RUB

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Current status of nuclear forces from chiral EFT

Bundesministerium für Bildung und Forschung
DFG Deutsche Forschungsgemeinschaft
erc European Research Council

From QCD to nuclei

QCD → **lattice**

symmetries (especially the chiral symmetry);
lost of information (LECs)

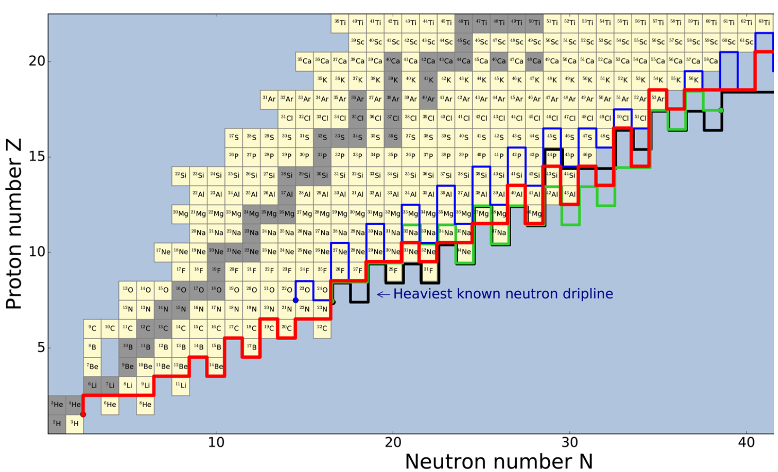
effective chiral Lagrangian $\mathcal{L}_{\text{eff}}(\pi, N)$

integrate out $|\vec{p}| \sim \sqrt{M_\pi m_N}$ (but retain $|\vec{p}| \sim M_\pi$):
Chiral Perturbation Theory

nuclear forces and currents

ab initio many-body methods:
lattice, FY, NCSM,…

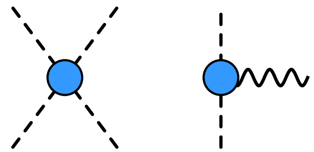
nuclear structure and dynamics



Chiral Effective Field Theory

GB dynamics

Weinberg, Gasser, Leutwyler, ...

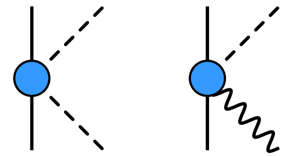


Chiral Perturbation Theory

$$Q = \frac{\text{momenta of particles or } M_\pi \sim 140\text{ MeV}}{\text{breakdown scale } \Lambda_b}$$

πN dynamics

Bernard-Kaiser-Meißner et al.



Effective Lagrangian:

$$\mathcal{L}_\pi = \frac{F^2}{4}\mathrm{Tr}(\nabla^\mu U \nabla_\mu U^\dagger + \chi_+) + \ldots ,$$

$$\mathcal{L}_{\pi N} = \bar{N}(iv \cdot D + g_A\, u \cdot S)N + \ldots ,$$

$$\mathcal{L}_{NN} = -\frac{1}{2}C_S(\bar{N}N)^2 + 2C_T(\bar{N}SN)^2 + \ldots$$

Chiral Effective Field Theory

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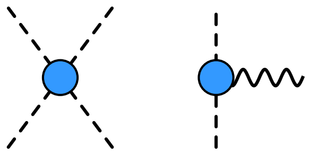

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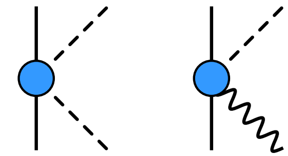

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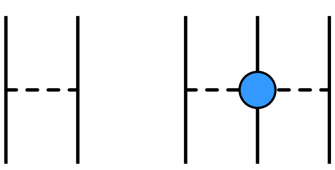
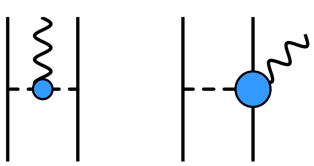
Nuclear forces
Weinberg, van Kolck, Kaiser, EGM, …

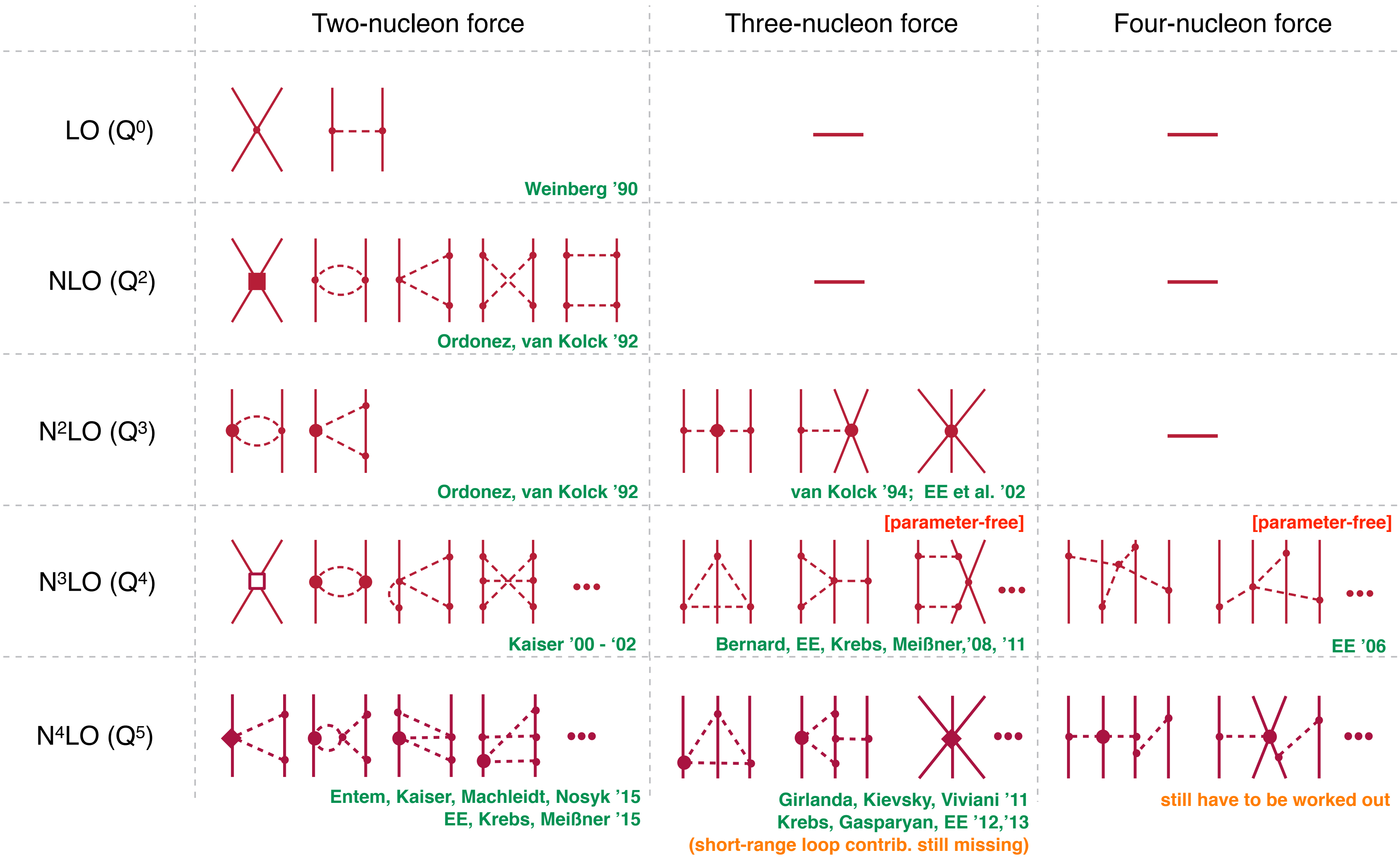


Auxilliary quantities (not observable):
More difficult to calculate than Feynman graphs
(renormalizability, off-shell consistency…)

