Progress in the Nucleon EDM Calculations in Lattice QCD

Sergey N. Syritsyn,
Stony Brook University & RIKEN / BNL Research Center
together with LHP and RBC collaborations

13th Conference on the Intersections of
Particle and Nuclear Physics
Palm Springs, CA, May 29–Jun 3, 2018

Outline

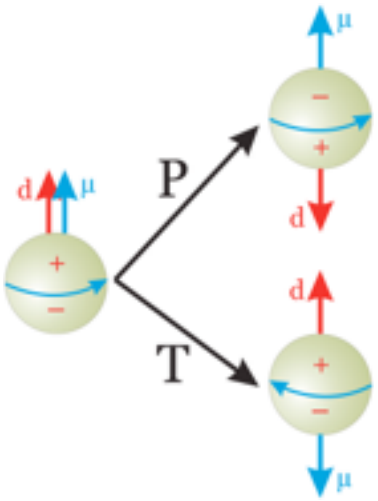
- θ_{QCD} -induced nucleon EDM:
  - *Previous calculations*
  - *New challenges*
  - *Improved techniques on a lattice*
  - *Outlook for θ_{QCD}-nEDM*

- Quark chromo-EDM-induced nucleon EDM
  - *Preliminary results at the physical point*

CP Violation: Electric Dipole Moments

EDMs are the most sensitive probes of CPv:

- Prerequisite for Baryogenesis
- Evidence for SM Extensions
- ($\theta_{\rm QCD}$ in particular) Strong ~~CP~~ problem

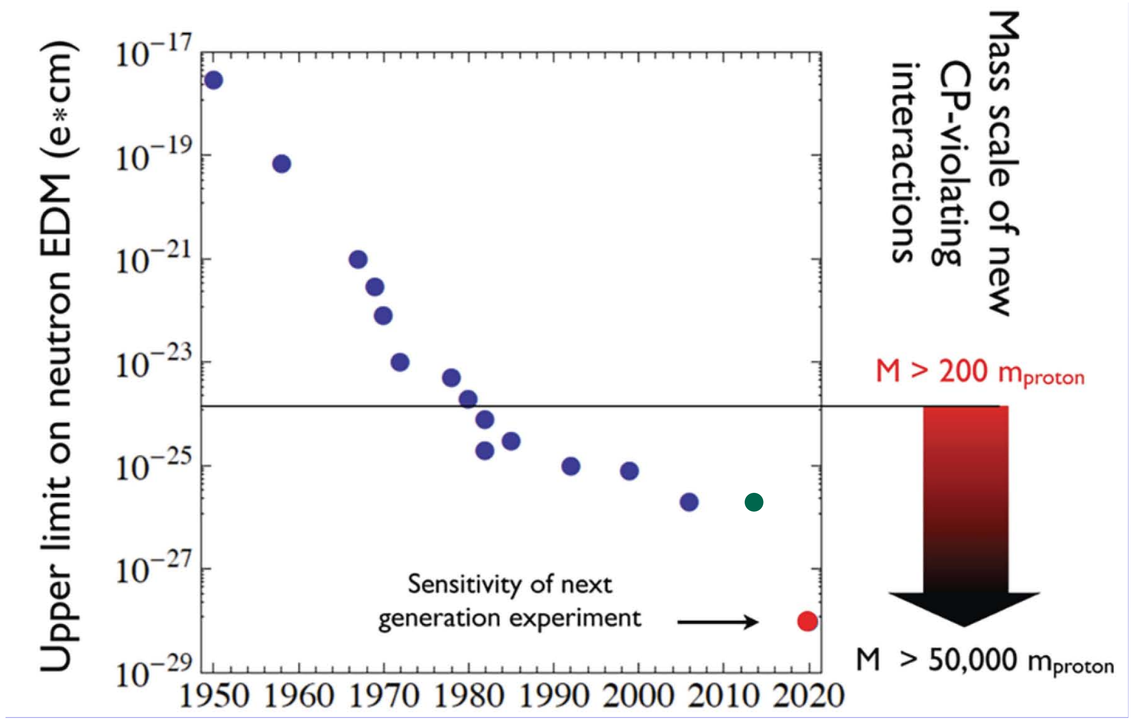


$$\vec{d}_N = d_N \frac{\vec{S}}{S} \qquad \mathcal{H} = -\vec{d}_N \cdot \vec{E}$$

OR
$$\begin{aligned} \mathcal{L}_{int} &= eA^{\rm em}_\mu \mathcal{V}^\mu && \text{(P,T-even)} \\ &+ eA^{\rm em}_\mu \mathcal{A}^\mu && \text{(P,T-odd)} \end{aligned}$$

Experimental Outlook: Neutron EDM

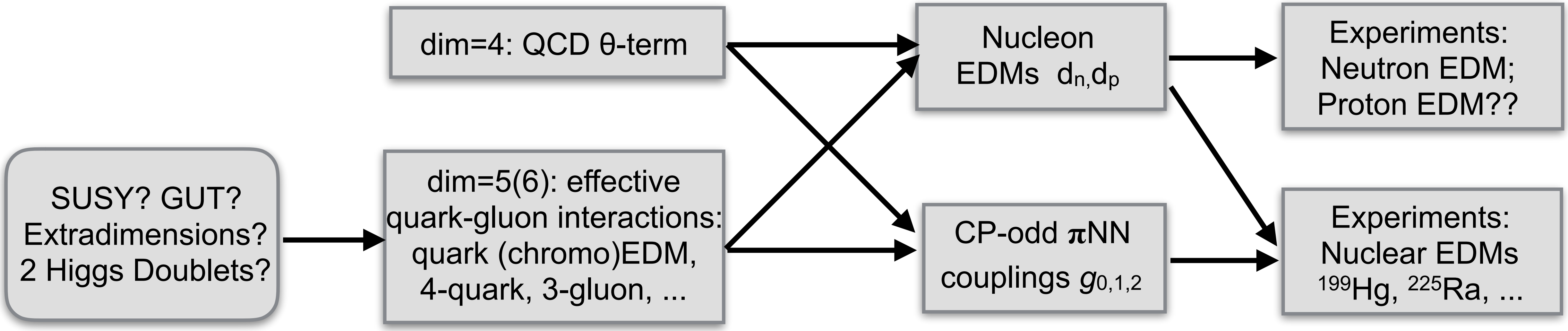
	10^{-28} *e* cm
CURRENT LIMIT	<300
Spallation Source @ORNL	**< 5**
Ultracold Neutrons @LANL	~30
PSI EDM	<50 (I), **<5 (II)**
ILL PNPI	<10
Munich FRMII	**< 5**
RCMP TRIUMF	<50 (I), **<5 (II)**
JPARC	**< 5**
Standard Model (CKM)	< *0.001*


[B.Filippone's talk, KITP 2016]

nEDM sensitivity :

- 1–2 years : next best limit
- 3–4 years : x10 improvement
- 7-10 years : x100 improvement

Nucleon EDMs: a Window into New Physics



SUSY? GUT? Extradimensions? 2 Higgs Doublets?

dim=4: QCD θ-term

dim=5(6): effective quark-gluon interactions: quark (chromo)EDM, 4-quark, 3-gluon, ...

Nucleon EDMs d_n, d_p

CP-odd πNN couplings $g_{0,1,2}$

Experiments: Neutron EDM; Proton EDM??

Experiments: Nuclear EDMs ^{199}Hg, ^{225}Ra, ...

- Effective quark-gluon CPv interactions organized by dimension

$$\mathcal{L}_{eff} = \sum_i \frac{c_i}{[\Lambda_{(i)}]^{d_i-4}} \mathcal{O}_i^{[d_i]}$$

[J.Engel, M. Ramsey-Musolf, U. van Kolck, Prog.Part.Nucl.Phys. 71 (2013), pp. 21-74]

d=4 : θ_{QCD}

d=5(6) : quark EDM, quark-gluon chromo EDM

d=6 : 4-fermion CPv, 3-gluon (Weinberg)

$$d_{n,p}$$
$$F_3^{n,p}(Q^2)$$

- $$d_{n,p} = d^{\theta}_{n,p}\theta_{\mathrm{QCD}} + d^{cEDM}_{n,p} c_{cEDM} + \dots$$

$$c_i \iff d_{n,p}\ ?$$

lattice QCD calculations are needed to relate to constrain θ_{QCD}, c_{cEDM}, ...

