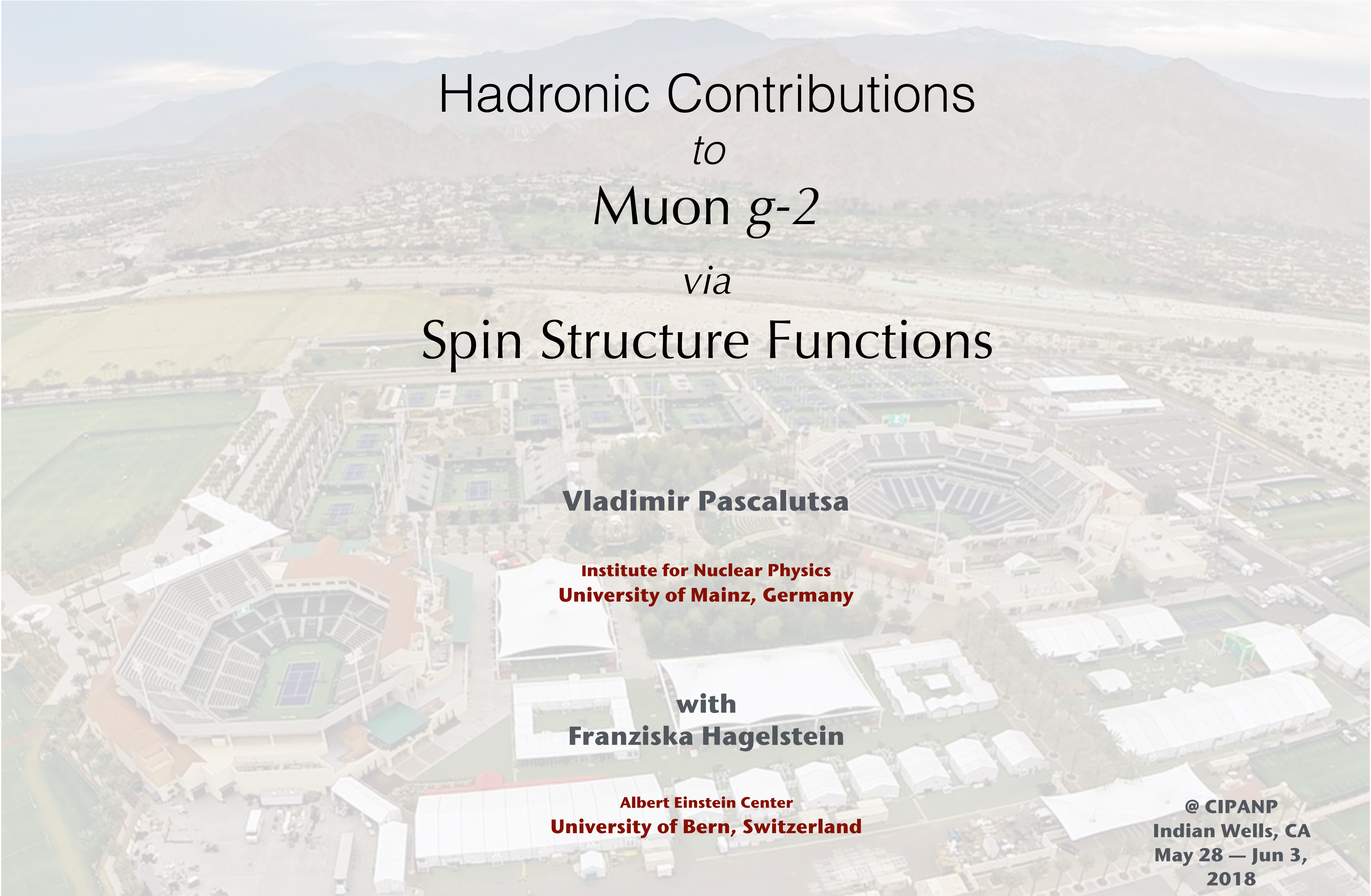

Hadronic Contributions *to* Muon *g*-2 *via* Spin Structure Functions

Vladimir Pascalutsa

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with
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Albert Einstein Center
University of Bern, Switzerland

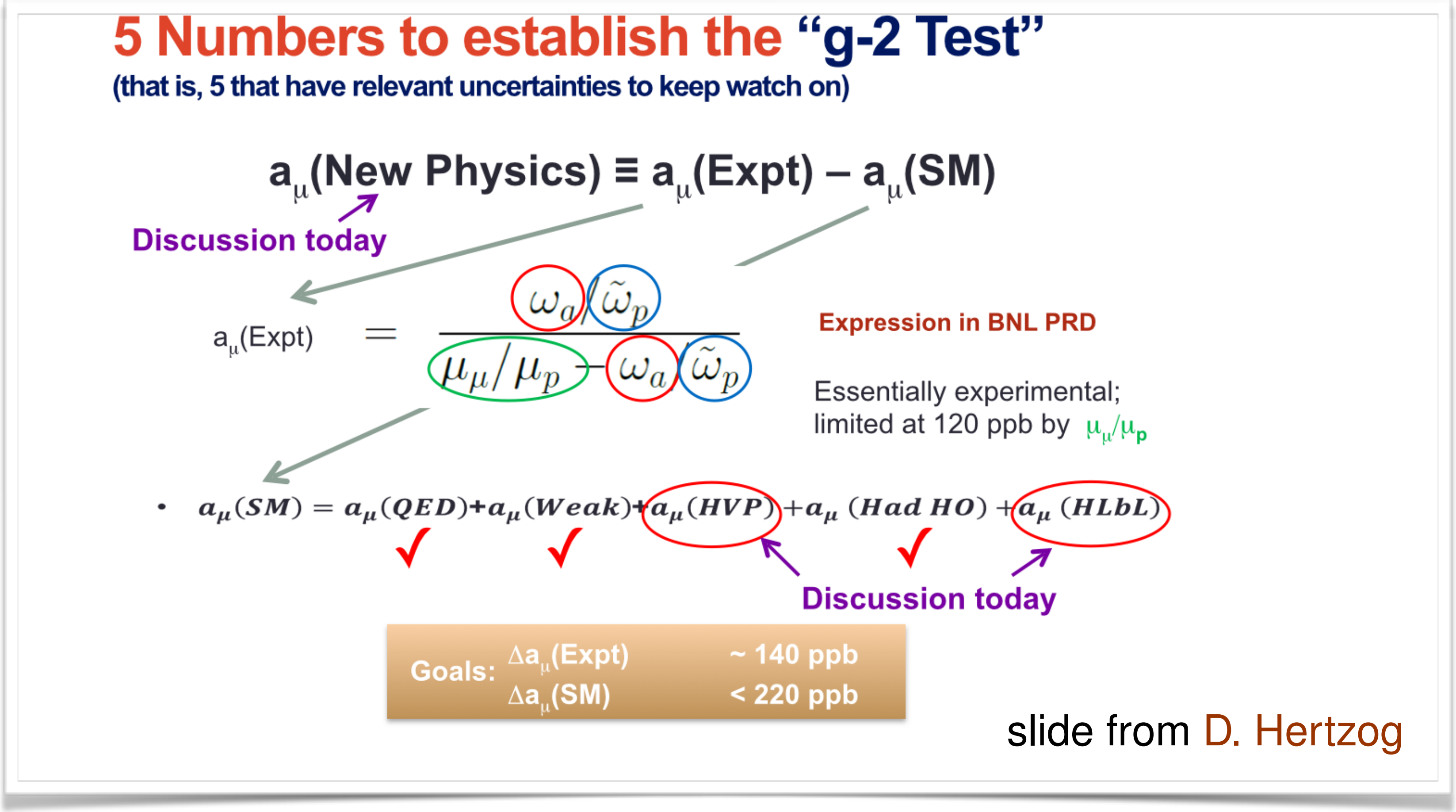
@ CIPANP
Indian Wells, CA
May 28 — Jun 3, 2018

JG|U

SFB 1044
THE LOW-ENERGY FRONTIER
OF THE STANDARD MODEL

Muon $g{-}2$

or, anomalous magnetic moment: $a_\mu \equiv (g-2)_\mu/2$



5 Numbers to establish the "g-2 Test"

(that is, 5 that have relevant uncertainties to keep watch on)

$$a_\mu(\text{New Physics}) \equiv a_\mu(\text{Expt}) - a_\mu(\text{SM})$$

Discussion today

$$a_\mu(\text{Expt}) = \frac{\omega_a/\tilde{\omega}_p}{\mu_\mu/\mu_p - \omega_a/\tilde{\omega}_p}$$

Expression in BNL PRD

Essentially experimental;
limited at 120 ppb by μ_μ/μ_p

- $a_\mu(SM) = \underset{\checkmark}{a_\mu(QED)} + \underset{\checkmark}{a_\mu(Weak)} + a_\mu(HVP) + \underset{\checkmark}{a_\mu\,(Had\ HO)} + a_\mu\,(HLbL)$

