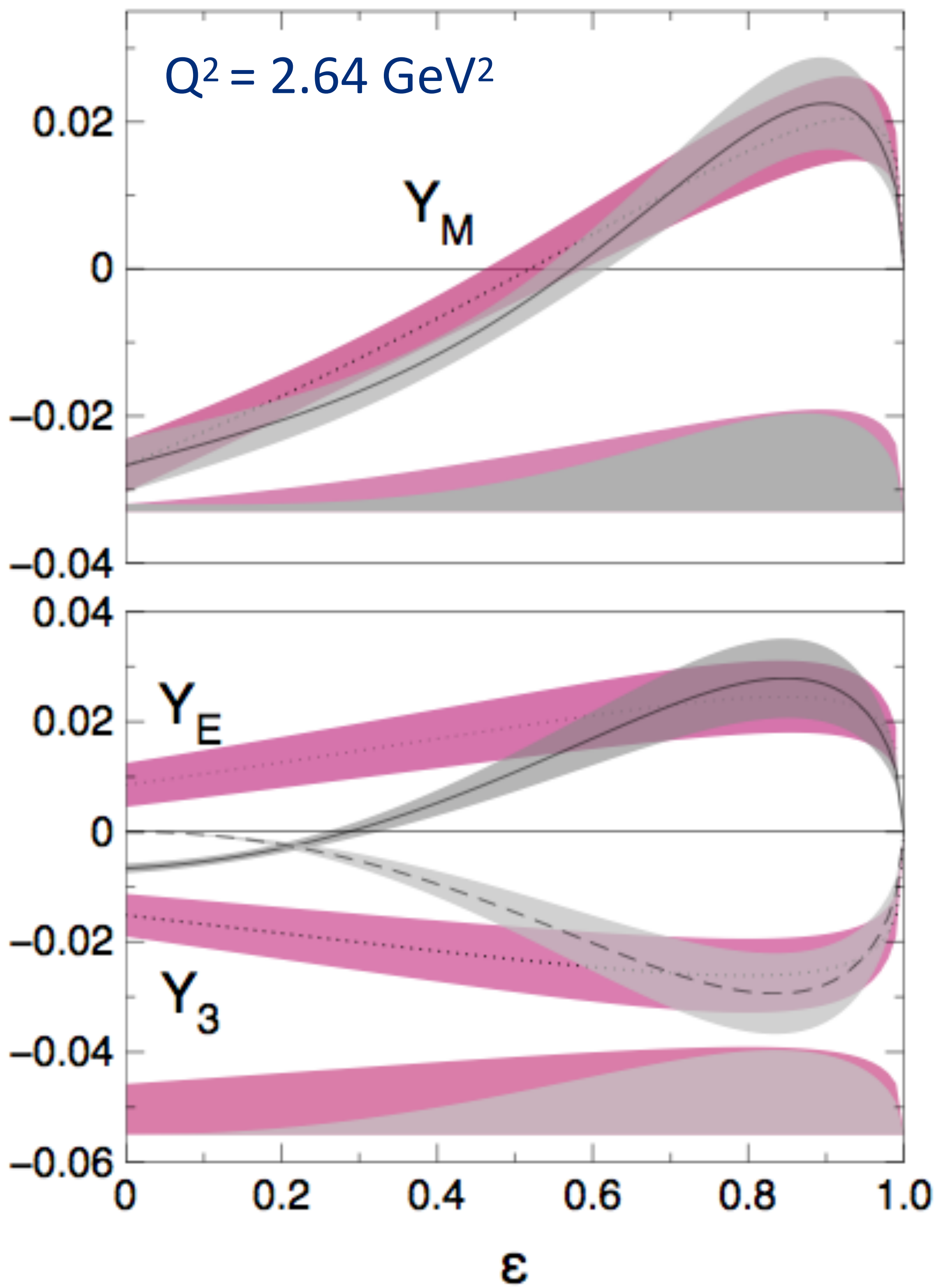
$Q^2 = 2.5\ GeV^2$

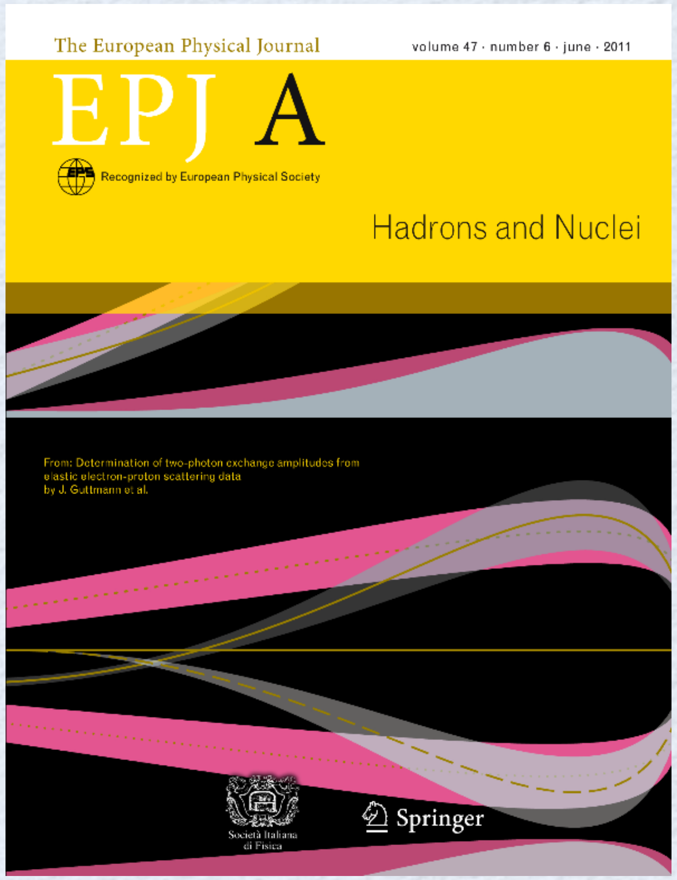




Meziane et al. (2011)

extraction of 2γ-amplitudes: fit



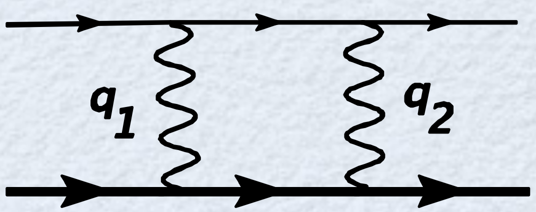


Guttmann, Kivel, Meziane, Vdh (2011)

extracted 2γ amplitudes are in the (expected) 2-3 % range

status of 2γ-exchange corrections

Tsai(1961), Mo & Tsai (1968)



box diagram calculated using only nucleon intermediate state

and using $q_1 \approx 0$ or $q_2 \approx 0$ in both numerator and denominator

-> gives correct IR divergent terms

Maximon & Tjon (2000)

same as above but make above approximation only in the numerator (calculate 4-point fct)

+ use on-shell nucleon form factors in loop integral

Blunden, Melnitchouk, Tjon (2003), Kondratyuk & Blunden (2007)

further improvement by keeping full numerator, insert higher resonances

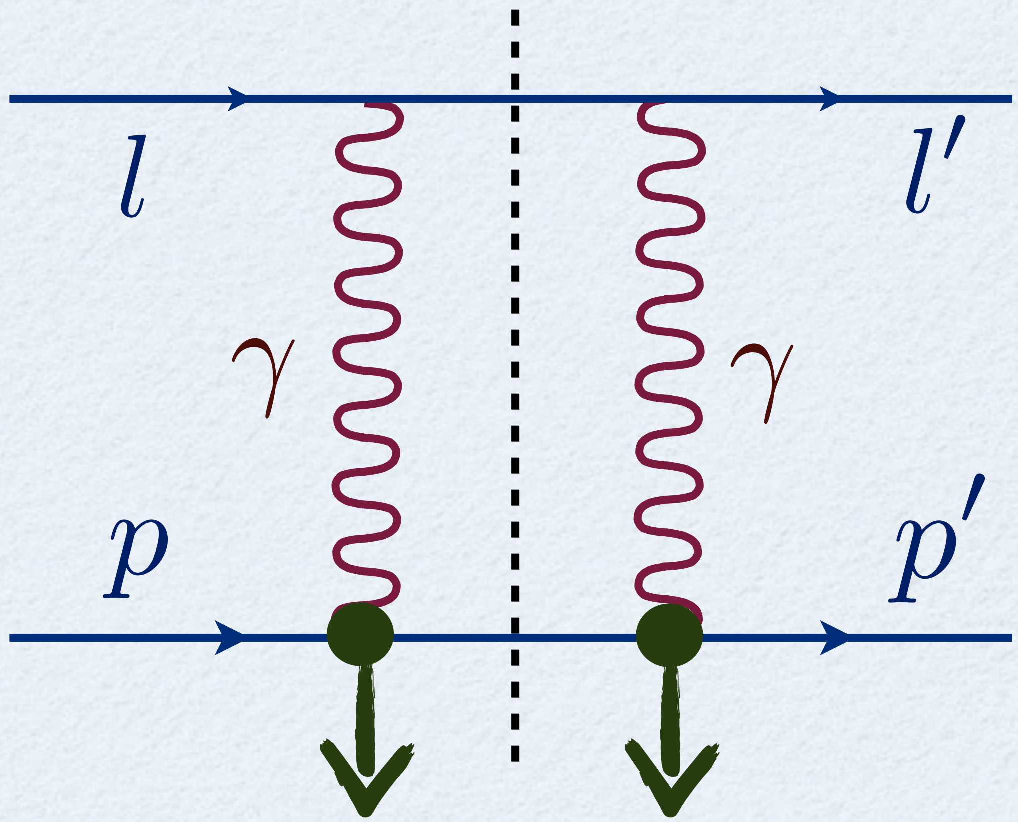
Borisyuk & Kobushkin (2006, 2007, 2008, 2012)

same as previous from dispersive approach

Bystritsky, Kuraev, Tomasi-Gustaffson (2006)

assumption that dominant region comes from $q_1 \approx q_2 \approx q/2$ (obtained TPE is very small) non-forward scattering

proton state



Dirac and Pauli form factors

box diagram

assumption about the vertex

Blunden, Melnitchouk, Tjon (2003)