CP-odd Nucleon Structure on a Lattice

CP-broken vacuum on a lattice:

- Linear response to CP-odd interaction (e.g., QCD θ-term)

$$\langle \mathcal{O}\ldots\rangle_{\cancel{CP}} = \langle \mathcal{O}\ldots\rangle_{CP-even} - i\theta\langle Q\cdot\mathcal{O}\ldots\rangle_{CP-even} + O(\theta^2)$$

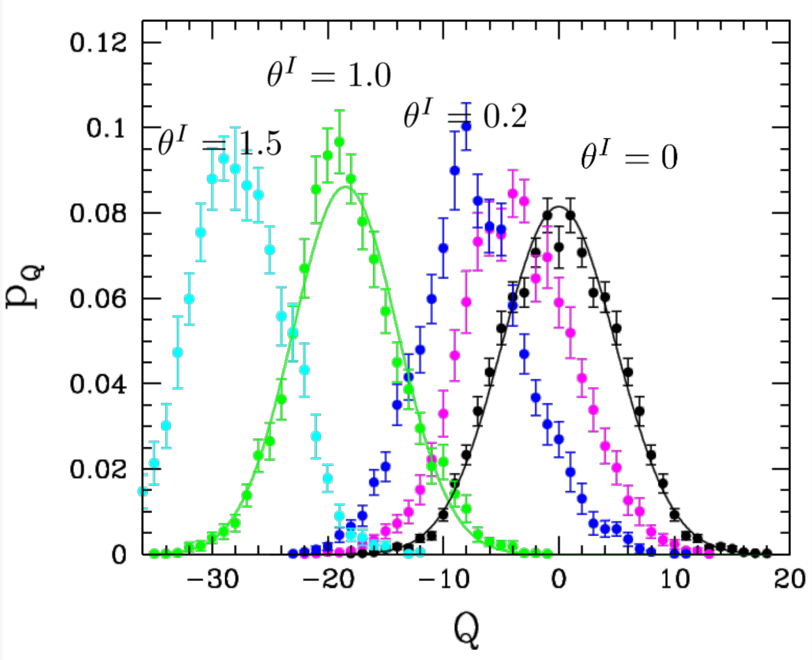
[S. Aoki et al (2005); F. Berruto et al (2005); A.Shindler et al (2015) ; C. Alexandrou et al (2015) ; E. Shintani et al (2016)]

- Simulation with dynamical (imaginary) θ^I_{QCD}

$$\langle \mathcal{O}\ldots\rangle_\theta \sim \int \mathcal{D}U\, e^{-S-\theta^I Q}\,(\mathcal{O}\ldots)$$

[R.Horsley et al (2008) ; F.K.Guo et al (2015)]

new gauge ensembles ⇒ better sampling of Q≠0 sectors

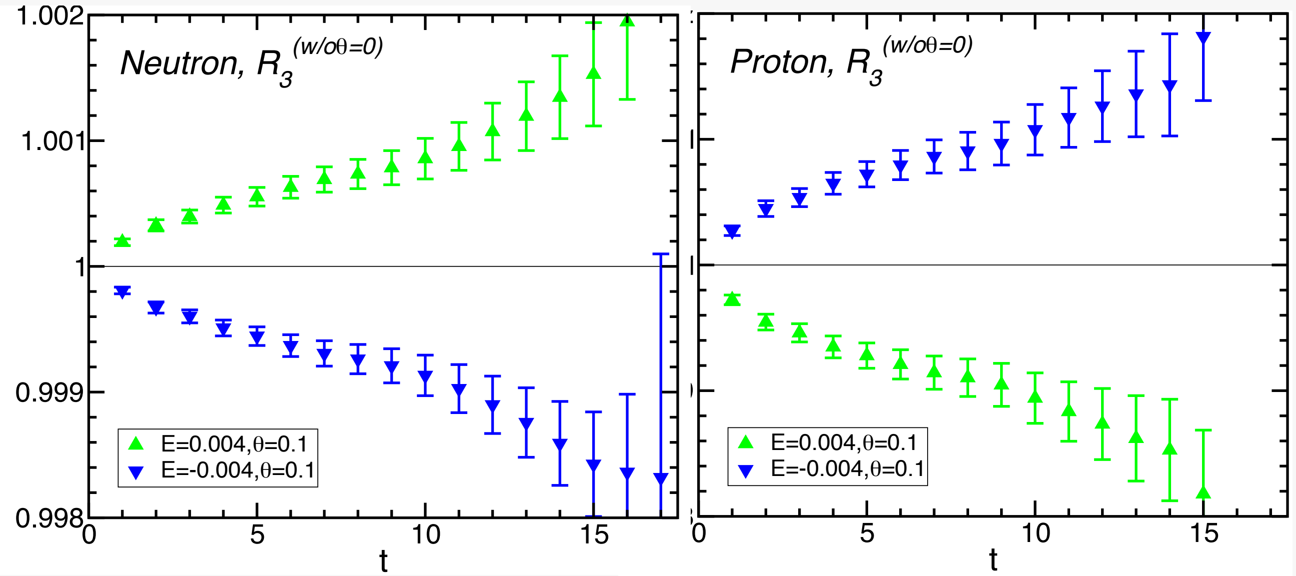


Extraction of d_N

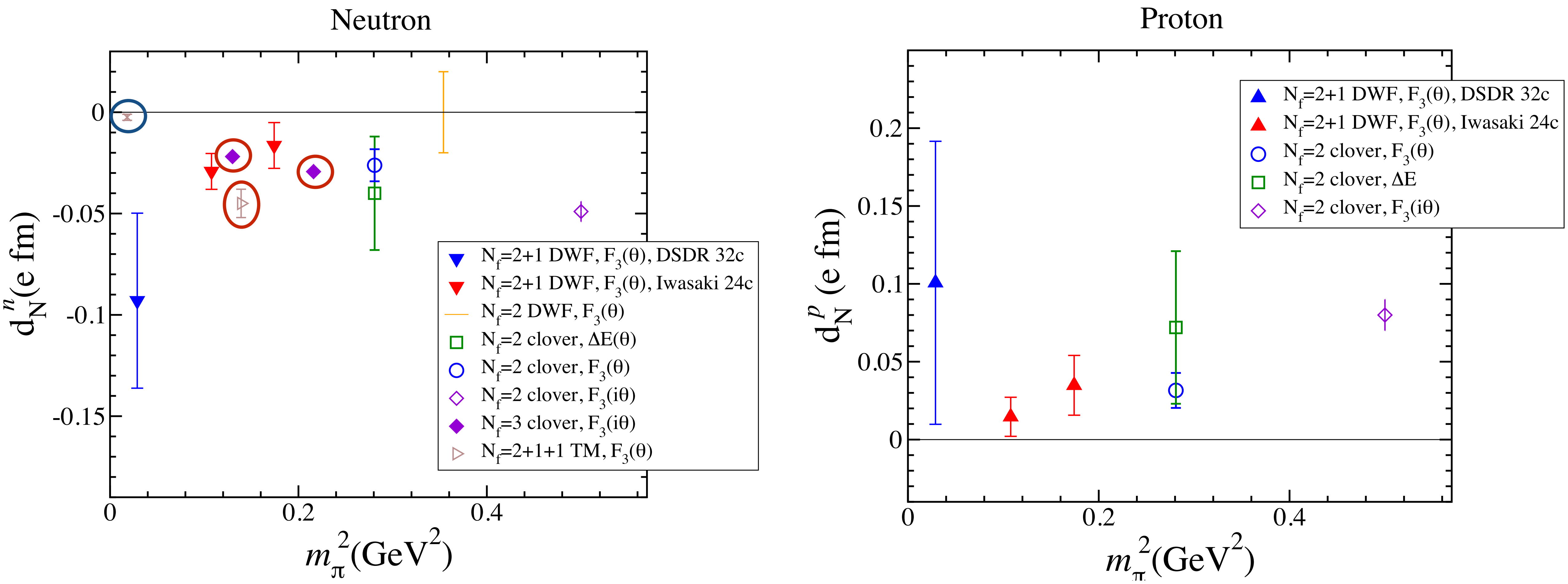
- Nucleon spectrum in the bg. electric field
  [S.Aoki et al '89 ; E.Shintani et al '06;
  E.Shintani et al, PRD75, 034507(2007)]

$$\langle N(t)\bar{N}(0)\rangle_{\theta,\vec{E}} \sim e^{-(E\pm\vec{d}_N\cdot\vec{E})t}$$

- P,T-odd Form Factor $d_N=F_3(0)/2m_N$
  [E.Shintani et al '05, '15 ; F.Berruto et al '05 ; A.Shindler et al '15 ; C.Alexandrou et al'15]
  Require extrapolation $F_3(Q^2\to 0)$



θ_{QCD}-induced Nucleon EDM



Summary of nEDM from LQCD circa 2015
[E.Shintani, T.Blum, T.Izubuchi, A.Soni, PRD93, 094503(2015)]

- Phenomenology: $|d_n| \simeq \theta_{QCD} \times (0.4\ ..\ 2.5)\cdot 10^{-3}\,e$ fm
- Lattice [Guo et al 2015] : $|d_n| \simeq \theta_{QCD} \times (4\cdot 10^{-3}\,e$ fm)

$\Longrightarrow$ tighter constraint on θ_{QCD} ?

