

Status of baryon-baryon interactions from lattice QCD

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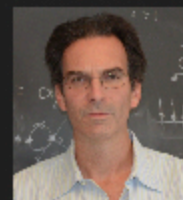
CIPANP Palm Springs

6/1/2018

NPLQCD: UNPHYSICAL NUCLEI

- ▶ Case study LQCD with unphysical quark masses ($m_\pi \sim 800$ MeV, 450 MeV)

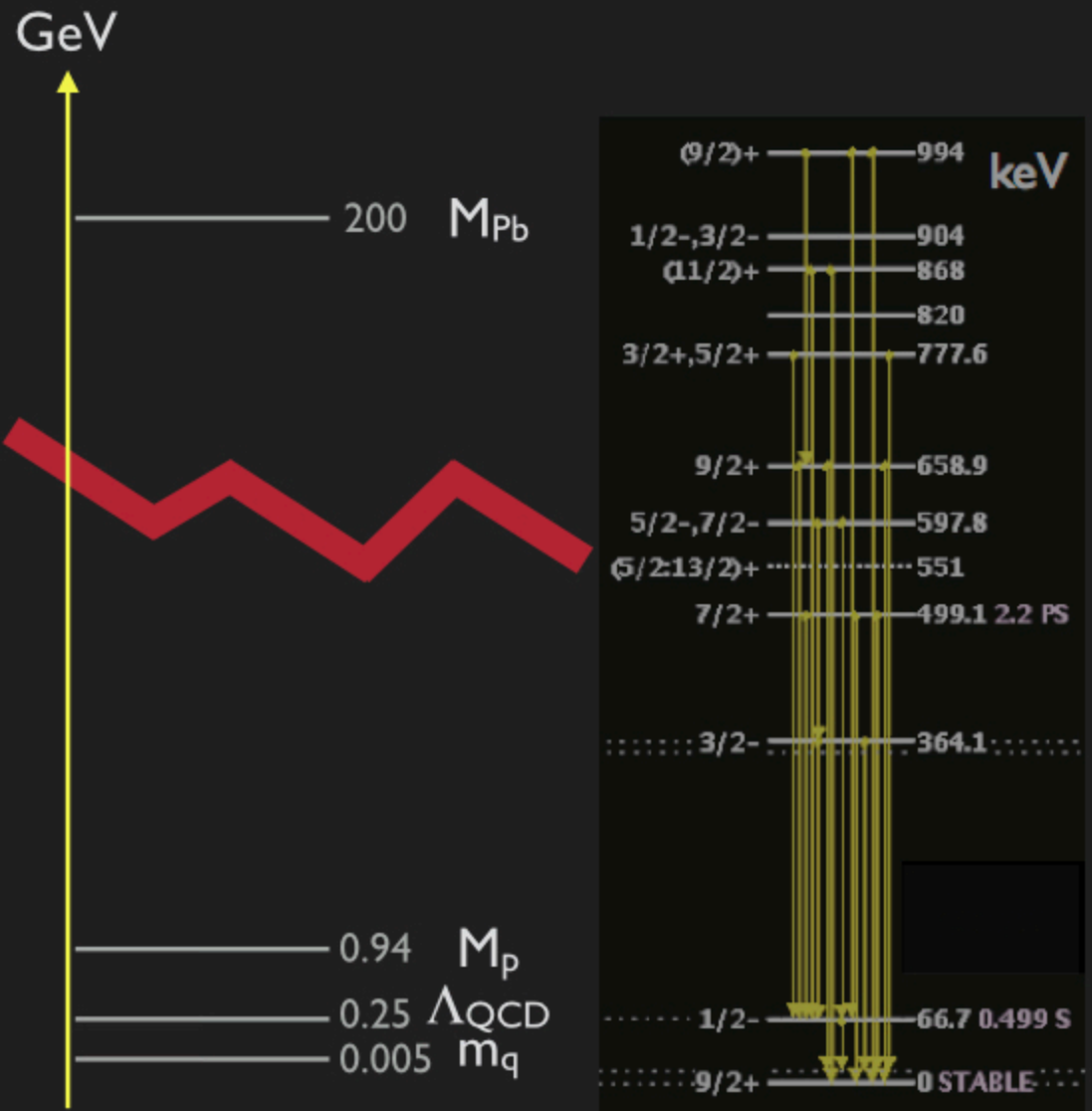
1. Spectrum and scattering of light nuclei ($A < 5$) [PRD 87 (2013), 034506]
2. Nuclear structure: magnetic moments, polarisabilities ($A < 5$) [PRL 113, 252001 (2014), PRL 116, 112301 (2016)]
3. Nuclear reactions: $np \rightarrow d\gamma$ [PRL 115, 132001 (2015)]
4. Gamow-Teller transitions: $pp \rightarrow d e \nu$, $g_A(^3\text{H})$ [PRL 119 062002 (2017)]
5. Double β decay: $pp \rightarrow nn$ [PRL 119, 062003 (2017)]
6. Gluon structure ($A < 4$) [PRD 96 094512 (2017)]
7. Scalar/tensor currents ($A < 4$) [PRL 120 152002 (2018)]



+ Arjun Gambhir (WM \rightarrow LLNL)

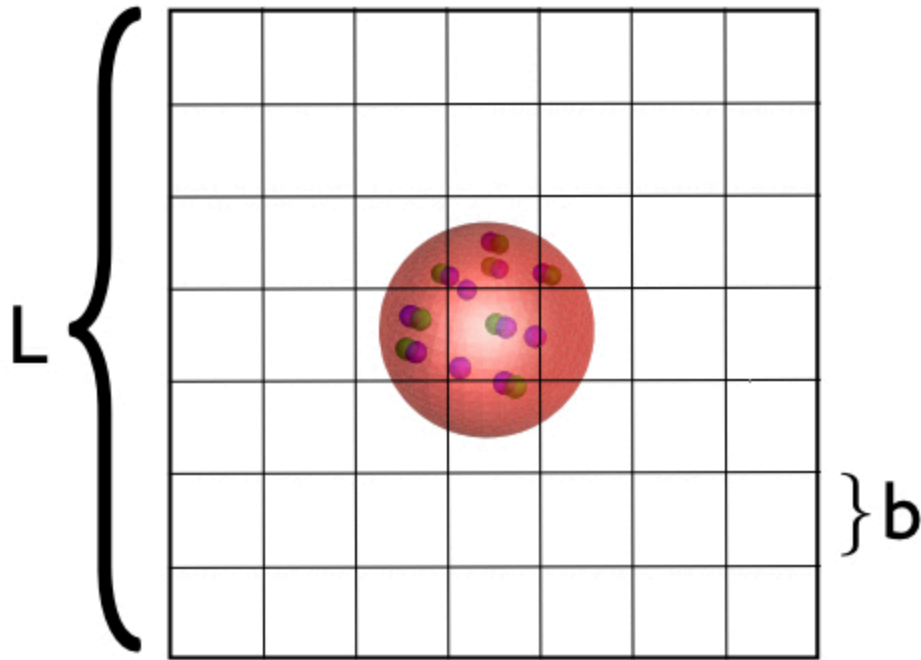
In principle can calculate properties of any nucleus from QCD and EW

Multi-scale physics with at least two exponentially difficult computational challenges



LATTICE QCD = QCD ON A GRID OR LATTICE

Non-perturbative definition of QCD



volume: $M_\pi L \gg 1$

infrared cutoff

lattice spacing: $b \ll M_N^{-1}$

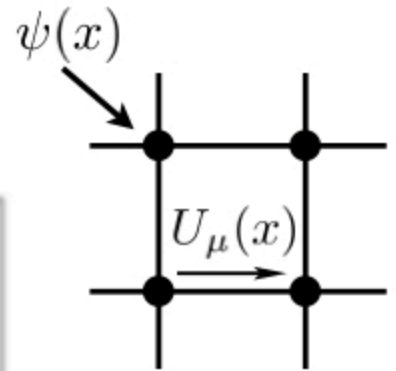
ultraviolet cutoff

Can use **Effective Field Theory** to extrapolate in L and b!

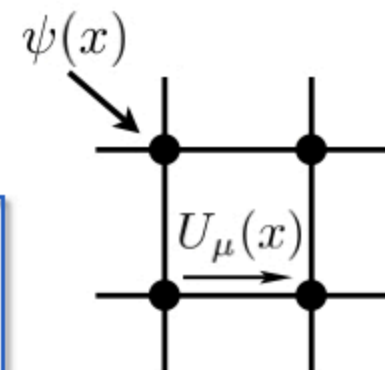
(systematic uncertainties from lattice artifacts are controlled)

QCD path integral with Montecarlo

$$\langle \mathcal{O} \rangle \sim \int dU_\mu d\bar{\psi} d\psi \mathcal{O}(U, \psi, \bar{\psi}) e^{-S_g(U) - \bar{\psi} D(U) \psi}$$



QCD path integral with Montecarlo



$$\langle \mathcal{O} \rangle \sim \int dU_\mu d\bar{\psi} d\psi \mathcal{O}(U, \psi, \bar{\psi}) e^{-S_g(U) - \bar{\psi} D(U) \psi}$$

propagators
(detector)

N gauge configurations
(accelerator)

$$\langle \mathcal{O} \rangle \sim \int dU_\mu \mathcal{O}(D(U)^{-1}) \det(f(U)) e^{-S_g(U)}$$

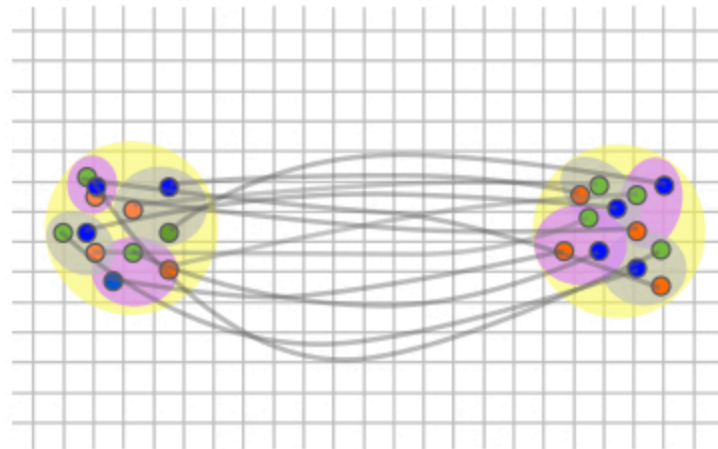
$$= \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{i=1}^N \mathcal{O}(D(U_i)^{-1})$$

Estimate of \mathcal{O} with $\sigma_{\mathcal{O}} \sim 1/\sqrt{N}$

Why is lattice QCD for nuclear physics hard ?

