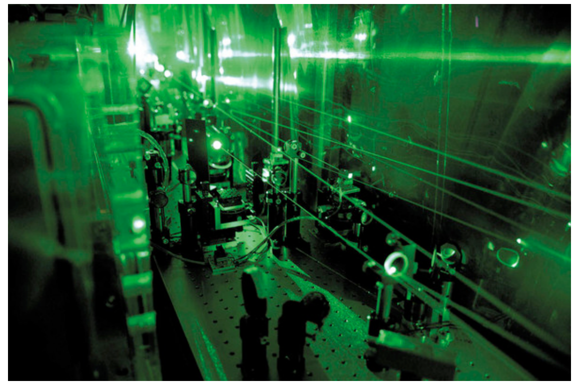
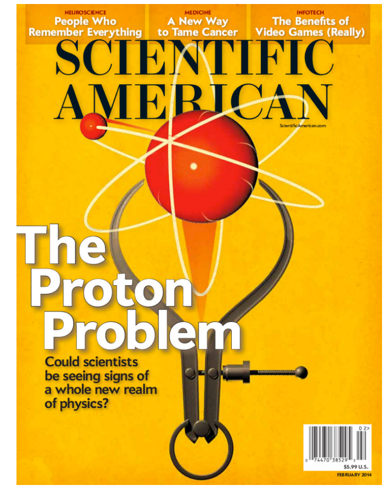
The Proton Radius Puzzle- Why we all should care

Gerald A. Miller, University of Washington

Feb. 2014

Pohl et al Nature 466, 213 (8 July 2010)





4 % Difference

$$r_p^2 \equiv -6 \frac{dG_E(Q^2)}{dQ^2}\bigg|_{Q^2=0}$$

Pohl, Gilman, Miller, Pachucki (ARNPS63, 2013)

C Carlson PPNP 82,59(2015)

muon H r_p =0.84184 (67) fm Small

electron H r_p =0.8768 (69)fm Large

~~electron-p scattering r_p =0.875 (10)fm~~

PRad at JLab- lower Q^2

C. Weiss

Weizhi Xiong

$$3 \times 10^4 \leq Q^2 \leq 5 \times 10^{-2}\ \mathrm{GeV}^2$$

$$7.7 \times 10^{-3} \leq Q^2 \leq 0.13\,\mathrm{fm}^{-2}$$

4 % in radius: why care?

- Can't be calculated to that accuracy?
  Sergey Syritsyn

Is the muon-proton interaction the same as the electron-proton interaction?
violation of universality
connections with muon g-2?
connections with LHCb ?

Something happening here ???

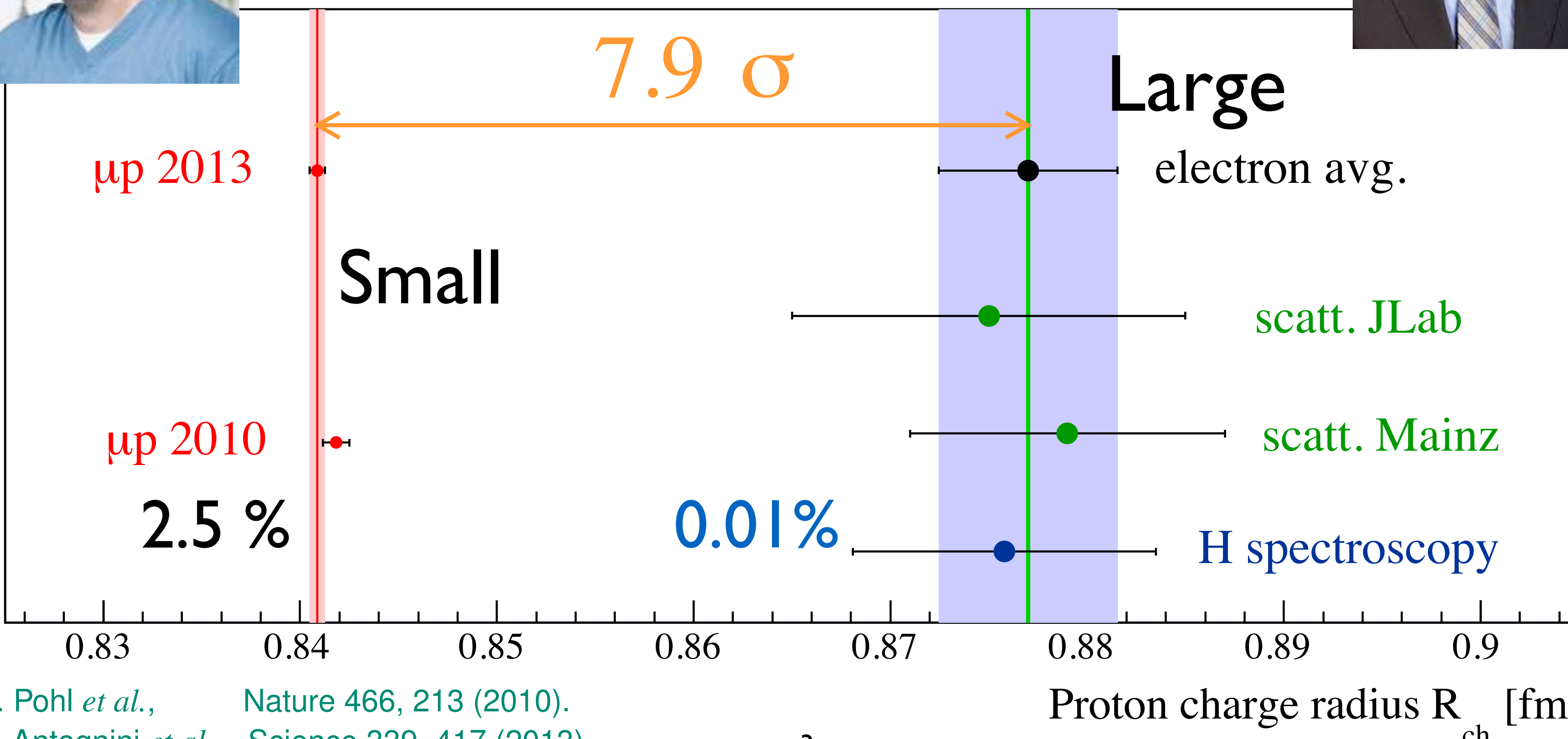
Outline - a) review history experiments
b) List & explain possible resolutions