Unfortunately, there was a problem...

Nucleon "Parity Mixing"

CPv interaction induces a chiral phase in fermion fields:

$$\langle \mathrm{vac}|\underset{\downarrow}{N}|p,\sigma\rangle_{\cancel{CP}} = e^{i\alpha\gamma_5}u_{p,\sigma} = \underset{\downarrow}{\tilde{u}_{p,\sigma}}$$

$$u\,[u^T C\gamma_5 d]$$

(P-even lattice nucleon field)

$$(\not{\partial} + m_N e^{-2i\alpha\gamma_5})\tilde{u}_p = 0$$

$$\sum_\sigma \tilde{u}_{p,\sigma}\bar{\tilde{u}}_{p,\sigma} \sim \big(-i\not{p}_{\mathcal{E}} + m_N e^{2i\alpha\gamma_5}\big)$$

Vector current M.E. has to be defined with positive-parity spinors to define $F_{2,3}$
[SNS, S.Aoki, *et al* (2017) arXiv:1701.07792]

$$\langle N_{p'}|\bar{q}\gamma^\mu q|N_p\rangle_{\cancel{CP}} = \bar{u}_{p'}\underbrace{\Big[F_1\gamma^\mu + (F_2 + i\boxed{F_3}\gamma_5)\frac{i\sigma^{\mu\nu}(p'-p)_\nu}{2m_N}\Big]}_{\Gamma^\mu_{\mathcal{E}}}u_p$$

$$\gamma_4 u = +u$$
$$\bar{u}\gamma_4 = +\bar{u}$$

... otherwise, $F_{2,3}$ mix under chiral rotation and lead to fake EDM/EDFF signal

$$e^{i\alpha\gamma_5}\Gamma^\mu e^{i\alpha\gamma_5} \leftrightarrow \Gamma^\mu$$

$$e^{2i\alpha}(\text{“}F_2\text{”} + i\text{“}F_3\text{”}) = (F_2 + iF_3)_{\mathrm{true}}$$

$$\text{“}F_3\text{”} \approx [F_3]_{\mathrm{true}} - 2\alpha[F_2]_{\mathrm{true}}$$

$$\text{“}d_{n,p}\text{”} \approx [d_{n,p}]_{\mathrm{true}} - 2\alpha\frac{\kappa_{n,p}}{2m_N}$$

The same issue is addressed correctly in EFT (ChPT) calculations

Nucleon "Parity Mixing" (2)

With proper definition of $F_{2,3}$ [SNS, S.Aoki, *et al* (2017) arXiv:1701.07792]

- coupling of E,B to spin in the forward limit

$$\langle H_{\rm int}\rangle = eA_\mu\langle J^\mu\rangle = -\frac{eG_M(0)}{2m_N}\vec{\Sigma}\cdot\vec{H} - \frac{eF_3(0)}{2m_N}\vec{\Sigma}\cdot\vec{E}$$

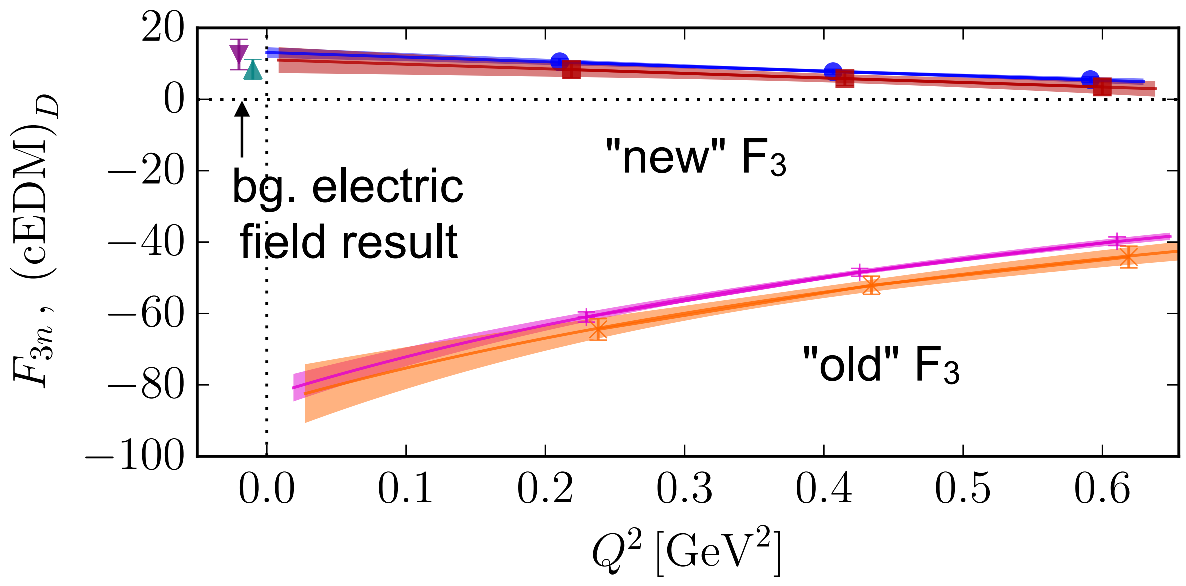
- poles of the Dirac operator in bg. electric & magnetic fields

$$\mathcal{L}_N = \bar{N}\Big[i\not{\partial} - m e^{-2i\alpha\gamma_5} - Q\gamma_\mu A^\mu - (\tilde{\kappa} + i\tilde{\zeta}\gamma_5)\frac{1}{2}F_{\mu\nu}\frac{\sigma^{\mu\nu}}{2m_N}\Big]N$$

$$\longrightarrow E_N(\vec{p}=0) - m_N = -\frac{\kappa}{2m_N}\vec{\Sigma}\cdot\vec{H} - \frac{\zeta}{2m_N}\vec{\Sigma}\cdot\vec{E} + O(\kappa^2,\zeta^2)$$

with $\kappa + i\zeta = e^{2i\alpha\gamma_5}(\tilde{\kappa} + i\tilde{\zeta})$

- Numerical test: compare EDFF with mass shift in uniform bg. electric field



d-cEDM induces large mixing
$\alpha_D \approx 30(0.2)$

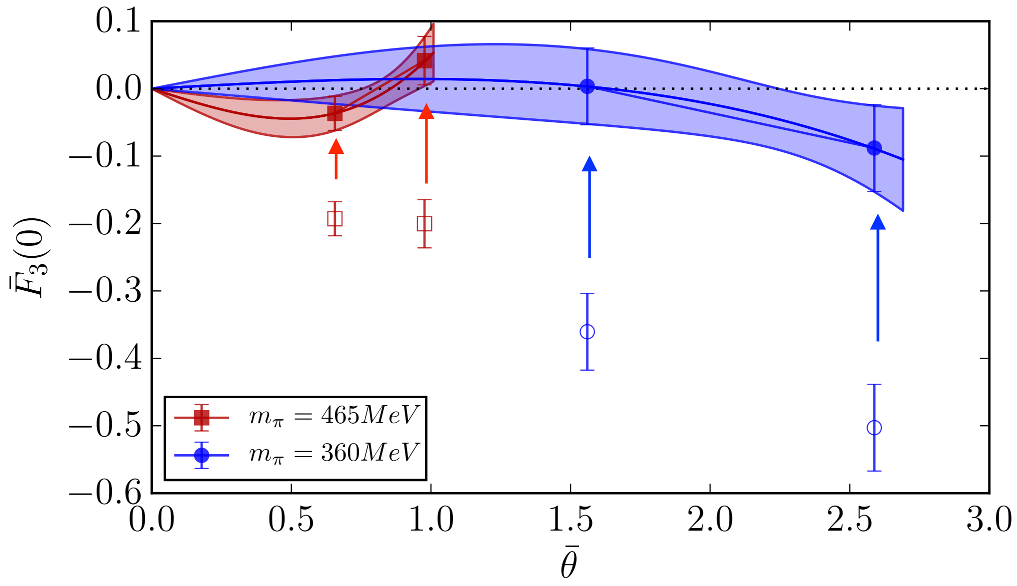
Large F_{2n} contribution to "F_{3n}"

$$\text{“}F_{3n}^D\text{”} = [F_{3n}^D]_{\rm true} - 2\alpha_D F_{2n}$$

Recent Lattice Results on θ_{QCD}-induced nEDM

Correction to previous results:

$$[F_3]_{\text{true}} = \text{“}F_3\text{”} + 2\alpha F_2$$



- [F. Guo *et al* (QCDSF), PRL115:062001 (2015)]
  dynamical calculations with finite imag. θ^I angle
- [C.Alexandrou *et al* (ETMC), PRD93:074503 (2016]
  d_n=–0.045(06) *e* fm (~7.5σ) → +0.008(6) *e* fm (1.3σ)
- *Uniform bg.electric field method is not affected by "parity mixing"*
  Precision in Ref. [E.Shintani et al, D78:014503 (2008)] is insufficient for comparison

		m_π [MeV]	m_N [GeV]	F_2	α	$\tilde{F}_3$	F_3
[ETMC 2016]	n	373	1.216(4)	$-1.50(16)^a$	$-0.217(18)$	$-0.555(74)$	0.094(74)
[Shintani et al 2005]	n	530	1.334(8)	$-0.560(40)$	$-0.247(17)^b$	$-0.325(68)$	$-0.048(68)$
	p	530	1.334(8)	0.399(37)	$-0.247(17)^b$	0.284(81)	0.087(81)
[Berruto et al 2006]	n	690	1.575(9)	$-1.715(46)$	$-0.070(20)$	$-1.39(1.52)$	$-1.15(1.52)$
	n	605	1.470(9)	$-1.698(68)$	$-0.160(20)$	0.60(2.98)	1.14(2.98)
[Guo et al 2015]	n	465	1.246(7)	$-1.491(22)^c$	$-0.079(27)^d$	$-0.375(48)$	$-0.130(76)^d$
	n	360	1.138(13)	$-1.473(37)^c$	$-0.092(14)^d$	$-0.248(29)$	$0.020(58)^d$

After removing spurious contributions,

- *no lattice signal for θ_{QCD}-induced nEDM ⇨ d_N is very small*
- *no conflict with phenomenology values or m_q scaling*

θ-Term Noise Reduction for EDM

Lattice signal for *θ*-nEDM $d_N \sim \langle Q \cdot \big(N(x) J_\mu \bar{N}(0)\big)\rangle_{CP-even}$

Top. charge Q is global $Q \sim \int_{V_4} (G\tilde{G})$ with $\langle |Q|^2 \rangle \sim V_4$

⇒ Variance of correlator ~ V_4

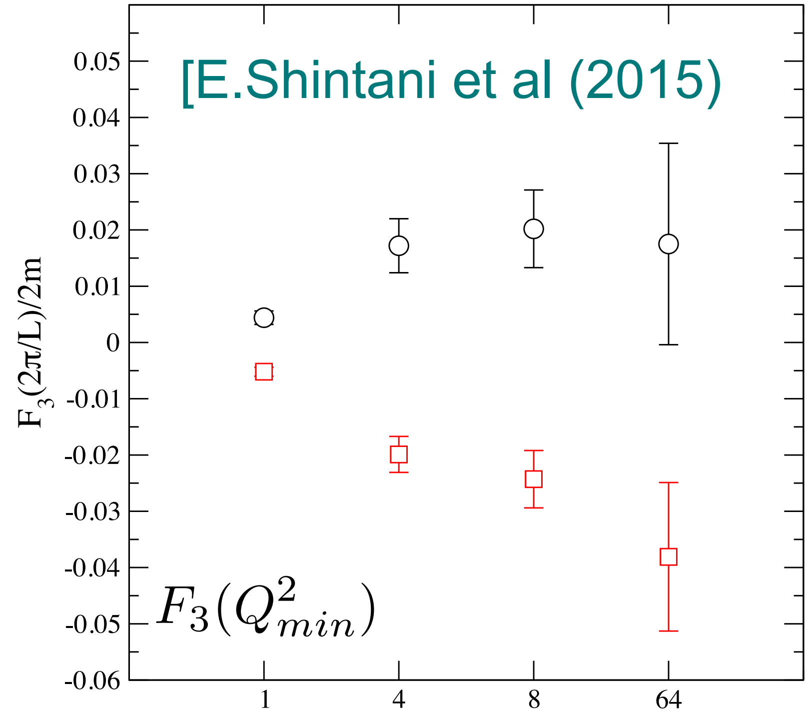
Constrain Q integral to the relevant volume

- constrain Q in time, $|t_Q - t_J| \leq \Delta t$

  [E.Shintani, T.Blum, T.Izubuchi, A.Soni, PRD93, 094503(2015)]

- "cluster decomposition" [K.-F.Liu et al, 1705.06358]:

  constrain Q in 4-d around "sink" within |r|<R



$$Q \approx \int_{V_Q} d^4z\, q(z)$$

Proper account of nucleon parity mixing is critical for correct determination of F₃

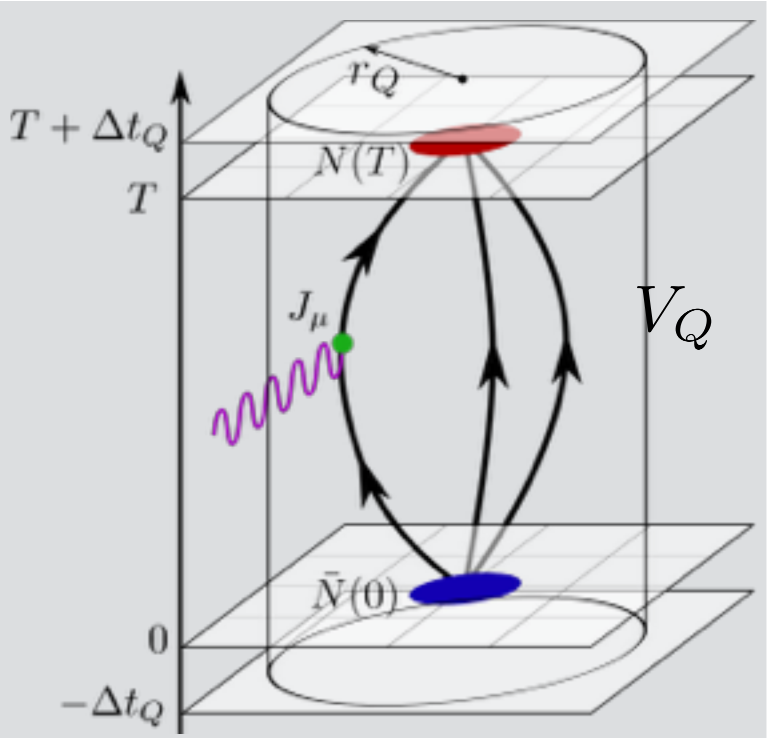
$\Longrightarrow$ *nucleon states must "settle" in the new vacuum*

$$N^{(+)} \rightarrow \tilde{N}^{(+)} \approx N^{(+)} + i\alpha N^{(-)}$$

$$N^{(-)} \rightarrow \tilde{N}^{(-)} \approx N^{(-)} - i\alpha N^{(+)}$$

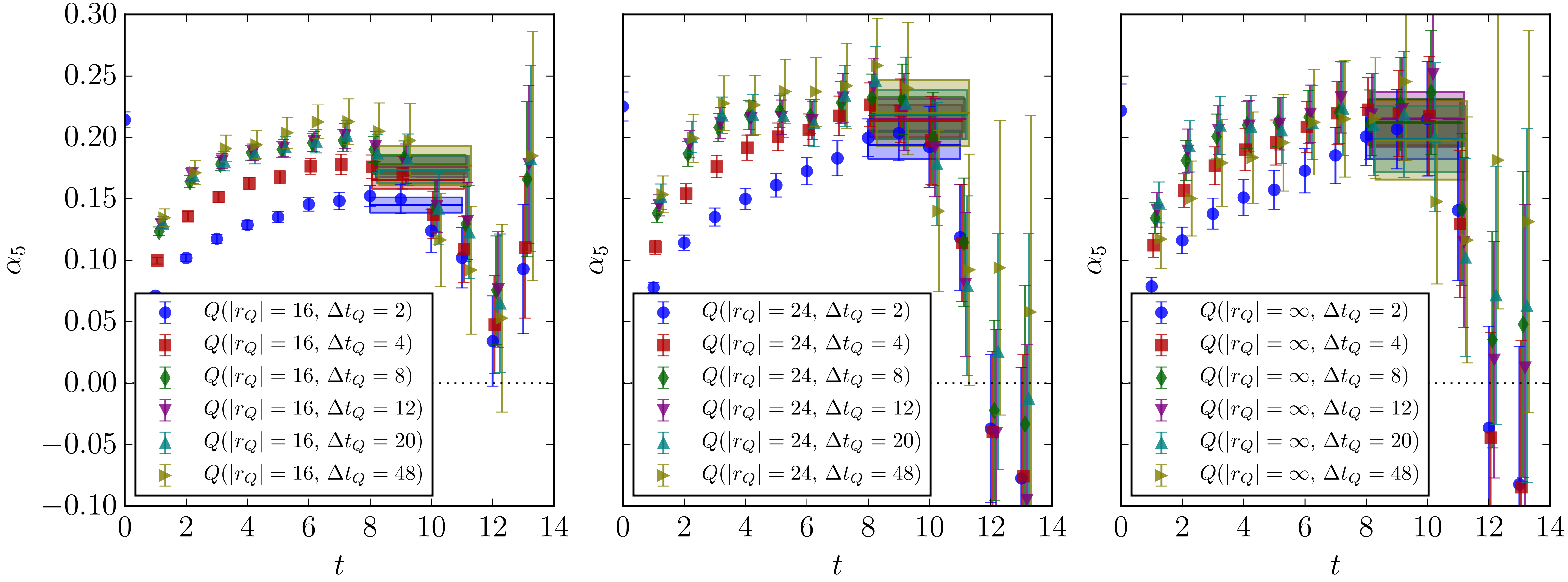
$\Longrightarrow$ *treat time differently from space:*