dispersion relations

based on on-shell information

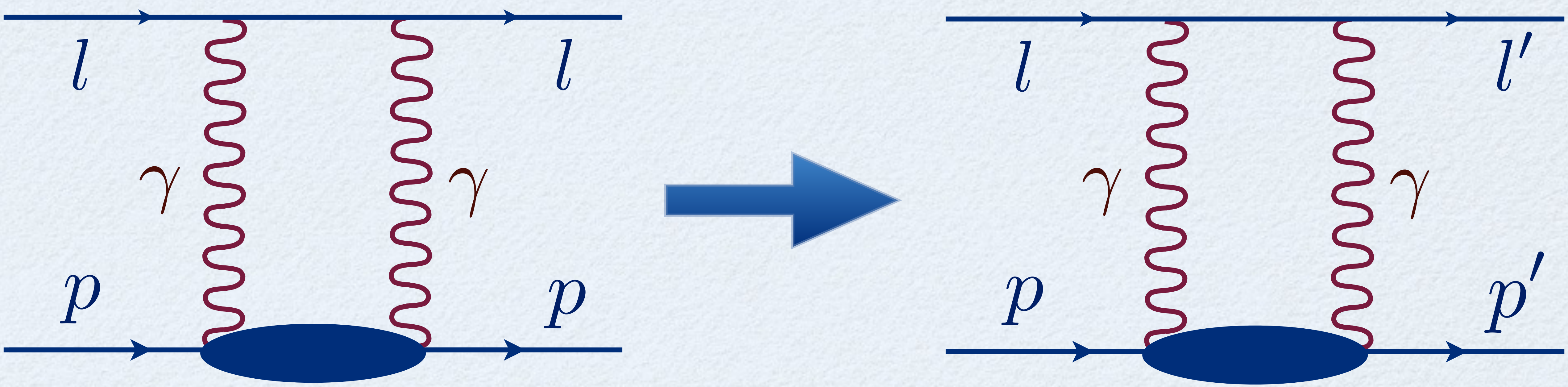
Borisyuk, Kobushkin (2008)

Tomalak, Vdh (2014)

forward scattering

near-forward scattering

account for all inelastic 2γ



2γ-exchange at low Q^2

2ɣ blob: near-forward virtual Compton scattering

Feshbach → a; inelastic → b; elastic → b, c

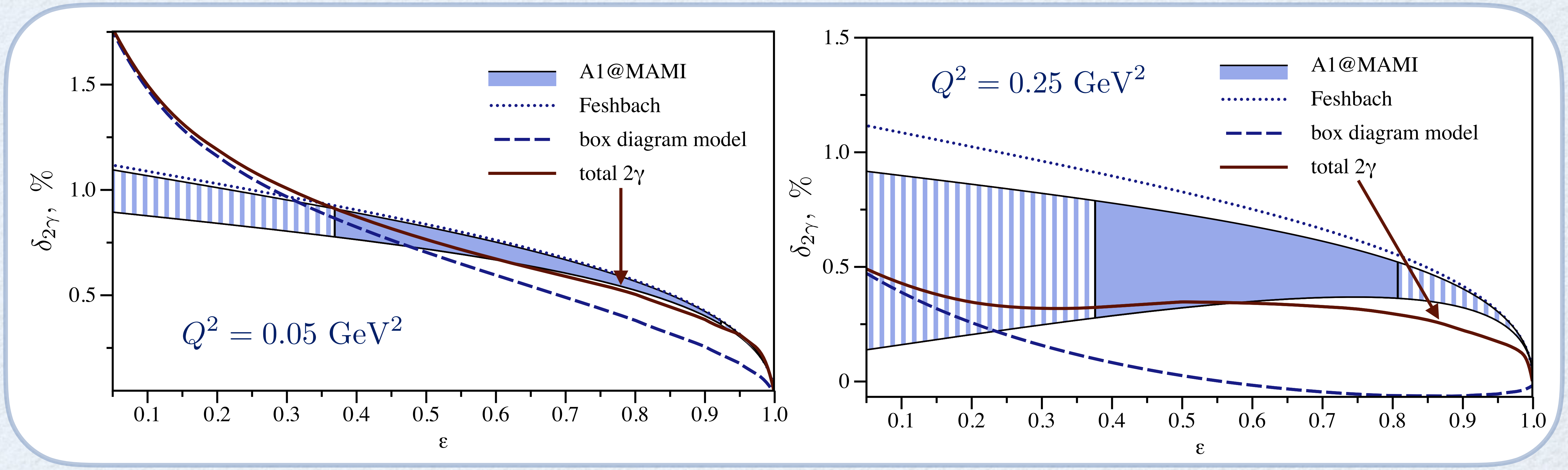
$$\delta_{2\gamma} \sim a\,\sqrt{Q^2} + b\, Q^2 \ln Q^2 + c\, Q^2 \ln^2 Q^2$$

McKinley, Feshbach (1948)
R.W.Brown (1970)
M. Gorchtein (2013)

unpolarized proton structure → F_1, F_2

Tomalak, Vdh (2016)

$$\delta_{2\gamma} = \int \mathrm{d}\nu_\gamma \mathrm{d}\tilde{Q}^2 (\omega_1(\nu_\gamma, \tilde{Q}^2) \cdot F_1(\nu_\gamma, \tilde{Q}^2) + \omega_2(\nu_\gamma, \tilde{Q}^2) \cdot F_2(\nu_\gamma, \tilde{Q}^2))$$

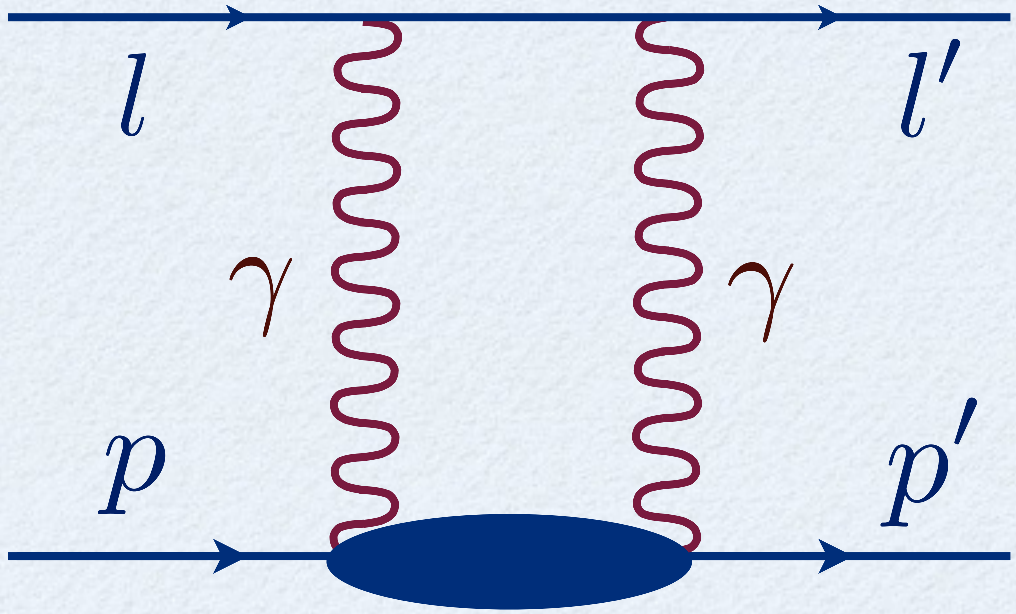


2ɣ at large ε agrees with empirical fit

r_E extraction ✓

near-forward scattering

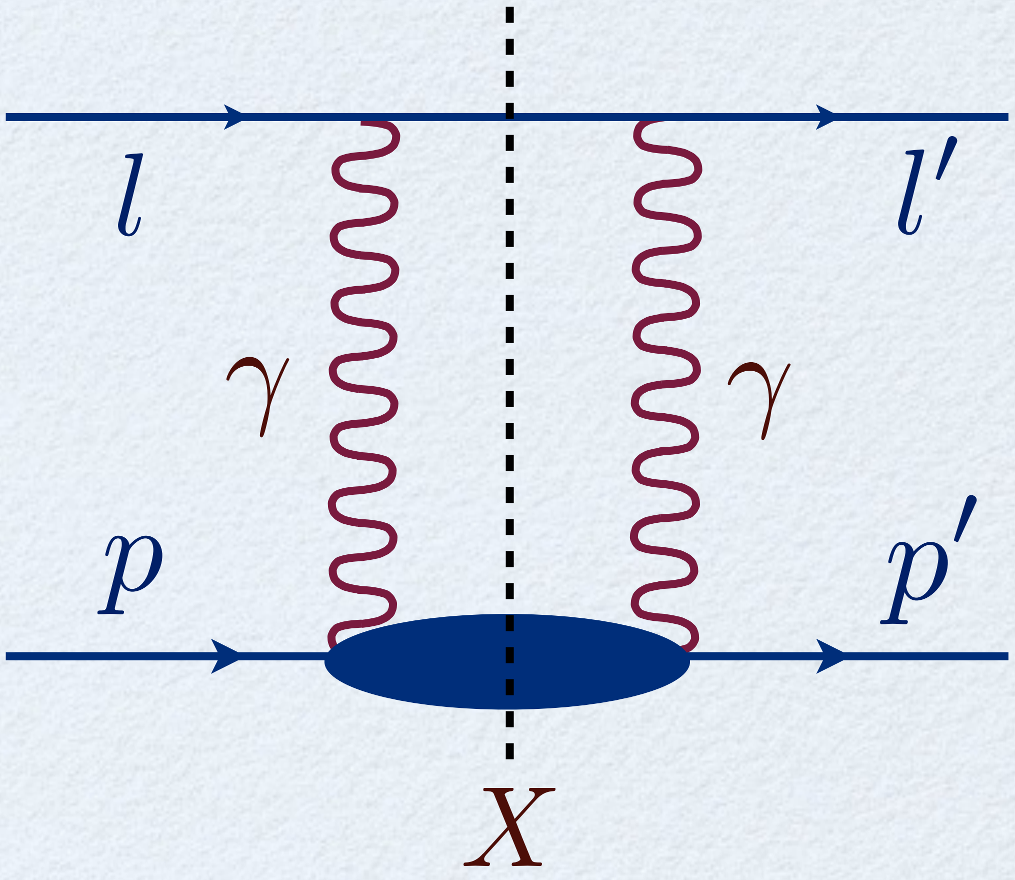
(large ε)