The proton radius puzzle In a picture

The proton rms charge radius measured with

electrons: 0.8770 ± 0.0045 fm

muons: 0.8409 ± 0.0004 fm



R. Pohl *et al.*, Nature 466, 213 (2010).
A. Antognini *et al.*, Science 339, 417 (2013).

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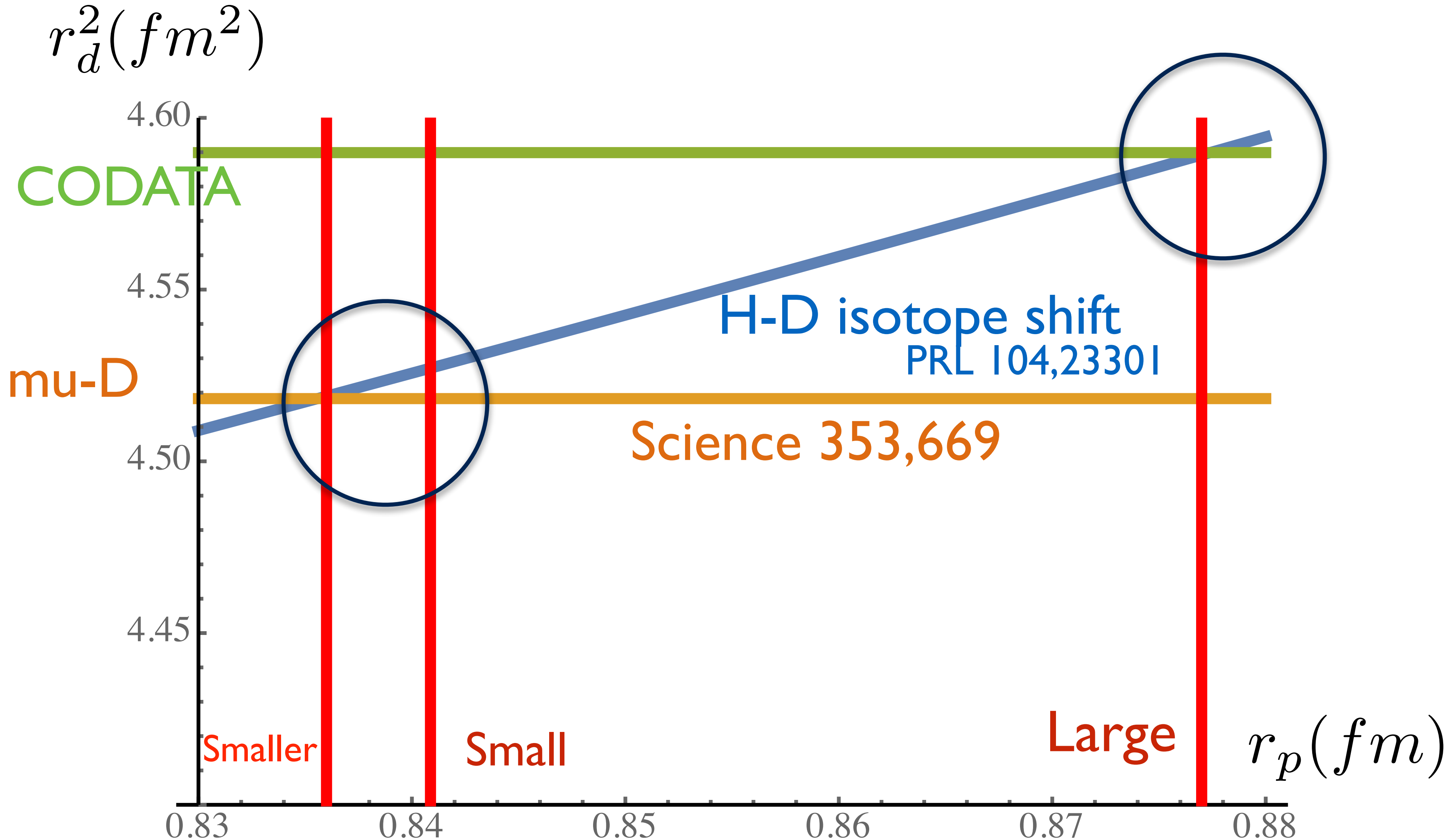
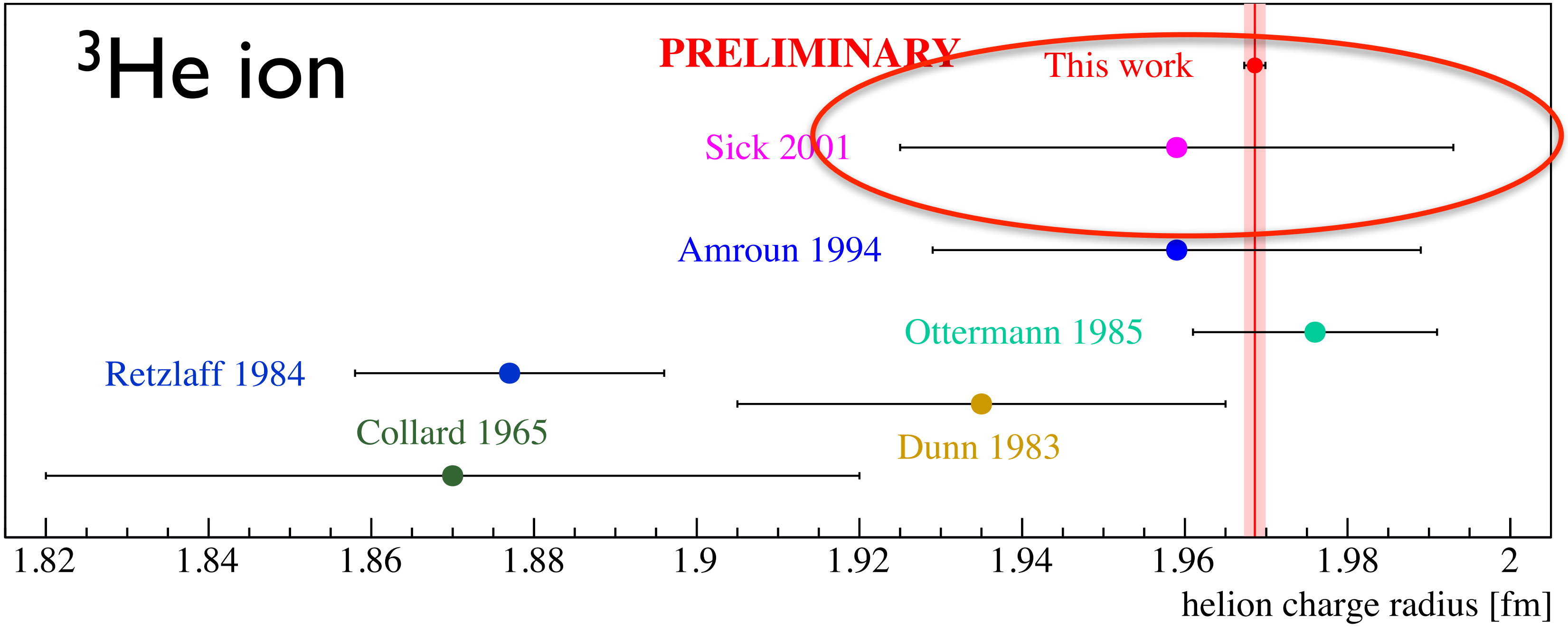
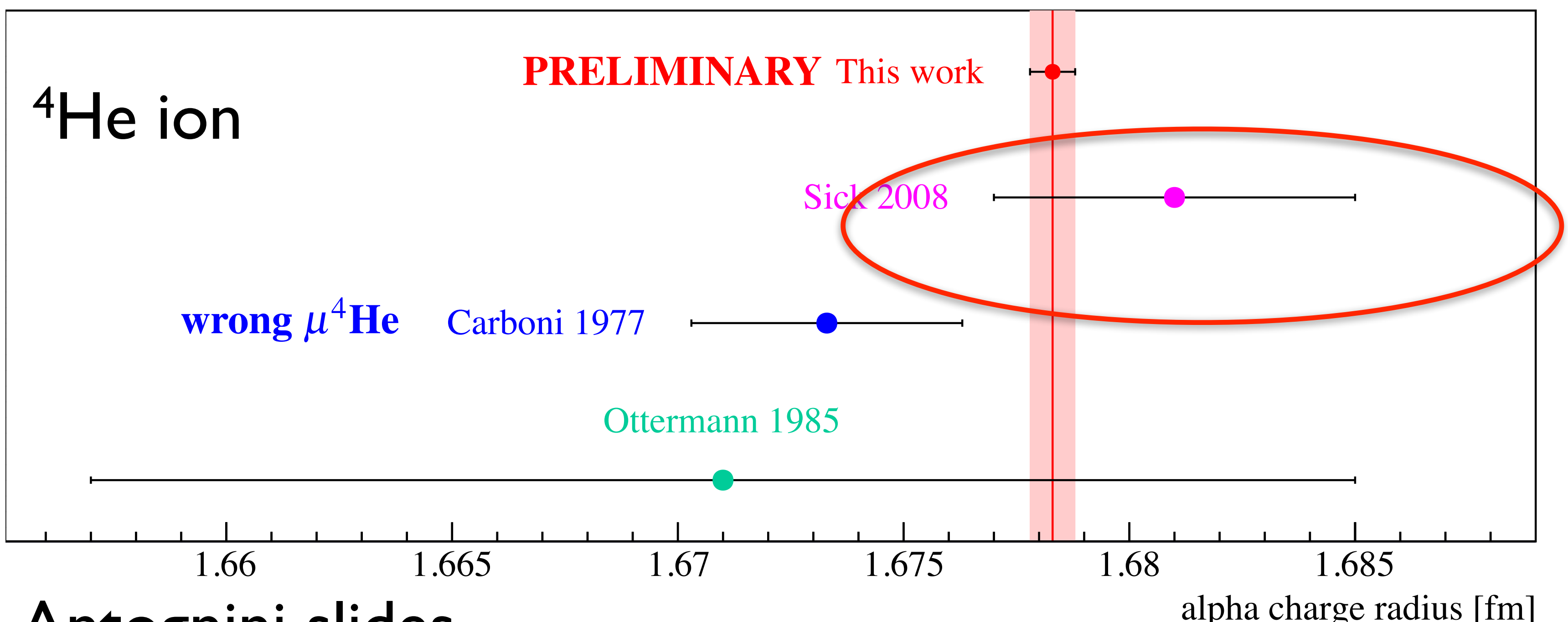
Deuteron is smaller too