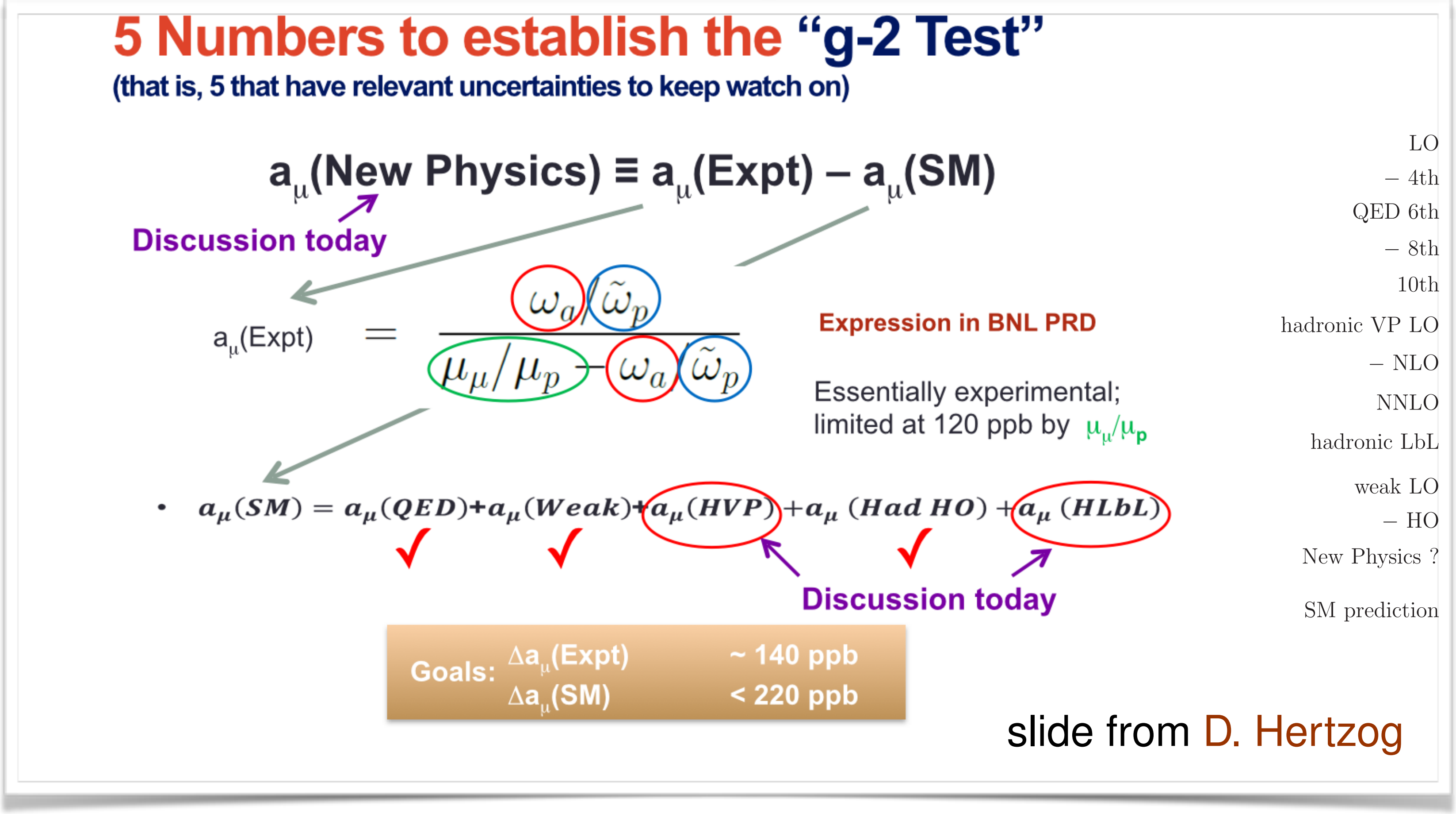
Discussion today

Goals:	Δa_μ(Expt)	~ 140 ppb
	Δa_μ(SM)	< 220 ppb

slide from D. Hertzog

Muon $g\!-\!2$

or, anomalous magnetic moment: $a_\mu \equiv (g-2)_\mu/2$



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✓ ✓ ✓

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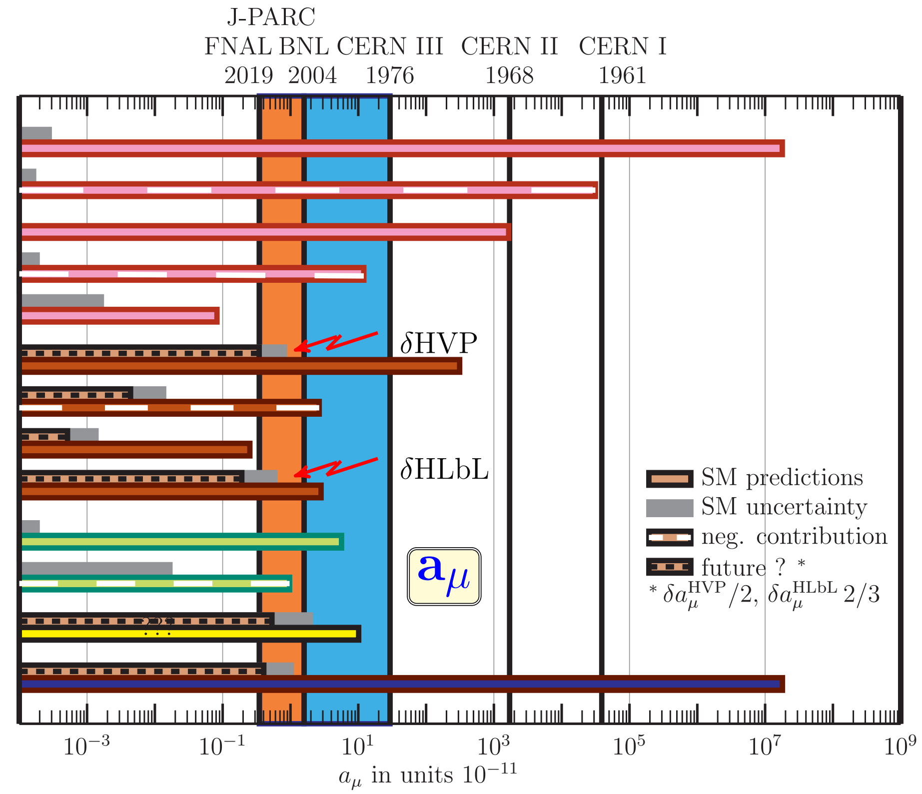

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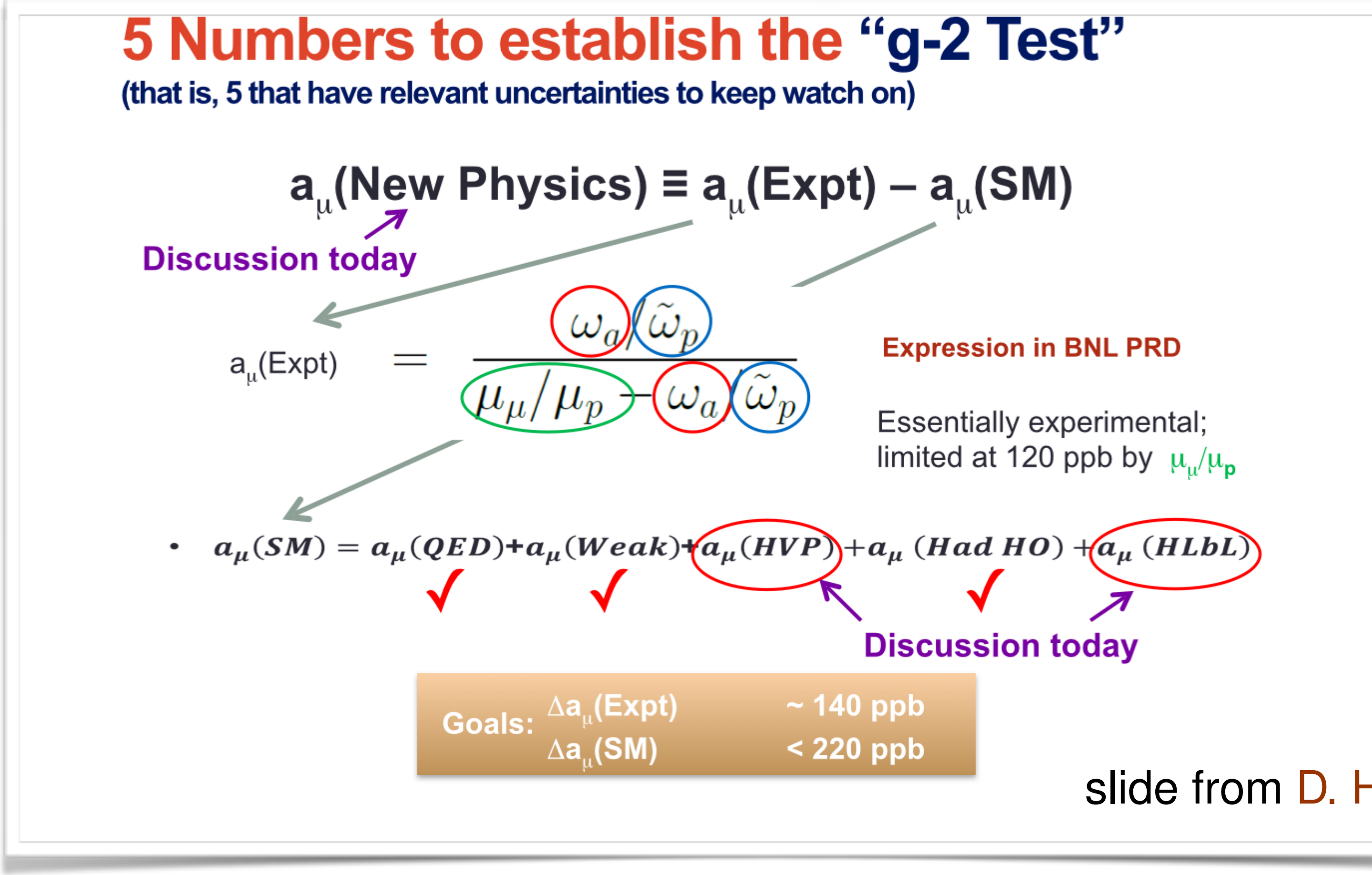

5 Numbers to establish the "g-2 Test"

(that is, 5 that have relevant uncertainties to keep watch on)

a_μ(New Physics) ≡ a_μ(Expt) – a_μ(SM)

Discussion today

$$a_\mu(\text{Expt}) = \frac{\omega_a/\tilde{\omega}_p}{\mu_\mu/\mu_p - \omega_a/\tilde{\omega}_p}$$