Nuclear currents
Park et al, Bochum-Bonn, JLab-Pisa



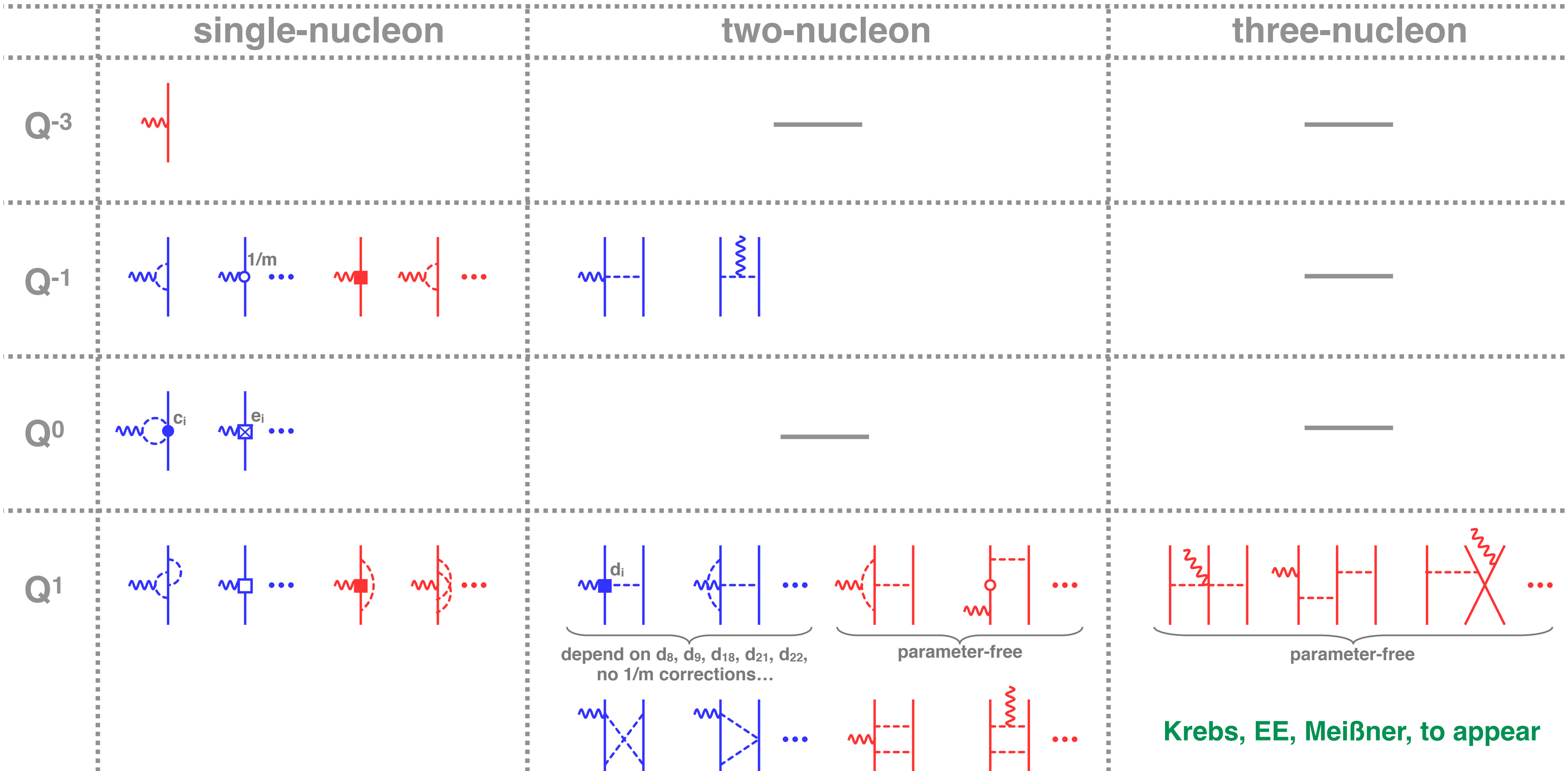
Chiral expansion of the nuclear forces [W-counting]

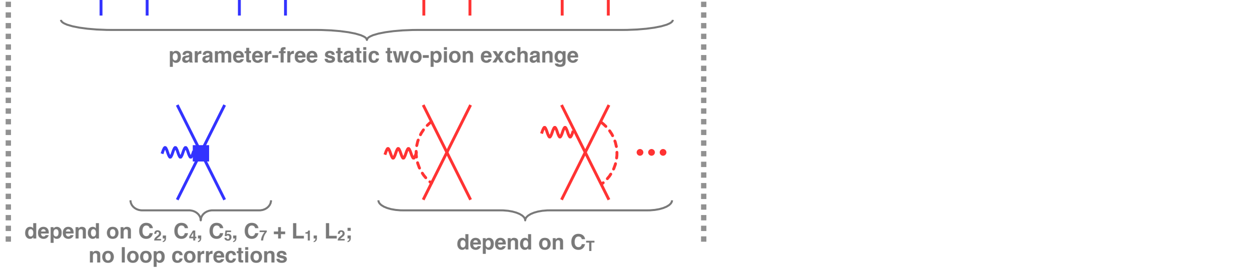


— Much more involved than just calculating Feynman diagrams…

— A similar program is being pursued for in chiral EFT with explicit Δ(1232) DOF

Chiral expansion of the electromagnetic **current** and **charge** operators



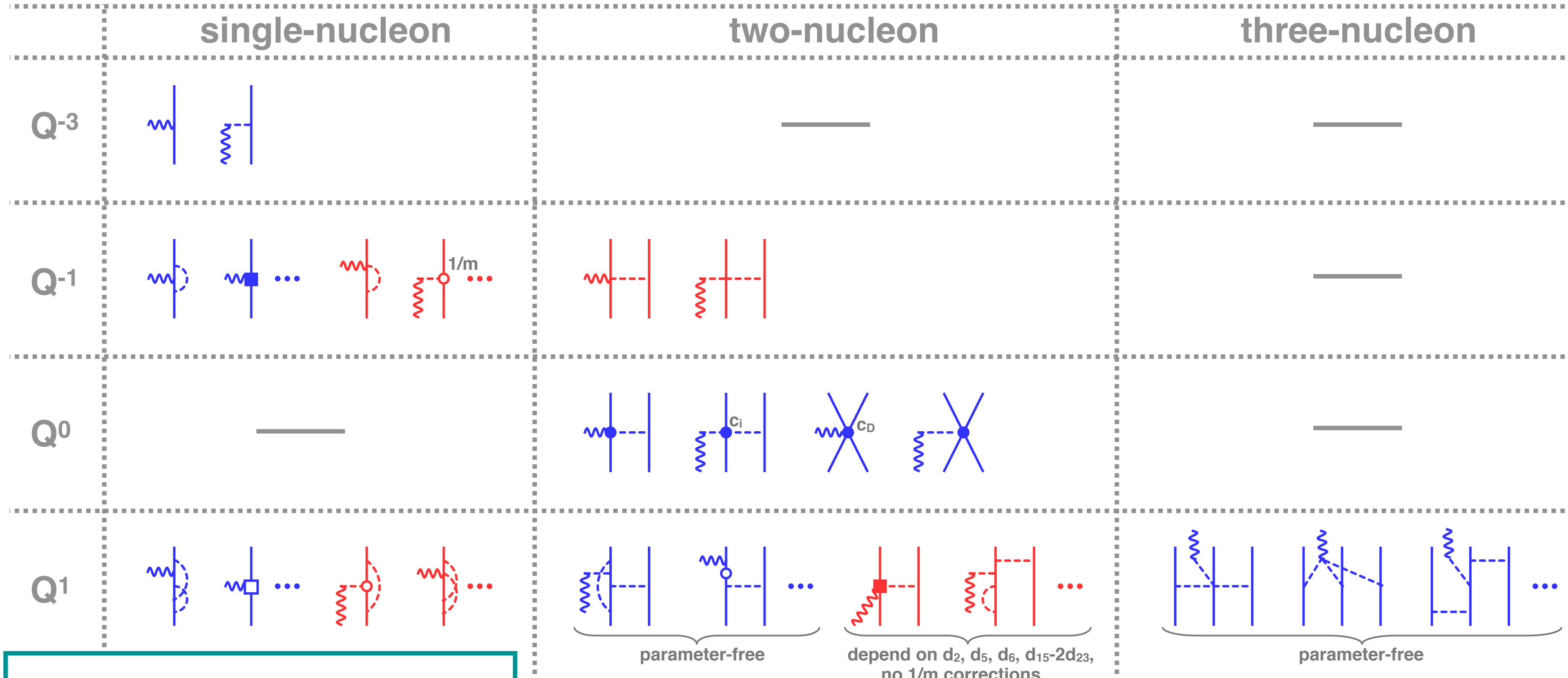


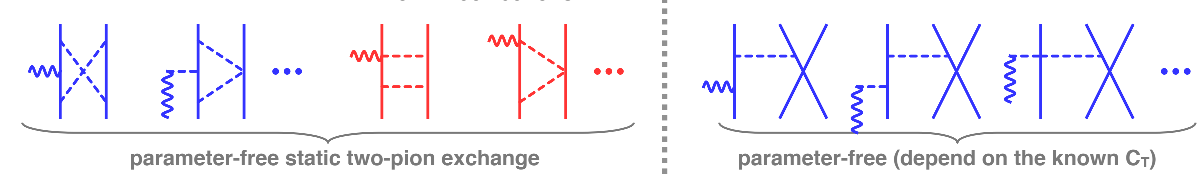
- **Our results differ from the ones of the JLab-Pisa group (Pastore et al., 08-11)**

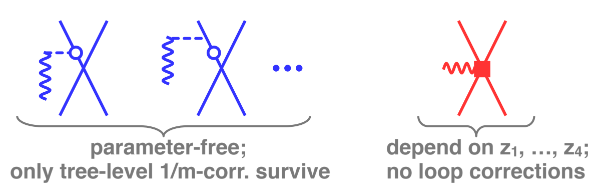
Axial currents

Krebs, EE, Meißner, Annals Phys. 378 (2017) 317

Chiral expansion of the axial current and charge operators







Comparison with Baroni et al. (TOPT)

- didn't consider 1/m-corrections at order Q^1
- looked only at irred. 3N graphs
- different results for π-exchange current contributions
- differences in tree-level 1π-terms

For more details:
review article by Hermann Krebs
(in preparation)