4d "cylinder" $V_Q \;:\; |\vec{z}| < r_Q, \quad -\Delta t_Q < z_0 < T + \Delta t_Q$



Noise Reduction: θ-induced Parity-mixing

PRELIMINARY 48c96 mpi=140MeV

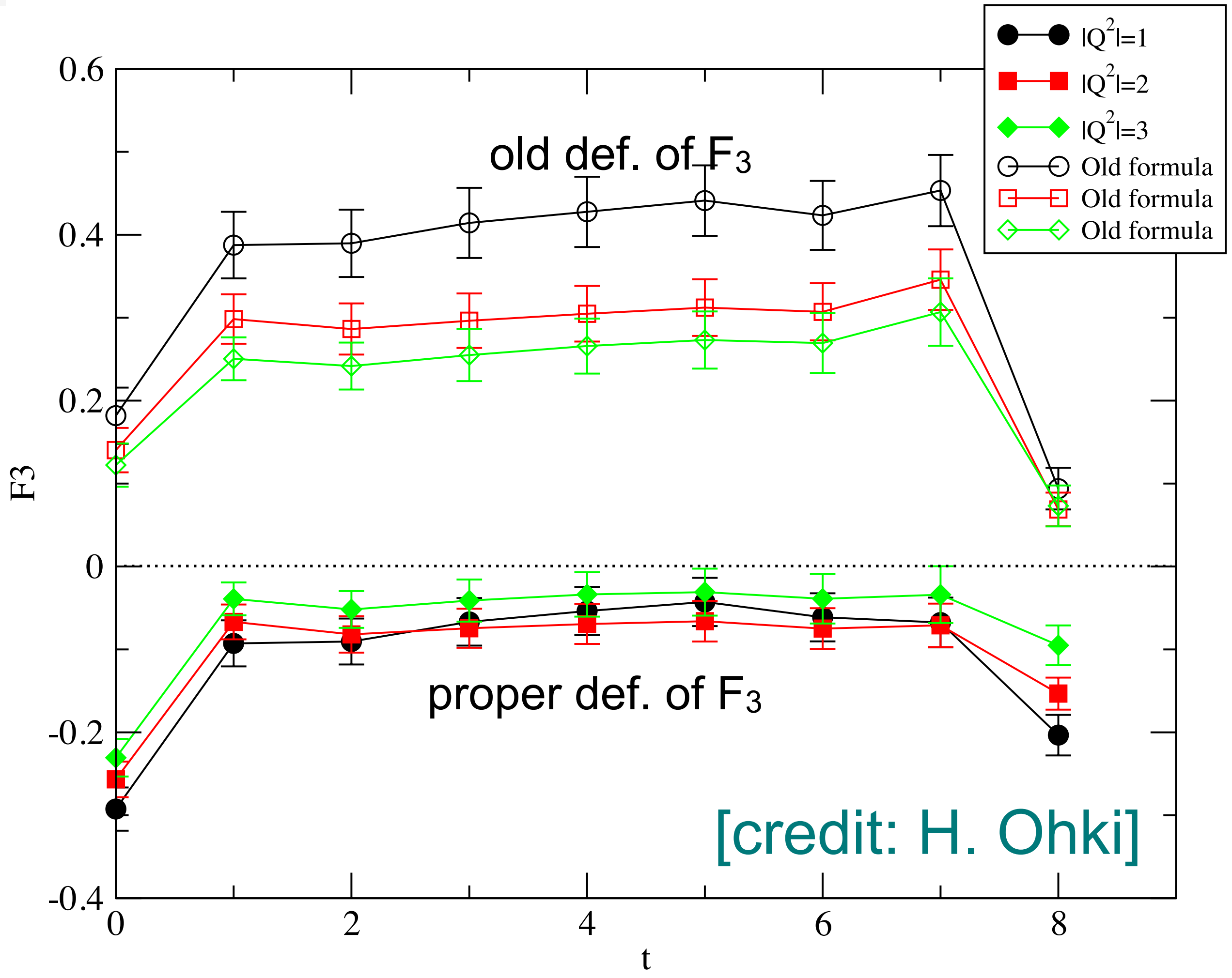


Parity-mixing angle from constrained Q sum

Reassuring results for noise reduction at the physical point

- required time region is small, $\Delta t_Q \gtrsim 8a \approx 1.2\,\mathrm{fm}$
- spatial region must be large, $r_Q \gtrsim 20a \approx 2.3\,\mathrm{fm}$

θ-nEDM Feasible at the Physical Point?



[credit: H. Ohki]

Preliminary Results with m_π=330 MeV

- Q sampled with $\Delta t_Q = 4a, \quad r_Q = \infty$

Best guess for neutron EDM d_n: extrapolation in $m_q \sim (m_\pi)^2$

- chiral fermions, m_π=330 MeV

  $\Longrightarrow$ phys.point $|F_3(0)| \approx 0.020$, $|dn| \approx 0.002$ *e* fm

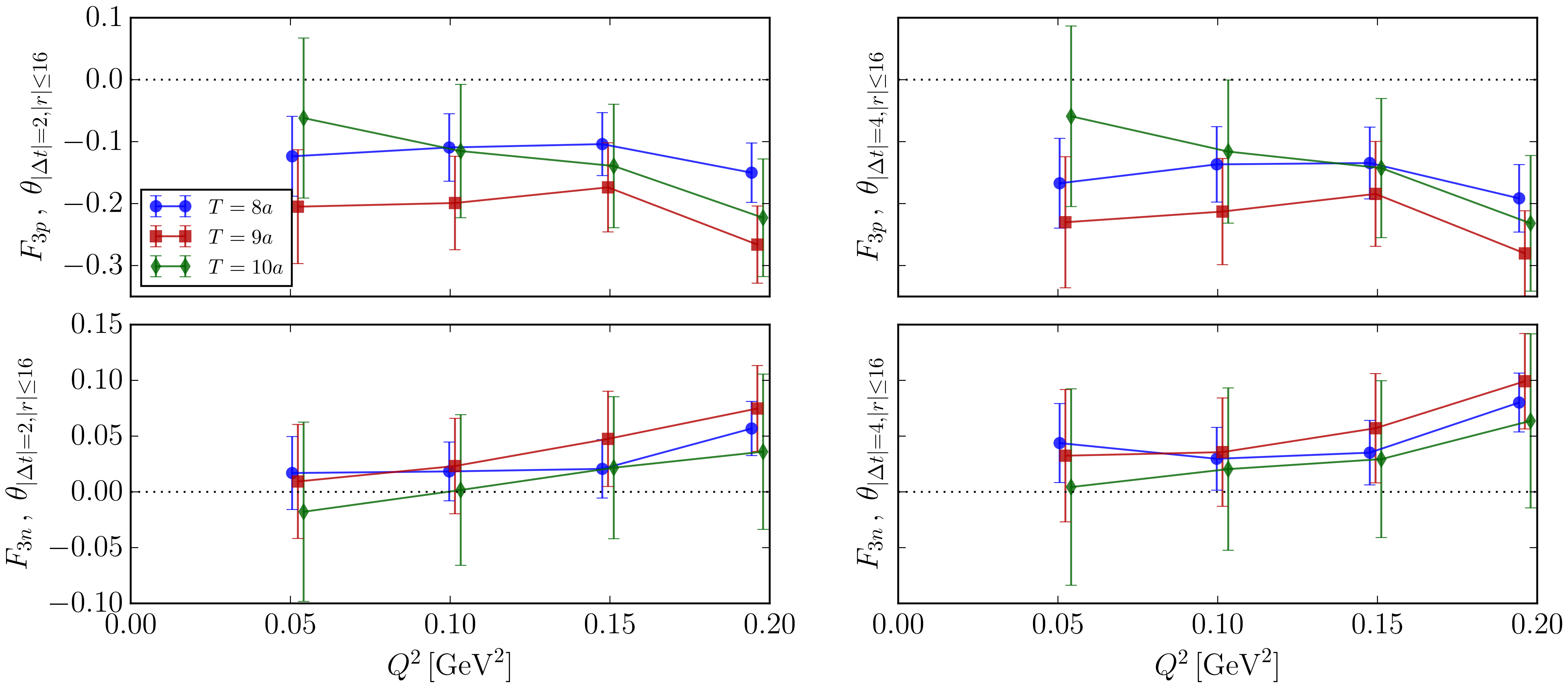
- Wilson fermions, m_π=360 MeV [Guo et al 2015]