Expression in BNL PRD

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✓ ✓ ✓

Discussion today

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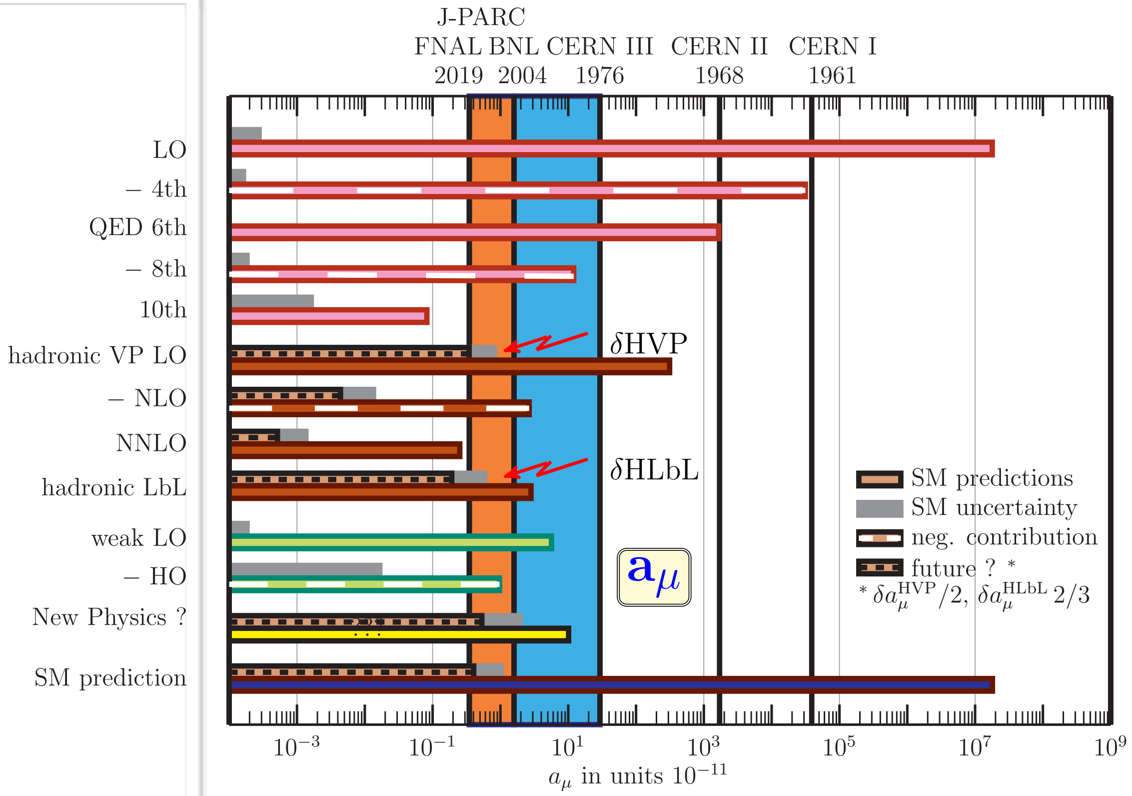
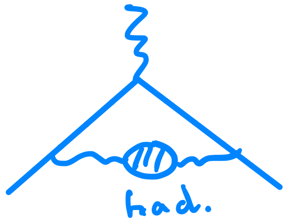


F. Jegerlehner Standard model theory and experiment comparison

Contribution	Value×10^{10}	Error×10^{10}	Reference
QED incl. 4-loops + 5-loops	11 658 471.886	0.003	Aoyama et al 12,Laporta 17
Hadronic LO vacuum polarization	689.46	3.25	
Hadronic light–by–light	10.34	2.88	
Hadronic HO vacuum polarization	-8.70	0.06	
Weak to 2-loops	15.36	0.11	Gnendiger et al 13
Theory	11 659 178.3	3.5	–
Experiment	11 659 209.1	6.3	BNL 04
The. - Exp. 4.3 standard deviations	-30.6	7.2	–

Hadronic Contributions to g-2

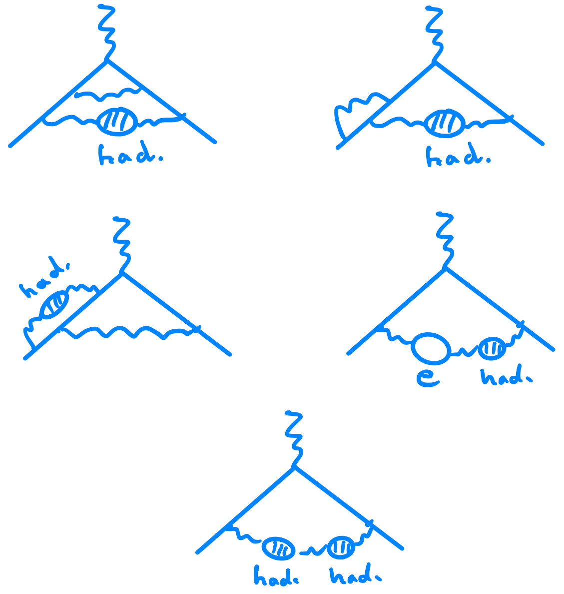
$O(\alpha^2)$



$a_\mu^{\text{HVP-LO}} \cdot 10^{10}$	situation today (from M. Knecht's talk in Feb'18):
$693.1(3.4)$	M. Davier et al., Eur. Phys. J. C 77, 827 (2017)
$693.27(2.46)$	A. Keshavarzi et al., arXiv:1802.02995 [hep-ph]
$688.07(4.14)$	F. Jegerlehner, arXiv:1705.00263 [hep-ph]

$\sim 0.4\%$

$O(\alpha^3)$

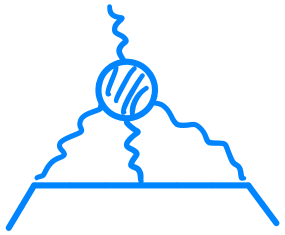


"had VP NLO"

$\sim -10(1) \cdot 10^{-10}$

VP or LbL

empirically in LO HVP

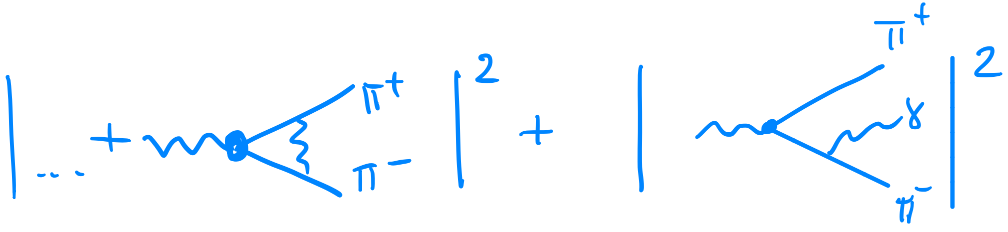


had LbL

$\sim 10 \cdot 10^{-10}$

$a_\mu^{\text{HLxL}} = +(10.3 \pm 2.9) \cdot 10^{-10}$

F. Jegerlehner, arXiv:1705.00263 [hep-ph]



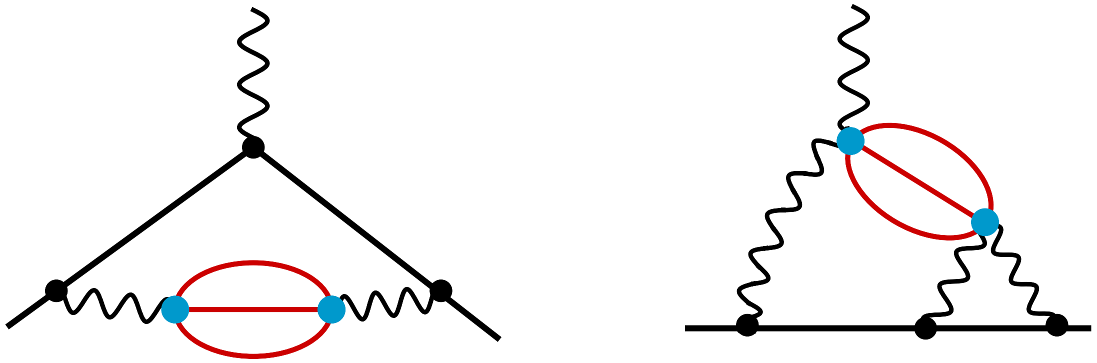
$\sim 4 \cdot 10^{-10}$ (sQED)

$\sim 4 \cdot 10^{-10}$ ($\pi^0\gamma$)

$a_\mu^{\text{HVP-NLO}} \cdot 10^{10}$	
$-9.84(7)$	K. Hagiwara et al., J. Phys. G 38, 085003 (2011)
$-9.93(7)$	F. Jegerlehner, arXiv:1705.00263 [hep-ph]
$-9.82(4)$	A. Keshavarzi et al., arXiv:1802.02995 [hep-ph]

Motivation

- Uncertainty of the SM prediction for the muon anomaly $(g-2)_\mu$ is dominated by hadronic contributions (HVP and HLbL)



- HVP is calculated with a data-driven dispersive approach:

$$\boxed{a^{\text{HVP}} = \frac{\alpha}{\pi^2} \int_{4m_\pi^2}^{\infty} \frac{ds}{s} \, \text{Im}\, \Pi^{\text{had}}(s)\, K(s/m^2)}$$

$$\text{Im}\,\Pi^{\text{had}}(s) = \frac{s}{4\pi\alpha}\,\sigma(\gamma^* \to \text{anything})$$

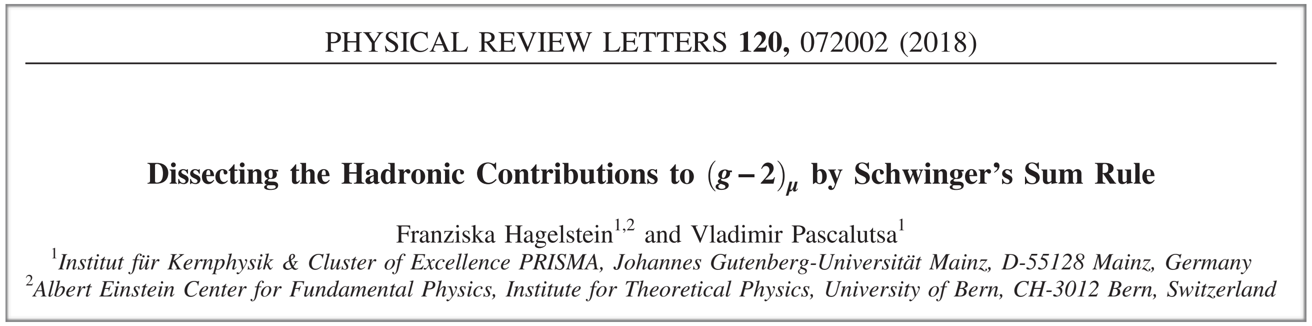
F. Jegerlehner, Springer Tracts Mod. Phys. 274 (2017).

M. Davier, Nucl. Part. Phys. Proc. 287-288, 70 (2017)

- HLbL is not as simple, data-driven, systematic

- Is there an exact dispersive formula which treats HVP and HLbL (and everything else) in the same way?