A new generation of accurate & precise chiral NN potentials

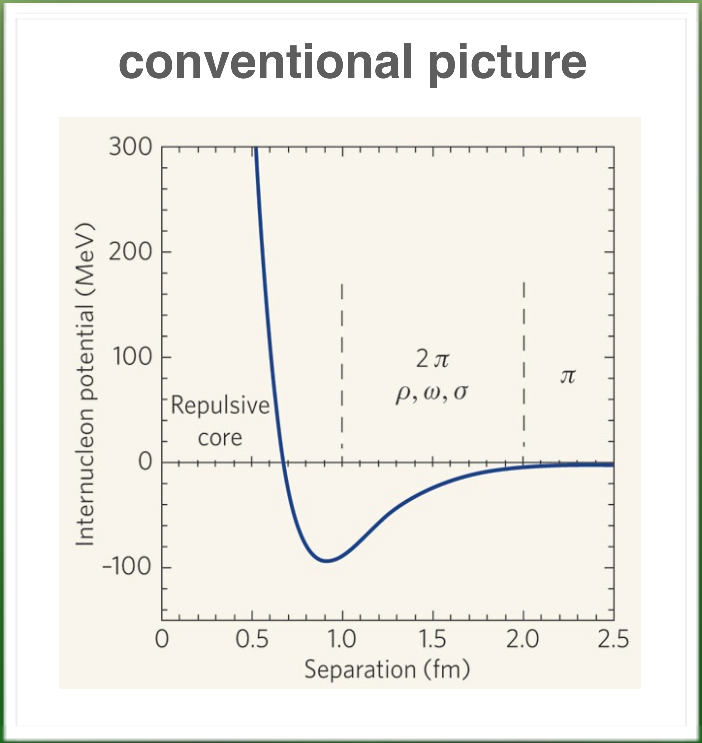
— semi-local, coordinate-space-regularized up to N^4LO
EE, Krebs, Meißner, EPJA 51 (2015) 53; PRL 115 (2015) 122301

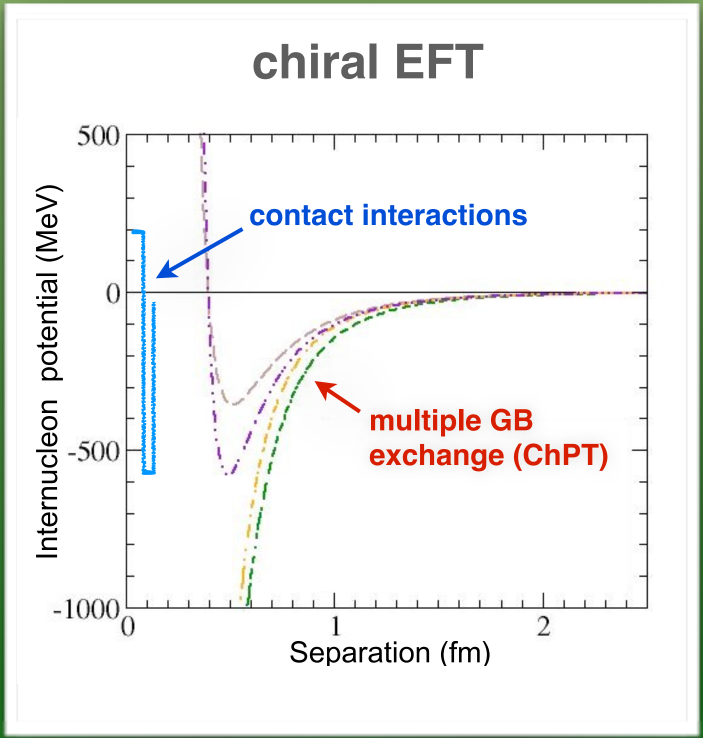
— semi-local, momentum-space-regularized up to N^4LO^+
Reinert, Krebs, EE, EPJA 54 (2018) 88

— nonlocal, momentum-space-regularized up to N^4LO^+
Entem, Machleidt, Nosyk, PRC 96 (2017) 024004

Other chiral EFT interactions on the market:
local potentials up to N^2LO **[Gezerlis et al. '14]**; minimally nonlocal N^3LO potential including N^2LO $\Delta(1232)$ contributions **[Piarulli et al.'15]**; N^2LO potentials tuned to heavier nuclei **[Ekström, Carlsson et al.]** …

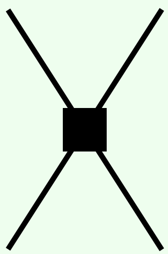
The long and short of nuclear forces





The long and short of nuclear forces

- Short-range interactions have to be tuned to experimental data. In the isospin limit, one has according to NDA:



LO [Q^0]:	2 operators (S-waves)
NLO [Q^2]:	+ 7 operators (S-, P-waves and ε_1)
N^{2}LO [Q^3]:	no new terms
N^{3}LO [Q^4]:	+ **12 operators** (S-, P-, D-waves and ε_1, ε_2)
N^{4}LO [Q^5]:	no new terms

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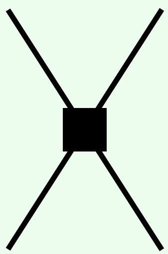

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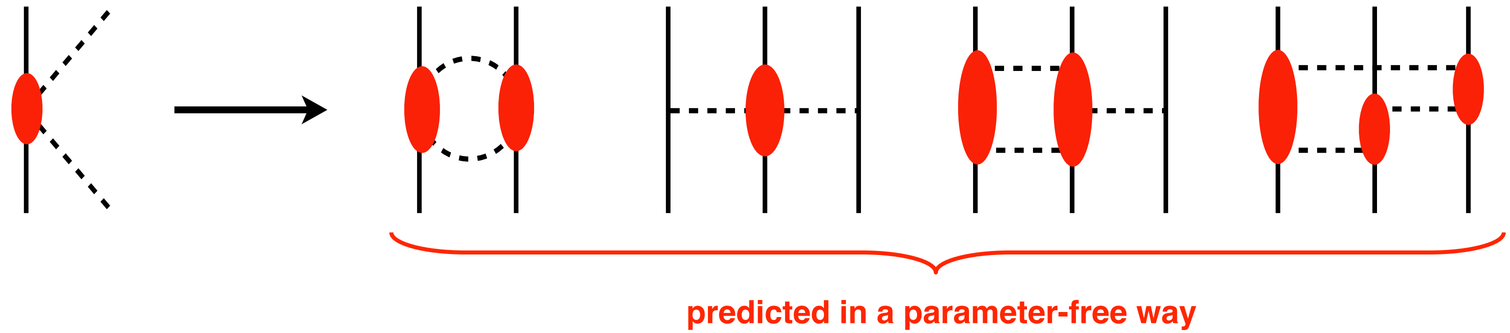
- The long-range part of nuclear forces and currents is completely determined by the chiral symmetry of QCD + experimental information on πN scattering



Determination of πN LECs

Pion-nucleon scattering up to Q^4 in heavy-baryon ChPT

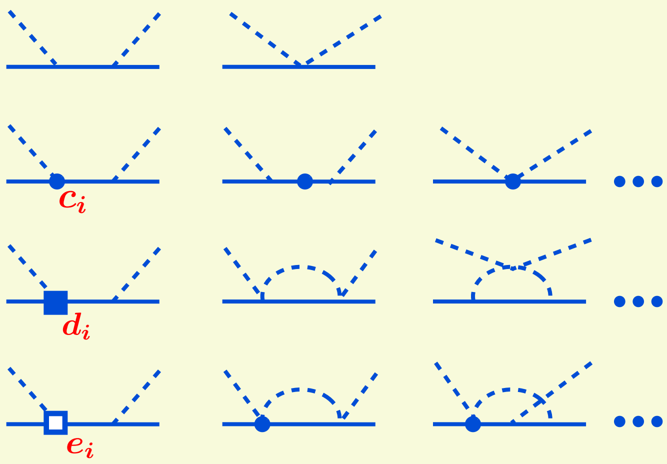
Fettes, Meißner '00; Krebs, Gasparyan, EE '12

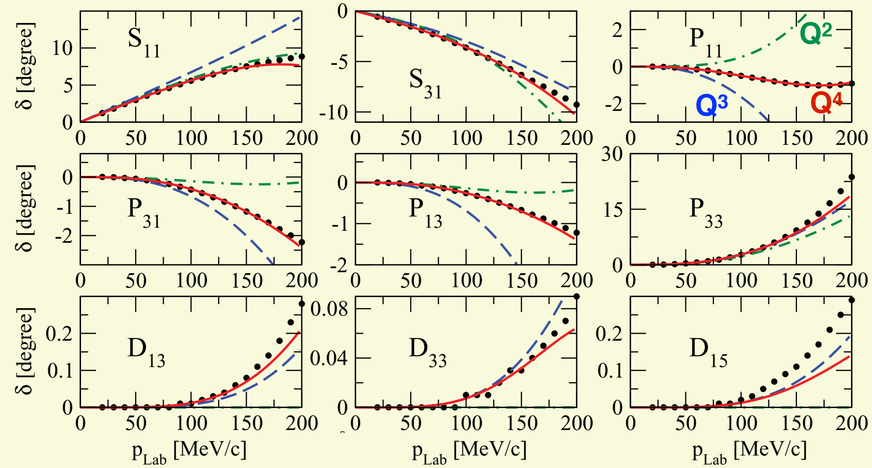
Order Q:

Order Q^2:

Order Q^3:

Order Q^4:





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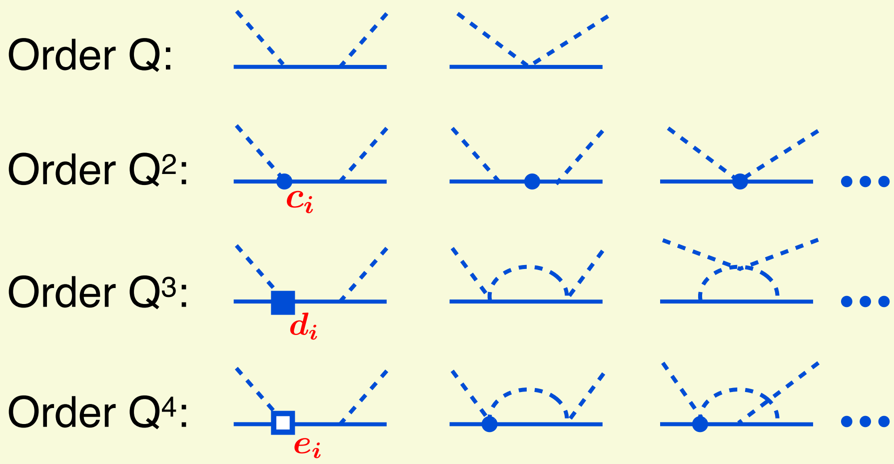

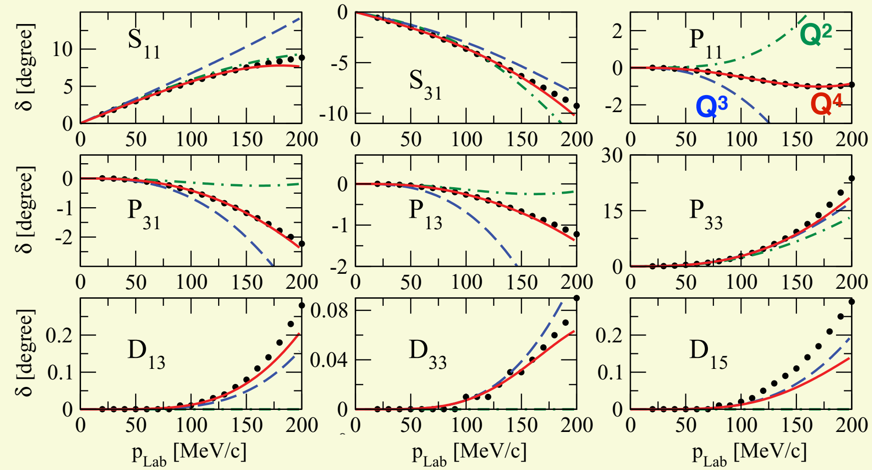

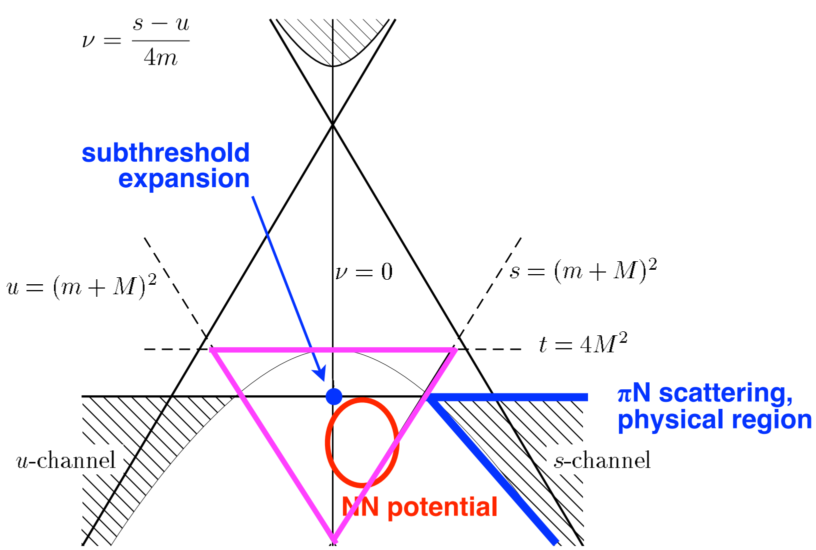

Matching ChPT to πN Roy-Steiner equations

Hoferichter, Ruiz de Elvira, Kubis, Meißner, PRL 115 (2015) 092301

- χ expansion of the πN amplitude expected to converge best within the Mandelstam triangle
- Subthreshold coefficients (from RS analysis) provide a natural matching point to ChPT

$$\bar{X} = \sum_{m,n} x_{mn}\, \nu^{2m+k} t^n, \qquad X = \{A^\pm, B^\pm\}$$

- Closer to the kinematics relevant for nuclear forces…

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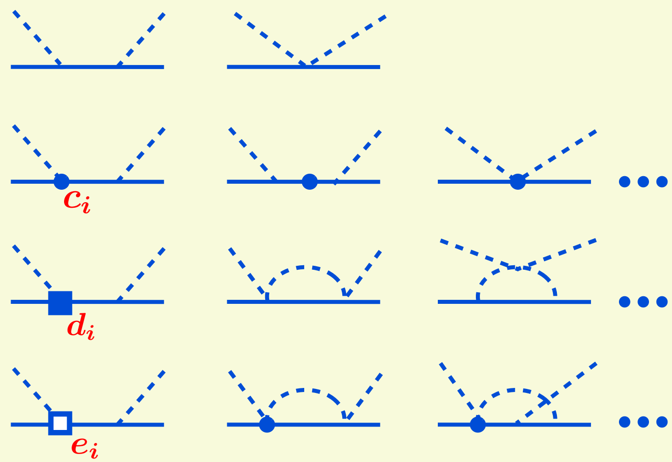

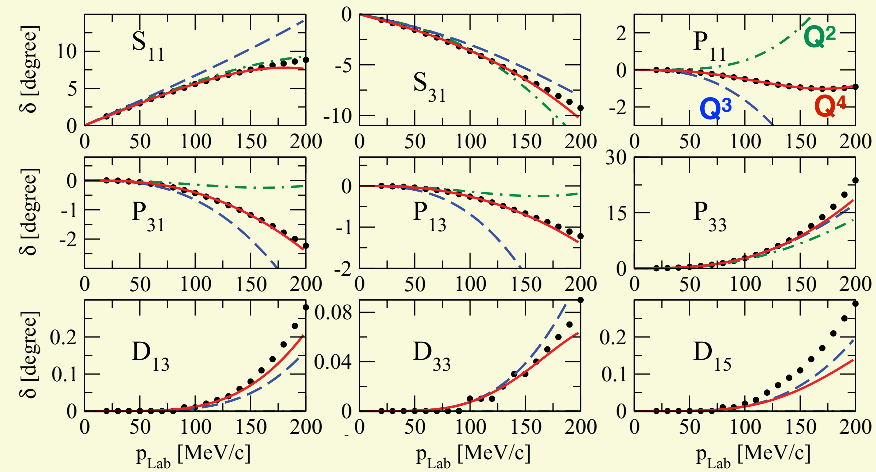

Relevant LECs (in GeV^{-n}) extracted from πN scattering

	c_1	c_2	c_3	c_4	$\bar{d}_1+\bar{d}_2$	$\bar{d}_3$	$\bar{d}_5$	$\bar{d}_{14}-\bar{d}_{15}$	$\bar{e}_{14}$	$\bar{e}_{17}$
$[Q^4]_{\mathrm{HB,NN}}$, GW PWA	-1.13	3.69	-5.51	3.71	5.57	-5.35	0.02	-10.26	1.75	-0.58
$[Q^4]_{\mathrm{HB,NN}}$, KH PWA	-0.75	3.49	-4.77	3.34	6.21	-6.83	0.78	-12.02	1.52	-0.37
$[Q^4]_{\mathrm{HB,NN}}$, Roy-Steiner	-1.10	3.57	-5.54	4.17	6.18	-8.91	0.86	-12.18	1.18	-0.18
$[Q^4]_{\mathrm{covariant}}$, data	-0.82	3.56	-4.59	3.44	5.43	-4.58	-0.40	-9.94	-0.63	-0.90

Krebs, Gasparyan, EE, PRC85 (12) 054006

Hoferichter et al., PRL 115 (15) 092301

Siemens et al., PRC94 (16) 014620

Notice:

— some LECs show sizable correlations (especially c_1 and c_3)…

— KH PWA and Roy-Steiner LECs lead to comparable results in the NN sector

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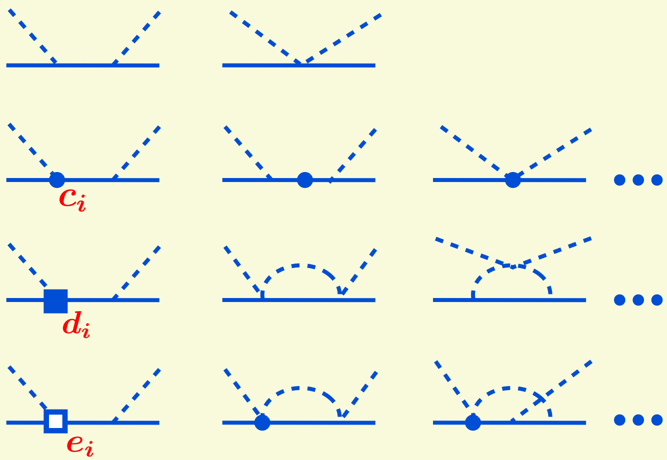