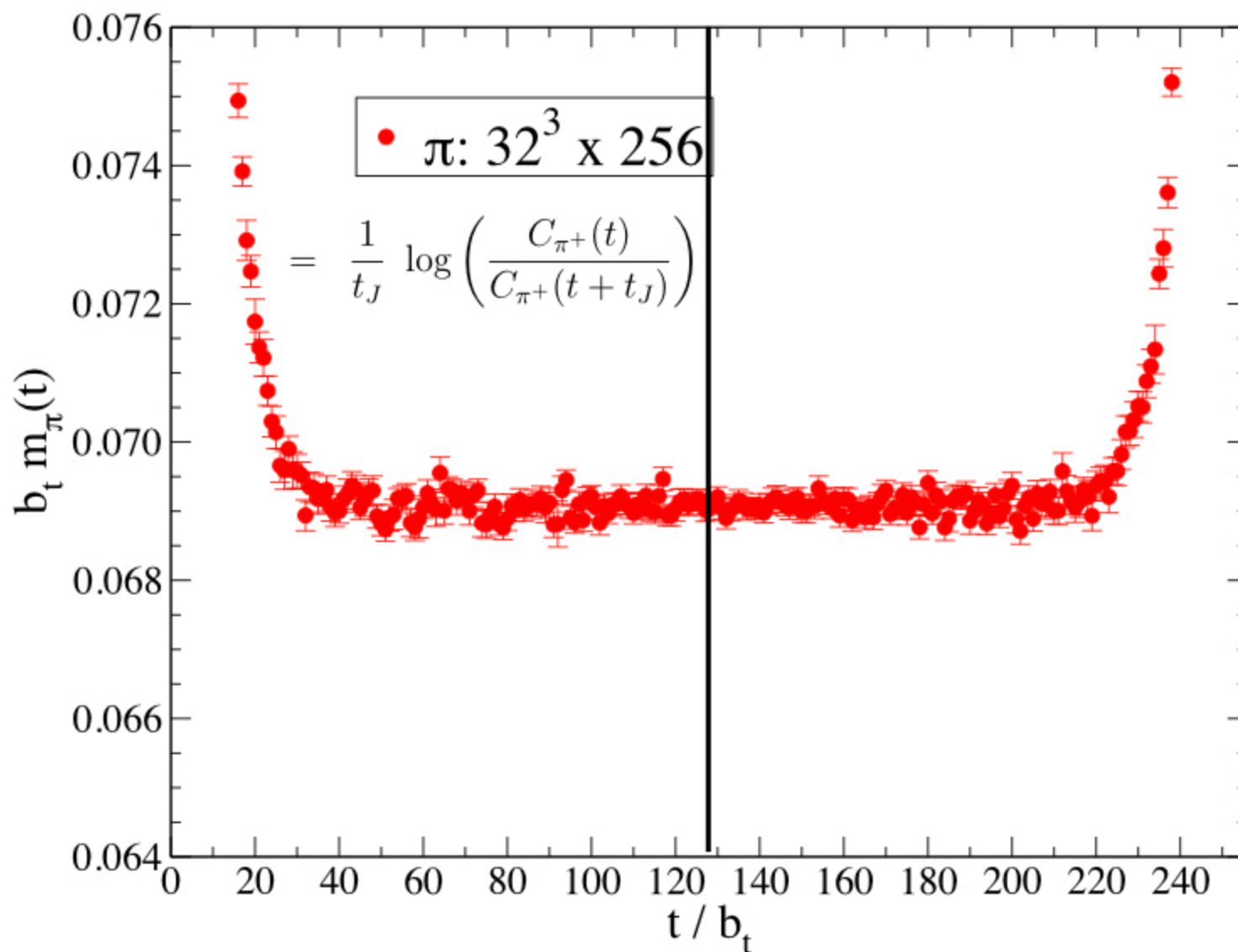
- Signal/noise (sign problem) and statistics
- Number of contractions

[Detmold,Orginos(2012),Doi,Endres(2012)]

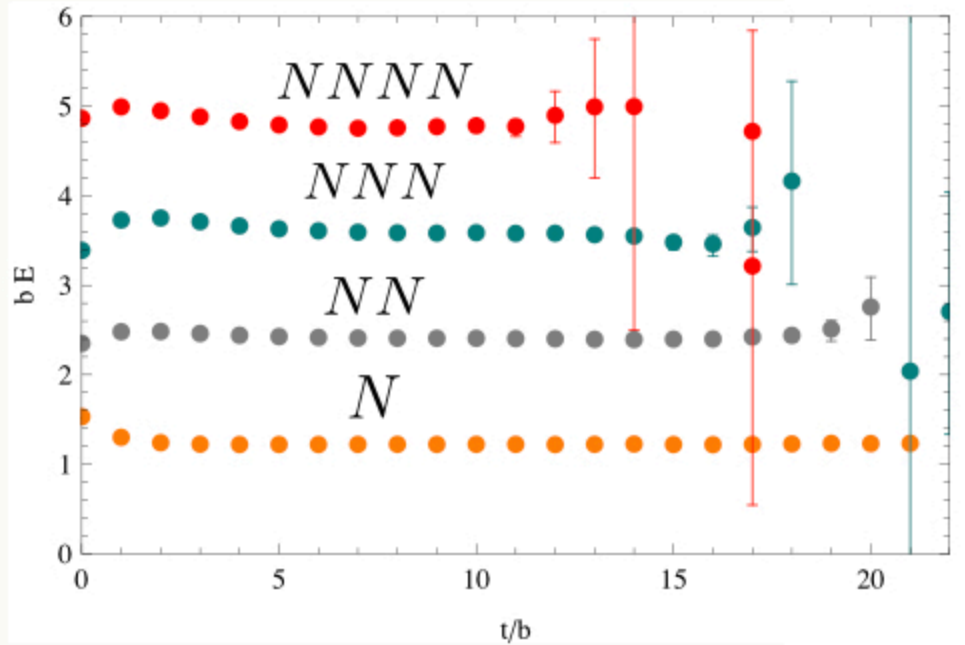
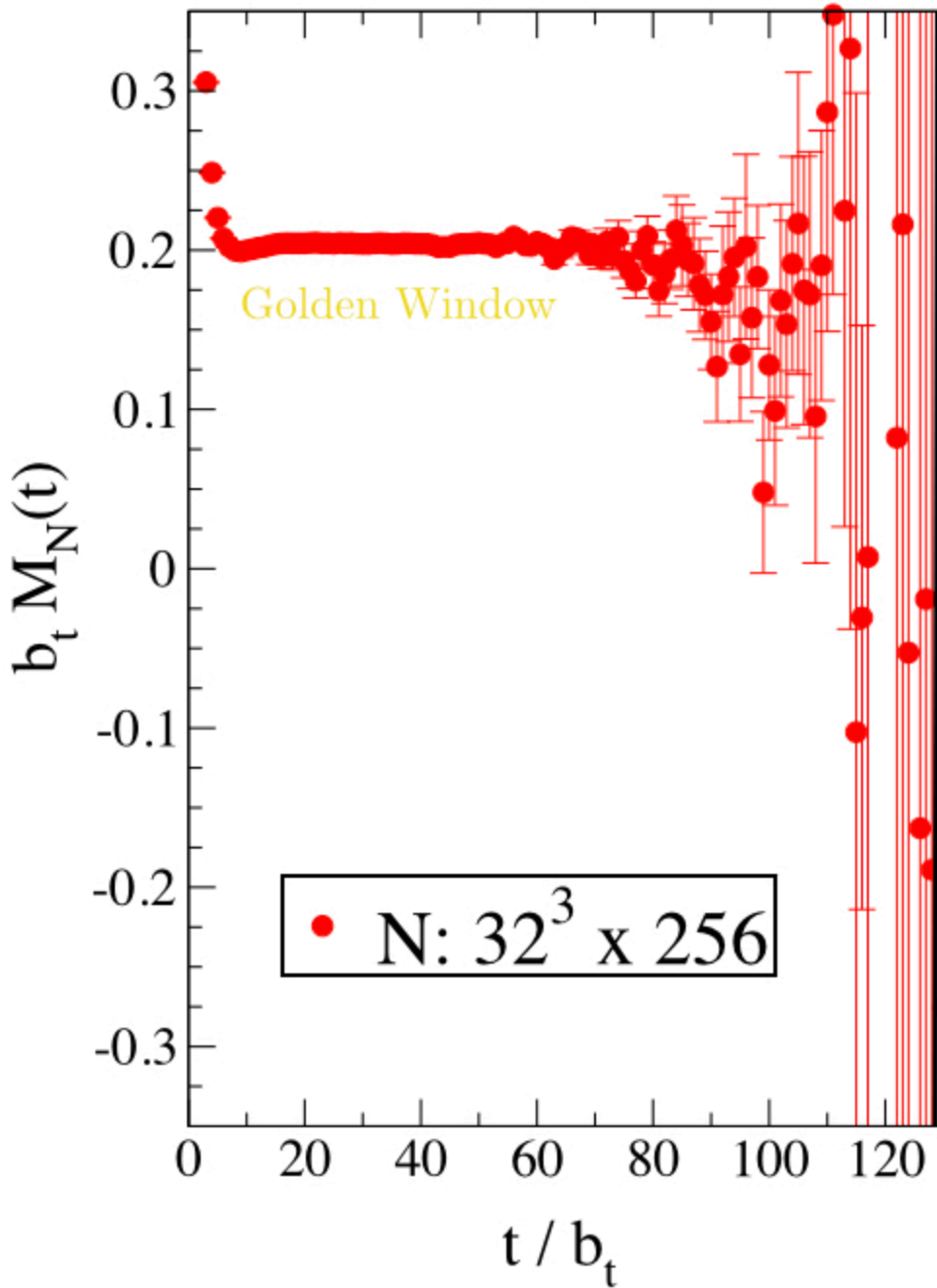


SIGNAL/NOISE PROBLEM

$$C_{\pi^+}(t) = \sum_{\mathbf{x}} \langle 0 | \pi^-(\mathbf{x}, t) \pi^+(\mathbf{0}, 0) | 0 \rangle \longrightarrow e^{-m_{\pi} t} \dots$$