Outline of this talk



PHYSICAL REVIEW LETTERS **120,** 072002 (2018)

Dissecting the Hadronic Contributions to $(g-2)_\mu$ by Schwinger's Sum Rule

Franziska Hagelstein[1,2] and Vladimir Pascalutsa[1]

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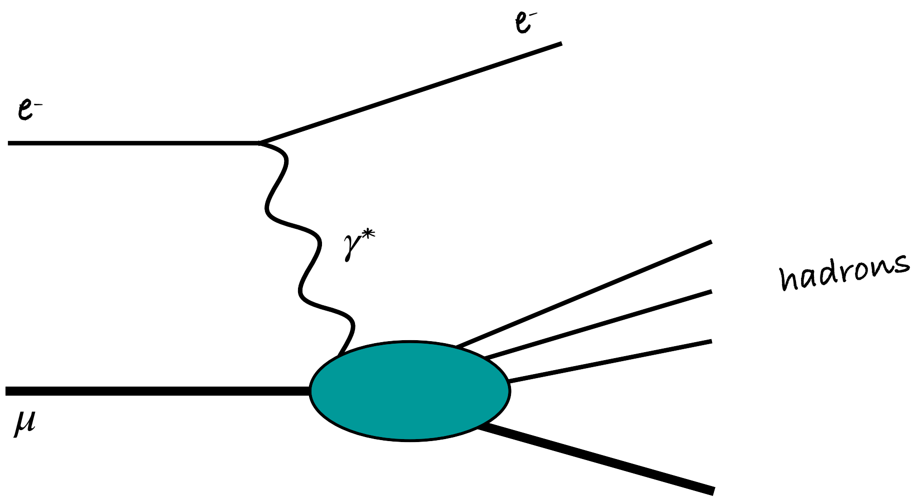
[2]*Albert Einstein Center for Fundamental Physics, Institute for Theoretical Physics, University of Bern, CH-3012 Bern, Switzerland*

$$a = \frac{m^2}{\pi^2\alpha}\int_{\nu_0}^{\infty} \mathrm{d}\nu \left[\frac{\sigma_{LT}(\nu,Q^2)}{Q}\right]_{Q^2=0}$$

$$= \lim_{Q^2\to 0}\frac{8m^2}{Q^2}\int_0^{x_0} \mathrm{d}x\, [\bar{g}_1+\bar{g}_2](x,Q^2)$$

- THE SCHWINGER SUM RULE:
- Reproducing $\alpha/2\pi$
- HADRONIC *VACUUM POLARIZATION* AND *LIGHT-BY-LIGHT* CONTRIBUTIONS ON THE SAME FOOTING
- PSEUDOSCALAR-MESON CONTRIBUTION
- *MUON STRUCTURE FUNCTIONS* FROM INELASTIC MUON-ELECTRON SCATTERING



Schwinger Sum Rule

J. S. Schwinger, Proc. Nat. Acad. Sci. 72, 1 (1975); ibid. 72, 1559 (1975) [Acta Phys. Austriaca Suppl. 14, 471 (1975)].
A. M. Harun ar-Rashid, Nuovo Cim. A 33, 447 (1976).

photon lab-frame energy ν
and virtuality $Q^2 = -q^2$

anomalous
magnetic moment
(a.m.m.)
$a_\mu = 1/2\,(g-2)_\mu$

muon mass m

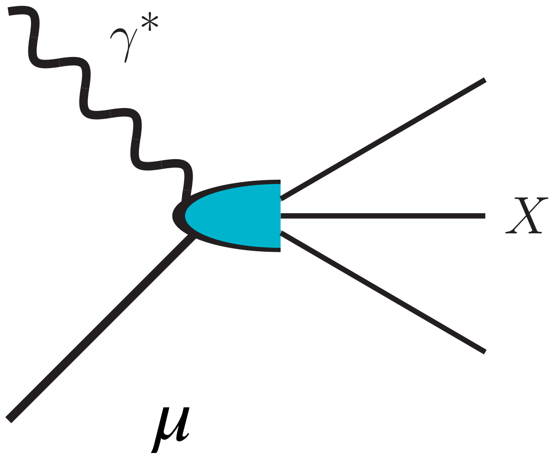
$$a = \frac{m^2}{\pi^2\alpha}\int_{\nu_0}^{\infty} d\nu \left[\frac{\sigma_{LT}(\nu, Q^2)}{Q}\right]_{Q^2=0}$$

fine-structure
constant $\alpha \approx 1/137$

photo-absorption threshold ν_0

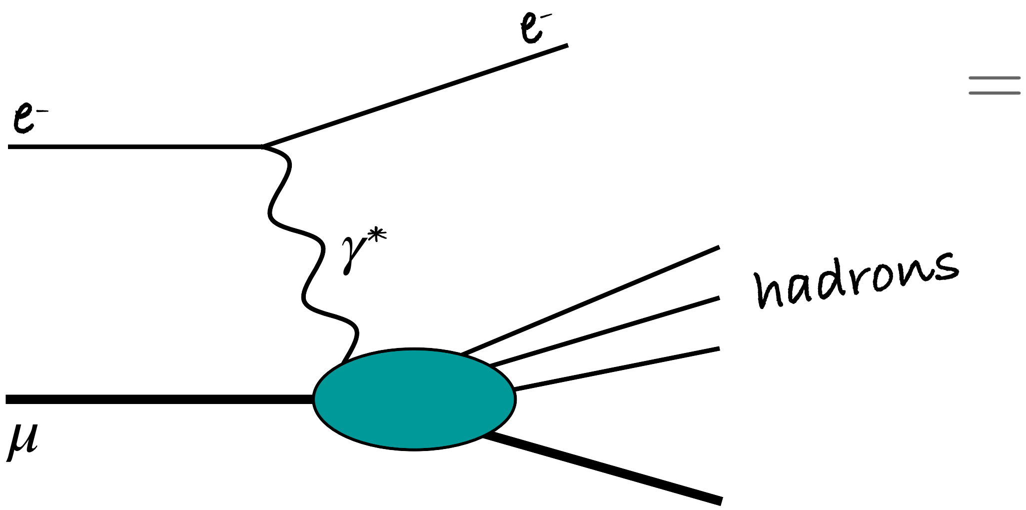
longitudinal-transverse
photo-absorption
cross section σ_{LT}

- photo-absorption on muon:



$= \gamma\mu,\ \gamma\gamma\mu,\ \pi^0\mu,\ \gamma\pi^0\mu,\ \ldots$

Spin structure functions



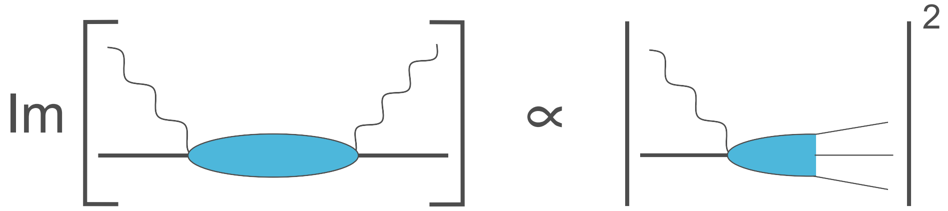
$$a = \frac{m^2}{\pi^2\alpha}\int_{\nu_0}^{\infty} \mathrm{d}\nu \left[\frac{\sigma_{LT}(\nu,Q^2)}{Q}\right]_{Q^2=0}$$

$$= \lim_{Q^2\to 0}\frac{8m^2}{Q^2}\int_0^{x_0} \mathrm{d}x \, [\bar{g}_1 + \bar{g}_2](x,Q^2)$$

muon spin structure functions
g_1 and g_2

- Spin-dependent forward doubly-virtual Compton scattering:

$$T_A^{\mu\nu}(q,p) = -\frac{1}{M}\gamma^{\mu\nu\alpha}q_\alpha\, S_1(\nu,Q^2) + \frac{Q^2}{M^2}\gamma^{\mu\nu}S_2(\nu,Q^2)$$



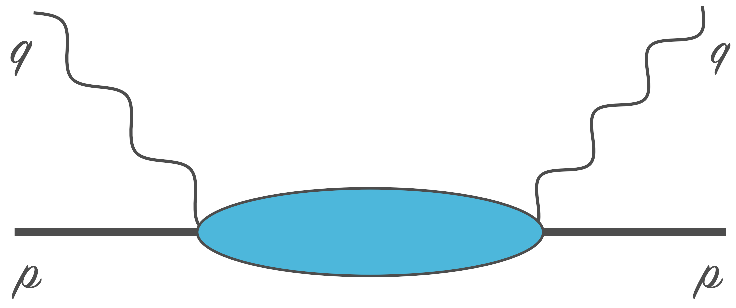
- Optical theorem:

$$\mathrm{Im}\, S_1(\nu,Q^2) = \frac{4\pi^2\alpha}{\nu}\, g_1(x,Q^2) = \frac{M\nu^2}{\nu^2+Q^2}\left[\frac{Q}{\nu}\sigma_{LT} + \sigma_{TT}\right](\nu,Q^2)$$

$$\mathrm{Im}\, S_2(\nu,Q^2) = \frac{4\pi^2\alpha M}{\nu^2}\, g_2(x,Q^2) = \frac{M^2\nu}{\nu^2+Q^2}\left[\frac{\nu}{Q}\sigma_{LT} - \sigma_{TT}\right](\nu,Q^2)$$

Origin

- Sum rules are model-independent relations based on general principles of:
  - Analyticity/causality (dispersion relations),
  - unitarity (optical theorem)
  - crossing symmetry



- Examples of sum rules include:

$$(1+a)\,a = \frac{m^2}{\pi^2\alpha}\int_{\nu_0}^{\infty} \mathrm{d}\nu \left[\frac{\sigma_{LT}}{Q} - \frac{\sigma_{TT}}{\nu}\right]_{Q^2=0}$$

Burkhardt—Cottingham sum rule (1970) $\int_0^1 \mathrm{d}x\, g_2(x,Q^2) = 0$

$$\ominus \qquad a^2 = -\frac{m^2}{\pi^2\alpha}\int_{\nu_0}^{\infty} \mathrm{d}\nu\, \frac{\sigma_{TT}(\nu)}{\nu}$$

Gerasimov—Drell—Hearn sum rule (1966)

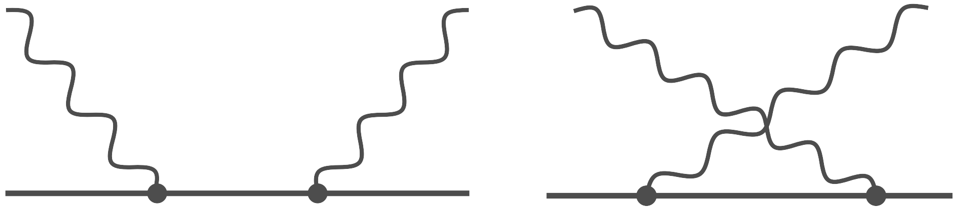
$$a = \frac{m^2}{\pi^2\alpha}\int_{\nu_0}^{\infty} \mathrm{d}\nu \left[\frac{\sigma_{LT}(\nu,Q^2)}{Q}\right]_{Q^2=0}$$

Schwinger sum rule (1975)