p + all inelastic

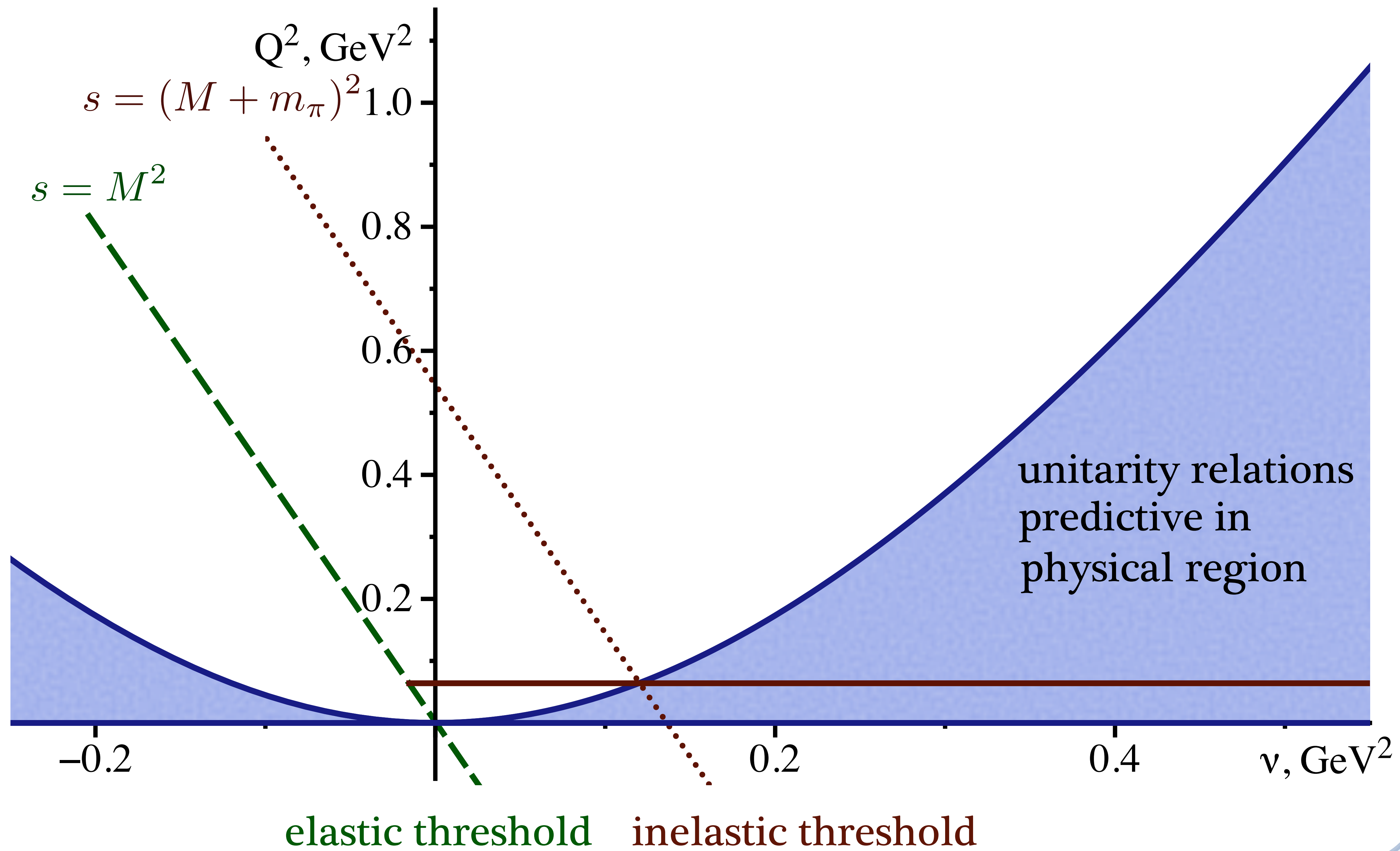
non-forward scattering
dispersion relations

(arbitrary ε)



$$X = p + \pi N$$

Mandelstam plane: ep scattering @ low Q²



proton intermediate state is outside physical region for $Q^2 > 0$

πN intermediate state is outside physical region for $Q^2 > 0.064$ GeV²

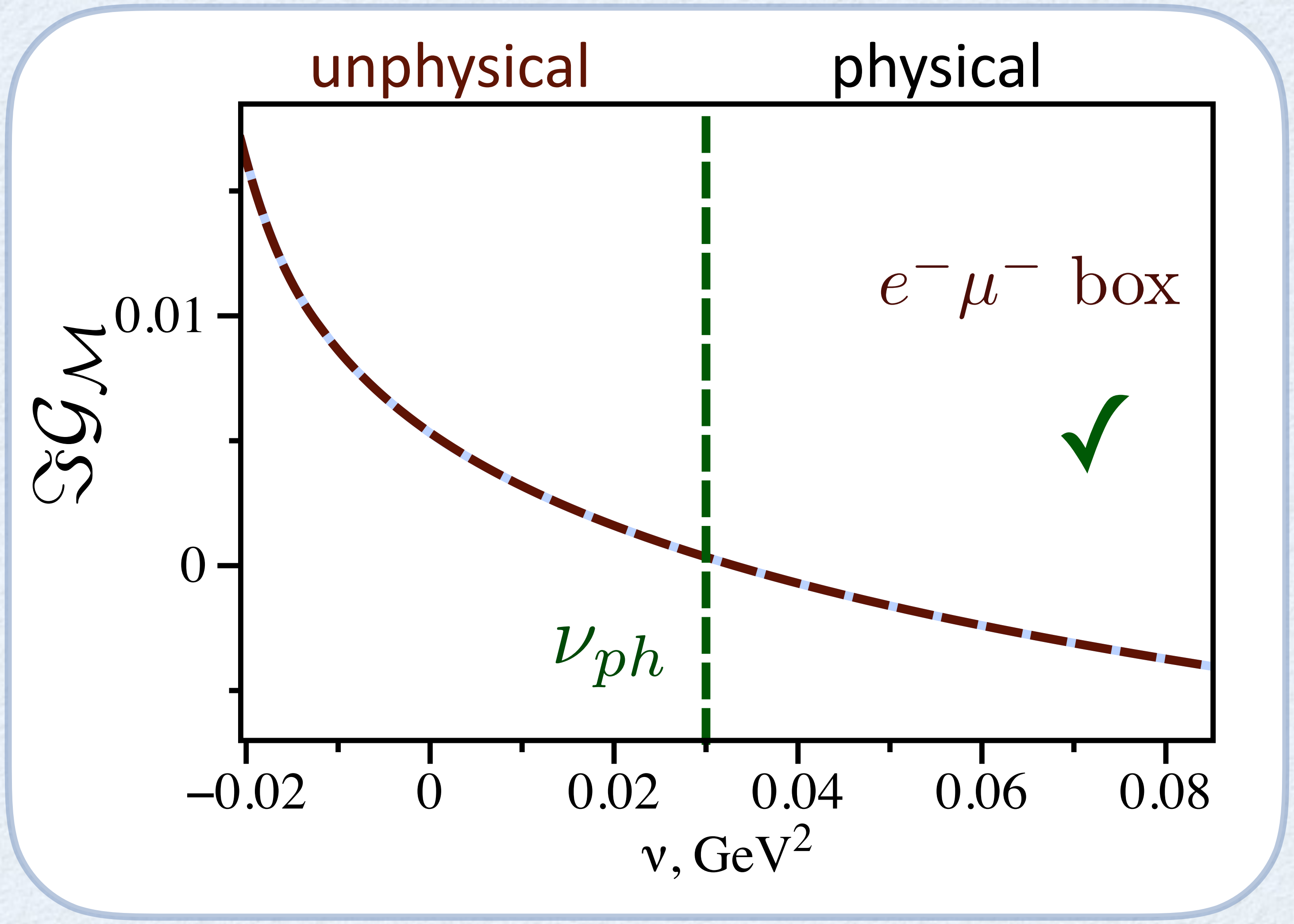
analytical continuation: proton state

contour deformation method:

Tomalak, Vdh (2014)

Blunden, Melnitchouk (2017)

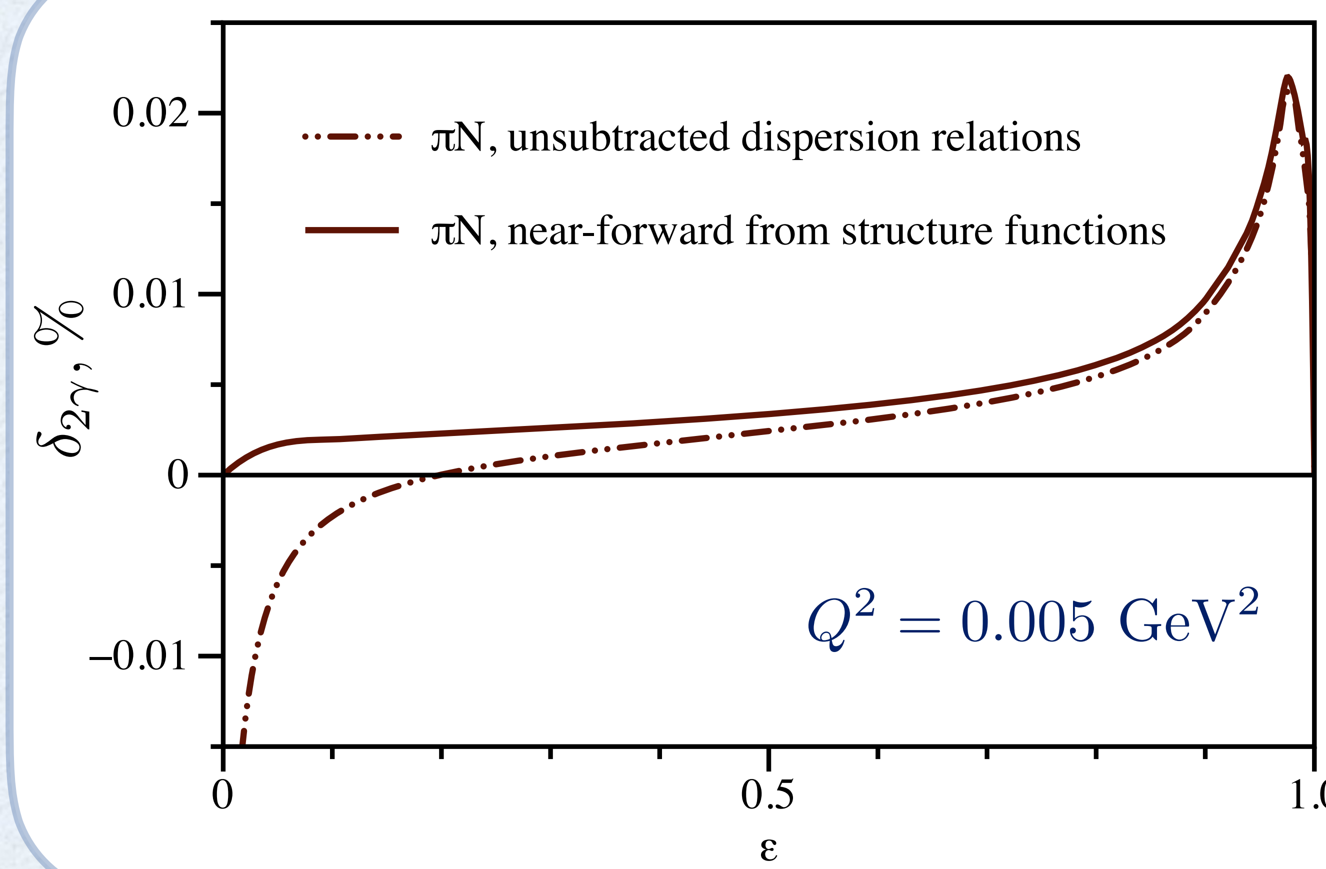
$\int d\Omega$ → angular integration to integration on curve in complex plane → deform integration contour keeping poles inside going to unphysical region