**pions are
easy!
(i.e.
cheap)**

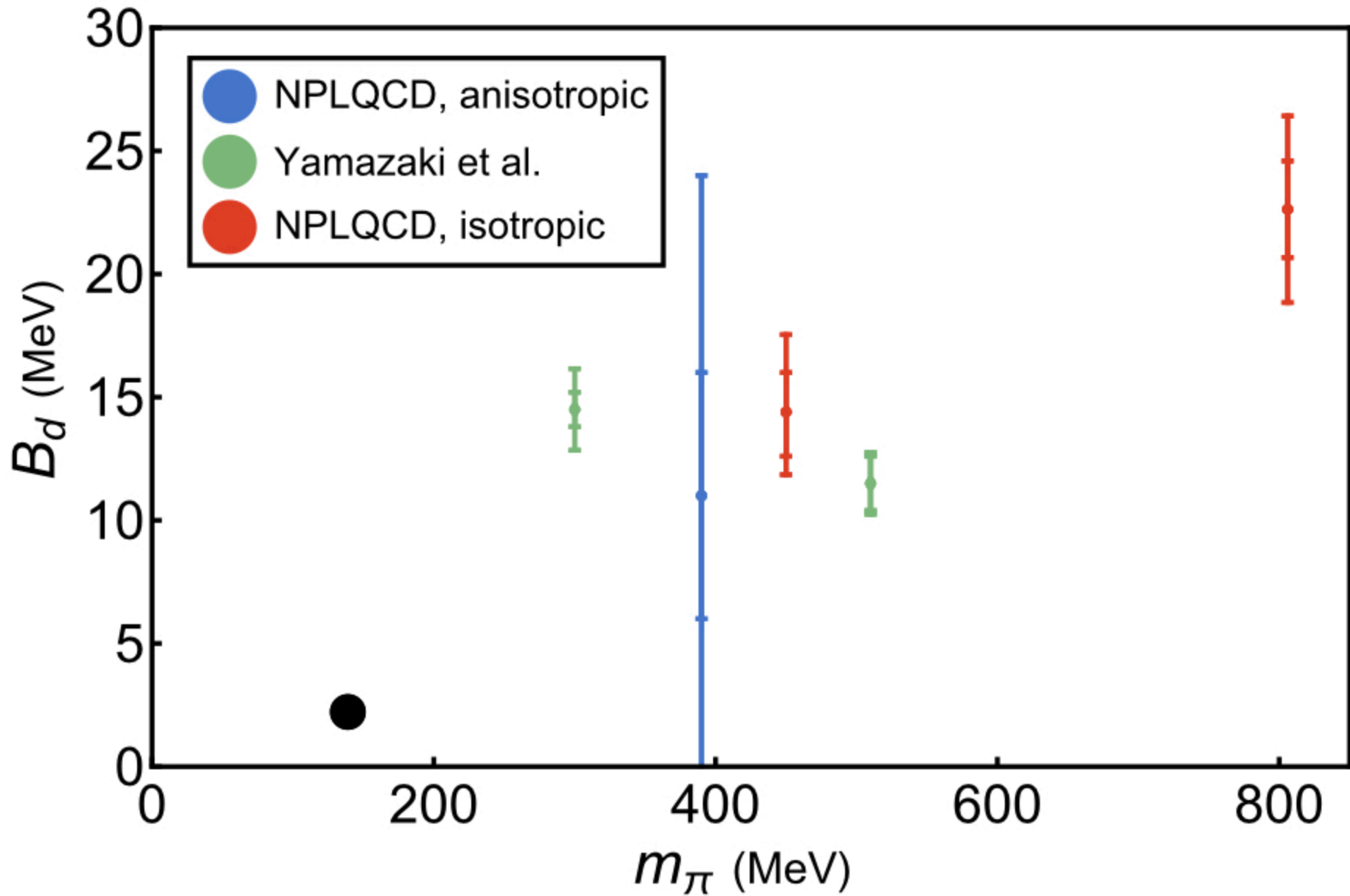


$$\frac{\text{noise}}{\text{signal}} \sim \frac{1}{\sqrt{N}} e^{A(m_p - \frac{3}{2}m_\pi)t}$$

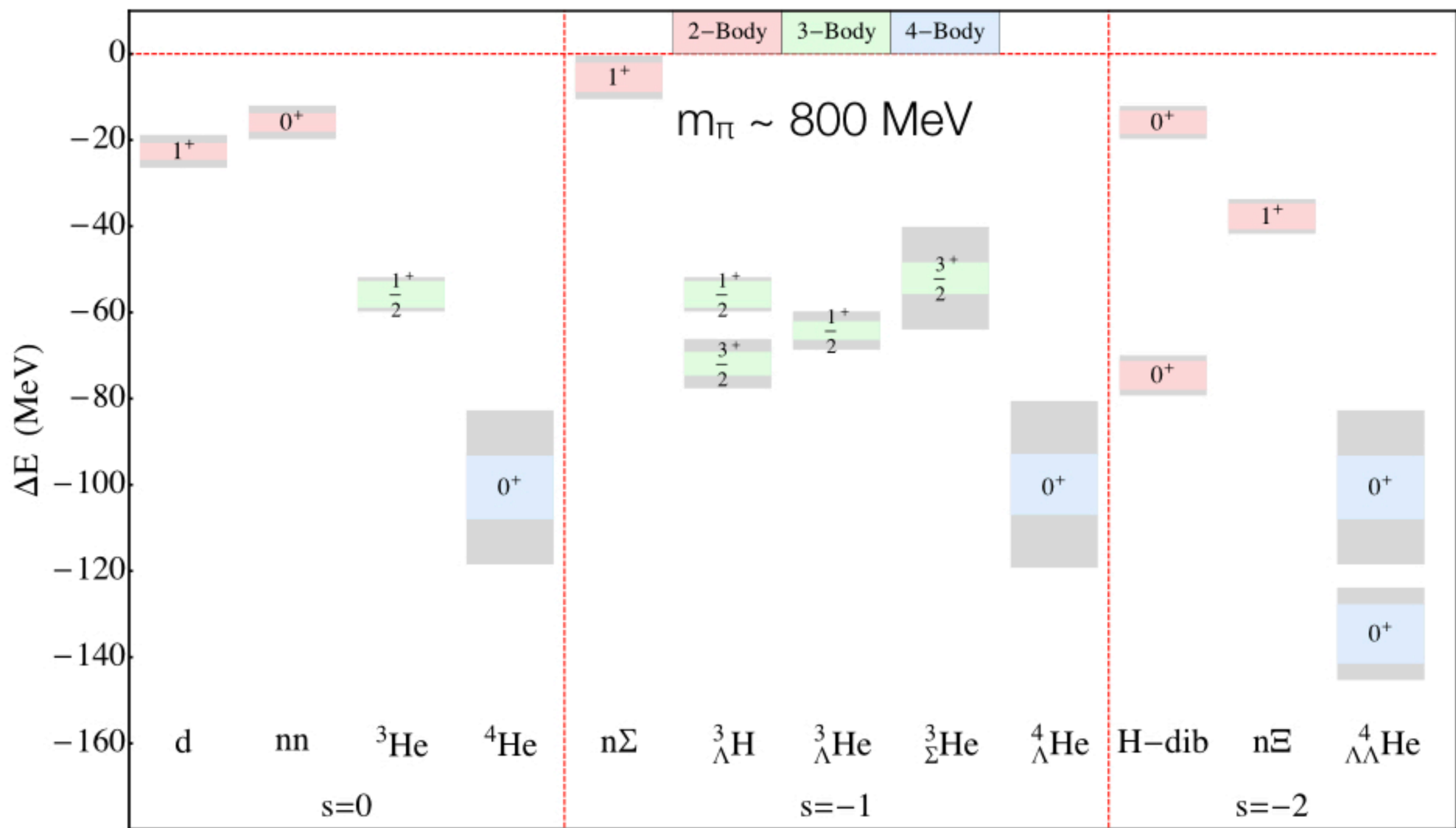
**baryons are
hard! (i.e. costly)**

(Need quantum computers?)

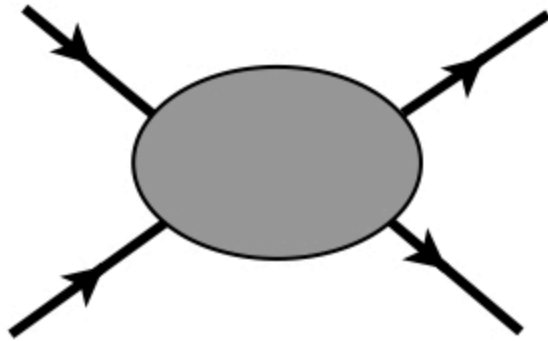
Deuteron binding energy from LQCD



LIGHT (HYPER)NUCLEI AT SU(3) POINT



Nucleon-nucleon scattering



$$k \cot \delta = -\frac{1}{a} + \frac{1}{2}r|\mathbf{k}|^2 + P|\mathbf{k}|^4 + \mathcal{O}(|\mathbf{k}|^6)$$



effective range:
range of interaction

scattering length: unbounded

EXPERIMENT:

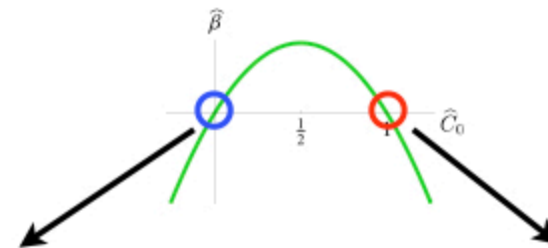
$$a^{(1S_0)} = -23.71 \text{ fm}$$

$$a^{(3S_1)} = 5.43 \text{ fm}$$

$$r^{(1S_0)} = 2.73 \text{ fm}$$

$$r^{(3S_1)} = 1.75 \text{ fm}$$

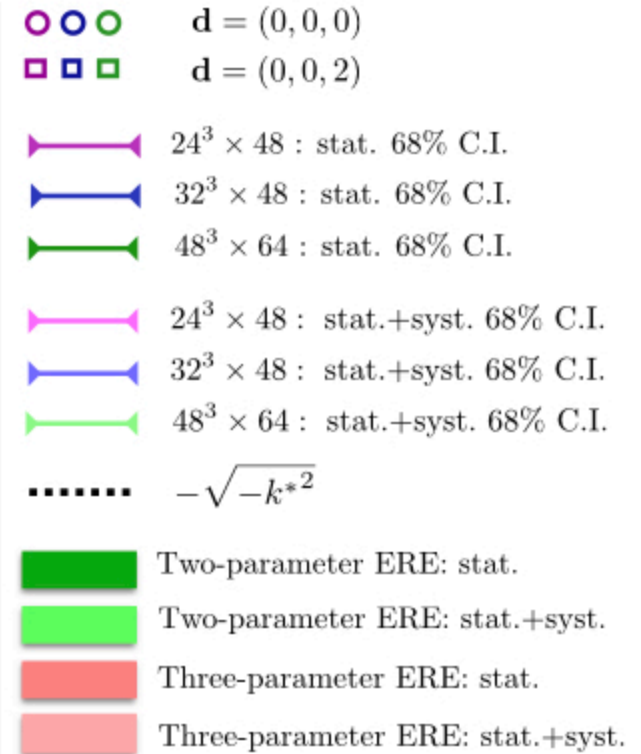
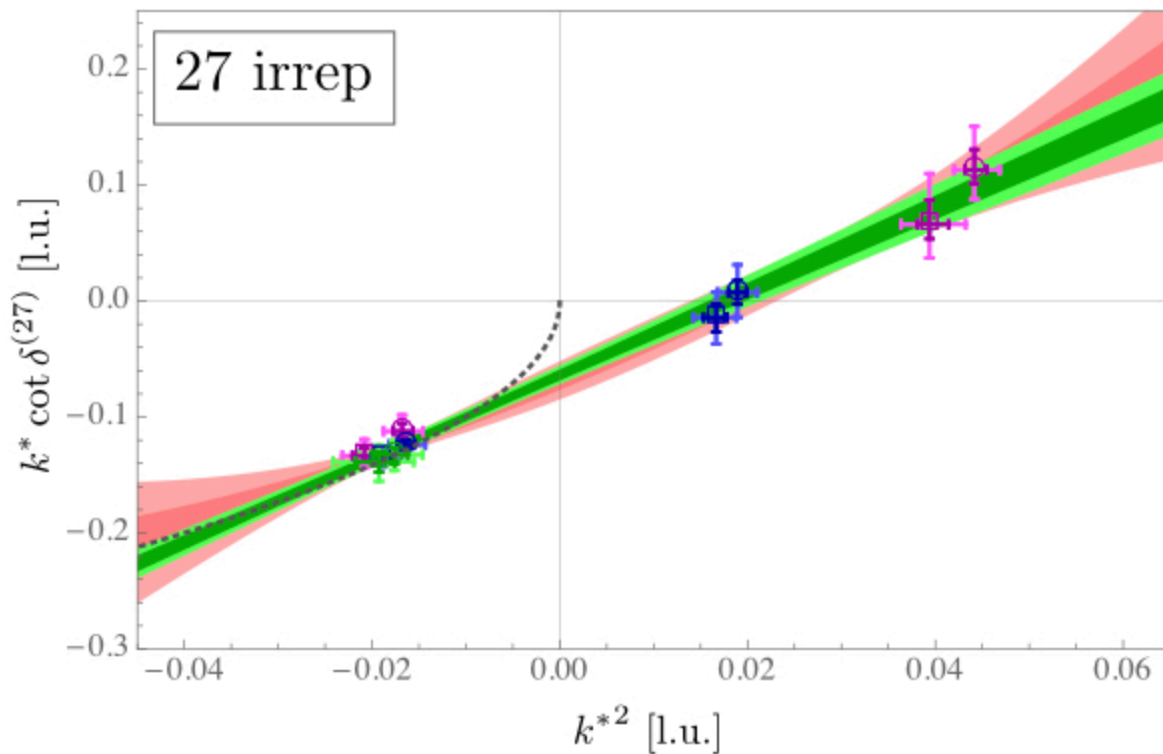
$$a^{(1S_0)} \gg \Lambda_{QCD}^{-1}$$