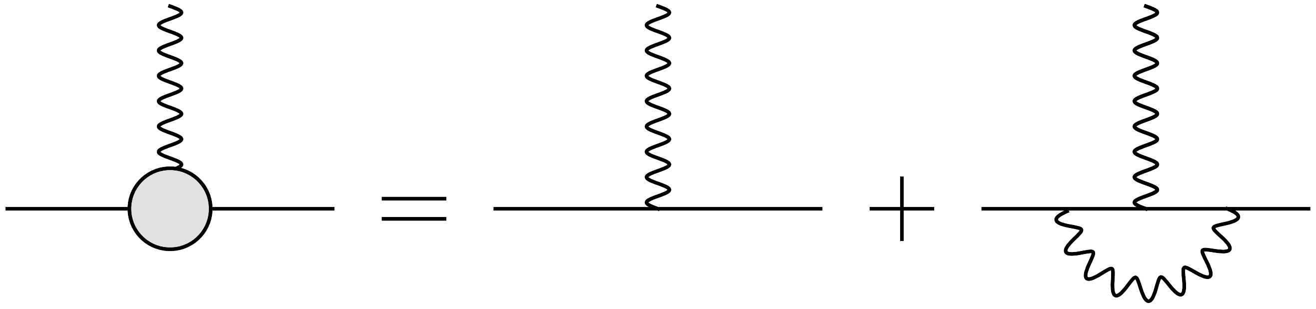
Reproducing the leading QED result

- Schwinger sum rule: $a = \frac{m^2}{\pi^2 \alpha} \int_{\nu_0}^{\infty} \mathrm{d}\nu \left[\frac{\sigma_{LT}(\nu, Q^2)}{Q} \right]_{Q^2=0}$
- Input: longitudinal-transverse photo-absorption cross section

tree-level QED
Compton scattering



$$\sigma_{LT}^{\gamma^* \mu \to \gamma \mu}(\nu, Q^2) = \frac{\pi \alpha^2 Q \,(s - m^2)^2}{4m^3 \nu^2 \left(\nu^2 + Q^2\right)} \left(-2 - \frac{m(m+\nu)}{s} + \frac{3m + 2\nu}{\sqrt{\nu^2 + Q^2}} \operatorname{arccoth} \frac{m+\nu}{\sqrt{\nu^2 + Q^2}} \right)$$

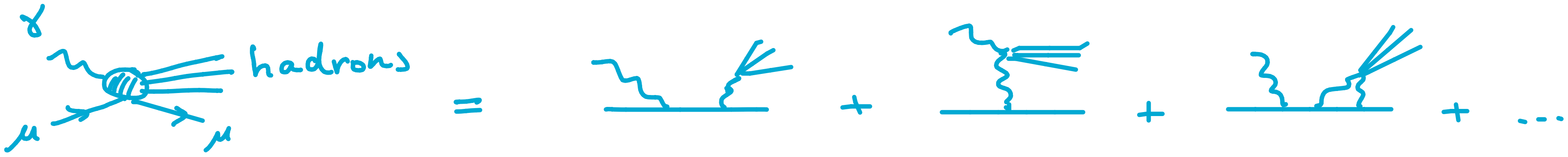


$F_2(0) = a$ $\quad a^{(0)} = 0$ $\quad a^{(1)} = \alpha / 2\pi$

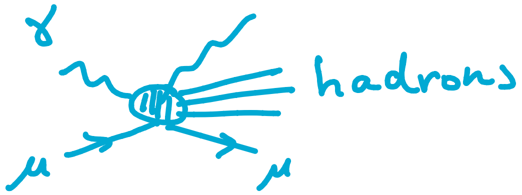
Hadronic Contributions:

4 channels to order α^3

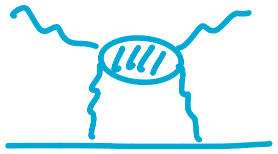
$\gamma\mu \to \mu + \text{hadrons}$



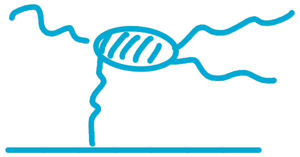
$\gamma\mu \to \gamma\mu + \text{hadrons}$



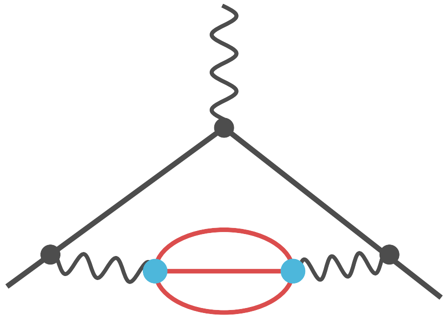
$\gamma\mu \to \gamma\mu$

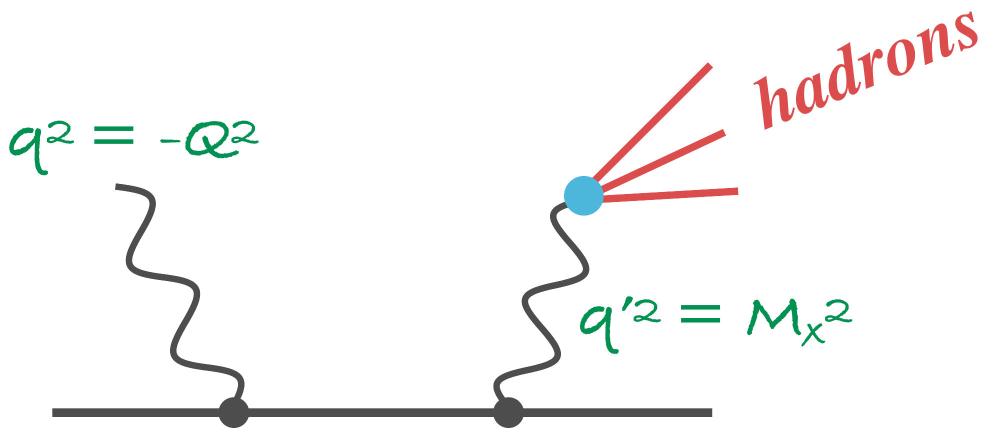


$\gamma\mu \to \gamma\gamma\mu$



Reproducing the HVP formula





+ crossed diagram

- Cross section of hadron production through timelike Compton scattering:

factorizes as:

$$\sigma(\gamma\mu \to \mu X) = \frac{1}{\pi}\int_{4m_\pi^2}^{\infty} \frac{\mathrm{d}M_X^2}{M_X^2}\, \sigma(\gamma\mu \to \gamma^*\mu)\, \mathrm{Im}\,\Pi_X(M_X^2)$$

timelike Compton scattering

virtual-photon decay into hadrons

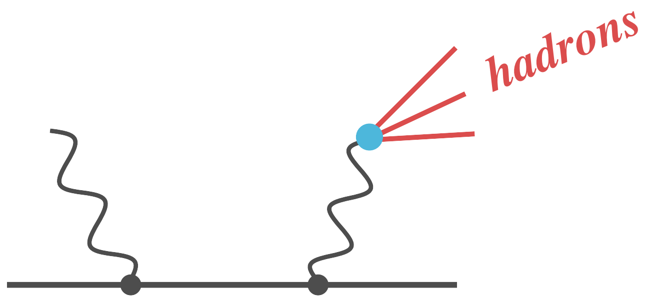
- Timelike Compton scattering cross section:

$$\boxed{\left[\frac{\sigma_{LT}^{\gamma\mu\to\gamma^*\mu}(\nu,Q^2)}{Q}\right]_{Q^2=0} = \frac{\pi\alpha^2}{2m^2\nu^3}\left[-(5s+m^2+M_X^2)\lambda + (s+2m^2-2M_X^2)\log\frac{\beta+\lambda}{\beta-\lambda}\right]}$$

$$\beta = (s+m^2-M_X^2)/2s \qquad s = m^2+2m\nu$$
$$\lambda = (1/2s)\sqrt{[s-(m+M_X)^2]\,[s-(m-M_X)^2]}$$

HVP from Schwinger sum rule

$$\boxed{a = \frac{m^2}{\pi^2\alpha}\int_{4m_\pi^2}^{\infty} \mathrm{d}M_X^2 \int_{\nu_0}^{\infty}\mathrm{d}\nu \left[\frac{1}{Q}\,\frac{\mathrm{d}\sigma_{LT}^{\gamma\mu\to\mu X}(\nu,Q^2)}{\mathrm{d}M_X^2}\right]_{Q^2=0}}$$

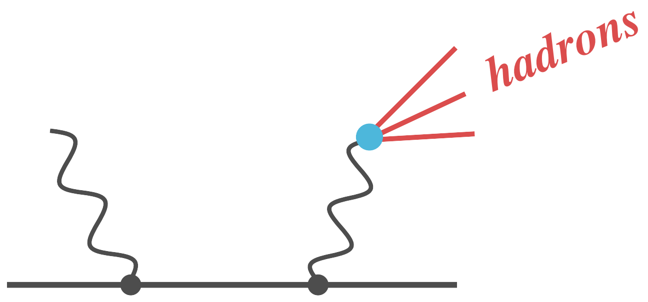


$$= \frac{1}{\pi}\int_{4m_\pi^2}^{\infty}\mathrm{d}M_X^2\,\frac{\mathrm{Im}\,\Pi^{\mathrm{had}}(M_X^2)}{M_X^2}\,\underbrace{\frac{m^2}{\pi^2\alpha}\int_{\nu_0}^{\infty}\mathrm{d}\nu\left[\frac{\sigma_{LT}^{\gamma\mu\to\gamma^*\mu}(\nu,Q^2)}{Q}\right]_{Q^2=0}}$$

kernel function: $= \frac{\alpha}{\pi}K(M_X^2/m^2) \equiv \frac{\alpha}{\pi}\int_0^1 \mathrm{d}x\,\frac{x^2(1-x)}{x^2+(1-x)(M_X^2/m^2)}$

for $M_X=0$, we find $K(0)=1/2$, and therefore
the Schwinger term: $a^{(1)} = \alpha/2\pi$