A. Antognini slides

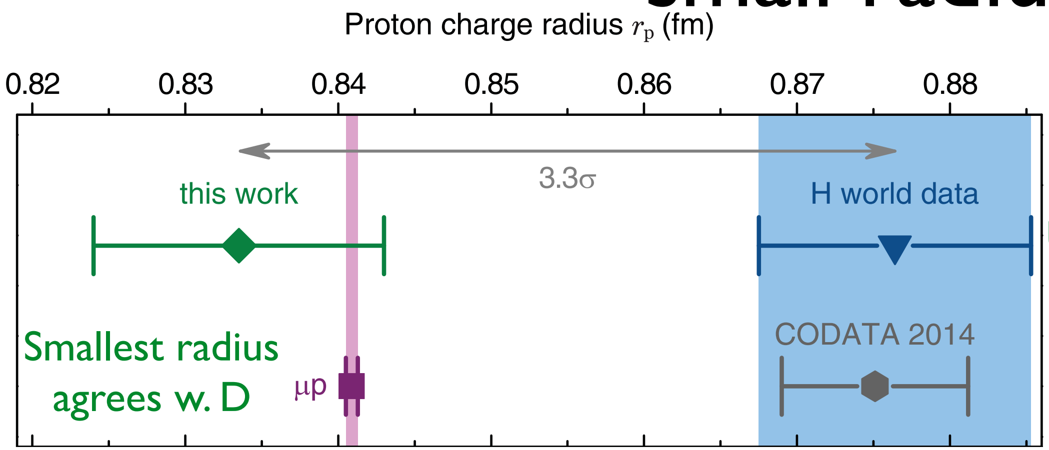
nuclear structure Barnea et al

So far

- difference between muon and electron hydrogen spectroscopy re the proton radius
- similar effect in the deuteron, implies effect on neutron
- no effect in 3He (large error bars)
- no effect in 4He (small error bars)
- any explanation must account for the above, assumes there is a difference between muon and electron hydrogen spectroscopy