Trivial IR fixed point:
"natural case"

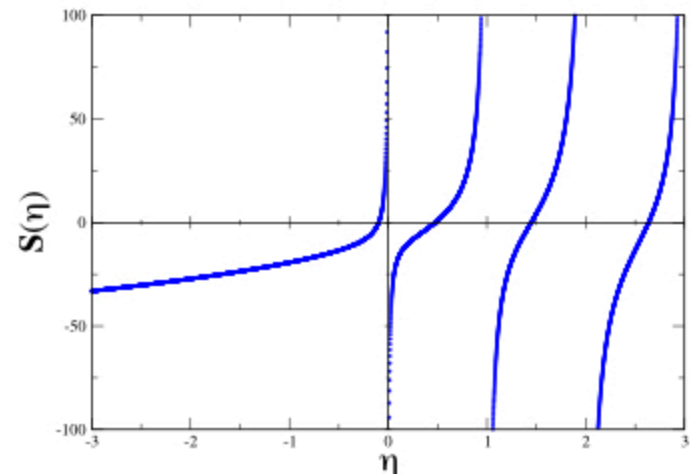
Nontrivial UV fixed point:
"unnatural case"

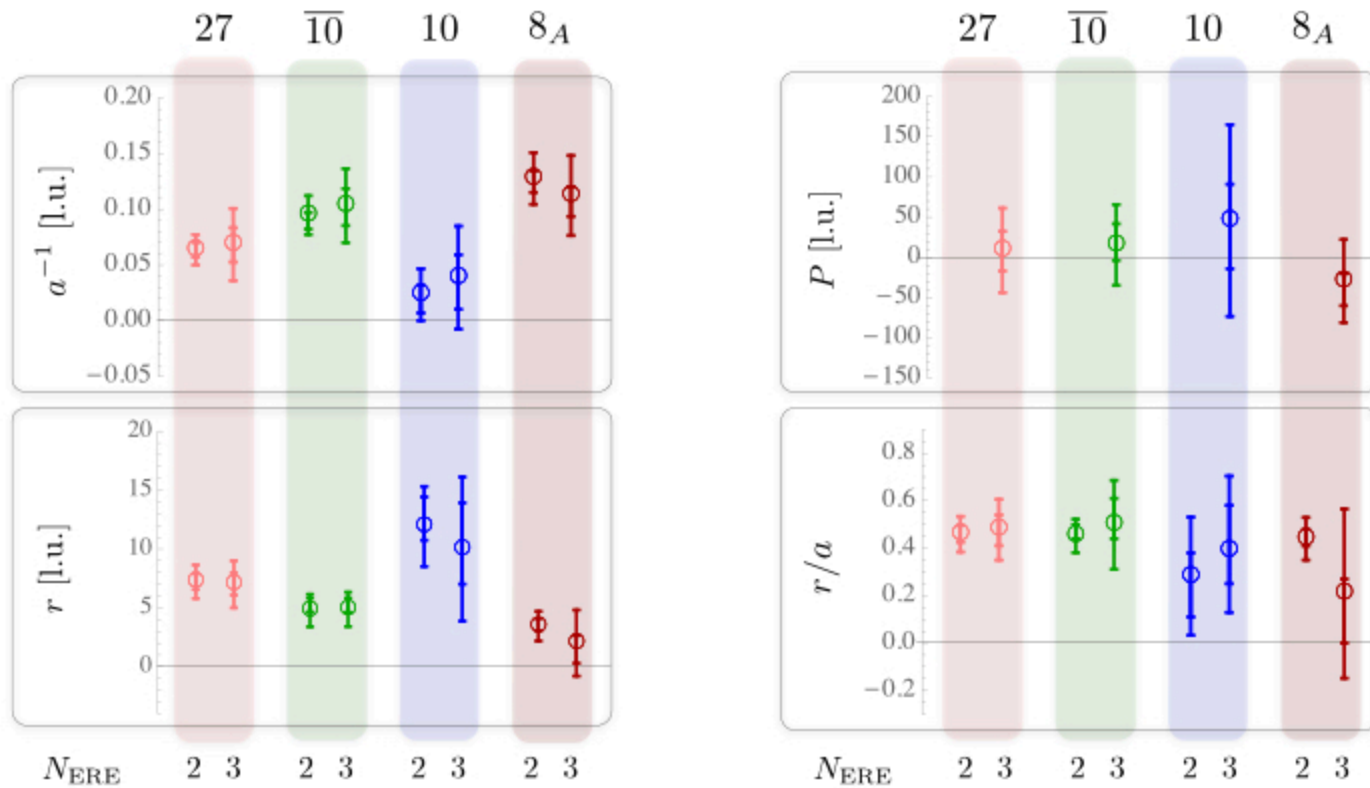
Baryon-Baryon S-wave phase shifts



$$p \cot \delta = \frac{1}{\pi L} \mathcal{S}(\tilde{p})$$

$$\mathcal{S}(x) \equiv \sum_{\mathbf{n}} \frac{\Lambda_{\mathbf{n}}}{|\mathbf{n}|^2 - x^2} - 4\pi \Lambda_{\mathbf{n}}$$





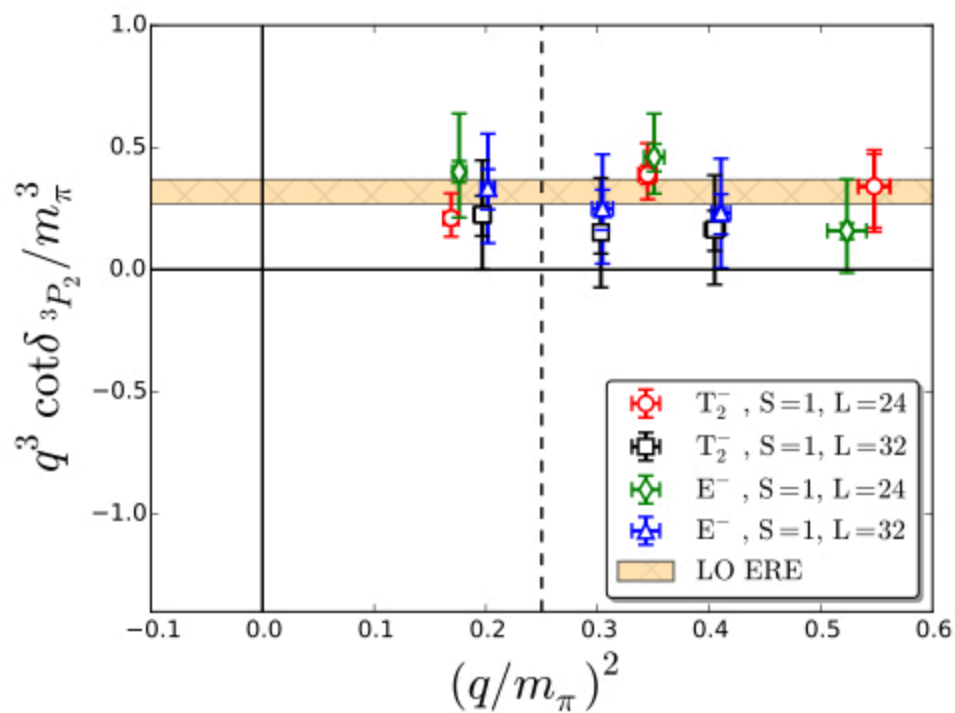
$SU(6)$ from large- N_c

$$\begin{aligned}
 \left[-\frac{1}{a^{(27)}} + \mu \right]^{-1} &= \frac{M_B}{2\pi} \left(a - \frac{b}{27} \right) + \mathcal{O} \left(\frac{1}{N_c^2} \right), & \left[-\frac{1}{a^{(\overline{10})}} + \mu \right]^{-1} &= \frac{M_B}{2\pi} \left(a - \frac{b}{27} \right) + \mathcal{O} \left(\frac{1}{N_c^2} \right), \\
 \left[-\frac{1}{a^{(10)}} + \mu \right]^{-1} &= \frac{M_B}{2\pi} \left(a + \frac{7b}{27} \right) + \mathcal{O} \left(\frac{1}{N_c} \right), & \left[-\frac{1}{a^{(8_A)}} + \mu \right]^{-1} &= \frac{M_B}{2\pi} \left(a + \frac{b}{27} \right) + \mathcal{O} \left(\frac{1}{N_c} \right), \\
 \left[-\frac{1}{a^{(8_S)}} + \mu \right]^{-1} &= \frac{M_B}{2\pi} \left(a + \frac{b}{3} \right) + \mathcal{O} \left(\frac{1}{N_c} \right), & \left[-\frac{1}{a^{(1)}} + \mu \right]^{-1} &= \frac{M_B}{2\pi} \left(a - \frac{b}{3} \right) + \mathcal{O} \left(\frac{1}{N_c} \right),
 \end{aligned}$$