$Q^2 = 0.1\ \mathrm{GeV}^2$

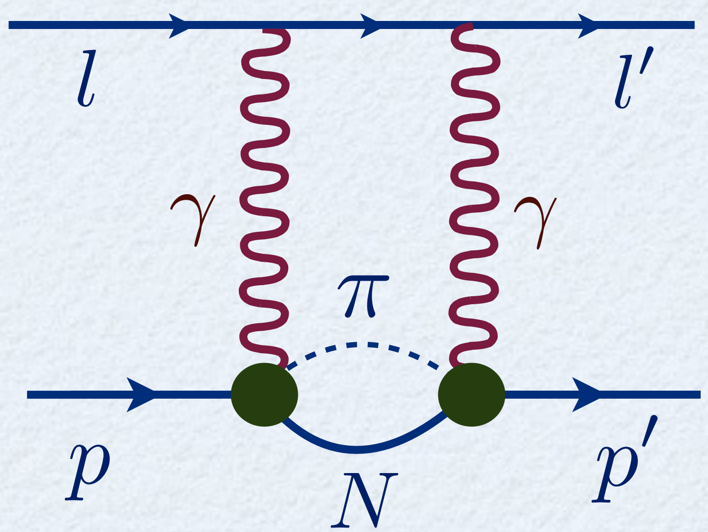
analytical continuation reproduces results in unphysical region

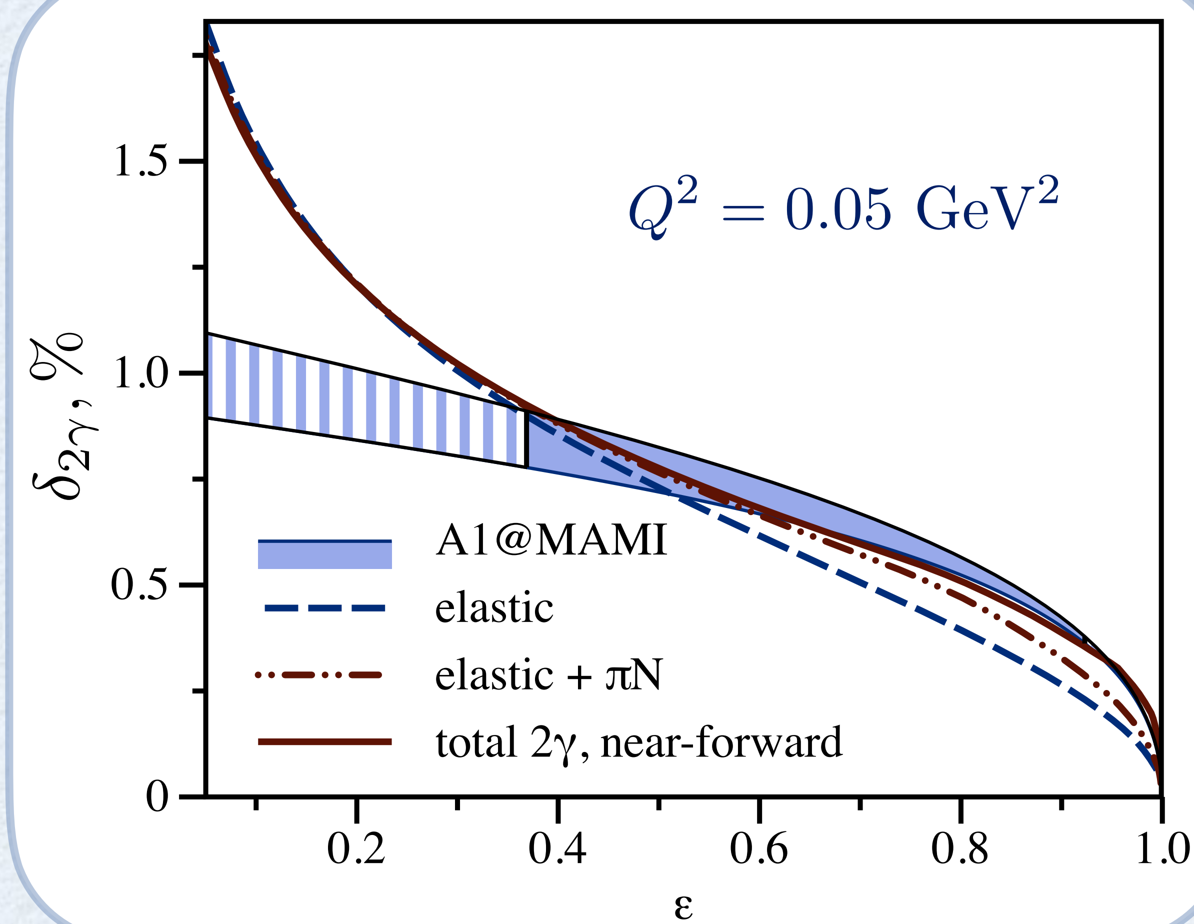
πN state contribution in dispersive framework



dispersion relations agree with near-forward at large ε

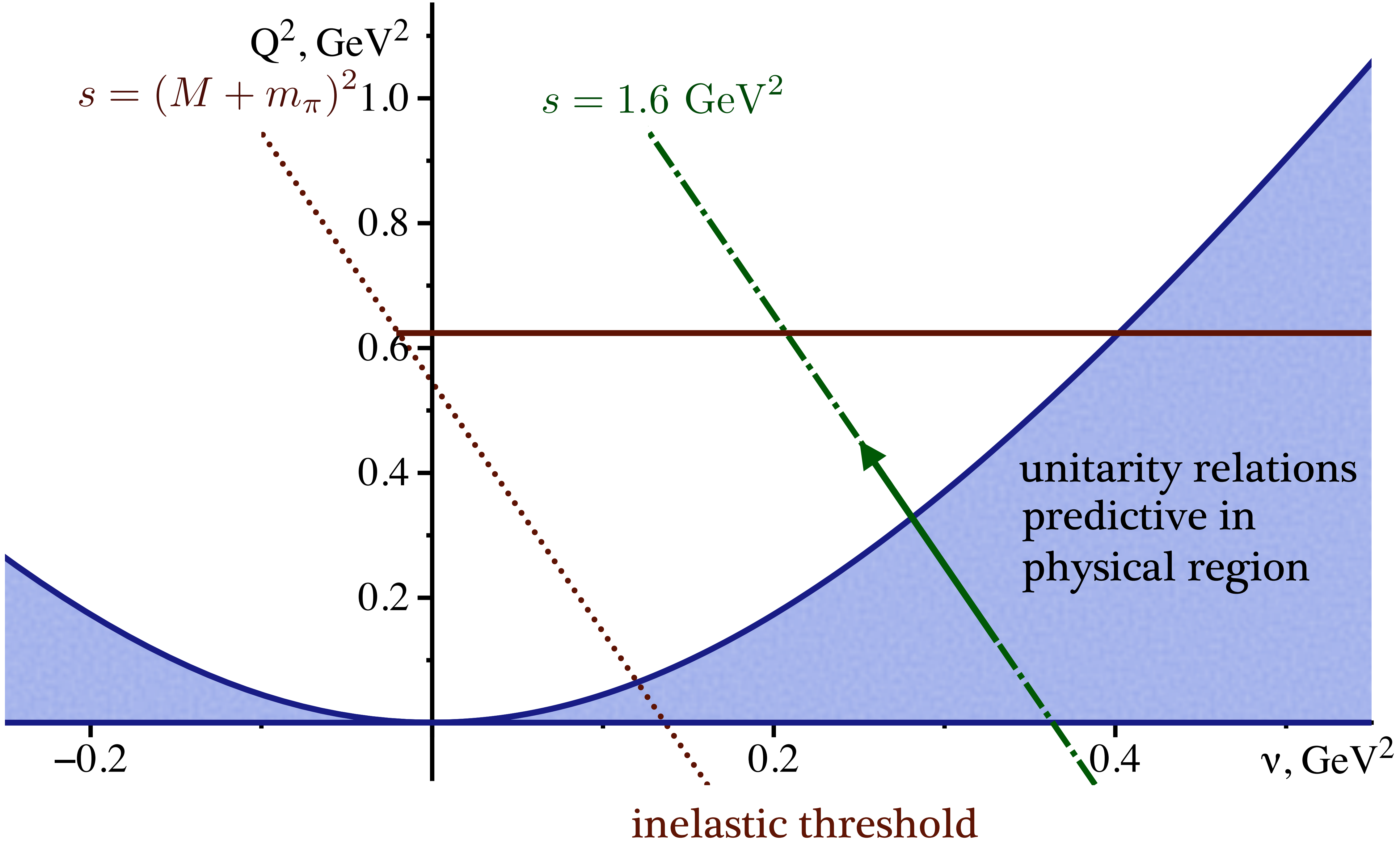
Tomalak, Pasquini, Vdh (2017)