HVP from Schwinger sum rule



$$\boxed{a = \frac{m^2}{\pi^2\alpha}\int\limits_{4m_\pi^2}^{\infty} \mathrm{d}M_X^2 \int\limits_{\nu_0}^{\infty} \mathrm{d}\nu \left[\frac{1}{Q}\frac{\mathrm{d}\sigma_{LT}^{\gamma\mu\to\mu X}(\nu,Q^2)}{\mathrm{d}M_X^2}\right]_{Q^2=0}}$$

$$= \frac{1}{\pi}\int\limits_{4m_\pi^2}^{\infty} \mathrm{d}M_X^2 \frac{\mathrm{Im}\,\Pi^{\mathrm{had}}(M_X^2)}{M_X^2} \underbrace{\frac{m^2}{\pi^2\alpha}\int\limits_{\nu_0}^{\infty}\mathrm{d}\nu \left[\frac{\sigma_{LT}^{\gamma\mu\to\gamma^*\mu}(\nu,Q^2)}{Q}\right]_{Q^2=0}}$$

kernel function: $\uparrow$

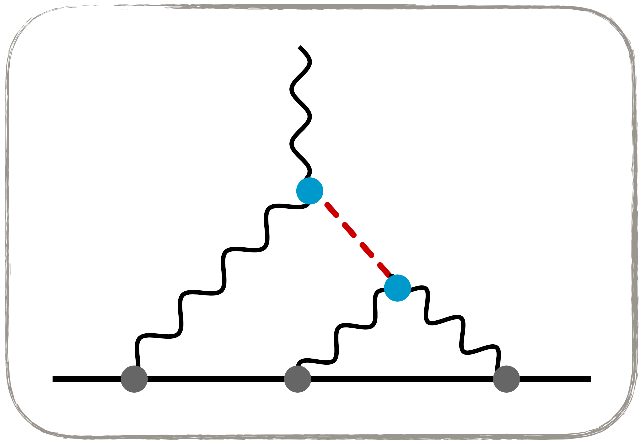
$$= \frac{\alpha}{\pi}K(M_X^2/m^2) \equiv \frac{\alpha}{\pi}\int_0^1 \mathrm{d}x \frac{x^2(1-x)}{x^2+(1-x)(M_X^2/m^2)}$$

for $M_X=0$, we find $K(0)=1/2$, and therefore the Schwinger term: $a^{(1)} = \alpha/2\pi$

- reproduces the HVP standard formula

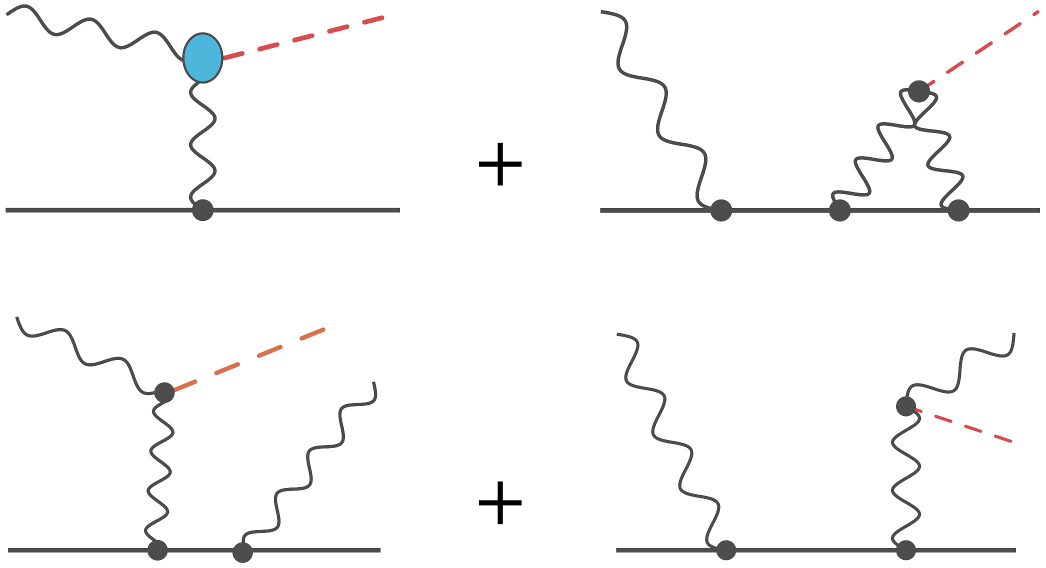
$$\boxed{a^{\mathrm{HVP}} = \frac{\alpha}{\pi^2}\int_{4m_\pi^2}^{\infty}\frac{\mathrm{d}s}{s}\,\mathrm{Im}\,\Pi^{\mathrm{had}}(s)\int_0^1 \mathrm{d}x \frac{x^2(1-x)}{x^2+(1-x)(s/m^2)}}$$

Light-by-Light contributions

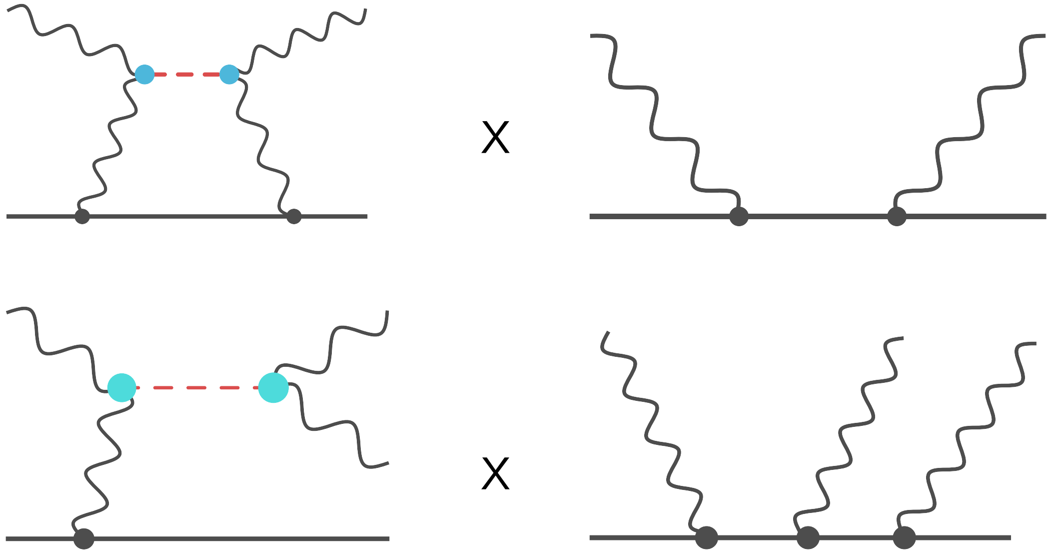


(pseudo-)scalar contribution

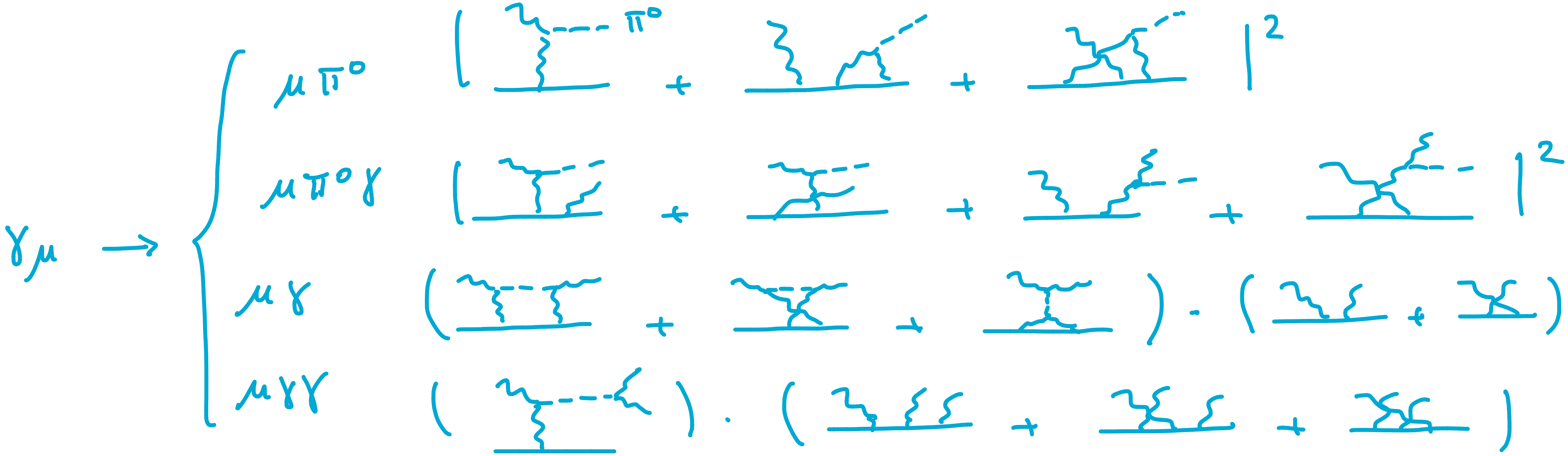
I. Hadron photo-production channels



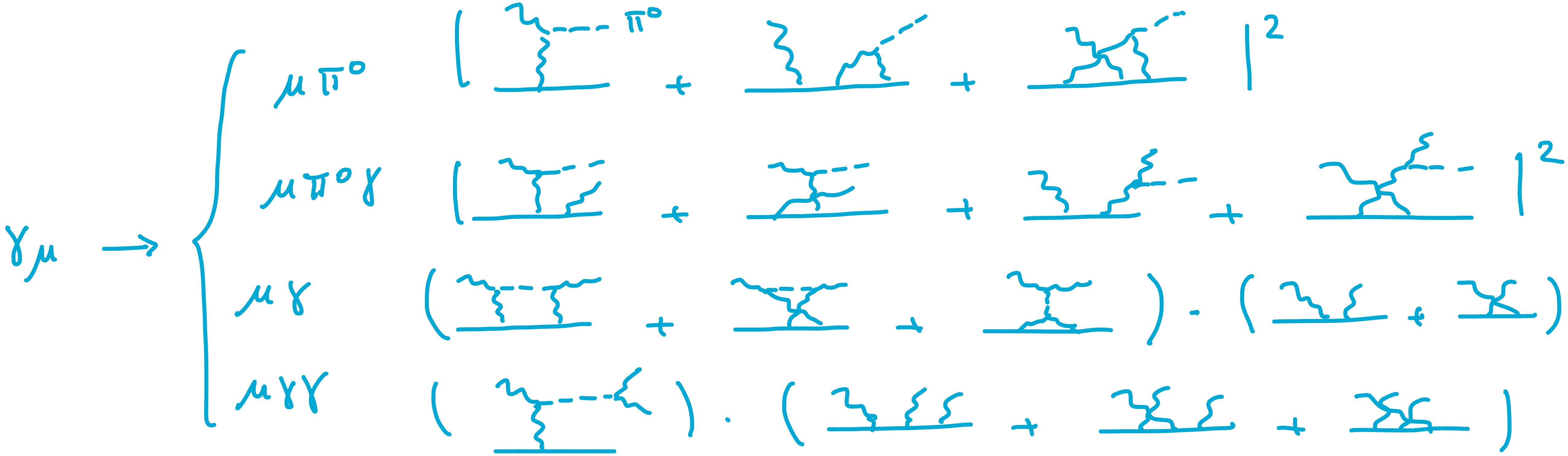
II. Electromagnetic channels



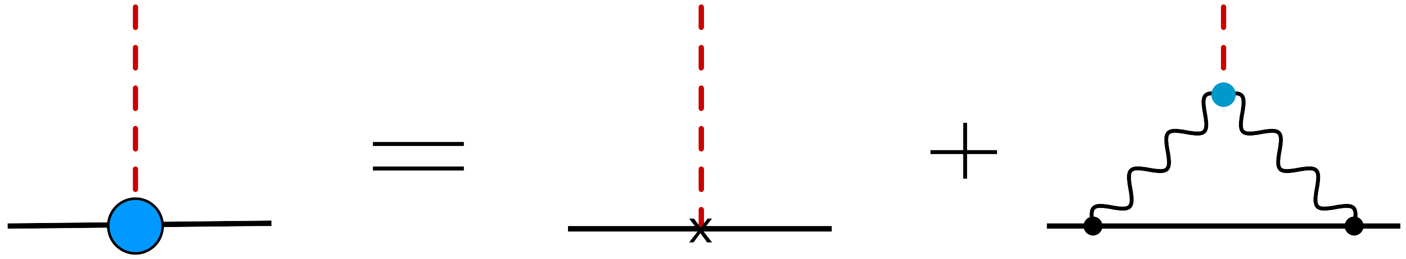
Pseudo-scalar contribution in full glory