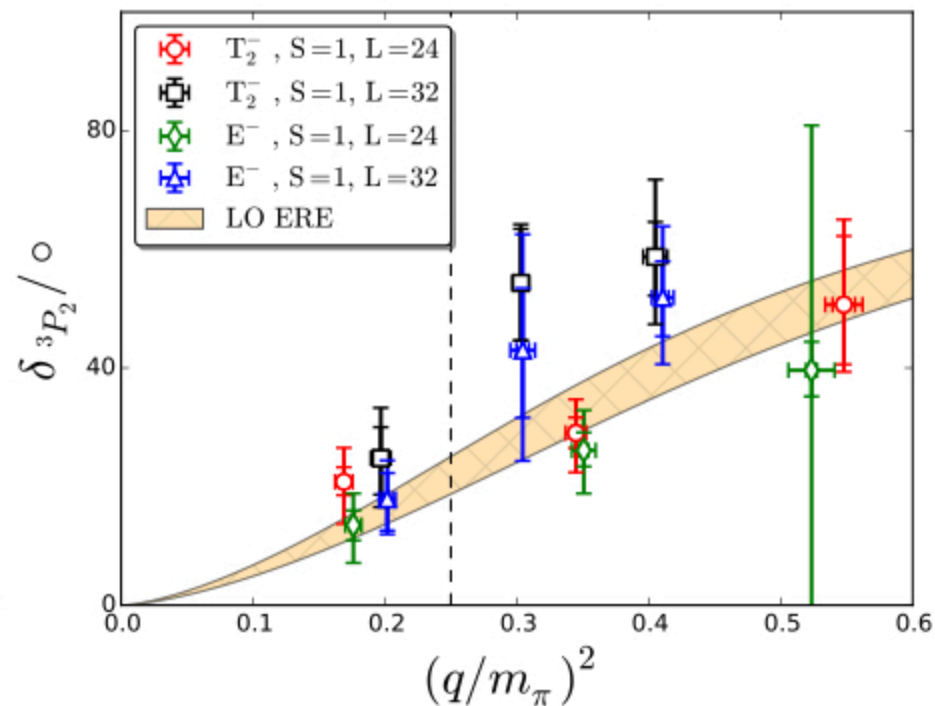
NN P-wave phase shifts

CalLat Collaboration (using NPLQCD resources)

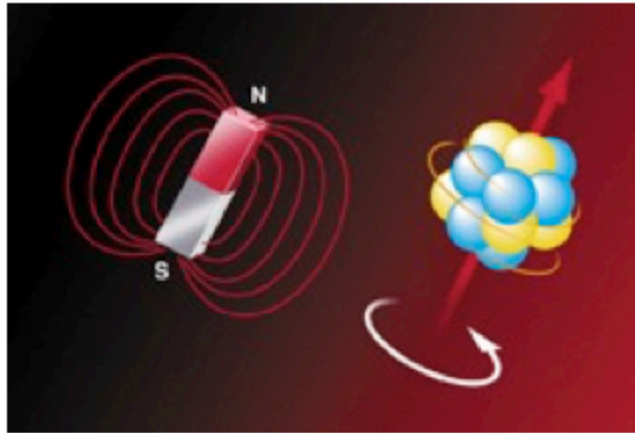
$m_\pi \sim 800$ MeV



Amy Nicholson *et al* (CalLatt)



Nuclear structure: magnetic moments



$U_Q(1)$ phase

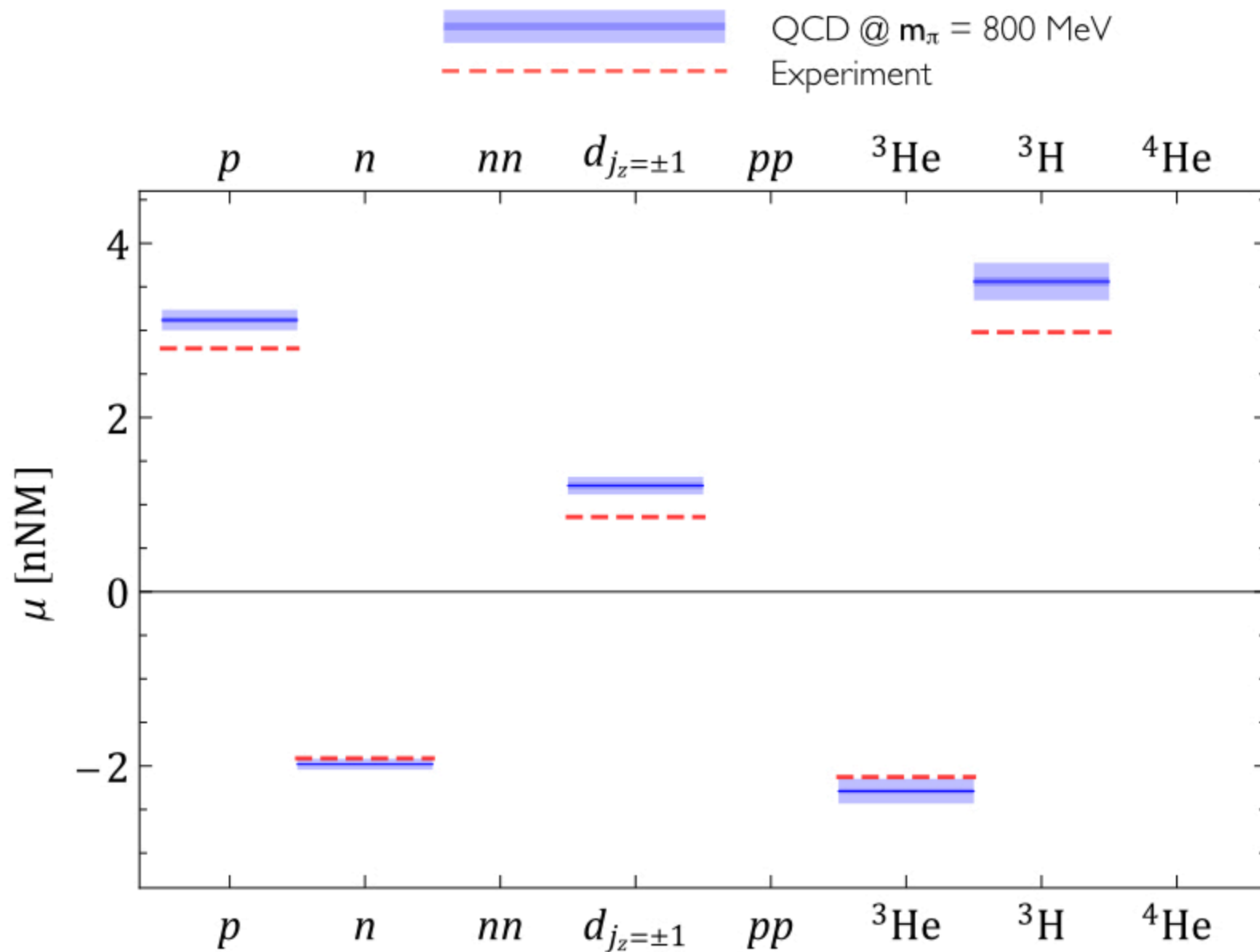
$$U_\mu(x) = e^{i\frac{6\pi Q_q \tilde{n}}{L^2} x_1 \delta_{\mu,2}} \times e^{-i\frac{6\pi Q_q \tilde{n}}{L} x_2 \delta_{\mu,1} \delta_{x_1, L-1}}$$

- ◆ Hadronic and nuclear correlation functions are modified in the presence of a background magnetic field:

$$E_{h;j_z}(\mathbf{B}) = \sqrt{M_h^2 + P_{\parallel}^2 + (2n_L + 1)|Q_h e \mathbf{B}|} - \boldsymbol{\mu}_h \cdot \mathbf{B} - 2\pi\beta_h^{(M0)} |\mathbf{B}|^2 - 2\pi\beta_h^{(M2)} \langle \hat{T}_{ij} B_i B_j \rangle + \dots$$

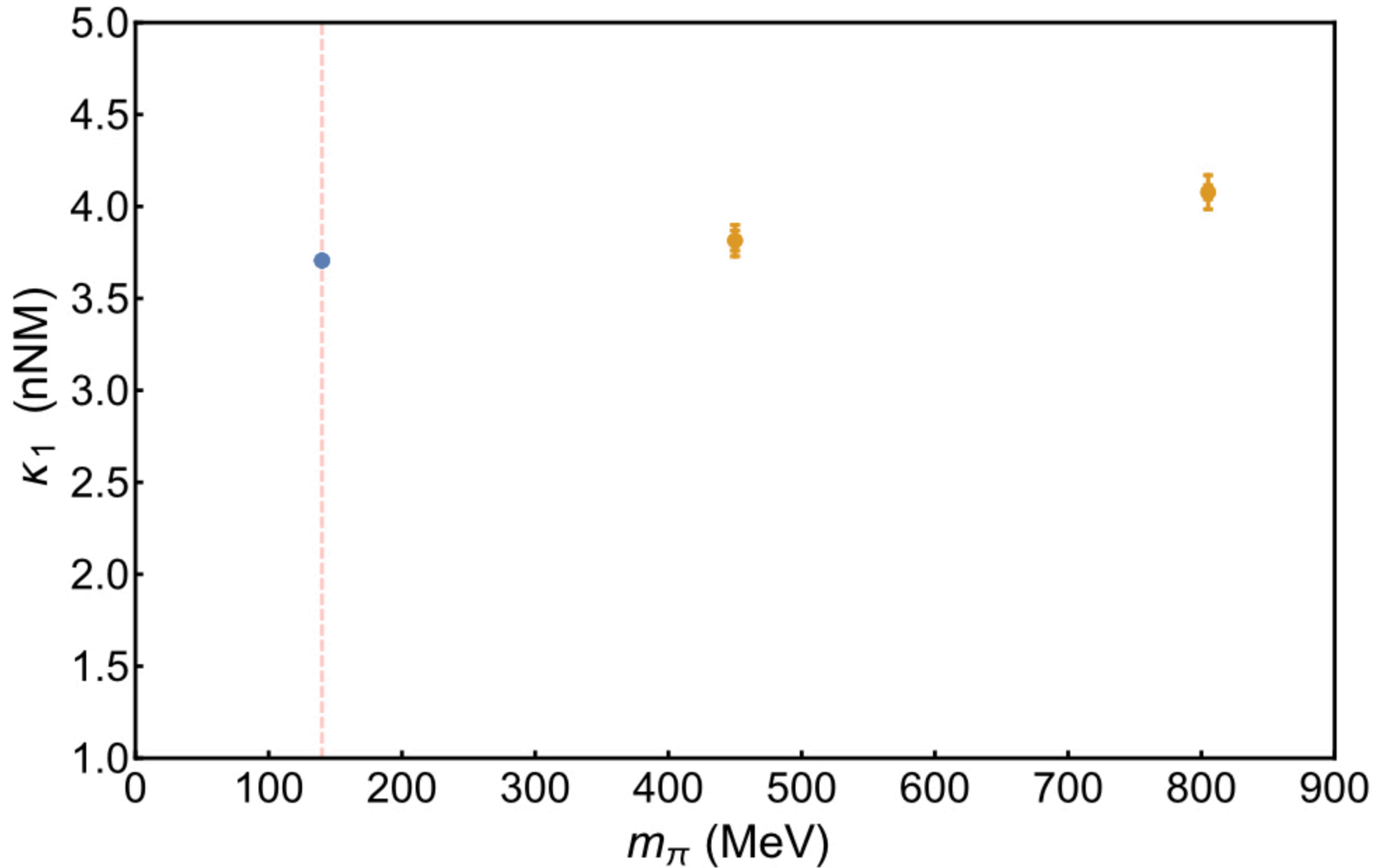
Landau level

- ◆ Can extract magnetic moments, polarizabilities, ...
- ◆ Extendable to external electric fields, etc.