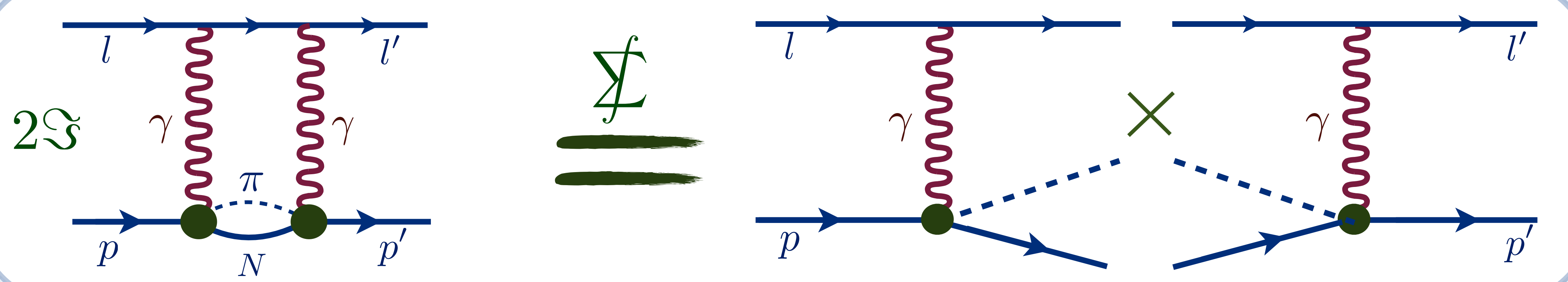
πN is dominant inelastic 2γ

Mandelstam plane: ep scattering @ larger Q^2



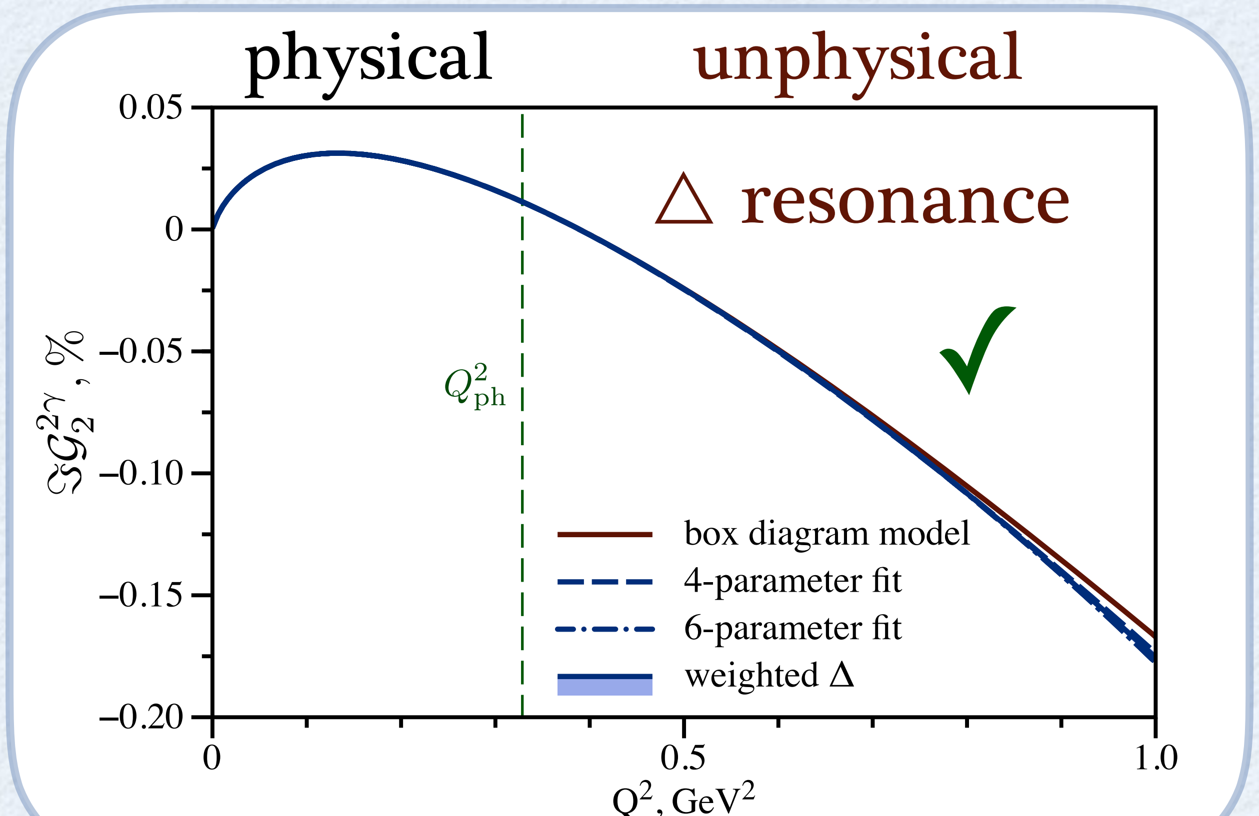
πN intermediate state is outside physical region for $Q^2 > 0.064$ GeV^2

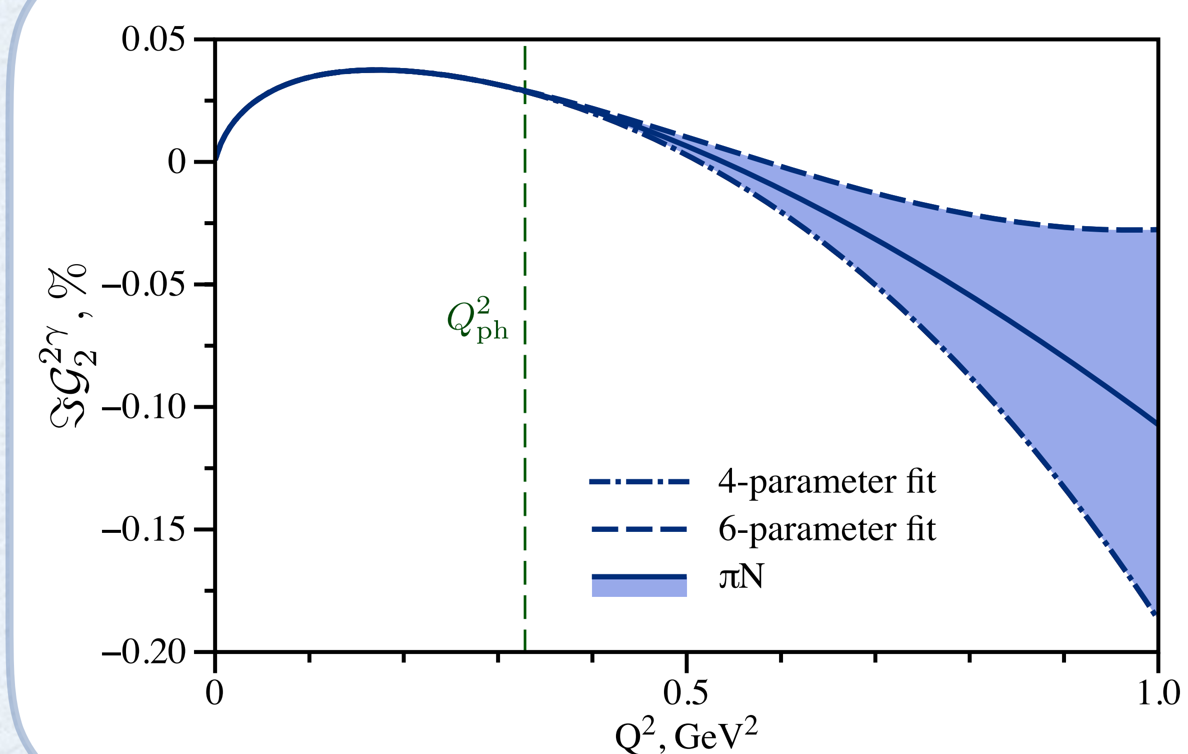
analytical continuation: πN states



- pion electroproduction amplitudes: MAID2007 Drechsel, Kamalov, Tiator (2007)
- analytical continuation: fit of low-Q^2 expansion in physical region

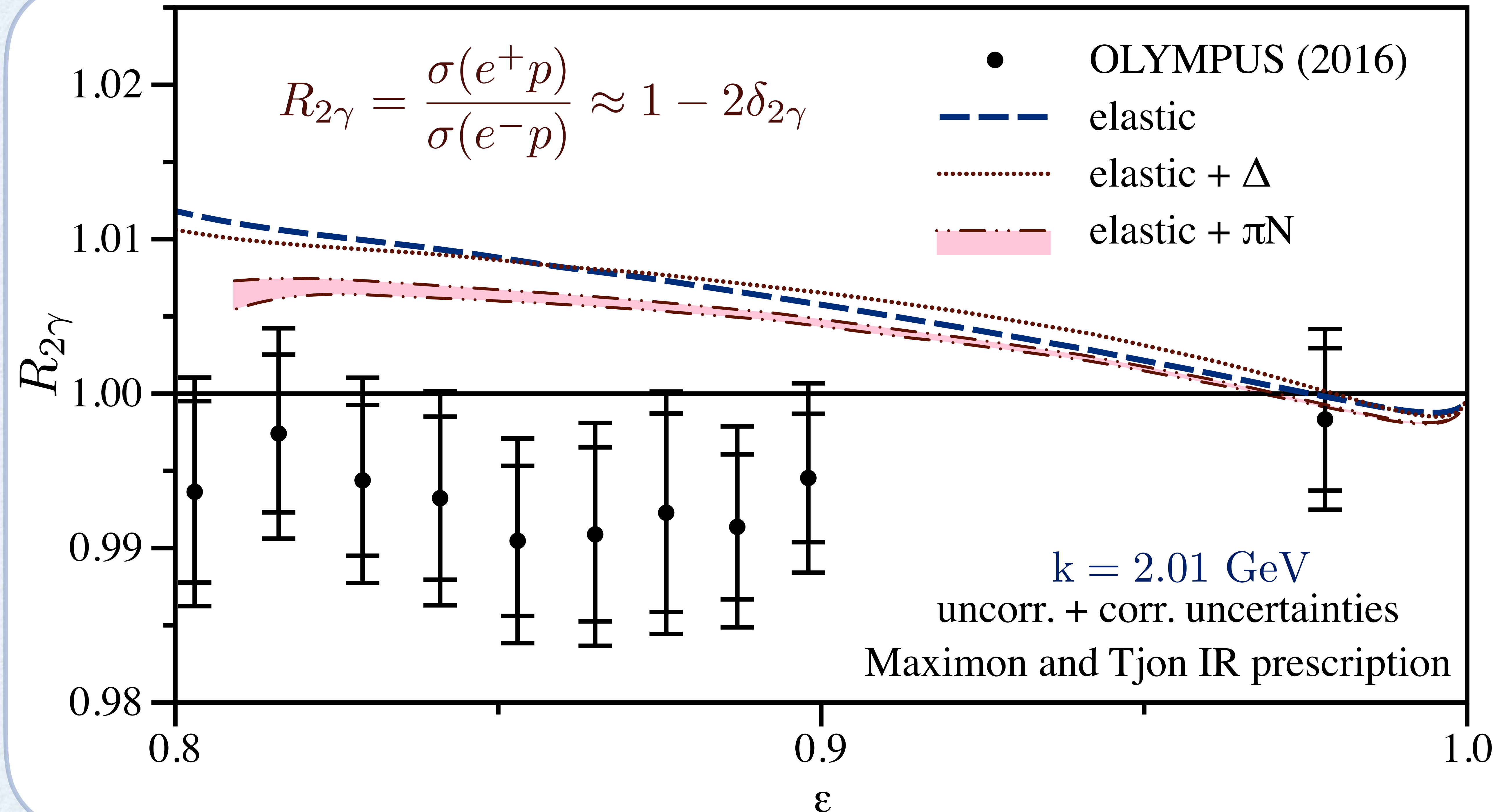
$$\mathcal{G}_{1,2}(s,\ Q^2),\ Q^2\mathcal{F}_3(s,\ Q^2) \sim a_1 Q^2 \ln Q^2 + a_2 Q^2 + a_3 Q^4 \ln Q^2 + \ldots$$