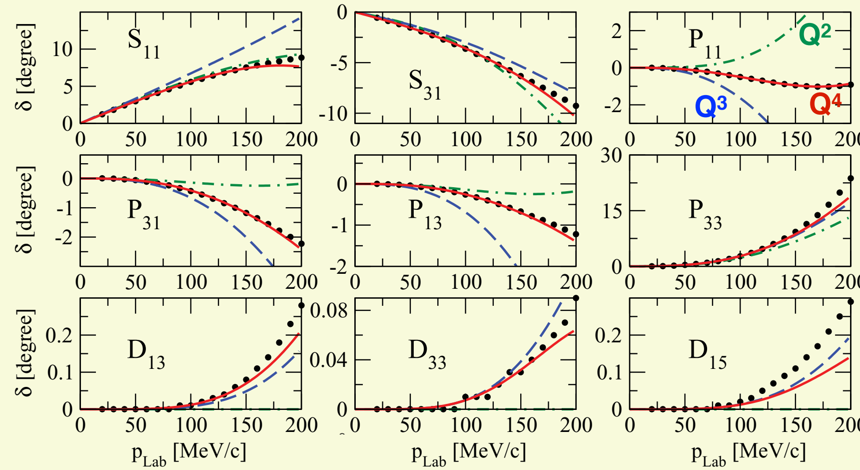

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Notice:

— some LECs show sizable correlations (especially c_1 and c_3)…

— KH PWA and Roy-Steiner LECs lead to comparable results in the NN sector

With the LECs taken from πN, the long-range NN force is completely fixed (parameter-free)

Regularization

The cutoff Λ has to be kept finite, $\Lambda \sim \Lambda_b$ (unless all counterterms are taken into account in the calculations) **[Lepage '97; EE, Gegelia '09]**. In practice, low values of Λ are preferred:

— many-body methods require soft interactions,

— spurious deeply-bound states for $\Lambda > \Lambda^{crit}$ make calculations for $A > 3$ unfeasible…

⟶ it is crucial to employ a regulator that minimizes finite-Λ artifacts!

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Nonlocal: $V_{1\pi}^{\rm reg} \propto \frac{e^{-\frac{p'^4+p^4}{\Lambda^4}}}{\vec{q}^{\,2}+M_\pi^2} \longrightarrow \frac{1}{\vec{q}^{\,2}+M_\pi^2} \underbrace{\left(1-\frac{p'^4+p^4}{\Lambda^4}+\mathcal{O}(\Lambda^{-8})\right)}_{\text{affect long-range interactions...}}$

EE, Glöckle, Meißner '04; Entem, Machleidt '03; Entem, Machleidt, Nosyk '17; …

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Local: [inspired by Thomas Rijken] $$V_{1\pi}^{\rm reg} \propto \frac{e^{-\frac{\vec q^{\,2}+M_\pi^2}{\Lambda^2}}}{\vec q^{\,2}+M_\pi^2} \longrightarrow \frac{1}{\vec q^{\,2}+M_\pi^2}\left(1+\text{short-range terms}\right)$$

Reinert, Krebs, EE '18;

⟶ does not affect long-range physics **at any order** in $1/\Lambda^2$-expansion

— Application to 2π exchange does not require re-calculating the corresponding diagrams:

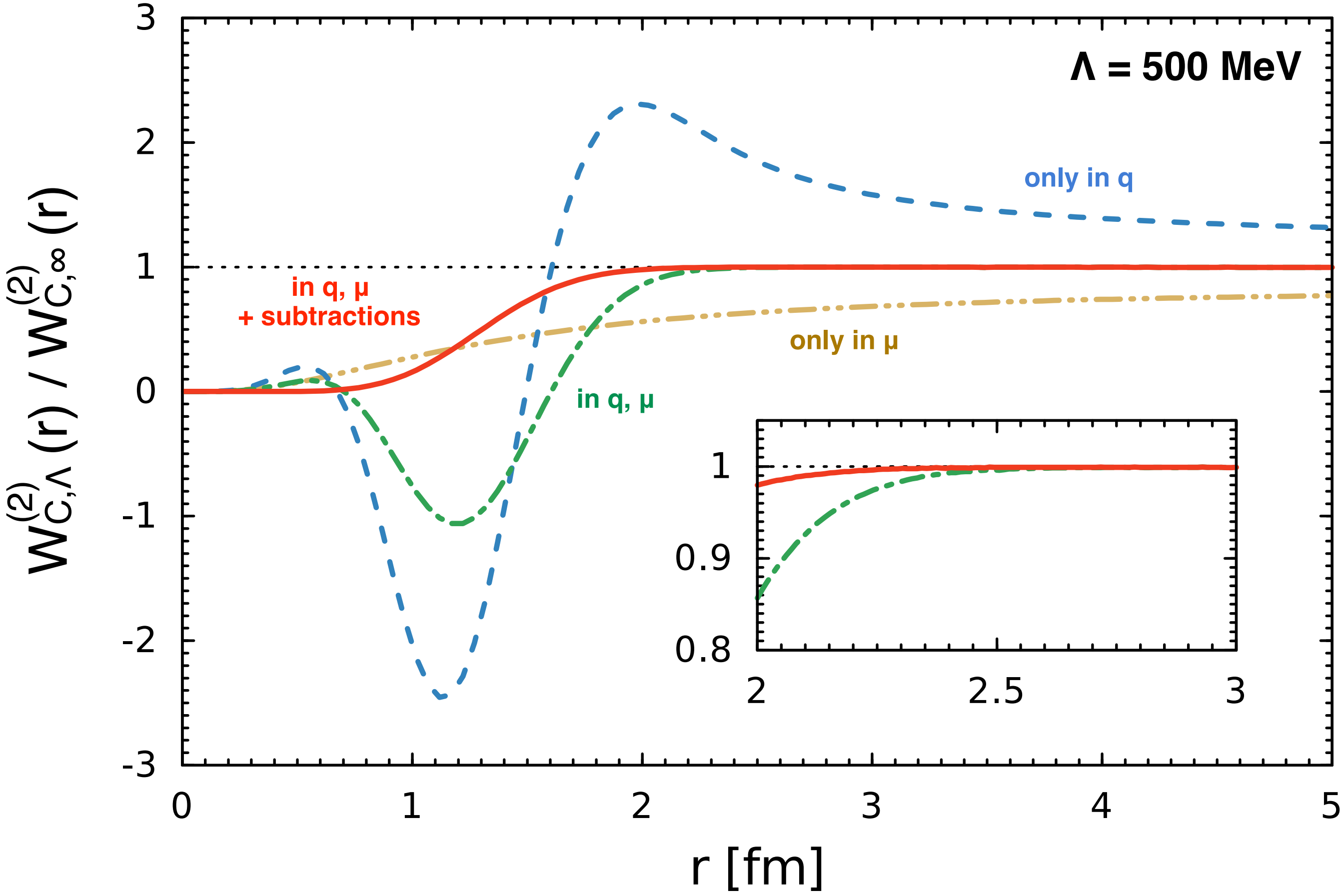
$$V(q)=\frac{2}{\pi}\int_{2M_\pi}^\infty \mu\, d\mu \frac{\rho(\mu)}{q^2+\mu^2}+\ldots \quad \xrightarrow{\text{reg.}} \quad V_\Lambda(q)=e^{-\frac{q^2}{2\Lambda^2}}\frac{2}{\pi}\int_{2M_\pi}^\infty \mu\, d\mu \frac{\rho(\mu)}{q^2+\mu^2}e^{-\frac{\mu^2}{2\Lambda^2}}+\underbrace{\ldots}_{\text{polynomial in } q^2,\, M_\pi}$$

— Convention: choose polynomial terms such that $\Delta^n V_{\Lambda,\,\rm long}(\vec r)\Big|_{r=0}=0$

Regularization

Regularized 2π-exchange potential: $W_{C,\Lambda}(q) = e^{-\frac{q^2}{2\Lambda^2}} \frac{2}{\pi} \int_{2M_\pi^2}^{\infty} \mu \, d\mu \, \frac{\rho(\mu)}{q^2 + \mu^2} \, e^{-\frac{\mu^2}{2\Lambda^2}}$

Various regularization approaches



Does it matter in practice?