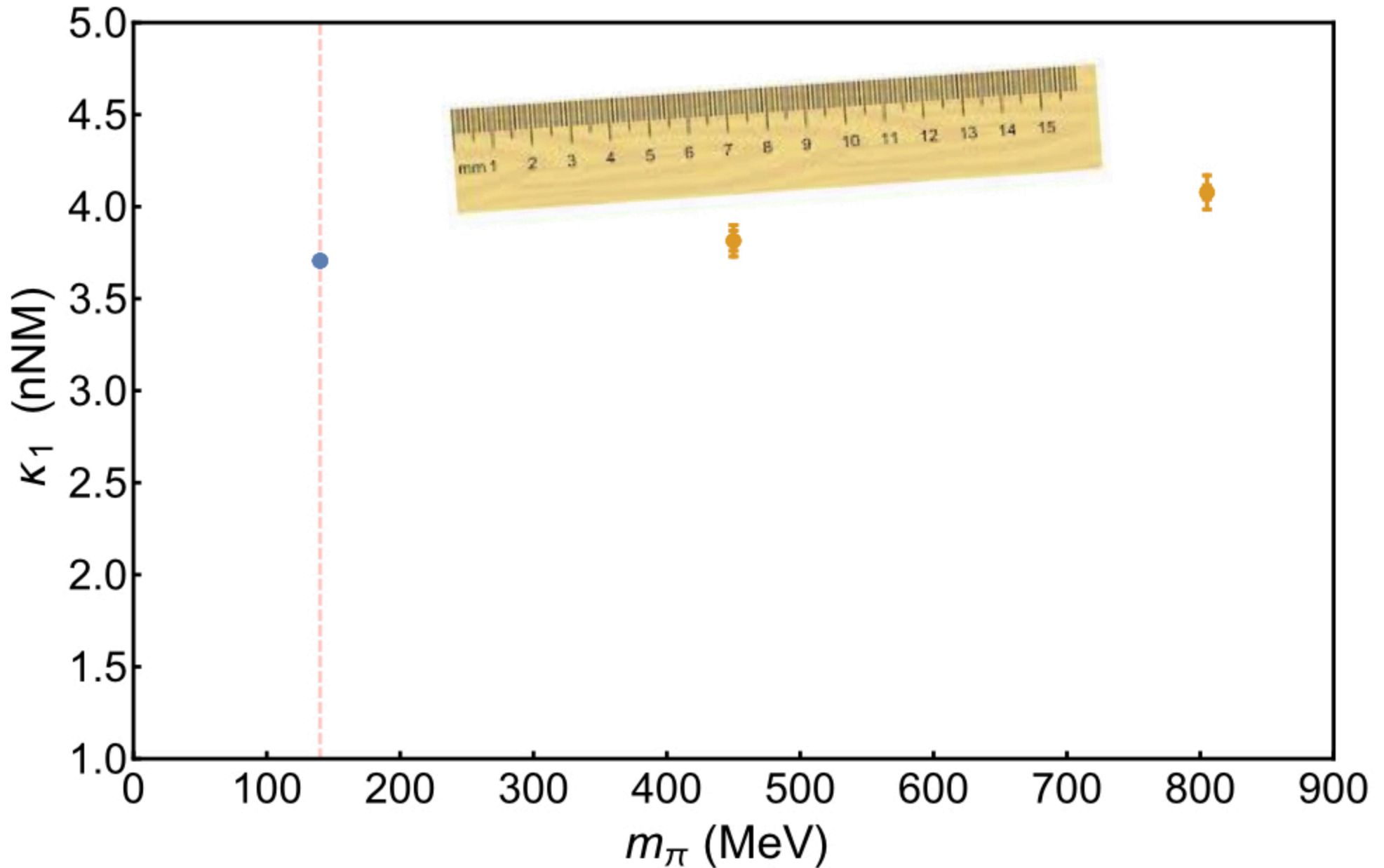


- ◆ Almost no quark mass dependence in units of $\frac{e}{2M(m_\pi)}$

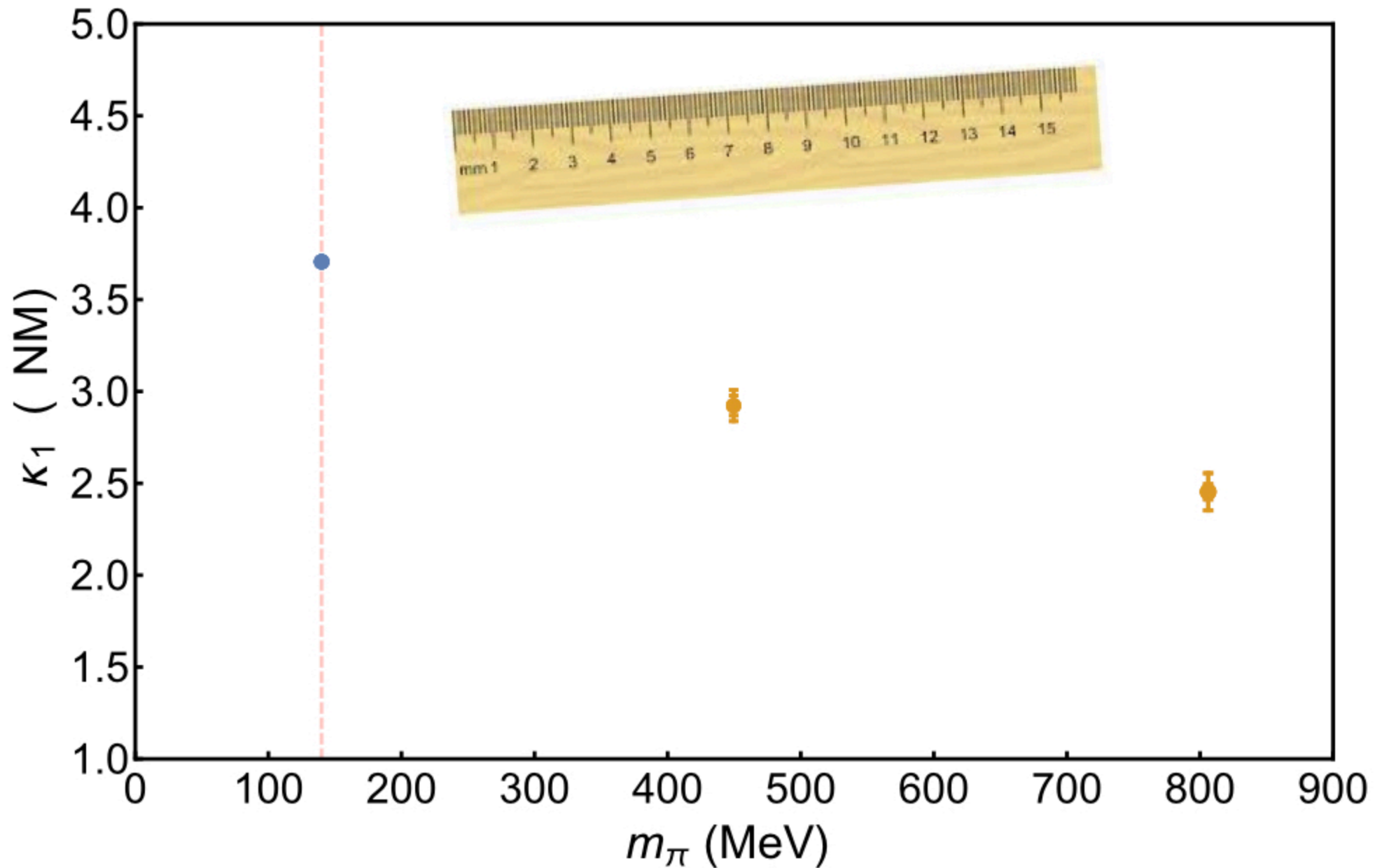
Nucleon isovector magnetic moment



Nucleon isovector magnetic moment



Nucleon isovector magnetic moment

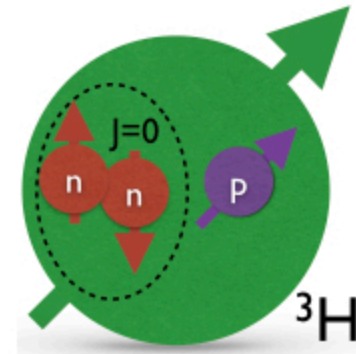


Nuclei as groupings of nucleons: shell model!