Beyer et al Science 358,79 electron H - small radius

Lothar Maisenbacher
2S-4P transition



Smallest radius agrees w. D

take 2017 result -no puzzle

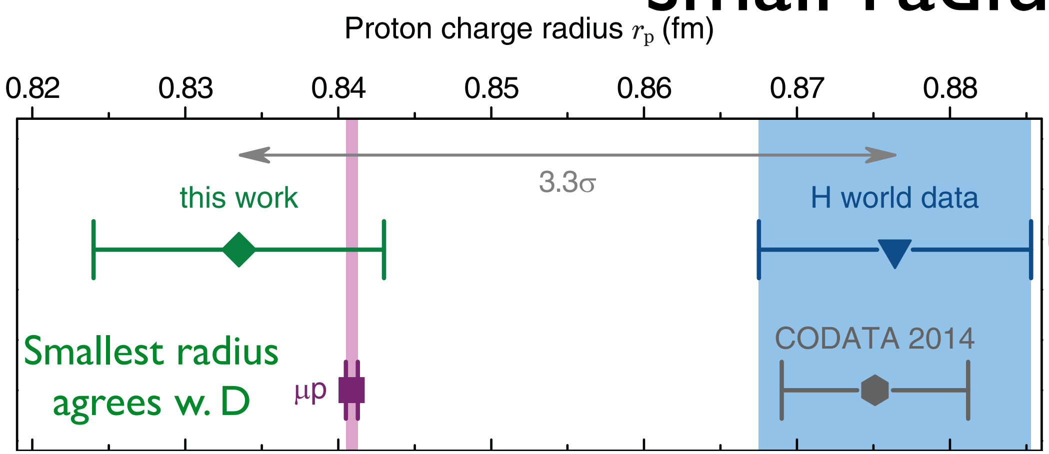
use old eH- puzzle as before

one more new eH needed

go halfway

Beyer et al Science 358,79 electron H - small radius

Lothar Maisenbacher
2S-4P transition

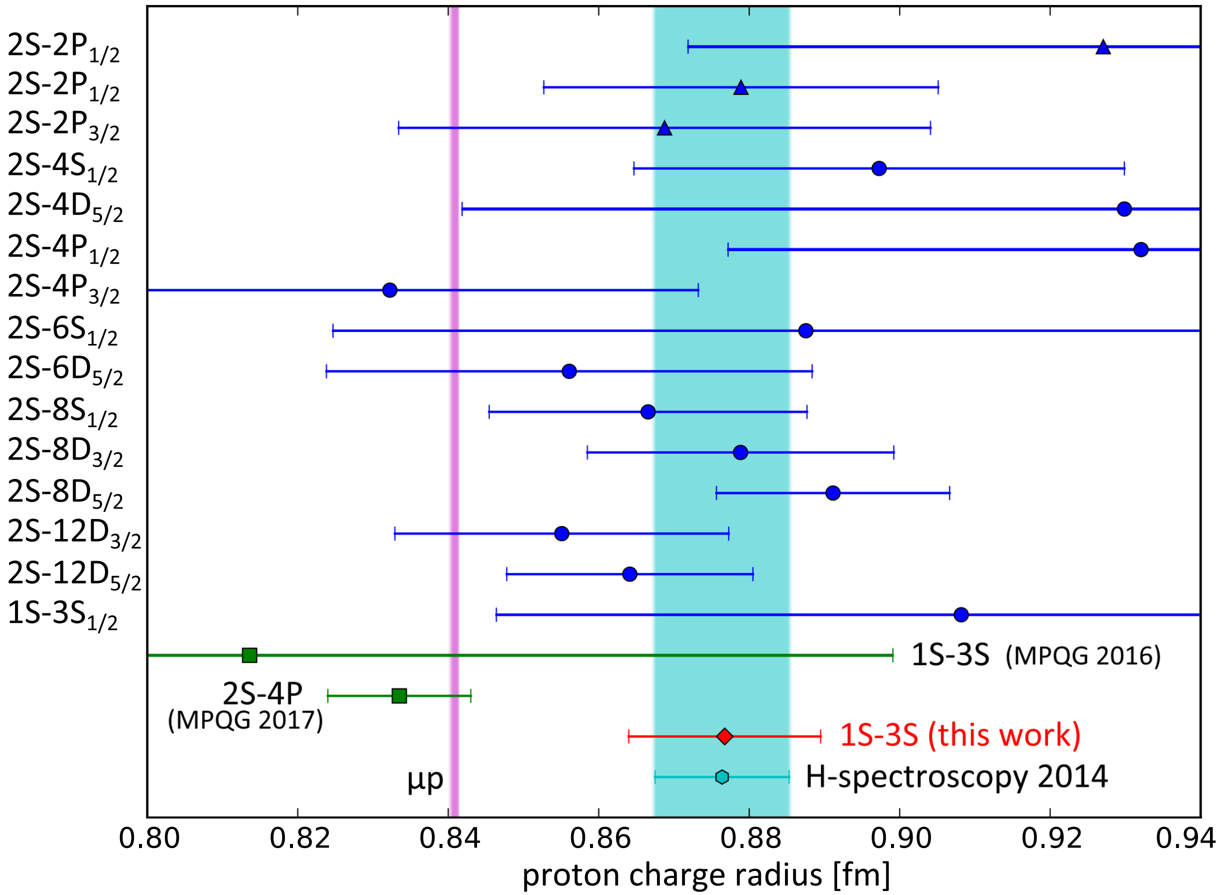


take 2017 result -no puzzle
use old eH- puzzle as before
one more new eH needed
go halfway

26 Jan., 2018 1801.08816 Paris

1S-3S hydrogen

Phys. Rev. Lett. 120, 183001 (2018)



Is there still a proton radius puzzle?

Is electron-hydrogen spectroscopy accurate enough?

Possible resolutions

- Electron H spectroscopy not so accurate
- Strong interaction effect in two photon exchange diagram
- Muon interacts differently than electron!- new scalar boson

The last two resolutions could be halved and not be in conflict with data

Shift from history to our efforts to explain

Suppose radii extracted in earlier H experiments is correct, some new muon effect is responsible

What energy difference corresponds to 4% in radius?

Measured =206.2949(32)= 206.0573(45)-5.2262 r_p^2+0.0347 r_p^3 meV

computed

Explain puzzle with radius as in earlier H atom measures: increase 206.0573 meV by 0.31 meV-attractive effect on 2S state

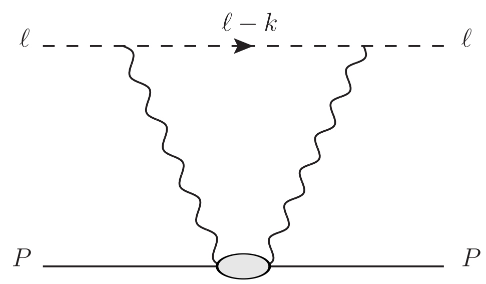
Can go half way and not disagree with data

Two photon exchange

Measured =206.2949(32)= 206.0573(45)-5.2262 r_p^2+0.0347 r_p^3 meV

computed

Miller PLB 2012



energy shift proportional to lepton mass^4

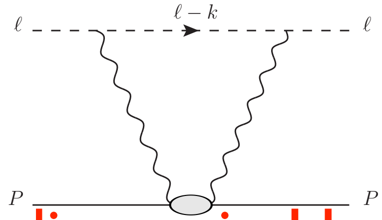
Re part of virtual Compton scattering

Im part is measured

use dispersion relations

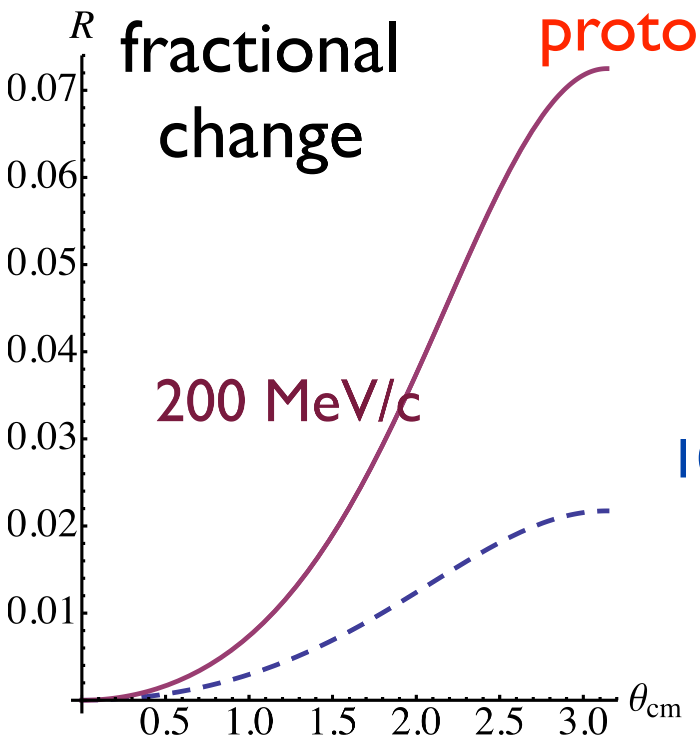
but unknown subtraction function is needed

Can account for 0.31 meV, no conflict with e-H



energy shift proportional to lepton mass^4

Explain puzzle with radius as in H atom increase 206.0573 meV by 0.31 meV-attractive effect on 2S state, reproduce Lamb shift in proton, deuterium



10-20% effect in scattering

100 MeV/c

Radians

fails in 4He

$$\delta E_L^{\mu A} \propto Z^3 (Z \delta E_L^p + N \delta E_L^n)$$

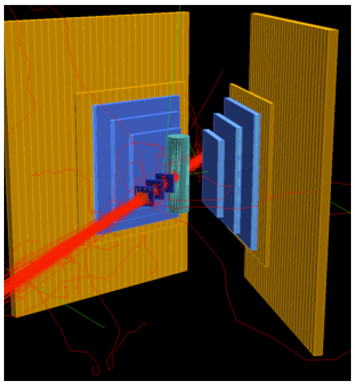
4 standard deviations off ^{4}He

or 1 st dev

e^+/e^- and μ^+/μ^- scattering on proton

So what? MUSE expt

http://www.physics.rutgers.edu/~rgilman/elasticmup/



PSI proposal R-12-01.1

Paul Reimer

- constrains two photon effect, which still survives at significant level
- if large radius correct and no two photon all leptons see large radius
- if small radius correct and no two photon all leptons see small radius
- will not see a new light particle, but all leptons see large radius

muon anomalous moment

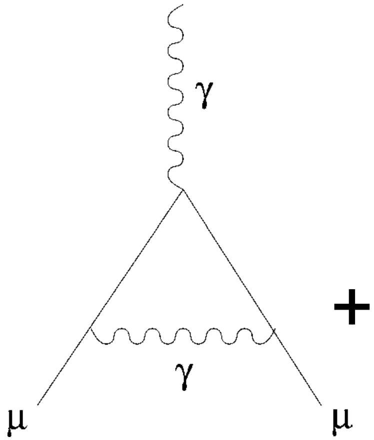


Figure 1 The first-order QED correction to g-2 of the muon.