  $\Longrightarrow$ phys.point $|F_3(0)| \leq 0.012$, $|dn| \leq 0.001$ *e* fm

$$|F_{3n}^{\mathrm{phys}}(0)| \sim O(10^{-2})\,\theta, \quad |d_n| \sim O(10^{-3})\,e\,\mathrm{fm}\,\theta$$

Noise Reduction: θ-induced EDFF F3

PRELIMINARY 48c96 mpi=140MeV



- EDFF F_3 from constrained Q sum: *the most aggressive* Q cuts
- 33k lattice samples, ~ 30 M core-hours on Argonne BlueGene/Q
- connected diagrams only
- result compatible with zero, $|F_{3n}| \leq 0.05$

Need to constrain $|F_{3n}| \approx 0.01..0.02$: θ-nEDM remains difficult at the physical point...

Outlook for θ-nEDM

Resort to simpler calculations

- heavier pion masses + EFT for extrapolations
- quenched calculations (see e.g. recent [J.Dragos et al,1711.04730])

Physical point calculations of θ-nEDM will be necessary to renormalize effects from other CPv sources of higher-dim. [T.Bhattacharya et at (2015)]

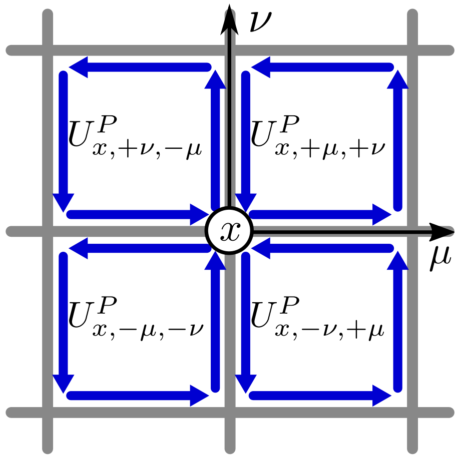
New lattice simulations at the physical point with dynamical θ^I-term

- coarse (a=0.2 fm) physical-point lattice $\Longrightarrow$ reduced cost due to lattice volume
- chiral lattice fermions allow independent $a\to 0$, $m_q\to 0$ limits
- enhance d_N signal with $\langle Q\rangle \neq 0$ – more critical at light quark masses

$\Longrightarrow$ *2018 ALCC award for 50 M BG/Q core-hours*

Ensembles with dynamical θ^I-term will be also useful for CPv πN coupling

Another Source of CPv: Quark Chromo-EDM



$$\mathcal{L}_{\rm cEDM} = \sum_{q=u,d} \frac{\tilde\delta_q}{2}\,\bar q\,[G_{\mu\nu}\sigma^{\mu\nu}\gamma_5]\,q$$

- O(a^{-2}) mixing with dim-3 pseudoscalar density
  ⇒ need non-perturbative subtractions

- Non-chiral (e.g.Wilson) fermions have a O(a) clover term ("chromo-magnetic DM")

$$\mathcal{L}^{\rm clover} = a\frac{c}{4}\bar q\,[G_{\mu\nu}\sigma^{\mu\nu}]\,q$$

Condensate realignment in presense of CPv $q \to e^{i\gamma_5\Omega}q$
assuring $\langle{\rm vac}|\mathcal{L}_m + \mathcal{L}_{\cancel{CP}}|\pi^a\rangle = 0$

mixes (chromo)EDM and (chromo)MDM:

$$\delta\mathcal{L}_{\rm cEDM} = \delta(\bar q\,[\tilde D_q G_{\mu\nu}\sigma^{\mu\nu}\gamma_5]\,q) = \bar q\,[\{\Omega,\tilde D_q\}G_{\mu\nu}\sigma^{\mu\nu}]\,q) \sim \delta\mathcal{L}_{\rm cMDM}$$

⇒ *Chirally-symmetric actions avoid these cMDM contributions*

Quark-Gluon EDM: Insertions of dim-5 Operators

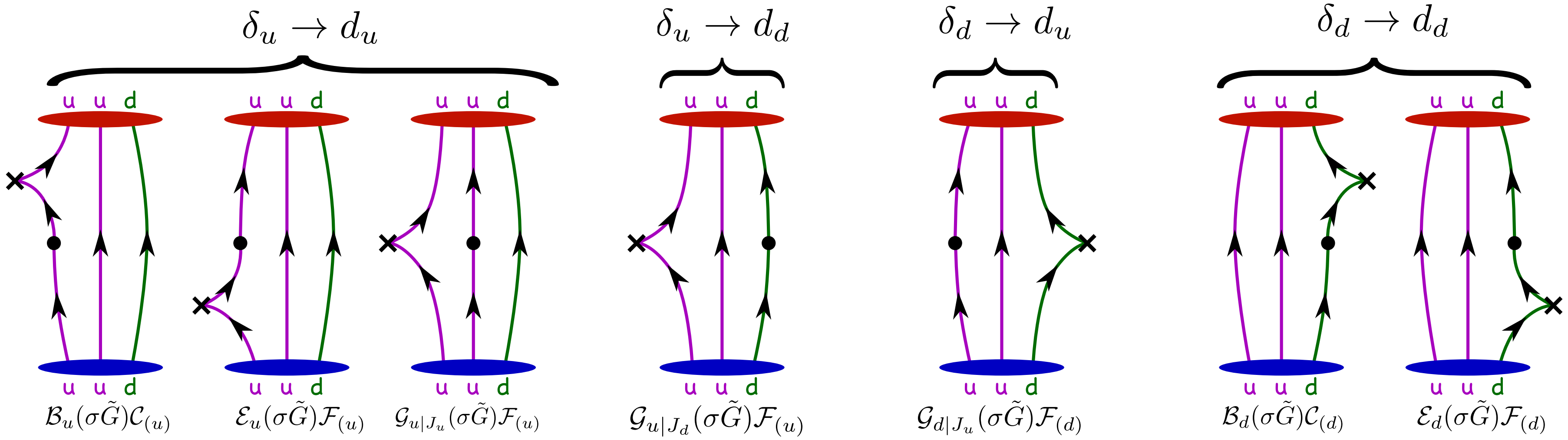
$$\mathcal{L}^{(5)} = \sum_q \tilde{d}_q\, \bar{q}(G\cdot\sigma)\gamma_5 q$$

$$\longrightarrow \langle N(y)\, \bar{N}(0) \int d^4x\, \bar{q}(G\cdot\sigma)\gamma_5 q\rangle$$

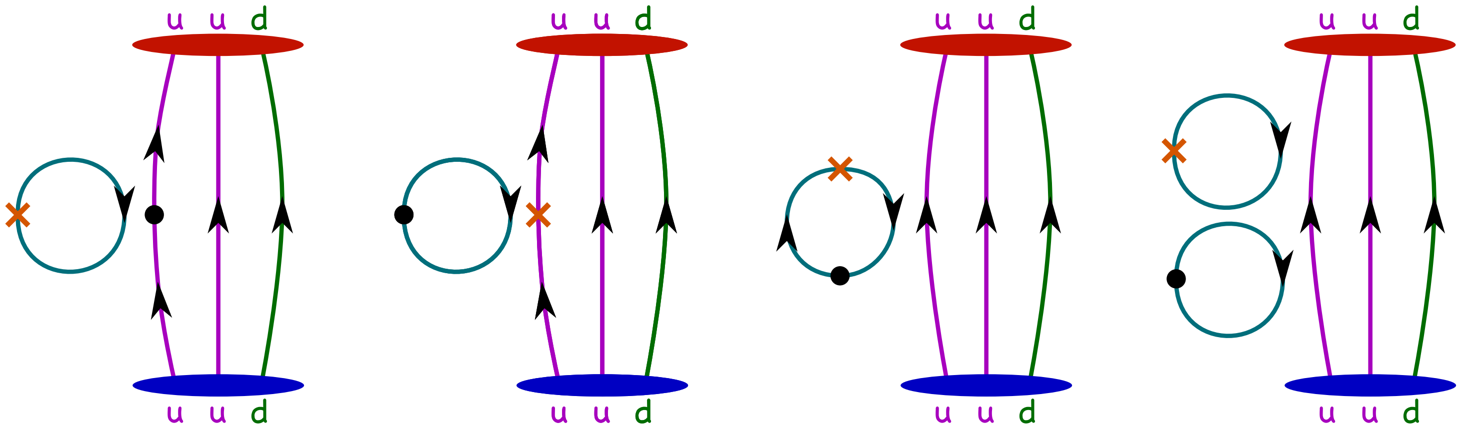
$$\longrightarrow \langle N(y)\, [\bar{\psi}\gamma^\mu\psi]_z\, \bar{N}(0) \int d^4x\, \bar{q}(G\cdot\sigma)\gamma_5 q\rangle$$