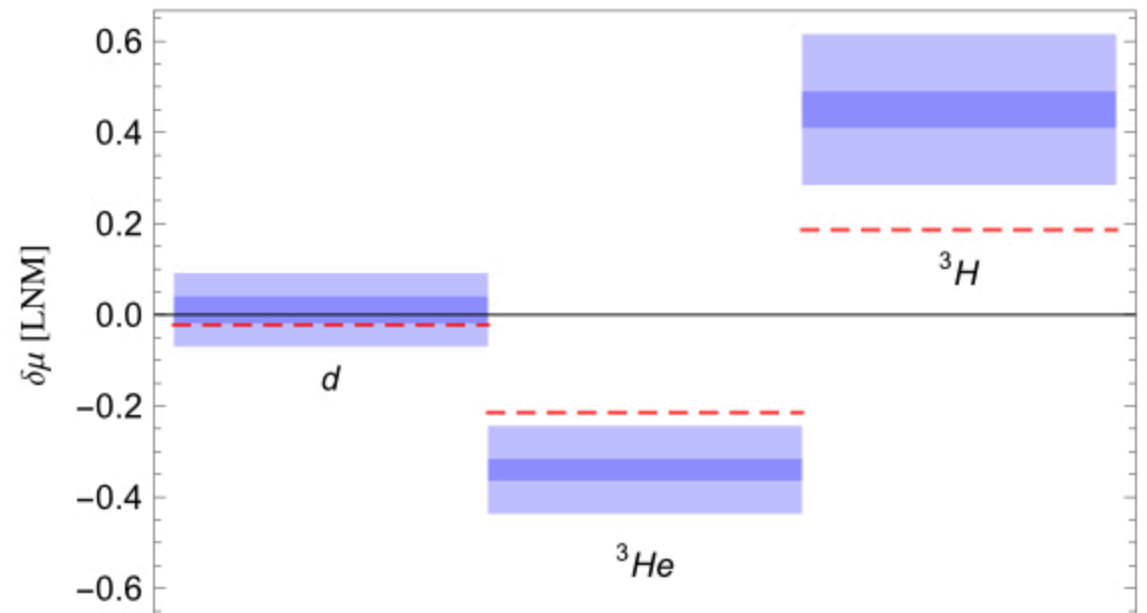
$$\mu^{3\text{H}} = \mu_p$$

$$\mu^{3\text{He}} = \mu_n$$

$$\mu_d = \mu_p + \mu_n$$

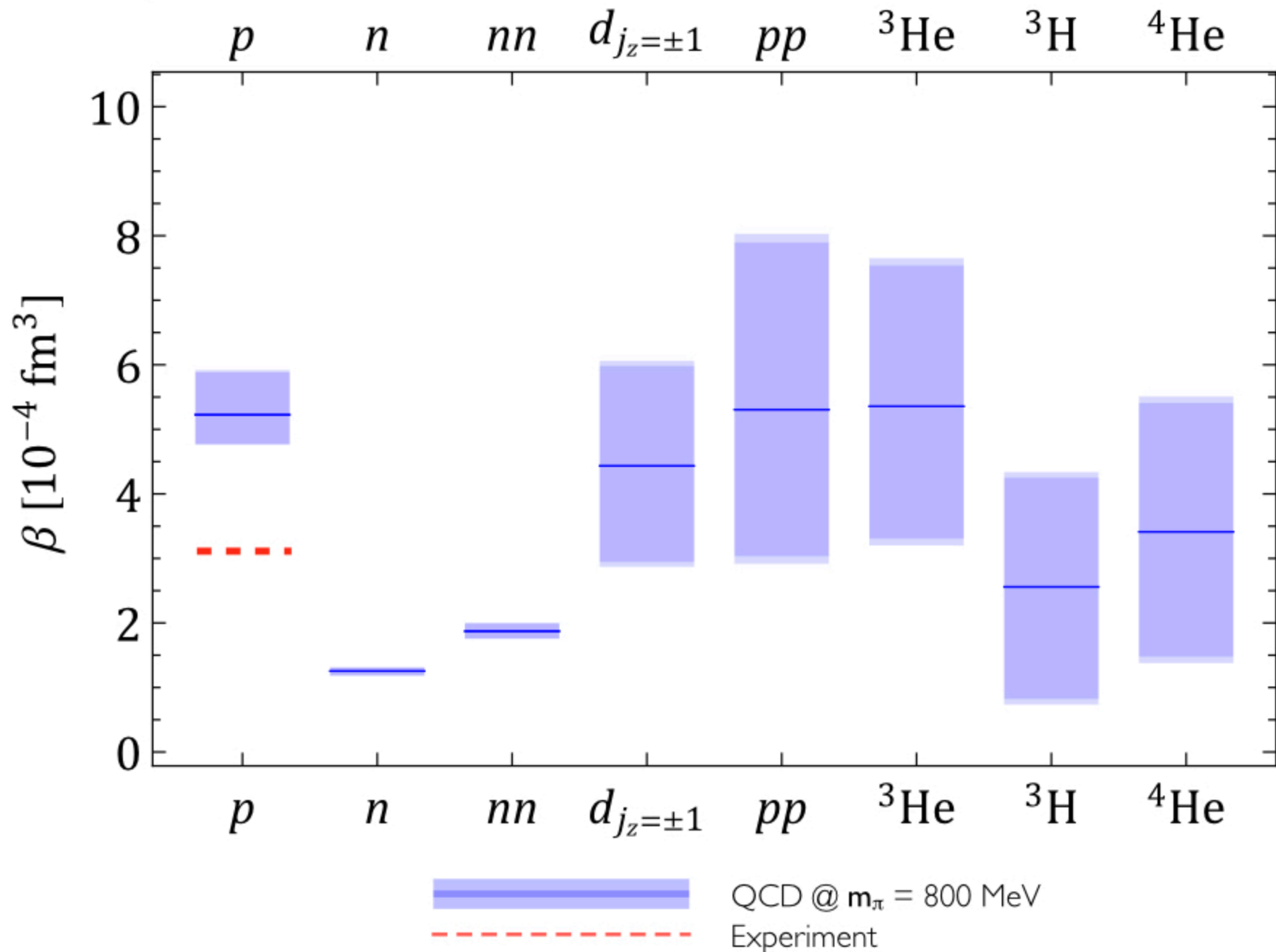


Difference between
nuclear magnetic
moments and shell
model predictions

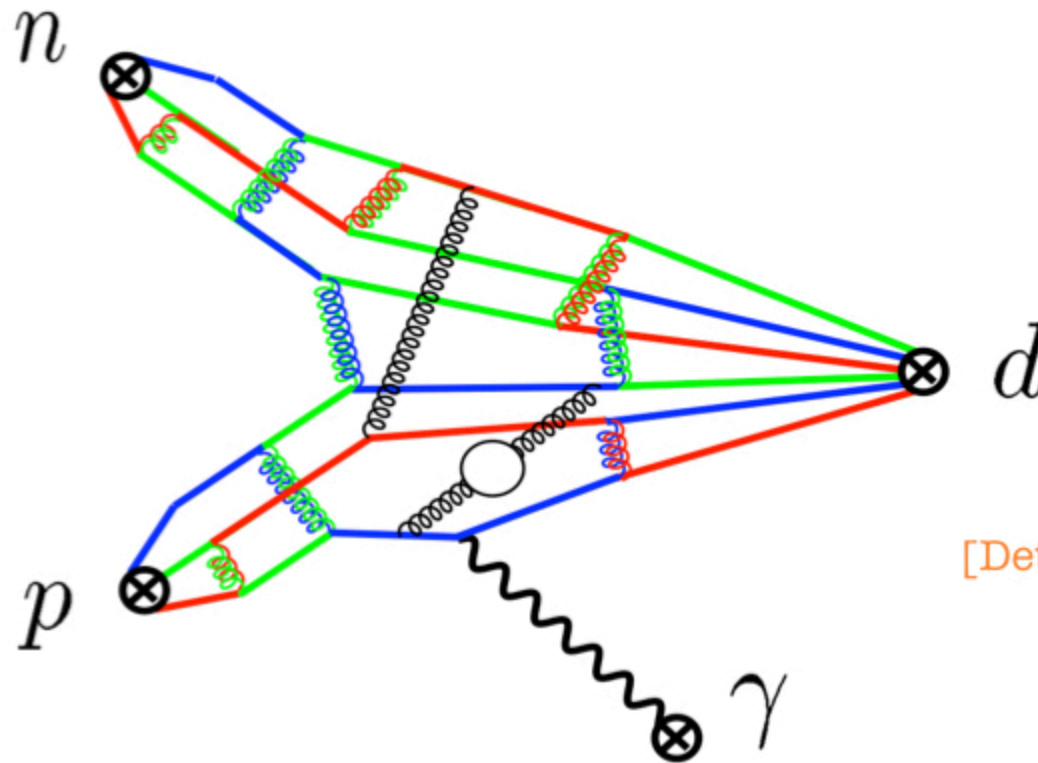


QCD @ $m_\pi = 800$ MeV
Experiment

Nuclear structure: polarizabilities



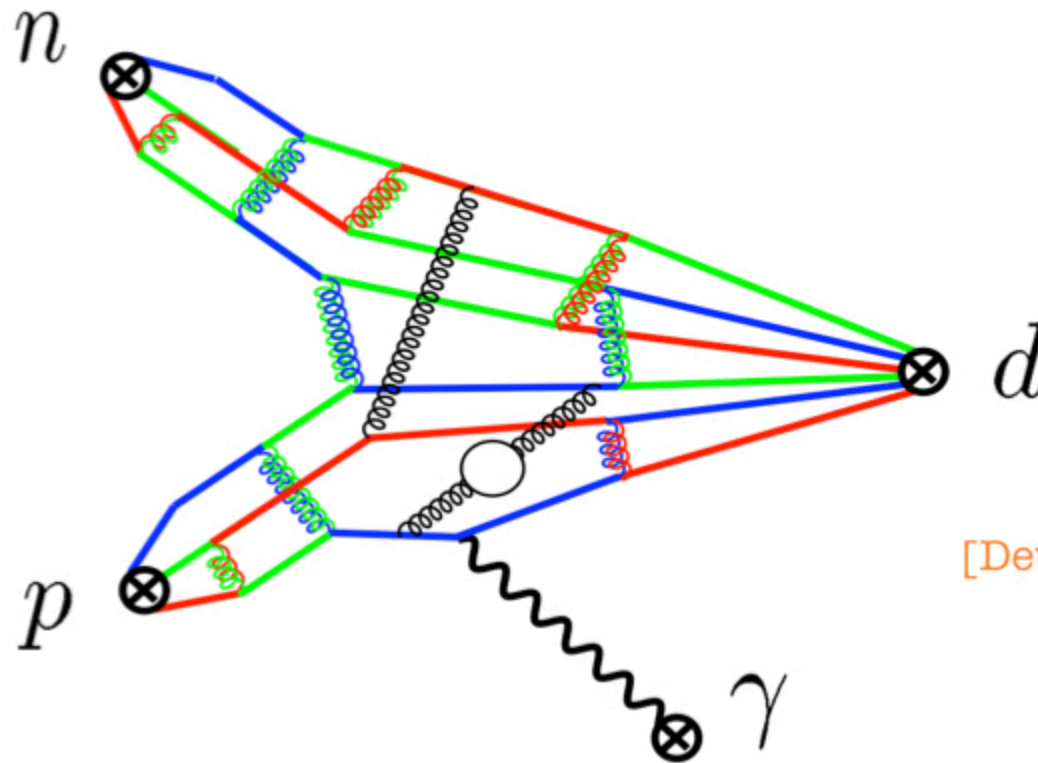
Nuclear reaction: $np \rightarrow d\gamma$



[Detmold and Savage (2004)]

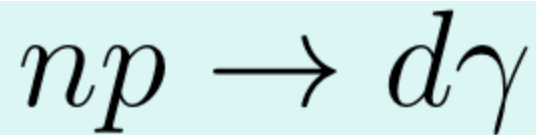
$$\Delta E_{3S_1, 1S_0} = \mp Z_d^2 (\kappa_1 + \gamma_0 l_1) \frac{|e\mathbf{B}|}{M} + \dots = \mp (\kappa_1 + \bar{L}_1) \frac{|e\mathbf{B}|}{M} + \dots$$