NN data analysis

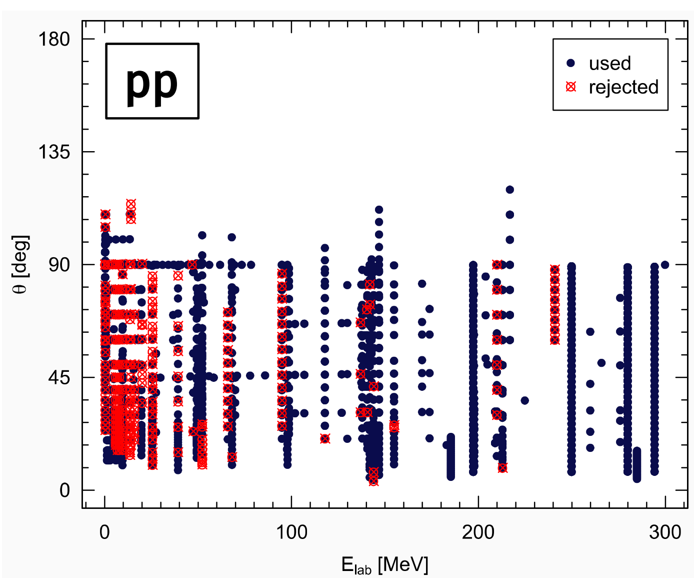
P. Reinert, H. Krebs, EE, EPJA 54 (2018) 88

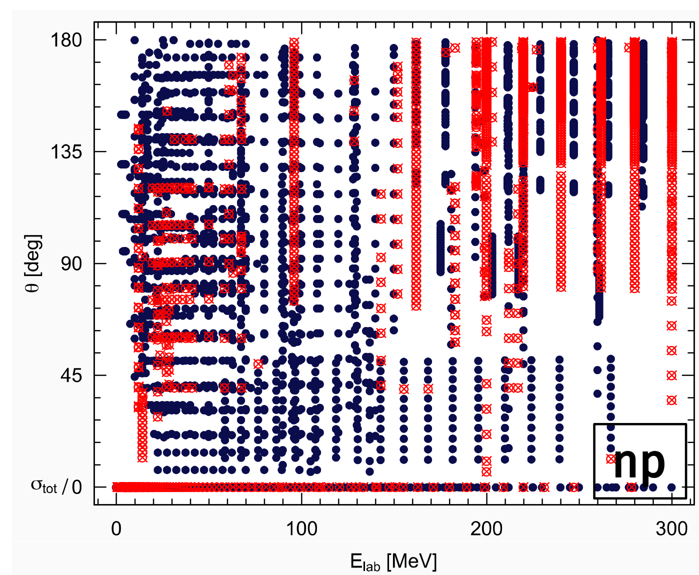
- To fix NN contact interactions, use scattering data together with $B_d = 2.224575(9)$ MeV and $b_{np} = 3.7405(9)$ fm.

- Since 1950-es, about 3000 proton-proton + 5000 neutron-proton scattering data below 350 MeV have been measured.

- However, certain data are mutually incompatible within errors and have to be rejected.
  2013 Granada database [Navarro-Perez et al., PRC 88 (2013) 064002], rejection rate: 31% np, 11% pp:

  2158 proton-proton + 2697 neutron-proton data below E_{lab} = 300 MeV





Low-energy constants

P. Reinert, H. Krebs, EE, EPJA 54 (2018) 88

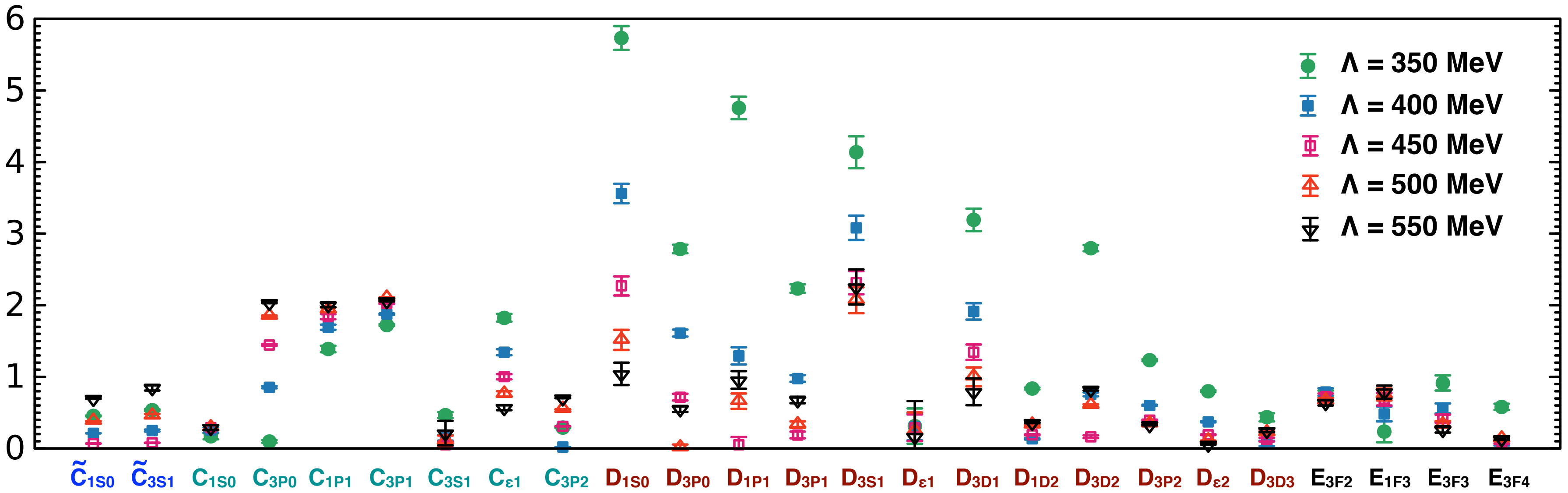
- Significant correlations within the $^1S_0$ and $^3S_1$-$^3D_1$ channels but little correlations otherwise. Still, all LECs can be accurately determined…

- Natural units for the LECs according to NDA:

$$|\tilde{C}_i| \sim \frac{4\pi}{F_\pi^2}, \qquad |C_i| \sim \frac{4\pi}{F_\pi^2 \Lambda_b^2}, \qquad |D_i| \sim \frac{4\pi}{F_\pi^2 \Lambda_b^4}, \qquad |E_i| \sim \frac{4\pi}{F_\pi^2 \Lambda_b^6}$$

Assuming $\Lambda_b = 600$ MeV [EE, Krebs, Meißner EPJA 51 (15) 53; Furnstahl, Klco, Phillips, Wesolowski, PRC 92 (15) 024005], the LECs come out of a natural size.