+

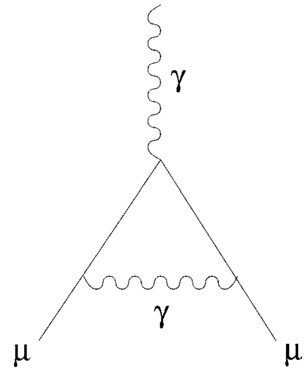
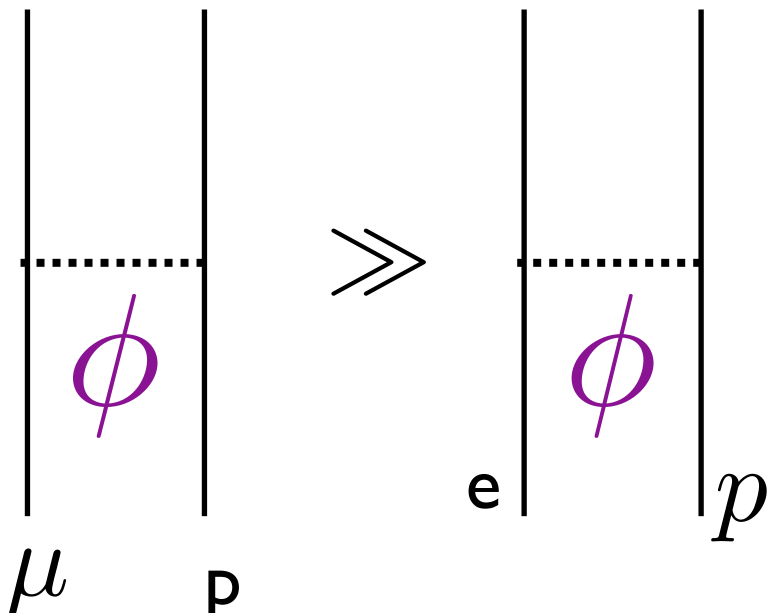


Figure 1 The first-order QED correction to g-2 of the muon. H

3.6 st. dev anomaly now
fix add heavy photon
interacts preferentially with
muon

Muon data is g-2 - BNL exp't,
Hertzog- FermiLab now...

Maybe dark matter, energy particles show up in muon physics!



Postulate new scalar boson!

look for violation
of 4 momentum conservation
elastic ep scattering
Dark Light interest

New scalar bosons

assumes puzzle exists

ϕ

- give μ-p Lamb shift
- almost no hyperfine in μ proton
- small effect for D, almost no effect ^{4}He
- consistent with g-2 of μ and electron
- avoid many other constraints
- be found

PRL **117**, 101801 (2016) PHYSICAL REVIEW LETTERS week ending 2 SEPTEMBER 2016

Electrophobic Scalar Boson and Muonic Puzzles

Yu-Sheng Liu,* David McKeen,† and Gerald A. Miller‡

New scalar bosons

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- give μ-p Lamb shift ✓
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PRL **117**, 101801 (2016) PHYSICAL REVIEW LETTERS week ending 2 SEPTEMBER 2016

Electrophobic Scalar Boson and Muonic Puzzles

Yu-Sheng Liu,[*] David McKeen,[†] and Gerald A. Miller[‡]

Constraining ϕ

Beam dump experiments do not see

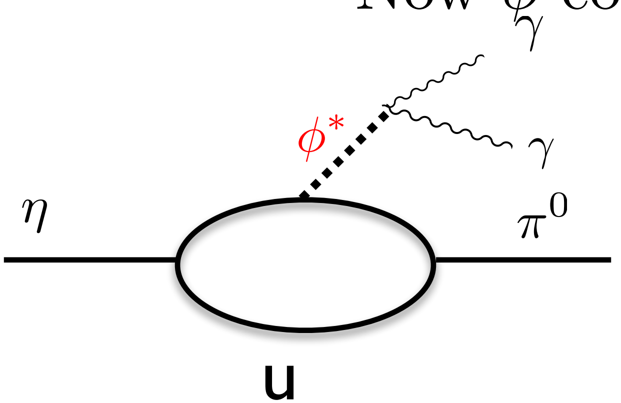
$$\phi \to e^+e^-,\ \phi \to \gamma\gamma$$

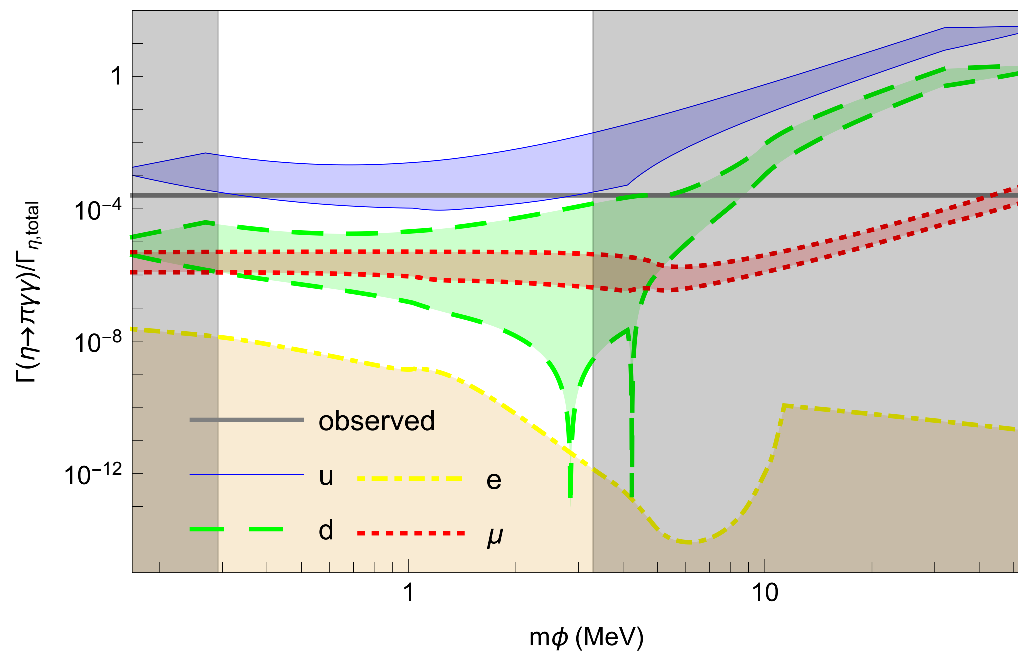
Eta decay and muonic puzzles 1805.01028

Yu-Sheng Liu, Ian Cloet, GAM

Previously ϕ couples to p, n, e, μ

Now ϕ couples to $u, d, e\mu$





Allowed mass range from 200 KeV to 3 MeV

Summary

- If the proton radius puzzle exists : new scalar boson of mass from 300 KeV to 3 MeV may exist- narrow target
- Direct detection is needed.