Nuclear reaction: $np \rightarrow d\gamma$

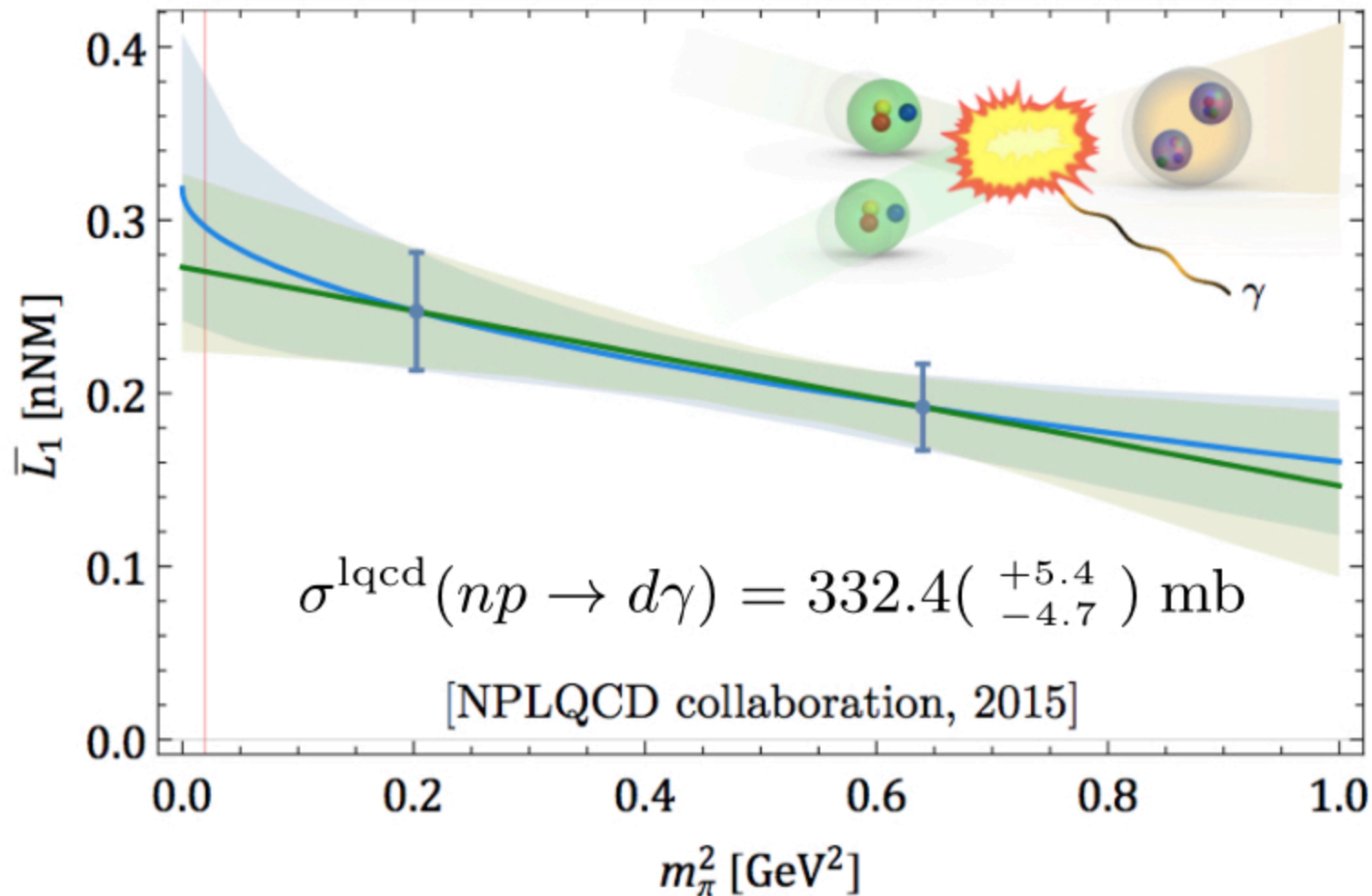


[Detmold and Savage (2004)]

$$\Delta E_{3S_1, 1S_0} = \mp Z_d^2 (\kappa_1 + \gamma_0 l_1) \frac{|e\mathbf{B}|}{M} + \dots = \mp (\kappa_1 + \bar{L}_1) \frac{|e\mathbf{B}|}{M} + \dots$$



$np \rightarrow d\gamma$ cross section from lattice QCD



$$\sigma^{\text{expt}}(np \rightarrow d\gamma) = 334.2(0.5) \text{ mb}$$

Nuclear structure: axial transitions

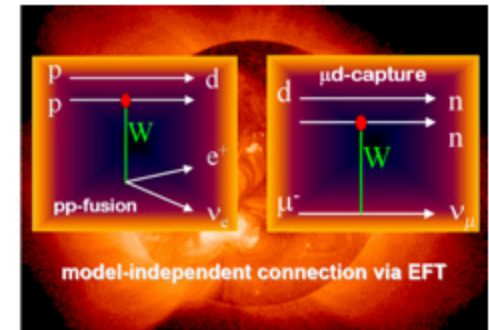
◆ Axial coupling to the NN system

◆ $\mu^- d \rightarrow nn\nu_\mu$: MuSun @ PSI

◆ $\nu d \rightarrow e^+ nn$: SNO

◆ “calibrating the sun” $pp \rightarrow de^+\nu_e$

MuSun



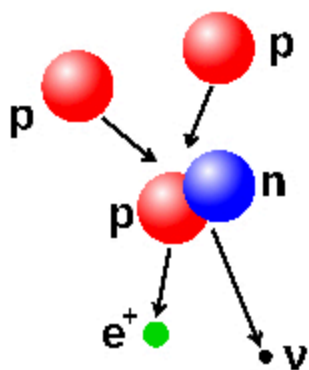
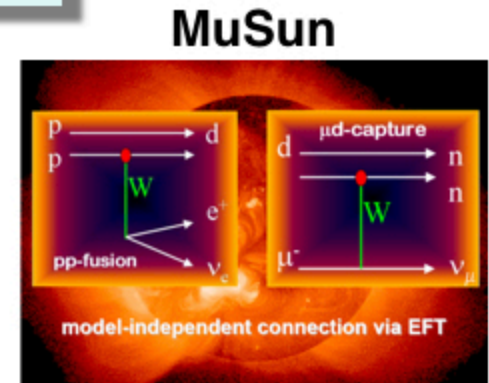
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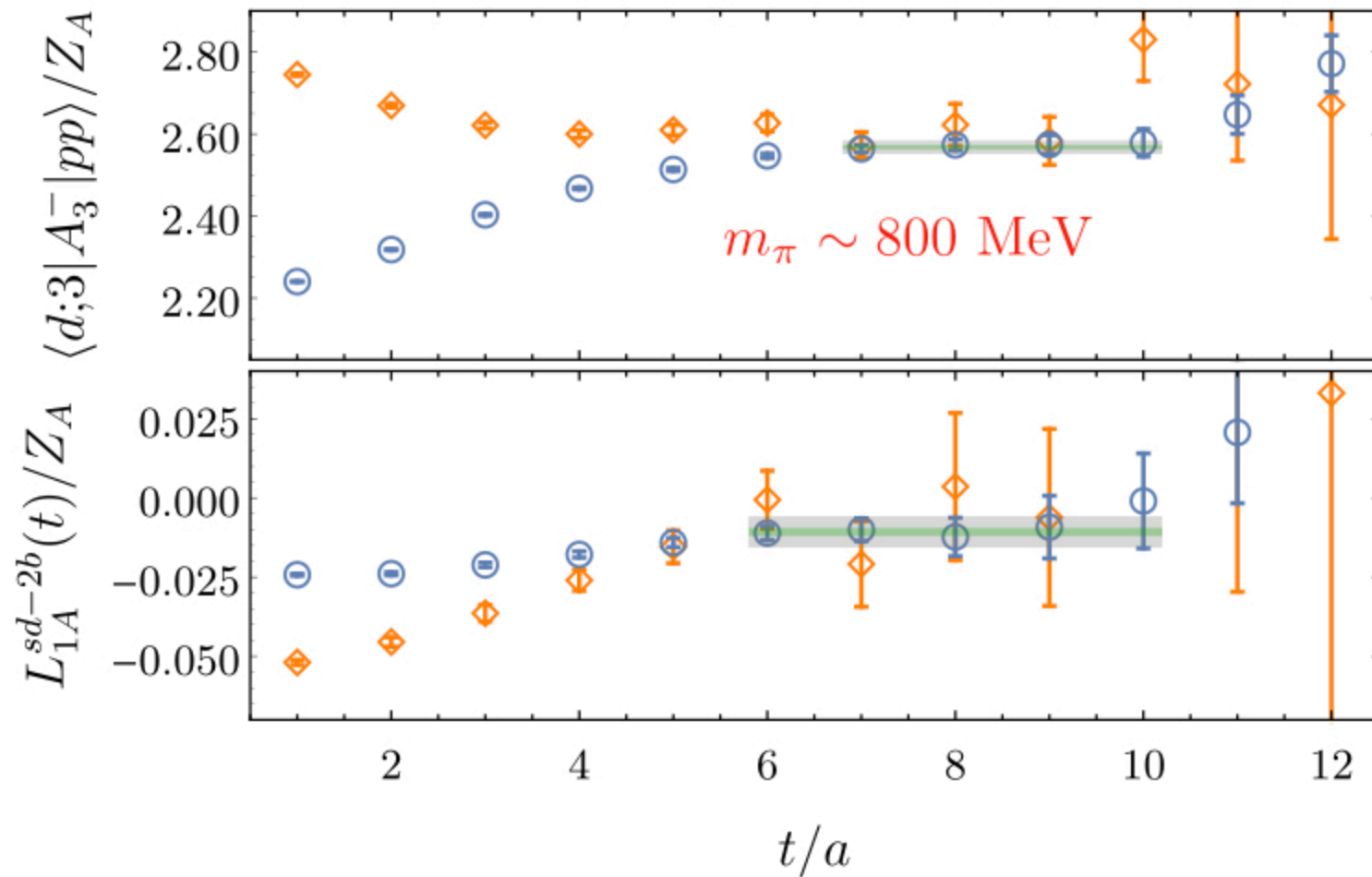
◆ $\nu d \rightarrow e^+ nn$: SNO

◆ “calibrating the sun” $pp \rightarrow de^+\nu_e$



$$|\langle d; j | A_k^- | pp \rangle| \equiv g_A C_n \sqrt{\frac{32\pi}{\gamma^3}} \Lambda(p) \delta_{jk}$$

$$\Lambda(0) = \frac{1}{\sqrt{1-\gamma\rho}} \{ e^\chi - \gamma a_{pp} [1 - \chi e^\chi E_1(\chi)] + \frac{1}{2} \gamma^2 a_{pp} \sqrt{r_1 \rho} \} - \frac{1}{2g_A} \gamma a_{pp} \sqrt{1-\gamma\rho} L_{1,A}^{sd-2b}$$



$$\frac{L_{1,A}^{sd,2b}}{Z_A} = \frac{\langle {}^3S_1; J_z = 0 | A_3^3 | {}^1S_0; I_z = 0 \rangle - 2g_A}{2Z_A} = -0.0107(12)(48)$$

$$\Lambda(0)_{lqcd} = 2.6585(6)(71)(25)$$

$$\Lambda(0)_{exp} = 2.652(2)$$

- ◆ Progress has been made in benchmarking lattice QCD calculations of nucleon-nucleon interactions.
- ◆ An important goal of lattice QCD is to generate hyperon-nucleon and hyperon-hyperon interactions with *fully-controlled* uncertainties.
- ◆ Prospect of a quantitative connection to QCD makes this an exciting time for nuclear physics.



US Lattice Quantum Chromodynamics



National Energy Research
Scientific Computing Center