Absolute values of the LECs in natural units



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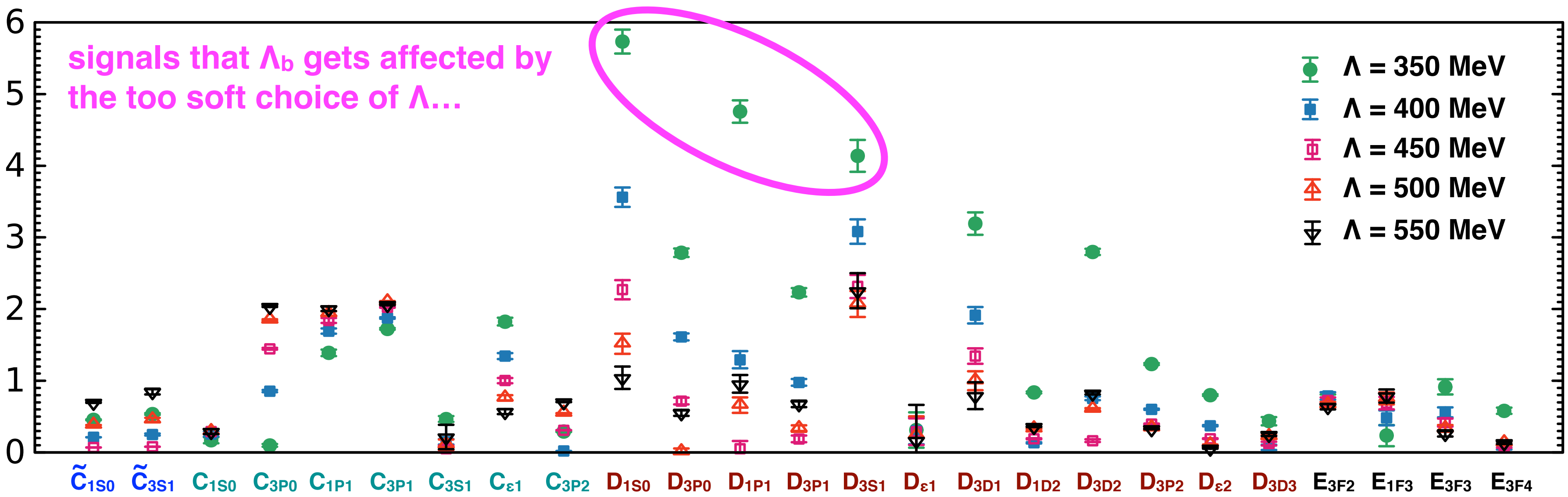
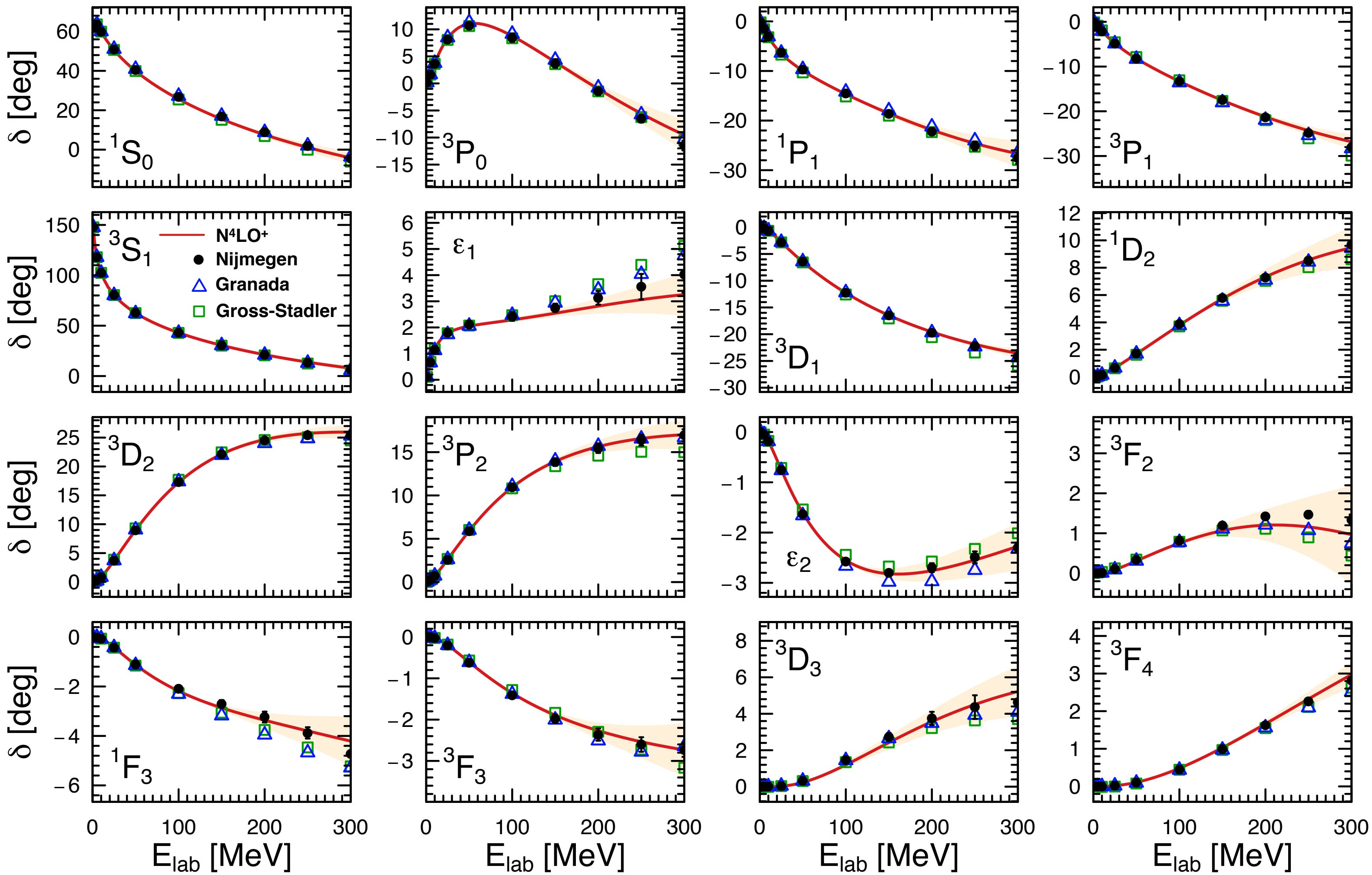


State-of-the-art NN potentials

P. Reinert, H. Krebs, EE, EPJA 54 (2018) 88



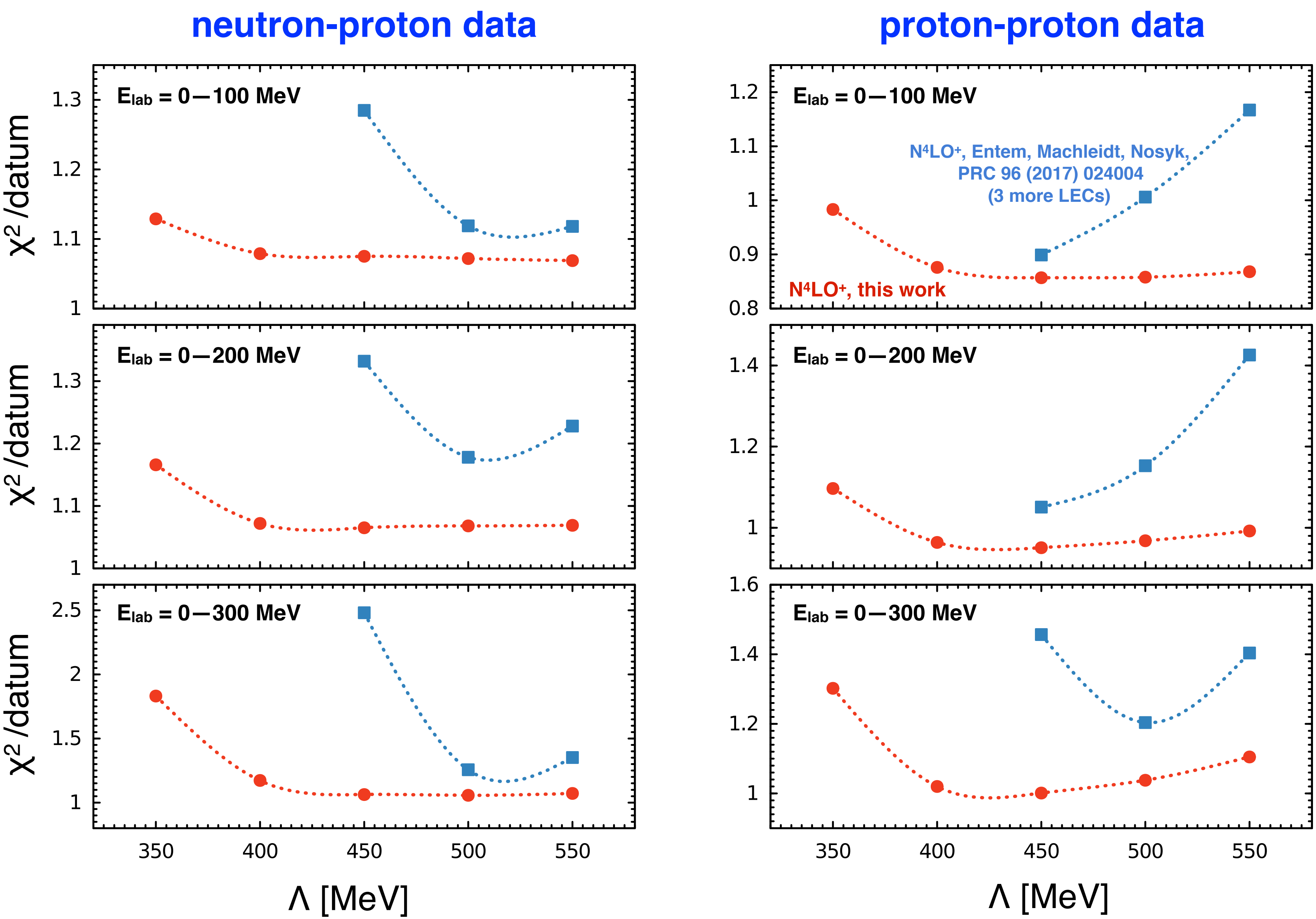
— N^4LO^+ yields currently the best description of the 2013 Granada database

— 40% less parameters (27+1) compared to high-precision potentials

— Clear evidence of the parameter-free chiral 2π exchange

State-of-the-art NN potentials

P. Reinert, H. Krebs, EE, EPJA 54 (2018) 88



Error analysis

P. Reinert, H. Krebs, EE, EPJA 54 (2018) 88

Careful error analysis: (i) truncation error [EE, Krebs, Meißner EPJ A51 (15)], (ii) statistical uncertainty (NN LECs), (iii) uncertainty due to πN LECs and (iv) choice of the energy range in the fits.