Tomalak, Pasquini, Vdh (2017)

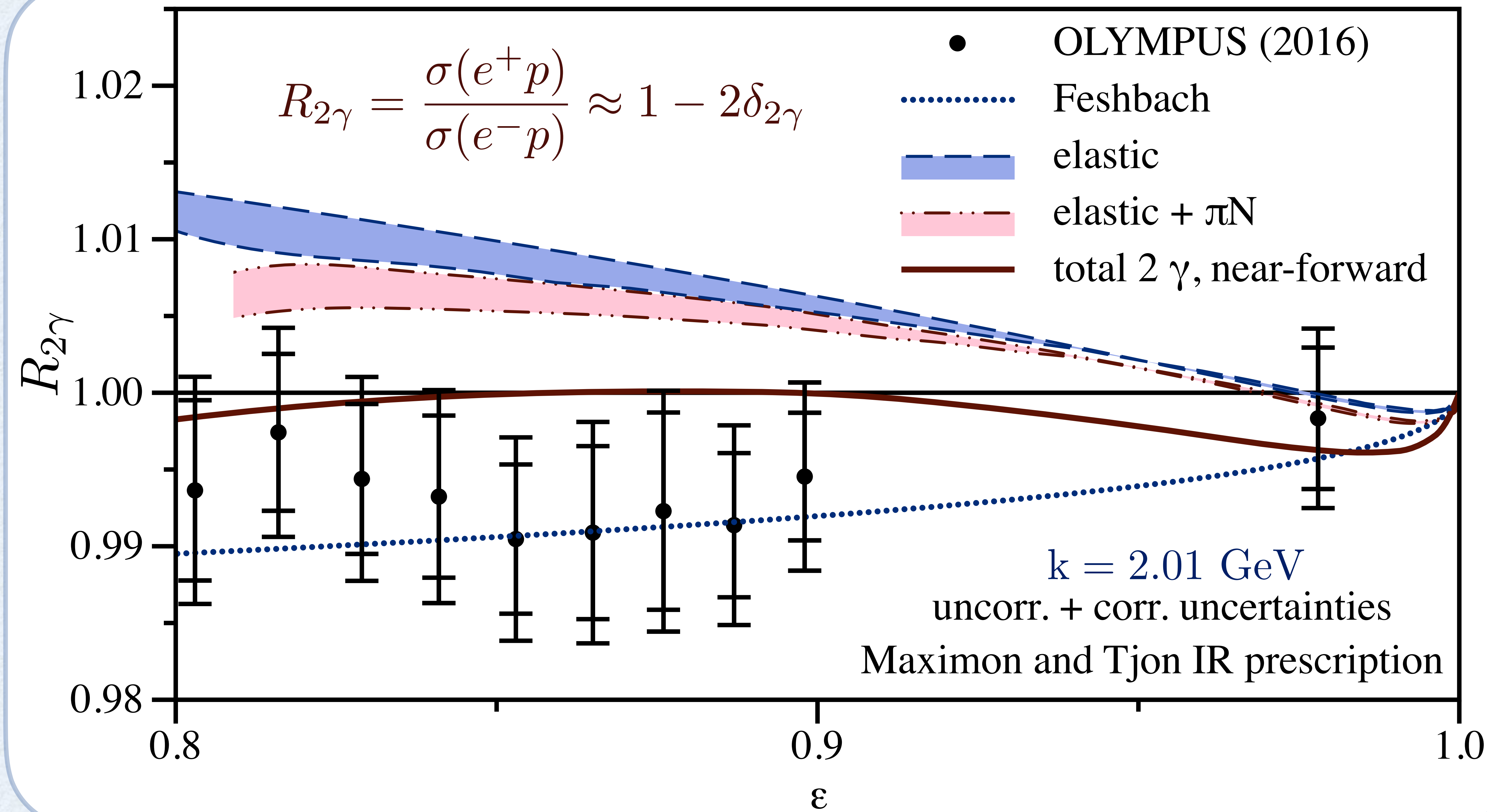
2γ-exchange: comparison with data



weighted △ is similar to narrow one of Blunden et al. (2017)
πN contribution is closer to data than △ only

Tomalak, Pasquini, Vdh (2017)