Pseudo-scalar contribution in full glory



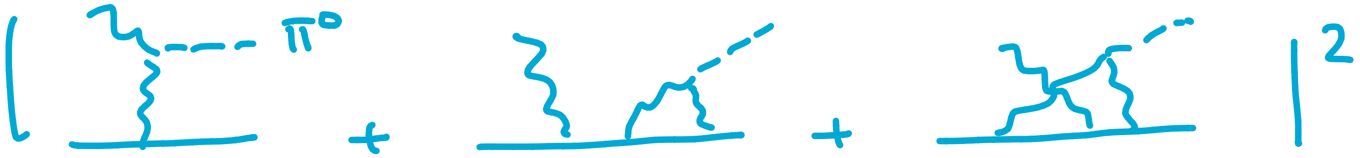
- **No doubly-virtual transition form factors needed, if hadronic channels are measured**

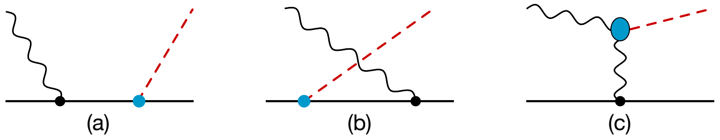


Pseudoscalar meson coupling to leptons.

Preliminary number for neutral-pion contribution

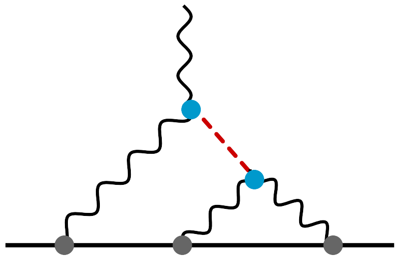
$\gamma\mu \to \mu\pi^0$





$11.9(9) \times 10^{-10}$ [Hagelstein et al., in prep.] (one out of four channels!)

Compare with the full pion contribution:



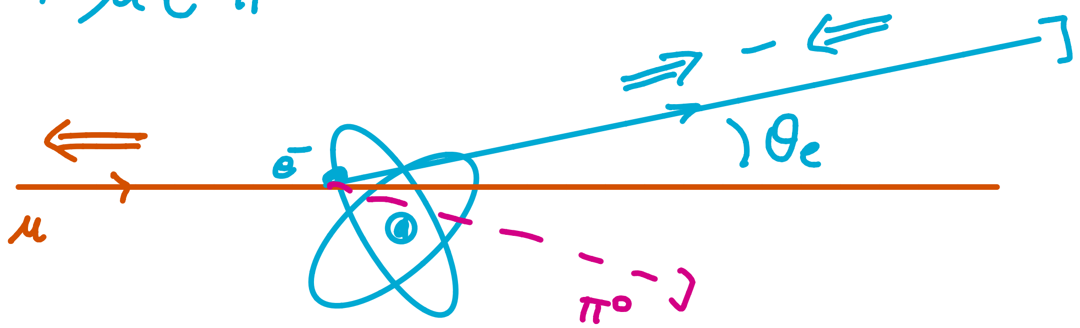
$(6 \pm 1) \times 10^{-10}$ [Knecht and Nyffeler (2002, 2009)]

7.7×10^{-10} [Melnikov and Vainshtein (2004)]

Feasibility of measurement at COMPASS as part of MUonE ?

cf. The Workshop on
Evaluation of the Leading Hadronic Contribution
to the Muon Anomalous Magnetic Moment
Mainz (Germany), 2 - 5 April 2017

$\mu e \to \mu e \pi^0$



$E_\mu = 150, 200$ GeV

$E_e' \simeq 1$ GeV

$\theta_e \simeq 10$ mrad

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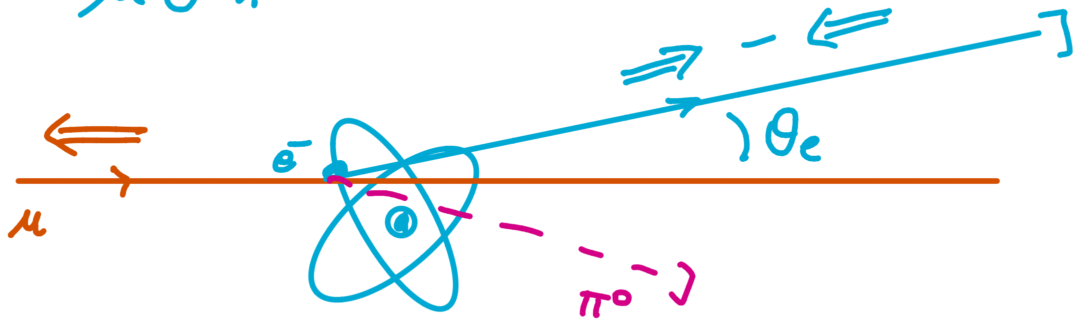

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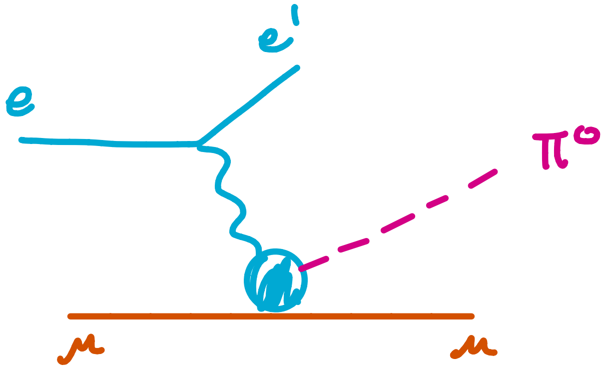
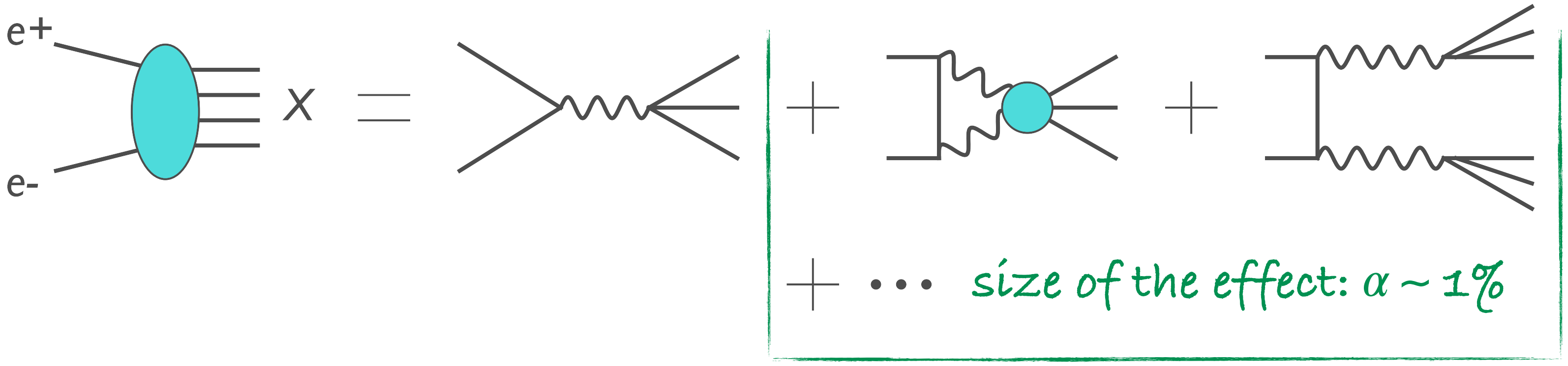


$$Q^2 \simeq 2 m_e E_e' \simeq 10^{-3}\,\mathrm{GeV}^2$$

$$\nu \simeq \frac{m_e E_\mu}{m_\mu}\left(1 - 2\frac{E_e'}{m_e}\sin^2\frac{\theta}{2}\right) = (\nu_{\pi^0},\ 1\,\mathrm{GeV})$$

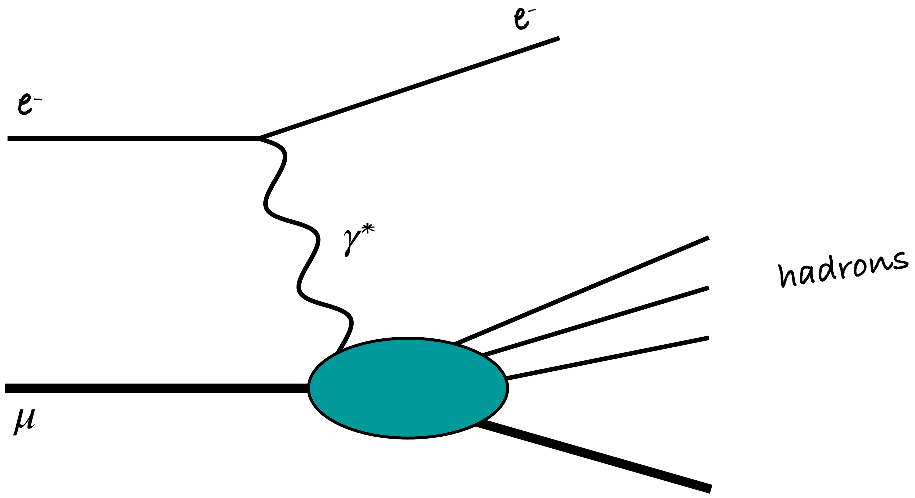
$$\nu_{\pi^0} = \frac{m_{\pi^0}}{m_\mu}\left(\tfrac{1}{2}m_{\pi^0} + m_\mu\right) \simeq 230\ \mathrm{MeV}$$

Possible refinements of the HVP



size of the effect: $\alpha \sim 1\%$

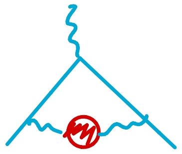
vs.