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Example: deuteron asymptotic normalizations (relevant for nuclear astrophysics)

Our determination:

truncation error, statistical error, πN LECs, variation of E_{max}

$$A_S = 0.8847^{(+3)}_{(-3)}(3)(5)(1)\ \text{fm}^{-1/2}$$

$$\eta \equiv \frac{A_D}{A_S} = 0.0255^{(+1)}_{(-1)}(1)(4)(1)$$

Exp: $A_S = 0.8781(44)\ \text{fm}^{-1/2}$, $\eta = 0.0256(4)$

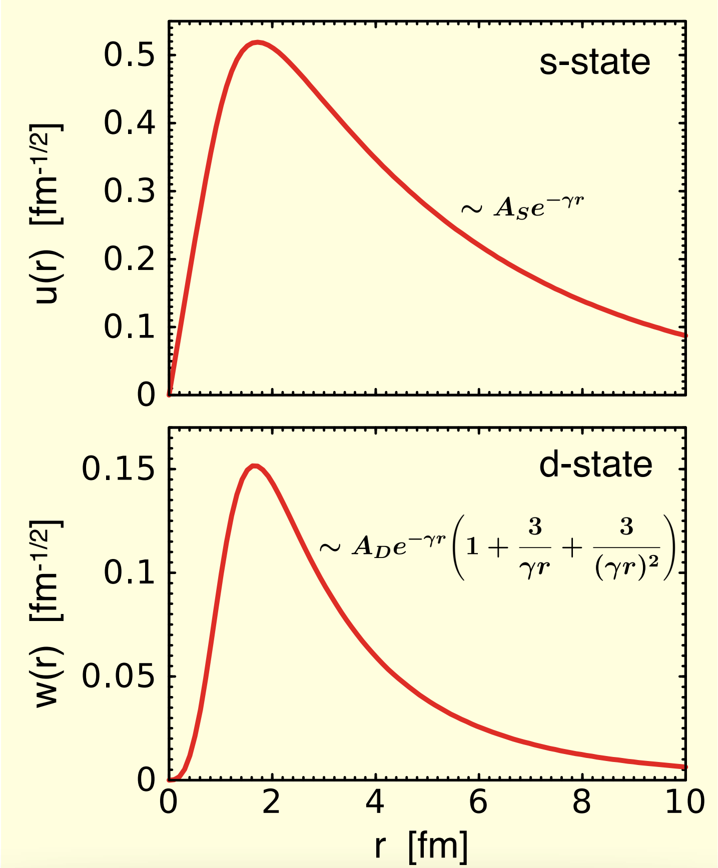
Borbely et al. '85 — Rodning, Knutson '90

Nijmegen PWA [errors are „educated guesses"] Stoks et al. '95

$$A_S = 0.8845(8)\ \text{fm}^{-1/2}, \quad \eta = 0.0256(4)$$

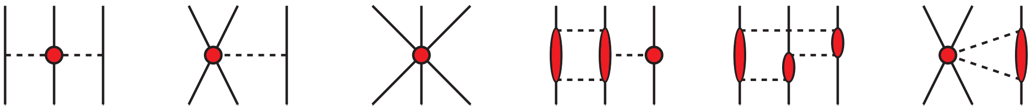
Granada PWA [errors purely statistical] Navarro Perez et al. '13

$$A_S = 0.8829(4)\ \text{fm}^{-1/2}, \quad \eta = 0.0249(1)$$



Three-nucleon forces

N²LO: tree-level graphs, 2 new LECs
van Kolck ’94; EE et al ’02



N³LO: leading 1 loop, parameter-free
Ishikawa, Robilotta ’08; Bernard, EE, Krebs, Meißner ’08, ’11

N⁴LO: full 1 loop, almost completely worked out, several new LECs
Girlanda, Kievski, Viviani ’11; Krebs, Gasparyan, EE ’12,’13; EE, Gasparyan, Krebs, Schat ’14

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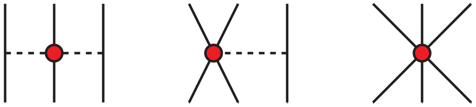

LENPIC: Low **E**nergy **N**uclear **P**hysics **I**nternational **C**ollaboration

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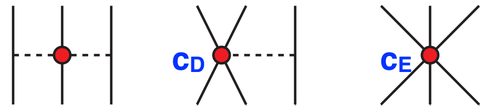
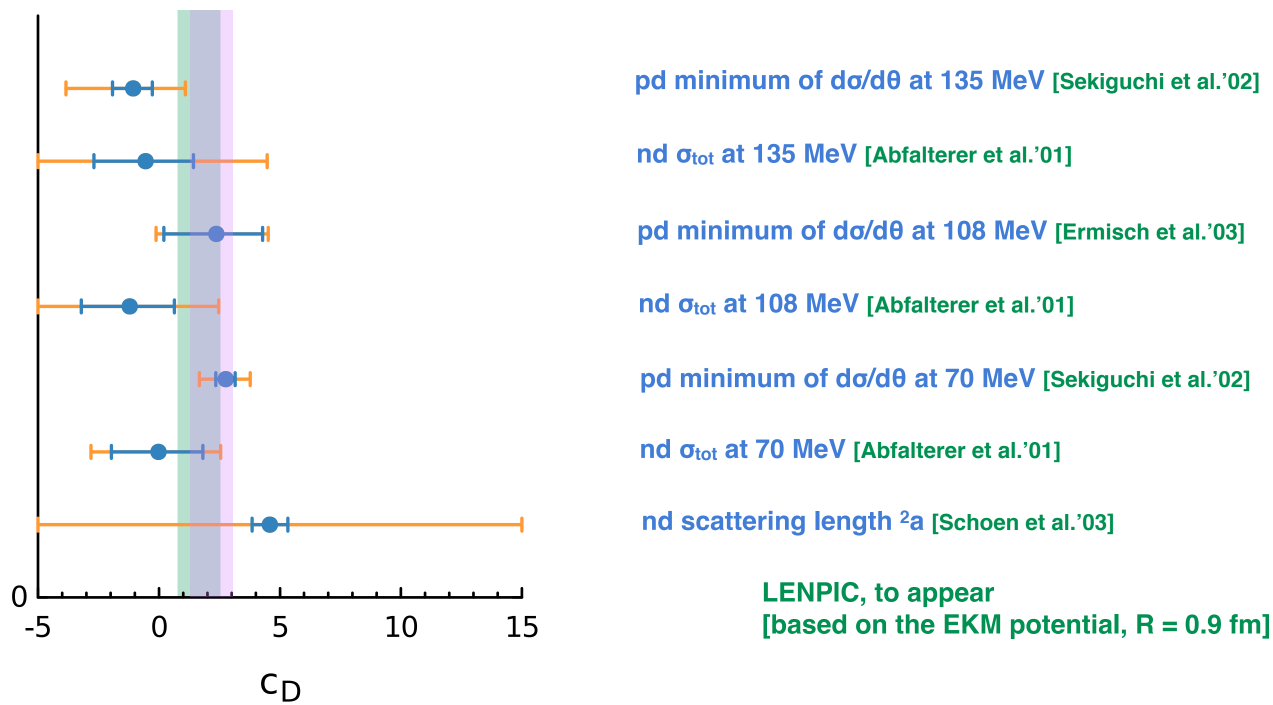


Determination of the LECs c_D, c_E

— Triton BE (c_D-c_E correlation)

— Explore various possibilities and let theory and/or data decide…



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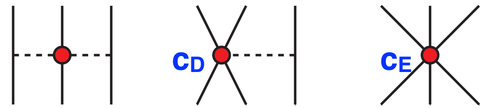

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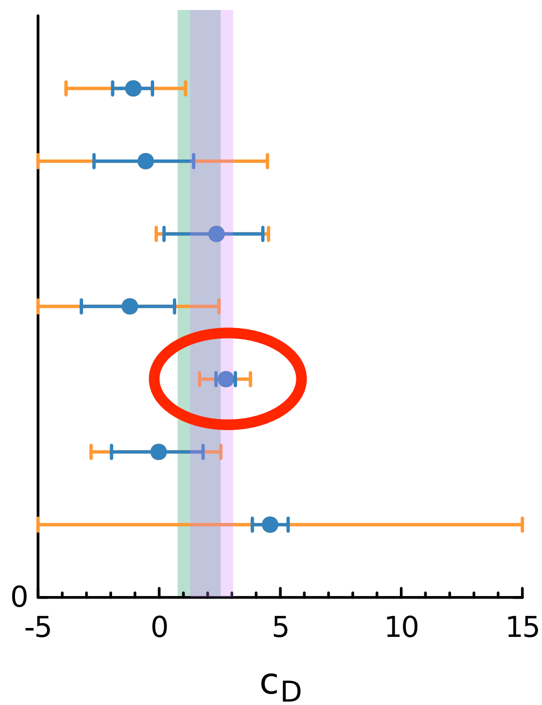

pd minimum of dσ/dθ at 135 MeV [Sekiguchi et al.'02]

nd σ_{tot} at 135 MeV [Abfalterer et al.'01]

pd minimum of dσ/dθ at 108 MeV [Ermisch et al.'03]

nd σ_{tot} at 108 MeV [Abfalterer et al.'01]

pd minimum of dσ/dθ at 70 MeV [Sekiguchi et al.'02]

nd σ_{tot} at 70 MeV [Abfalterer et al.'01]

nd scattering length ^2a [Schoen et al.'03]

LENPIC, to appear
[based on the EKM potential, R = 0.9 fm]

yields the strongest constraint…

LENPIC: Low Energy Nuclear Physics International Collaboration

Nd total cross section at 70 MeV (preliminary)

Light nuclei (preliminary)

Summary and outlook

Nuclear Hamiltonian:

— derivation of contributions up to N^3LO completed already in 2011; derivation of N^4LO corrections done for V_{2N} and almost done for V_{3N} (new LECs…) and V_{4N}

— accurate & precise 2N potentials at N^4LO^+ are available,

— promising results for few-N systems based on 2NF + 3NF@N^2LO **[LENPIC]**

Electroweak current operators:

— have been worked out completely to N^3LO

— some πN LECs in 1π axial charge at N^3LO are unknown…
[lattice QCD? ν-induced π-production? resonance saturation? large-N_c?…]

Work in progress:

— regularization of 3NF & currents beyond N^2LO (nontrivial to maintain χ-symm!)

Next steps:

— Precision tests of the theory for ^3H β decay & μ capture (validation)

— Extension to other processes, heavier nuclei, N^4LO, explicit Δ's, …