2γ-exchange: comparison with data

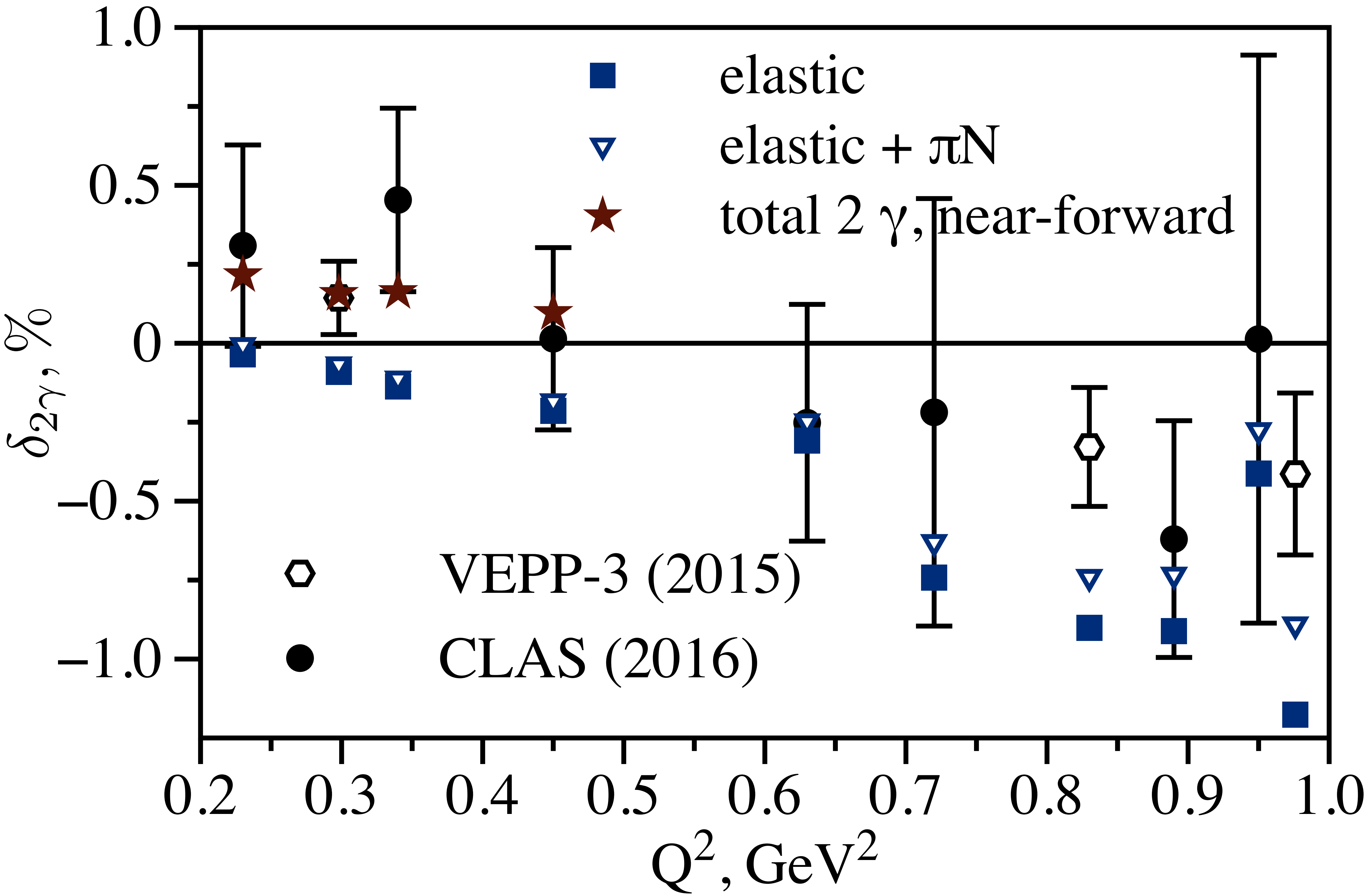


near-forward 2γ agree with data

multi-particle 2γ, e.g. ππN, is important

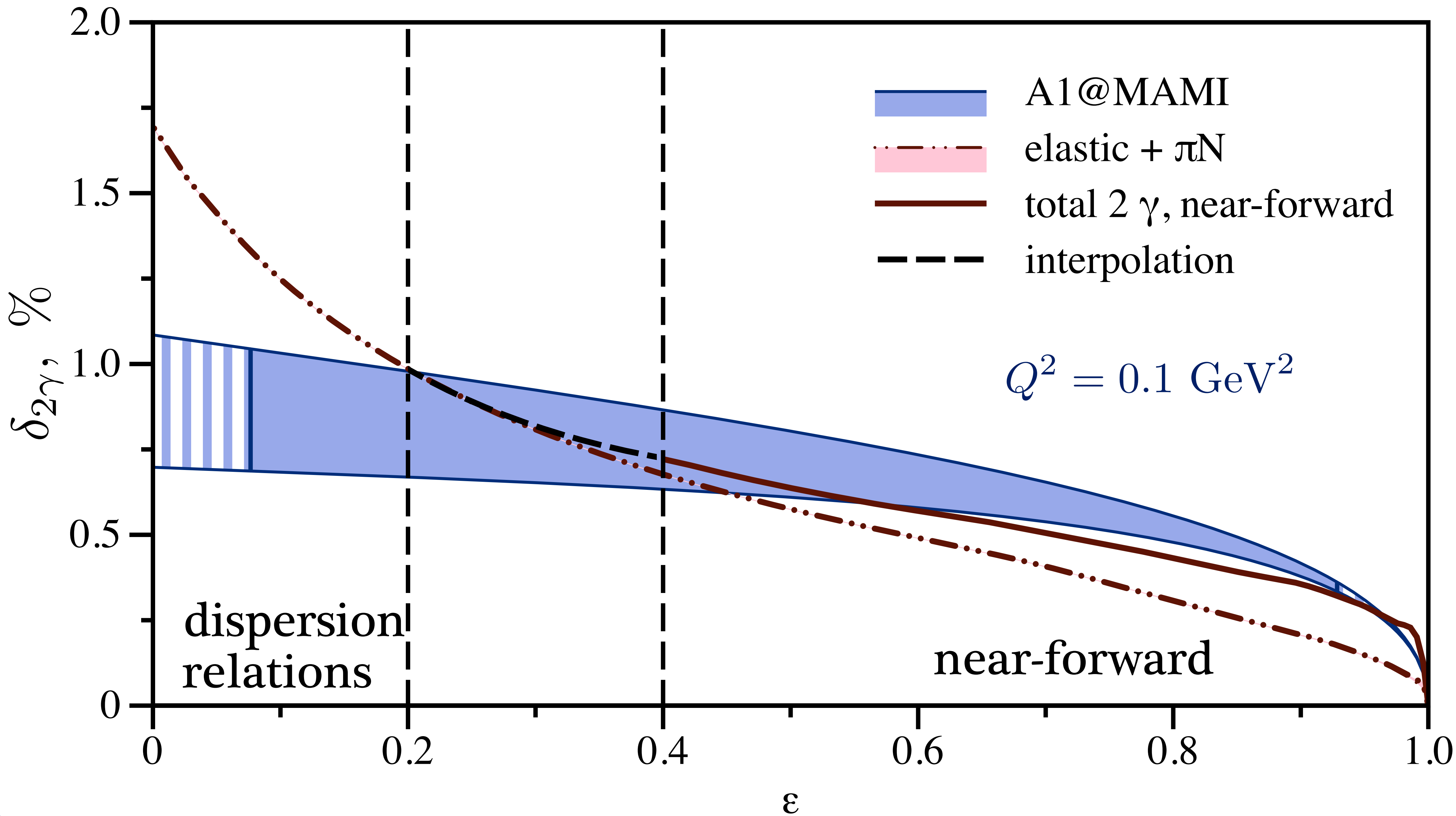
Tomalak, Pasquini, Vdh (2017)

2γ-exchange: comparison with data



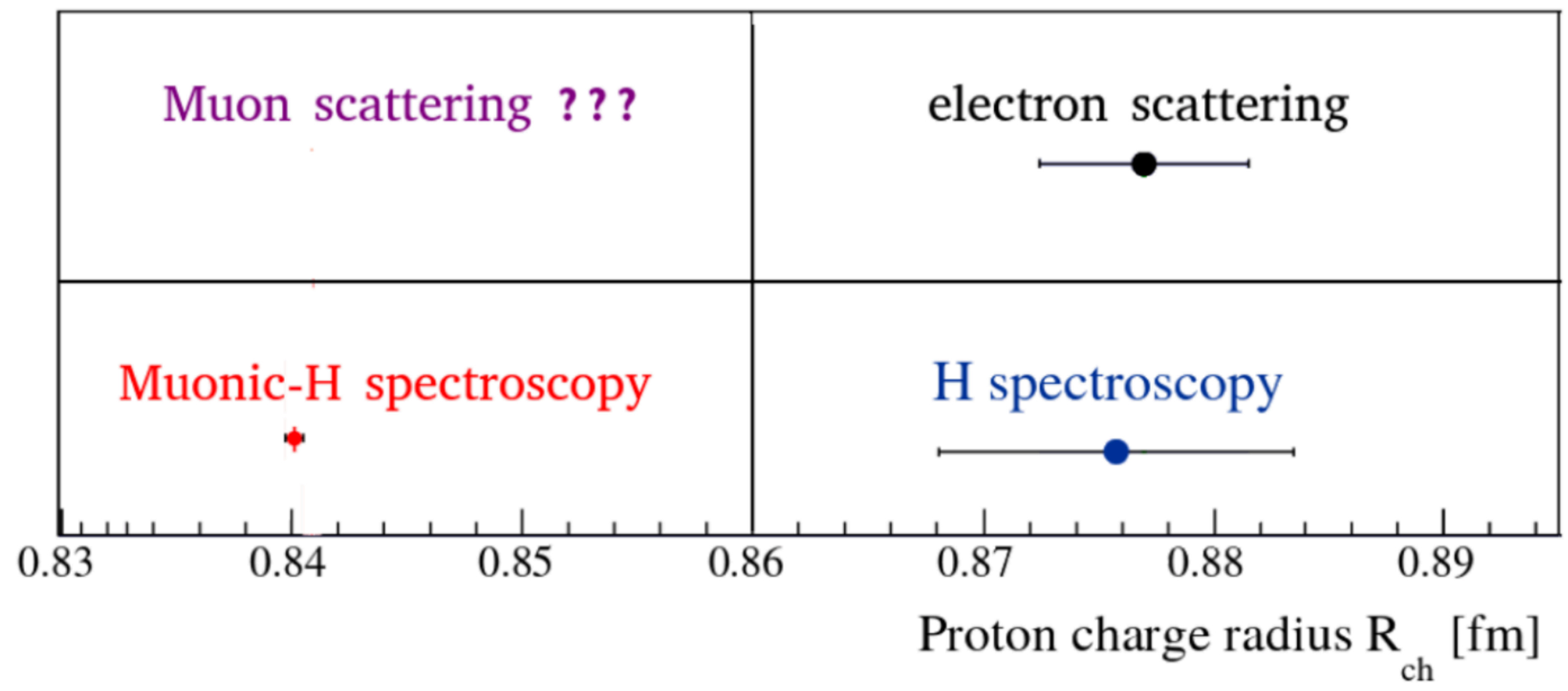
TPE calculation agrees with CLAS data

our best 2γ-exchange amplitudes at low Q²



small Q²: near-forward at large ε, all inelastic states
Q²≲1 GeV²: elastic+πN within dispersion relations
intermediate range: interpolation

μp scattering



- **MUSE@PSI** (2018/19), simulataneous measurement of e^- / e^+ and μ^- / μ^+ scattering on proton beam momenta 115, 153, 210 MeV/c: R_E difference to 0.005 fm, determination of TPE effects

  talk P. Reimer (Fri, 5:30pm)

- μ^+ scattering @**COMPASS** (M2 beam line), first test 2018, aim: R_E to 0.01 fm

- e^-e^+ vs $\mu^-\mu^+$ photoproduction @**MAMI** (LOI): test lepton universality test through ratio measurement

μ⁻ scattering: 2γ-exchange correction

$$T^{non-flip} = \frac{e^2}{Q^2}\bar{l}(k',h')\gamma_\mu l(k,h).\bar{N}(p',\lambda')[\mathcal{G}_M(\nu,t)\gamma^\mu - \mathcal{F}_2(\nu,t)\frac{P^\mu}{M} + \mathcal{F}_3(\nu,t)\frac{\hat{K}P^\mu}{M^2}]N(p,\lambda)$$

$m_l \neq 0$ →

$$T^{flip} = \frac{e^2}{Q^2}\frac{m_l}{M}\bar{l}(k',h')l(k,h).\bar{N}(p',\lambda')[\mathcal{F}_4(\nu,t) + \mathcal{F}_5(\nu,t)\frac{\hat{K}}{M}]N(p,\lambda) + \frac{e^2}{Q^2}\frac{m_l}{M}\mathcal{F}_6(\nu,t)\bar{l}(k',h')\gamma_5 l(k,h).\bar{N}(p',\lambda')\gamma_5 N(p,\lambda)$$

Gorchtein, Guichon, Vdh (2004)

$$\delta_{2\gamma} = \frac{2}{G_M^2 + \frac{\epsilon}{\tau}G_E^2}\left\{G_M \Re\mathcal{G}_1 + \frac{\epsilon}{\tau}G_E\Re\mathcal{G}_2 + \frac{1-\epsilon}{1-\epsilon_0}(\frac{\epsilon_0}{\tau}G_E\Re\mathcal{G}_4 - G_M\Re\mathcal{G}_3)\right\}$$

Tomalak, Vdh (2014)

$$\mathcal{G}_1 = \mathcal{G}_M + \frac{\nu}{M^2}\mathcal{F}_3 + \frac{m_l^2}{M^2}\mathcal{F}_5$$

$$\mathcal{G}_2 = \mathcal{G}_M - (1-\tau)\mathcal{F}_2 + \frac{\nu}{M^2}\mathcal{F}_3$$

$$\mathcal{G}_3 = \frac{\nu}{M^2}\mathcal{F}_3 + \frac{m_l^2}{M^2}\mathcal{F}_5$$

$$\mathcal{G}_4 = \frac{\nu}{M^2}\mathcal{F}_4 + \frac{\nu^2}{M^4(1+\tau)}\mathcal{F}_5$$