First calculations : [T.Bhattacharya et al(LANL, LATTICE'15,'16)]

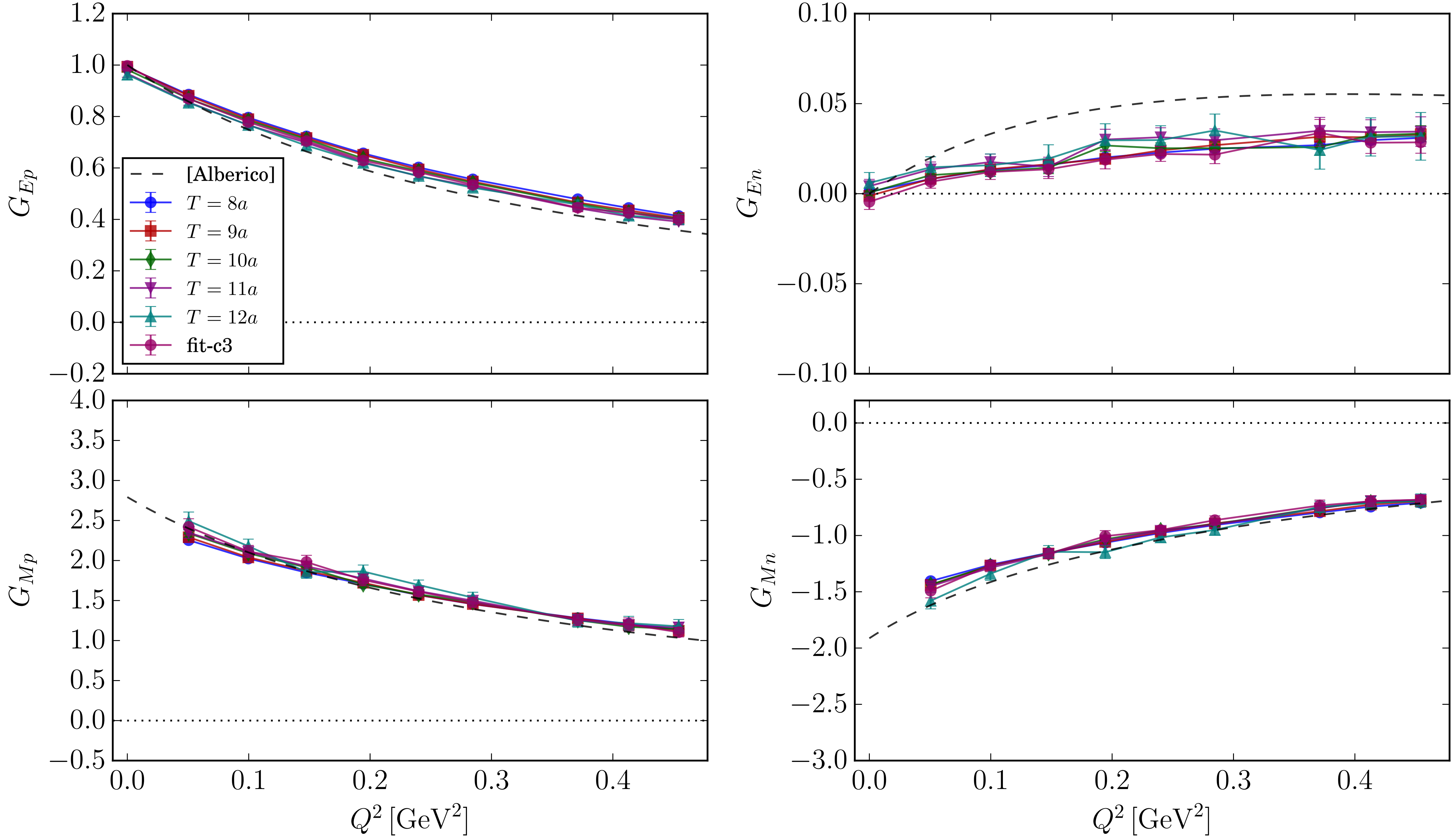
- This work: Only quark-connected insertions



- In future: Single- and double-disconnected diagrams (contribute to isosinglet cEDM, mix with θ-term)