Does 4% matter?

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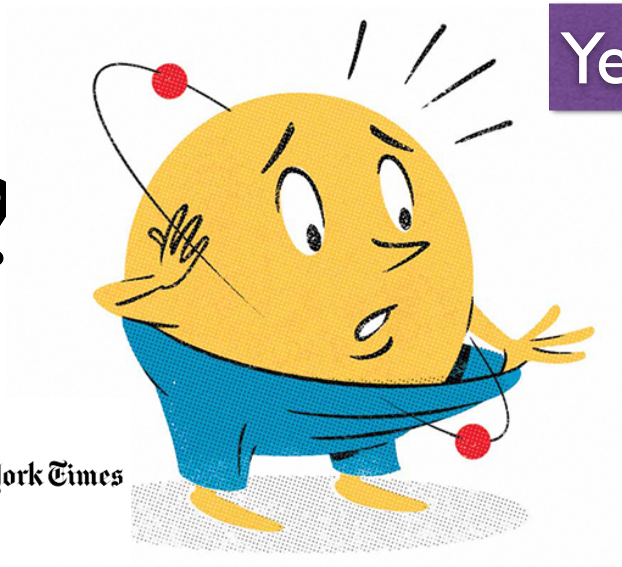
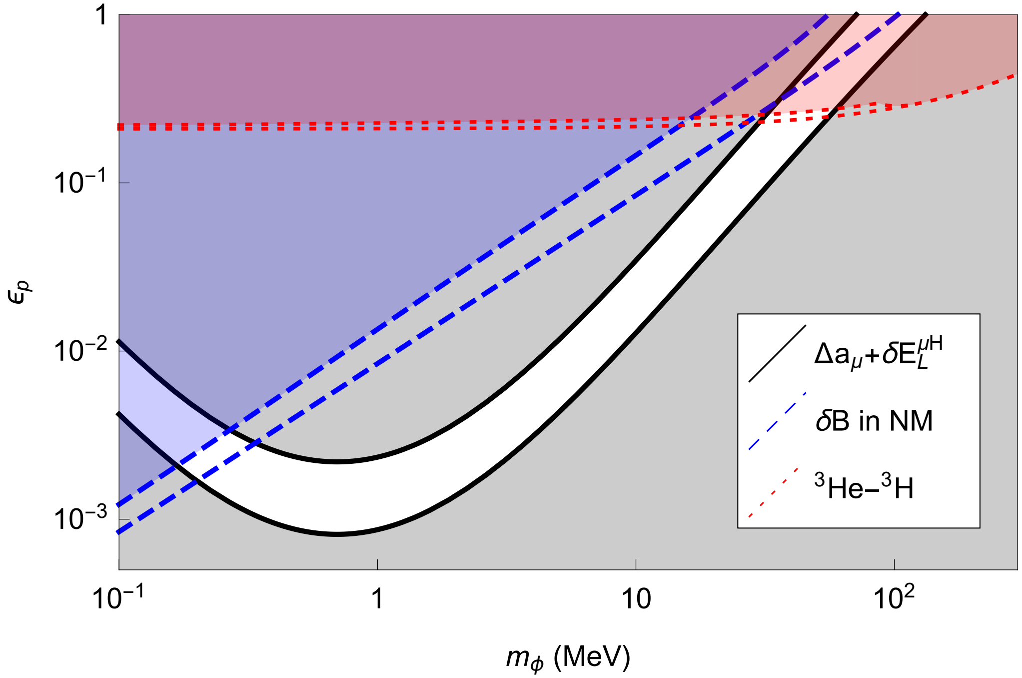
Does 4% matter?