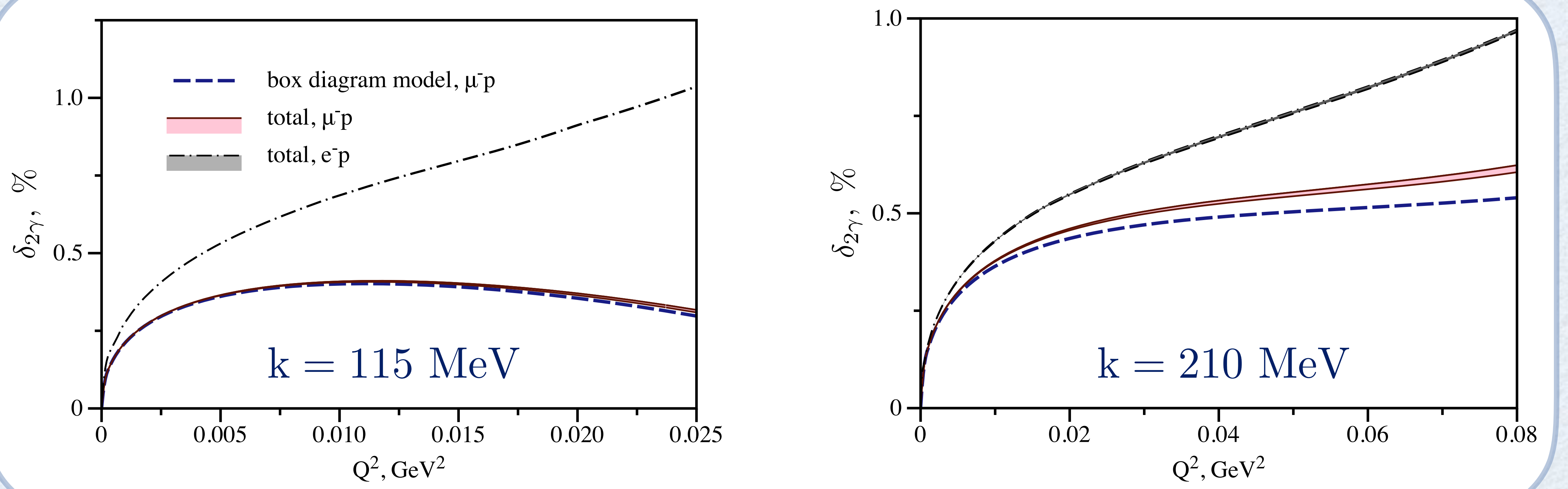
$$\epsilon = \frac{16\nu^2 - Q^2(Q^2+4M^2)}{16\nu^2 - Q^2(Q^2+4M^2) + 2(Q^2+4M^2)(Q^2-2m_l^2)}$$

$$\epsilon_0 = \frac{2m_l^2}{Q^2}$$

$\mu^{-}p$ experiment (MUSE) estimates

proton box diagram model

+ inelastic 2ɣ (near forward structure function calculation)

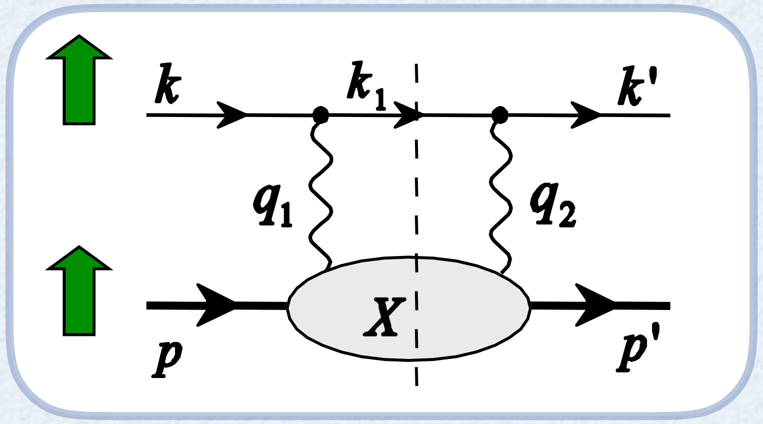


Tomalak, Vdh (2014, 2016)

In MUSE kinematics: small inelastic 2ɣ → small 2ɣ uncertainty

spin-off: TPE in normal spin asymmetries

- Beam or target normal spin asymmetries:

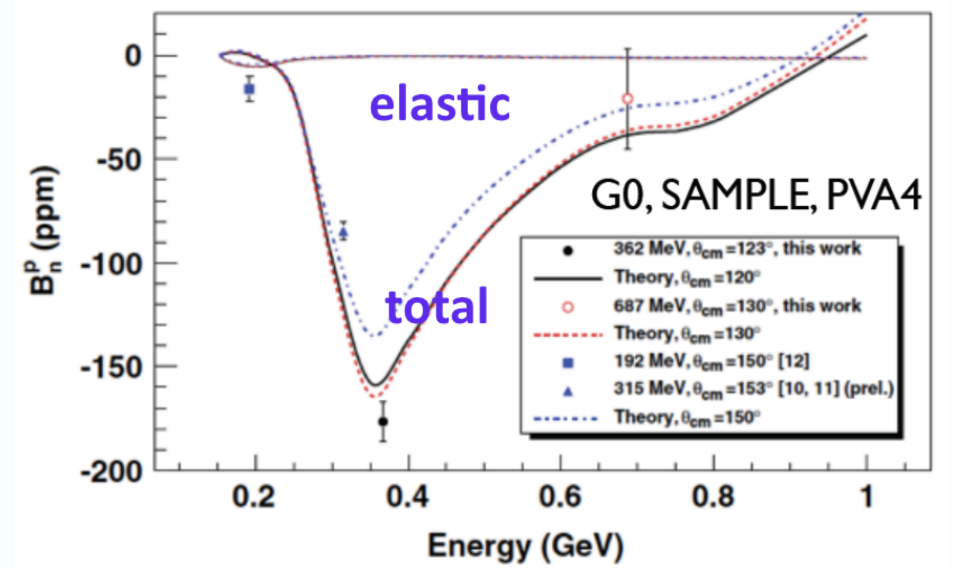


directly proportional to Im part of TPE

target: $A_n \sim \alpha_{em} \sim 10^{-2}$

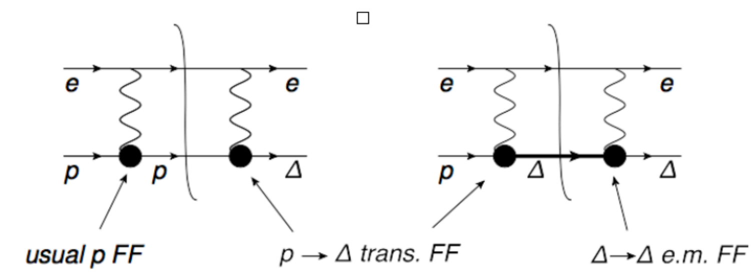
beam: $B_n \sim \alpha_{em} \frac{m_e}{E_e} \sim 10^{-6} - 10^{-5}$

B_n ep -> ep Phys.Rev.Lett. 107 (2011) 022501



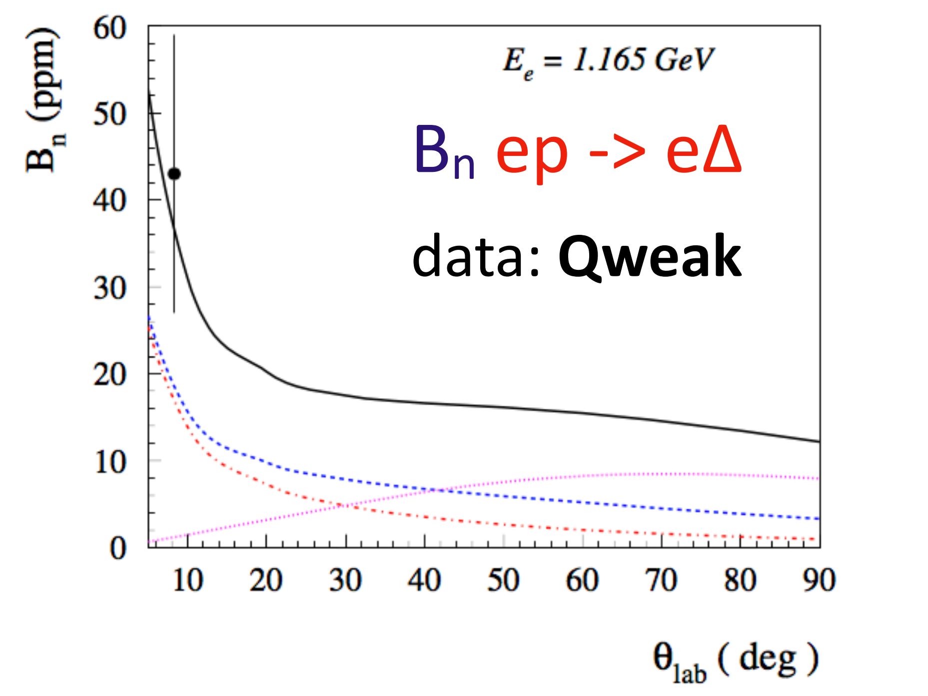
- B_n for ep -> eΔ accesses Δ e.m. FFs

Carlson, Pasquini, Pauk, Vdh (2017)



Results for QWeak kinematics

- Nucleon = dash-dot red line
- Δ = dashed blue line
- $S_{11} + D_{13}$ = dotted purple line
- Total = solid black line



Summary and outlook

- 2ɣ-exchange corrections are the largest **hadronic corrections** to Lamb shift in **muonic atoms**
  - **μH:** present TPE accuracy (2 μeV) is comparable with present Lamb shift accuracy (2.3 μeV)
  - **μD, μ $^3He^+$:** present TPE accuracy (15 μeV) is 5 times worse than Lamb shift accuracy (3.4 μeV)
  - crucial input for hyperfine splitting experiments (G_M, polarized structure functions)

- **electron scattering** has reached level of precision where TPE effects are clearly visible
  - cross section (Rosenbluth separation) vs Pt/Pl
  - epsilon dependence of Pl (needed for a full separation of TPE effects)
  - beam and target normal spin asymmetries (are zero in absence of TPE)
  - e^-/e^+ cross section ratios
  - high precision data forthcoming: PRad@JLab, MAGIX@MESA

- **muon scattering** experiments started/planned: MUSE@PSI will quantify TPE

- **Theoretical understanding**: dispersive calculations based on empirical input

  Low Q^2: - good quantitative understanding emerging

  - may be used to provide an improved extraction of R_M

  Larger Q^2: a quantitative understanding is still a challenge

- TPE effects in normal spin asymmetries -> spin offs: tool to access Δ e.m. FFs