Nucleon Sachs Form Factors

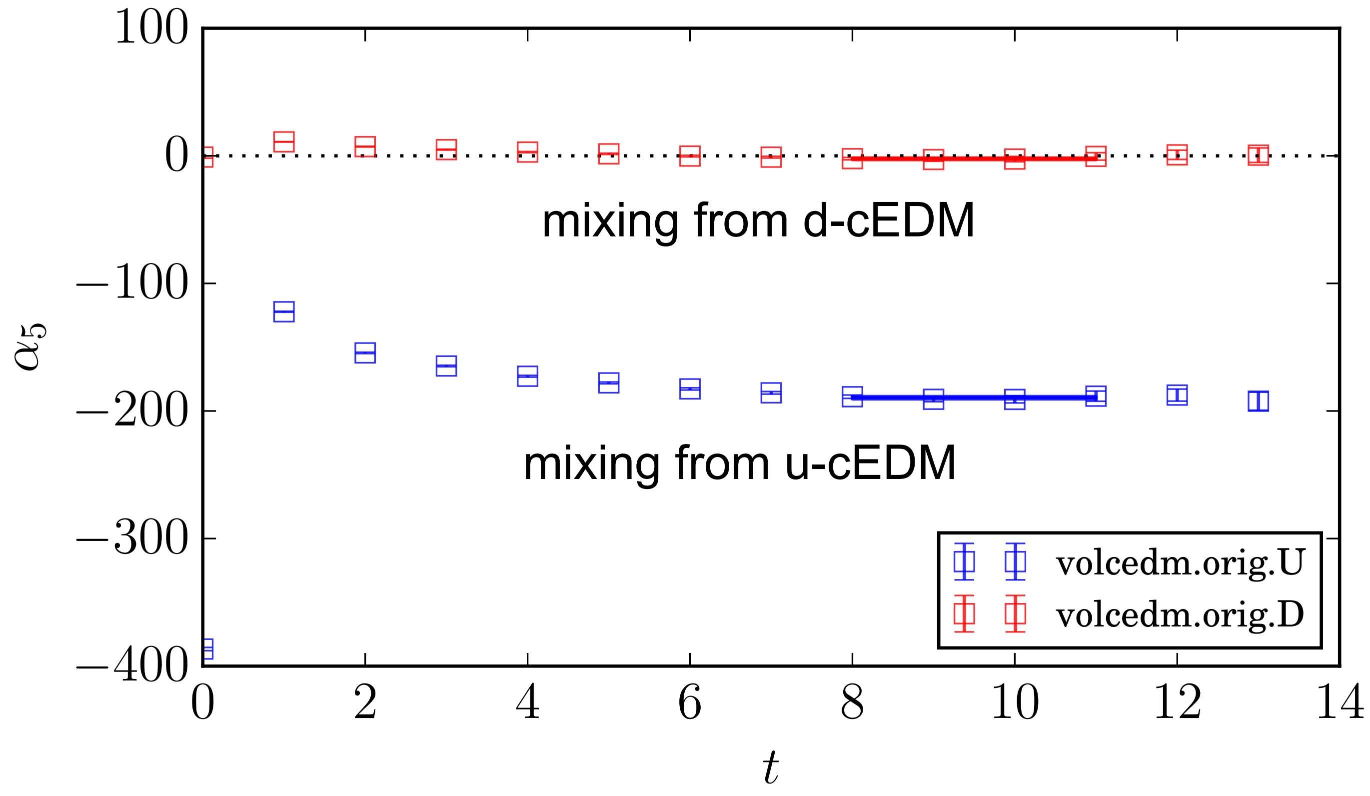


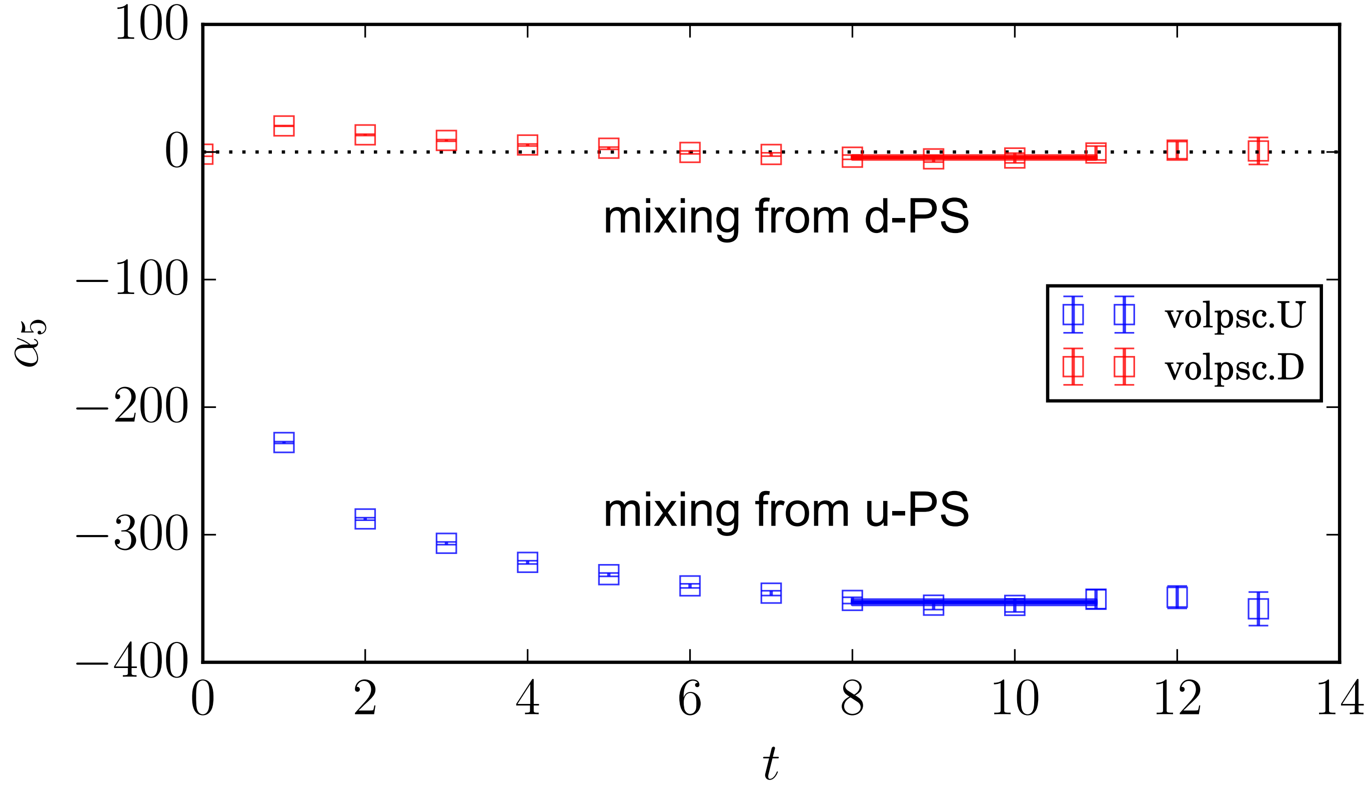
$$G_E = F_1 - \frac{Q^2}{4m_N^2} F_2$$

$$G_M = F_1 + F_2$$

- (5.5 fm)^{3}x(11 fm) box, mπ=140 MeV
- connected-only contractions

Parity Mixing (Proton)





$$N_\delta = \epsilon^{abc}\, u^a_\delta\, (u^{aT} \mathcal{C}\gamma_5 d^c)$$

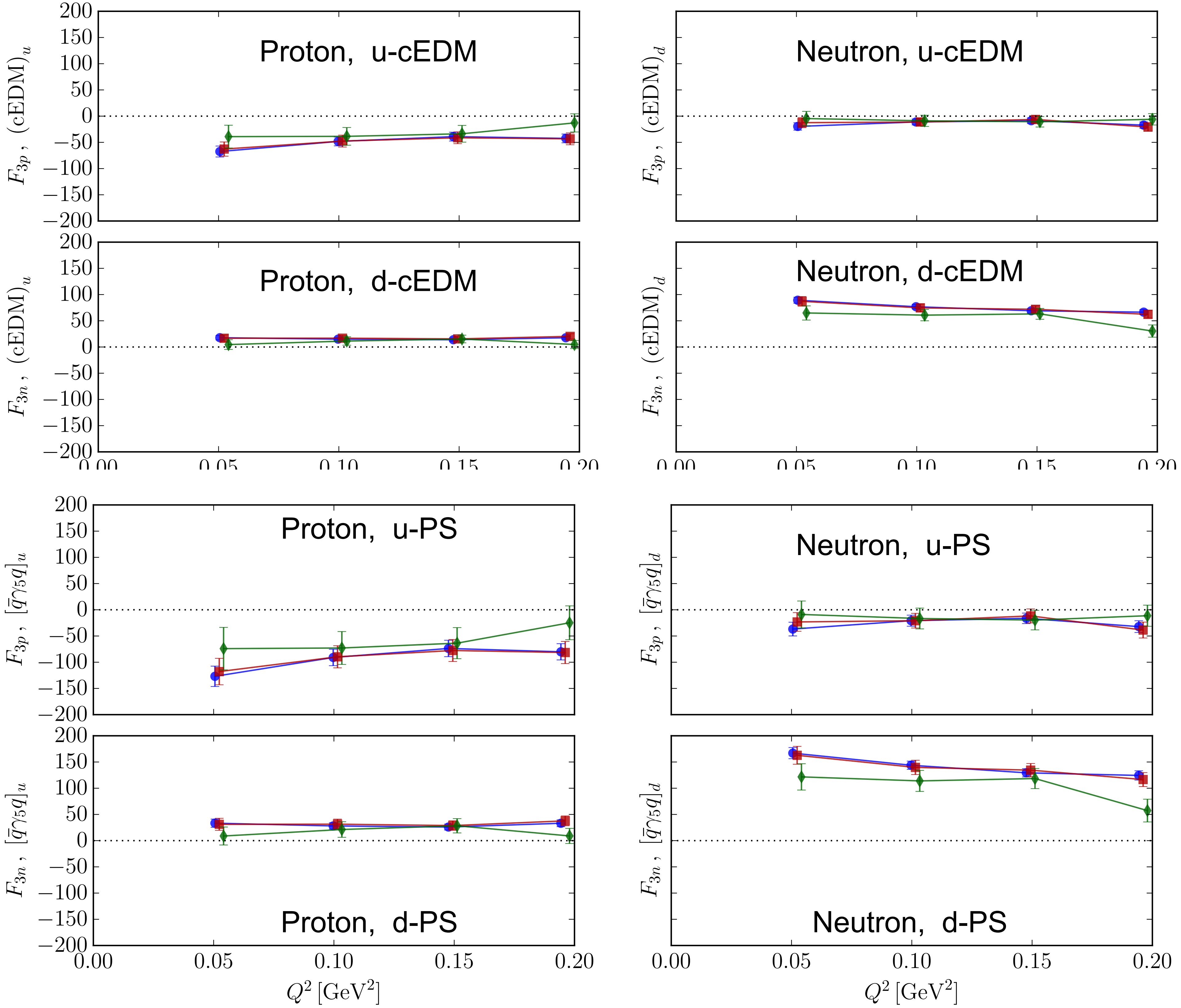
$$\langle N(t)\bar{N}(0)\rangle_{\cancel{CP}} = \frac{-i\cancel{p} + m_N e^{2i\alpha_5\gamma_5}}{2m_N} e^{-E_N t}$$

$$\hat{\alpha}_5 = \frac{\alpha_5}{\tilde{d}} = -\frac{\mathrm{ReTr}\left[T^+\gamma_5 \cdot C_{2pt}^{\overline{CP}}(t)\right]}{\mathrm{ReTr}\left[T^+ \cdot C_{2pt}^{CP}(t)\right]}\,, \qquad t \to \infty$$

(flavors labeled
for the proton)

similarity effect on nucleon likely due to mixing between cEDM and PS

Proton & Neutron EDFF Form Factors (bare)



- $(5.5\ \text{fm})^3$x(11 fm) box
- m_π=140 MeV
- connected-only
- no renormalization

Nucleon EDM : Summary

- Previously reported lattice results for θ_{QCD}-induced nEDM contain spurious contributions from mixing with the anomalous mag.moment
- Corrected θ_{QCD}-nEDM lattice values are small, consistent with zero
  - Disagreement with phenomenology/EFT is eliminated
  - Much higher lattice statistics are required to constrain of θ_{QCD}
- Based on *preliminary* analysis at a heavier pion mass (330 MeV), at the physical point expect $|d_n| \approx (1..2) \times 10^{-3}\, e$ fm
  - Even with variance-reduction techniques, O(300) M core*hours may be required
- Promising results for *quark* cEDM-induced EDFF
  - *Renormalization & mixing subtractions are underway*