Yes

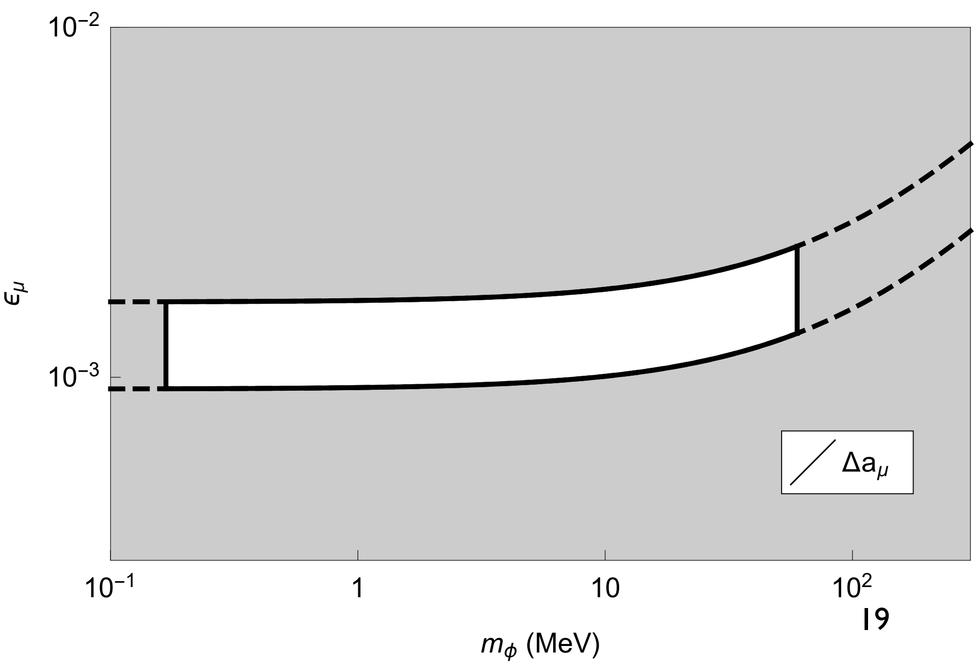
The New York Times

Spares follow

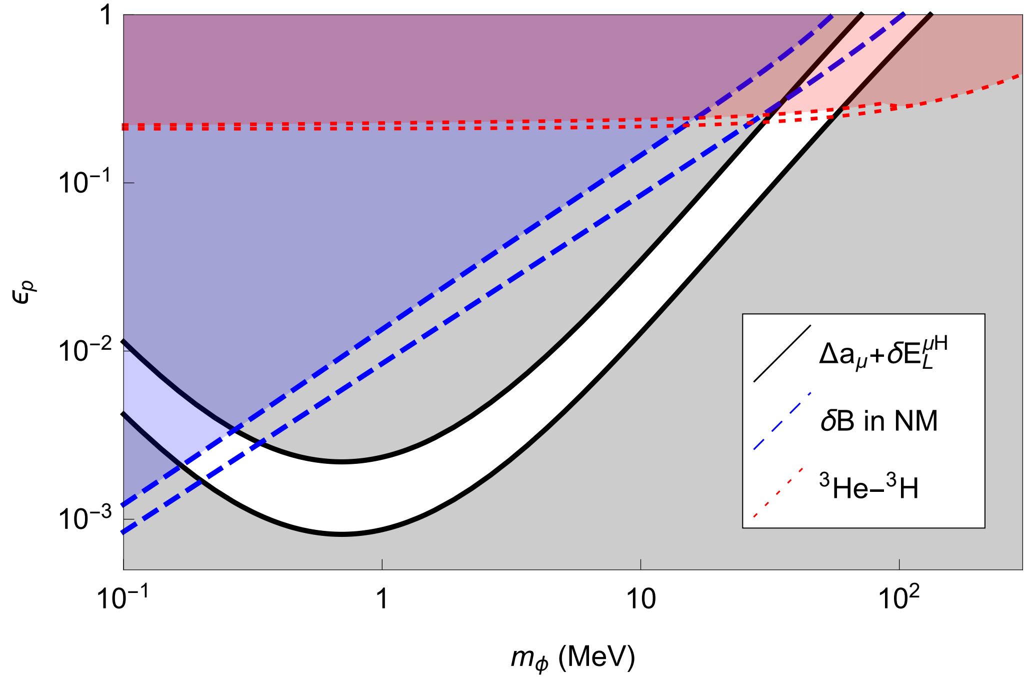


Unshaded allowed by muon g-2 and muon-p Lamb shift

Two anomalies have same scale



Unshaded allowed by muon g-2



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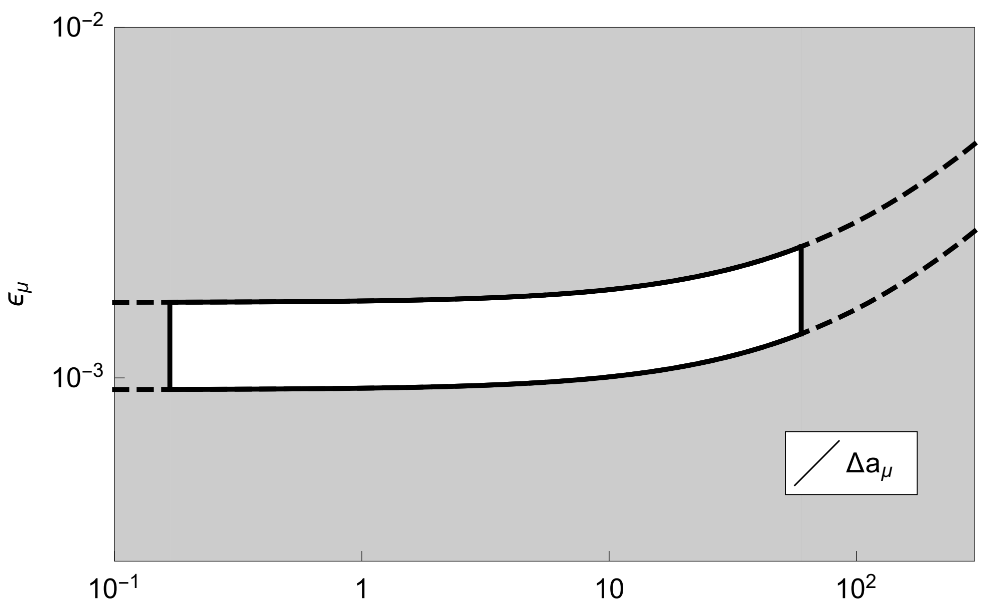


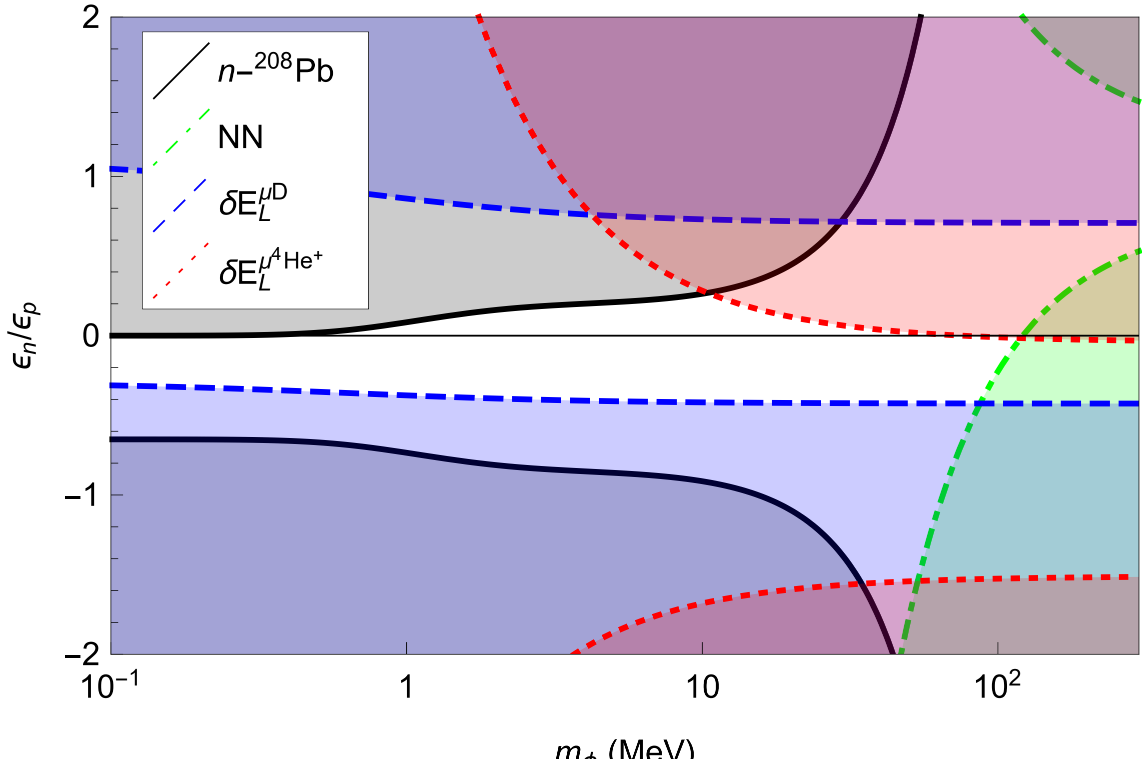
Unshaded allowed by muon g-2

Using new eH experiment
$\epsilon_\mu \epsilon_p$ is reduced by
a factor of 3:
barely visible in loglog plot

Nuclear Physics constraints

ϵ_n/ϵ_p

If $\epsilon_n = \epsilon_p$ scalar is ruled out by n-^{208}Pb scattering
If ϵ_n has opposite sign as ϵ_p parameter space widens
Other constraints:
NN scattering, nuclear matter & $^3\mathrm{He} - ^3\mathrm{H}$ binding
muonic D, ^{3}He