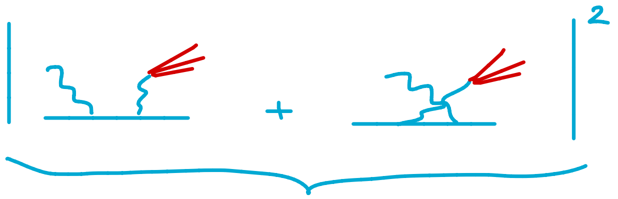
Summary and Conclusions

1. Schwinger sum rule — dispersive formula applying equally to HVP and HLbL

2. Reproduces $\alpha/2\pi$ and HVP formula:



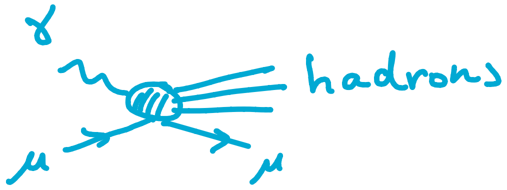
$$= \frac{m_\mu^2}{\alpha\,\pi^2}\int d\nu$$



$$a_\mu^{HVP} = \frac{m_\mu^2}{\alpha\,\pi^2}\int d\nu \int dM_X^2\; \sigma_{LT}(\gamma\mu \to \gamma_X^*\mu)\;\Gamma(\gamma_X^* \to \text{hadrons})$$

3. Splits contributions into hadron production and e.m. (LbL) channels

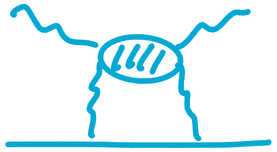
$\gamma\mu \to \mu$+hadrons



measurable

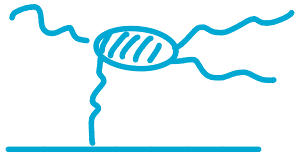
spin structure functions

$\gamma\mu \to \gamma\mu$



LQCD ?

$\gamma\mu \to \gamma\gamma\mu$



direct LbL scattering (ATLAS)

Summary and Conclusions

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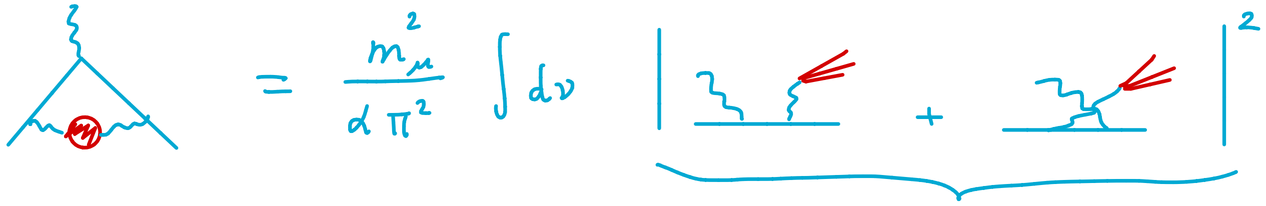

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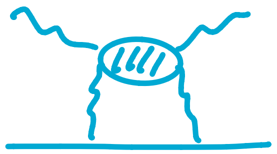

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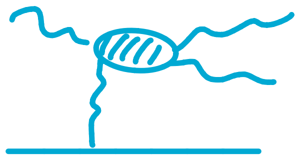


direct LbL scattering (ATLAS)

4. Partial calculation of pi0 contribution is a factor of 2 larger than the conventional model calculations.

to be continued…

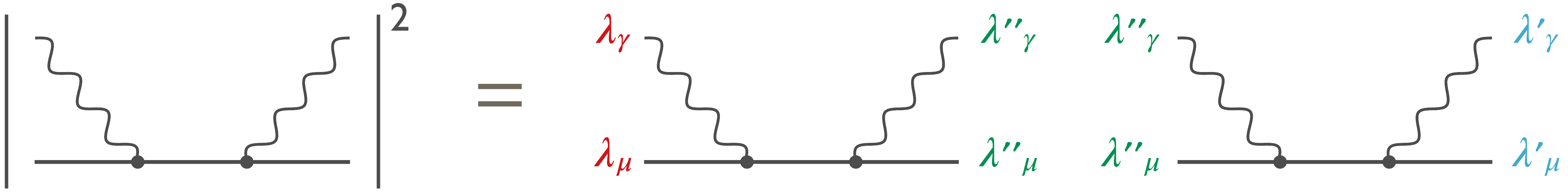
Backup slides

The Cross section σ_{LT}

- Example: tree-level QED Compton scattering cross section

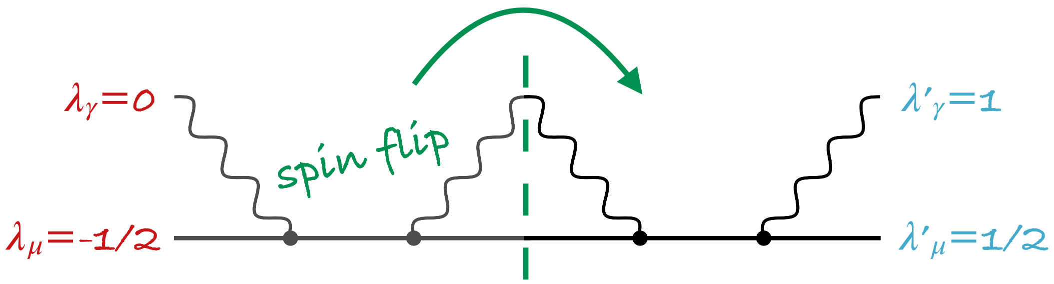
$$d\sigma_{\lambda'_\gamma \lambda'_\mu \lambda_\gamma \lambda_\mu} = (2\pi)^4 \delta^{(4)}(p_f - p_i) \sum_{\lambda''_\gamma, \lambda''_\mu} \frac{\mathcal{M}^\dagger_{\lambda'_\gamma \lambda'_\mu \lambda''_\gamma \lambda''_\mu} \mathcal{M}_{\lambda''_\gamma \lambda''_\mu \lambda_\gamma \lambda_\mu}}{4I} \prod_a \frac{d^3 p'_a}{(2\pi)^3 2E'_a},$$

with conserved helicity: $H = \lambda'_\gamma - \lambda'_\mu = \lambda_\gamma - \lambda_\mu$

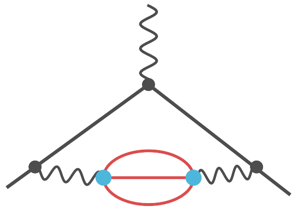


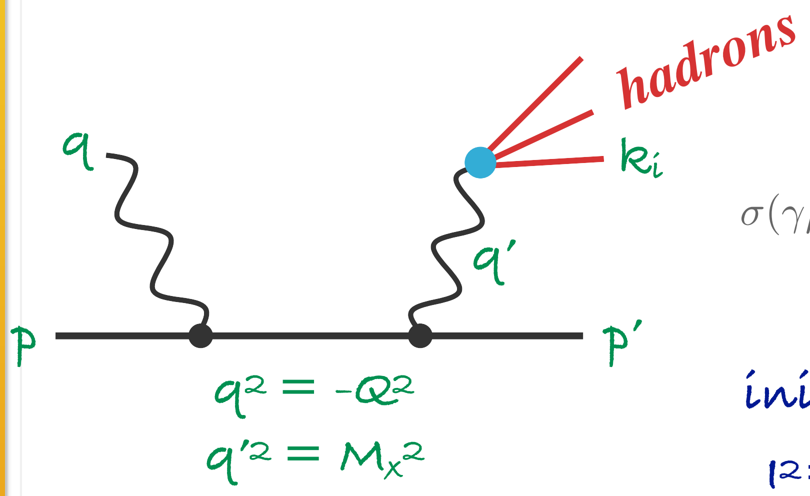
- helicity difference photo-absorption cross section: $\boxed{\sigma_{TT} = {}^1/_2\,(\sigma_{1/2} - \sigma_{3/2})}$
- longitudinal-transverse photo-absorption cross section:

$\gamma^*(\lambda_\gamma=0) + \mu(\lambda_\mu=-1/2) \to \gamma(\lambda'_\gamma=1) + \mu(\lambda'_\mu=1/2)$



Timelike CS mechanism





+ crossed diagram

Λ^ν: virtual-photon decay vertex

$$\sigma(\gamma\mu\to\mu X) = \frac{(2\pi)^4}{4I}\int \mathrm{d}^4q' \underbrace{\int\prod_i \frac{\mathrm{d}^3\boldsymbol{k}_i}{2E_{k_i}(2\pi)^3}\int\frac{\mathrm{d}^3\boldsymbol{p'}}{2E_{p'}(2\pi)^3}}\left[\frac{\Lambda^{\dagger\mu}\Lambda^\nu\rho_{\mu\nu}}{(-q'^2)^2}\right]\delta^4(q'-\sum_i k_i)\,\delta^4(p+q-p'-q')$$

initial flux factor

$I^2 = (p\cdot q)^2 - p^2 q^2$

phase space of the final state

$\rho_{\mu\nu}$: squared matrix element of timelike CS

- Virtual-photon decay width into hadronic state X:

$$[\Gamma(\gamma^*\to X)]^{\mu\nu} = \int\prod_i \frac{\mathrm{d}^3\boldsymbol{k_i}}{2E_{k_i}(2\pi)^3}\,\frac{\Lambda^{\dagger\mu}\Lambda^\nu}{2E_{q'}}(2\pi)^4\delta^4(q'-\sum_i k_i)$$

$$= -\frac{1}{\sqrt{q'^2}}\,(q'^2 g^{\mu\nu} - q'^\mu q'^\nu)\,\mathrm{Im}\,\Pi_X(q'^2)$$

$\mathrm{Im}\,\Pi_X$: contribution of state X to the VP

- Combine into: $\sigma(\gamma\mu\to\mu X) = -\frac{1}{2I}\int \mathrm{d}^4q'\int\frac{\mathrm{d}^3\boldsymbol{p'}}{2E_{p'}(2\pi)^3}\,\rho^\mu_\mu\,\frac{\mathrm{Im}\,\Pi_X(q'^2)}{q'^2}\,\delta^4(p+q-p'-q')$

- Final factorized cross section: $\boxed{\sigma(\gamma\mu\to\mu X) = \frac{1}{\pi}\int_{4m_\pi^2}^\infty\frac{\mathrm{d}M_X^2}{M_X^2}\,\sigma(\gamma\mu\to\gamma^*\mu)\,\mathrm{Im}\,\Pi_X(M_X^2)}$