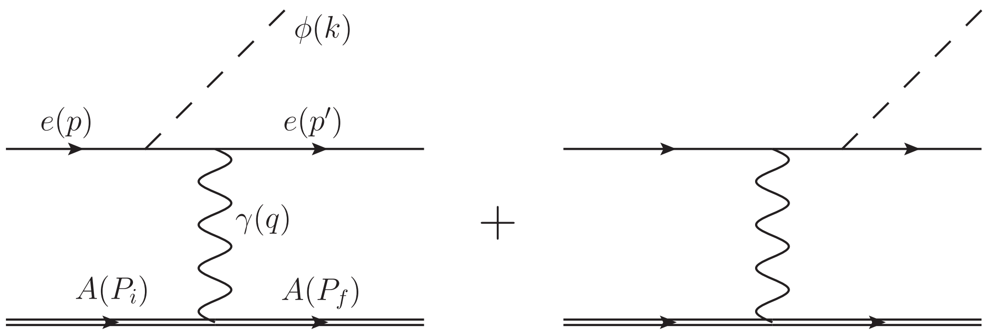


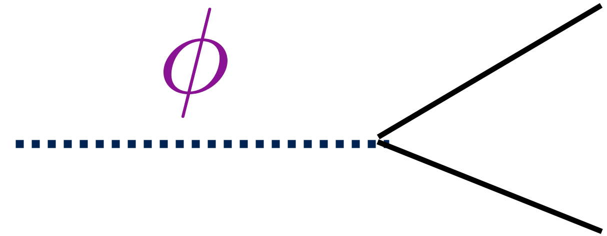
Beam dump experiments

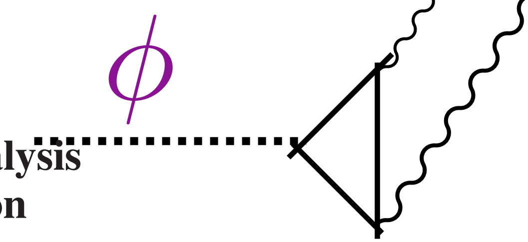
Beam dumps absorb beam of charged particles

e+^{27}Al to dissipate energy

hope e+^{27}Al makes penetrating new particles







PHYSICAL REVIEW D **95,** 036010 (2017)

Validity of the Weizsäcker-Williams approximation and the analysis of beam dump experiments: Production of a new scalar boson

Yu-Sheng Liu,* David McKeen,† and Gerald A. Miller‡

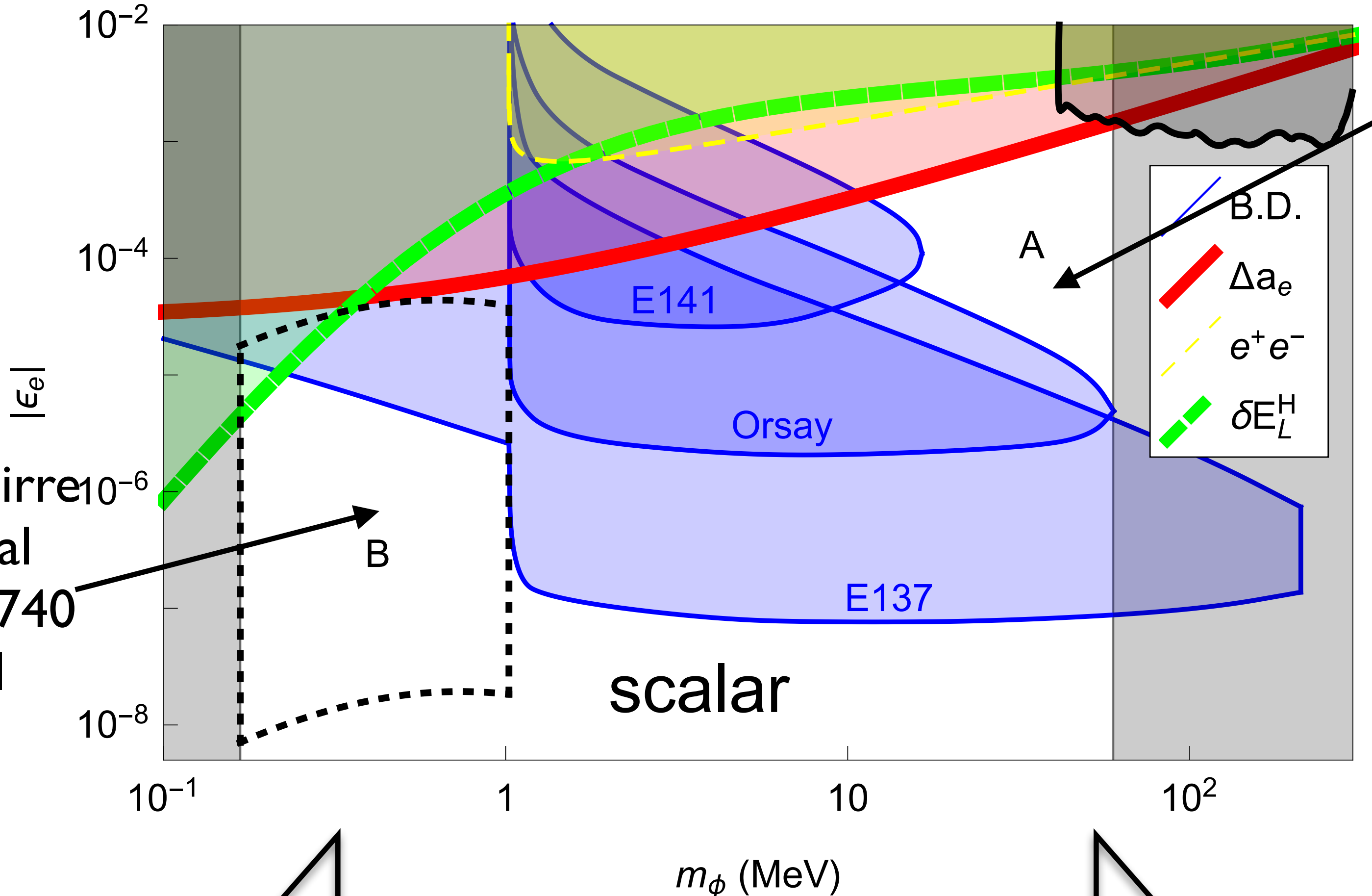
Better analysis if discovery

PHYSICAL REVIEW D **96,** 016004 (2017)

Validity of the Weizsäcker-Williams approximation and the analysis of beam dump experiments: Production of an axion, a dark photon, or a new axial-vector boson

Yu-Sheng Liu* and Gerald A. Miller†

electron Exclusion plot

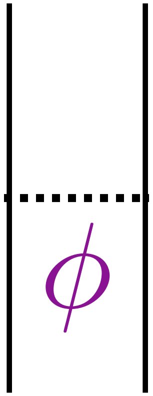


APEX
HPS
DarkLight
VEPP-3
MESA

Izaguirre et al
PLB740
61

allowed mass range

Our approach

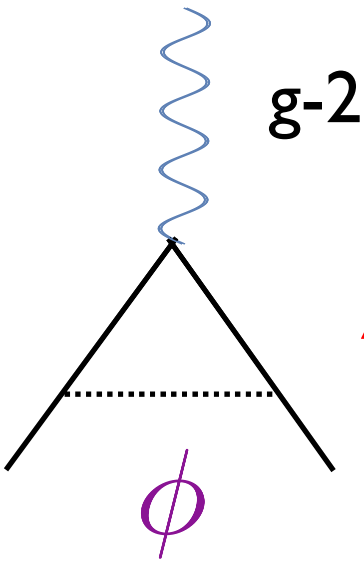


any signs

$$V(r) = -\epsilon_{f_1}\epsilon_{f_2}\alpha\frac{e^{-m_\phi r}}{r}. \qquad |\epsilon_e| \ll \epsilon_\mu$$

Bohr radius

$$\delta E_L^{\ell \mathrm{N}} = -\frac{\alpha}{2a_{\ell \mathrm{N}}}\epsilon_\ell[Z\epsilon_p + (A-Z)\epsilon_n]\frac{(a_{\ell \mathrm{N}} m_\phi)^2}{(1+(a_{\ell \mathrm{N}} m_\phi))^4}$$



$\Delta a_\mu = 287(80)\times 10^{-11}$, $\Delta a_e = 1.5\times 10^{-12}$ From ^{87}Rb

For light mass- evades 4He data

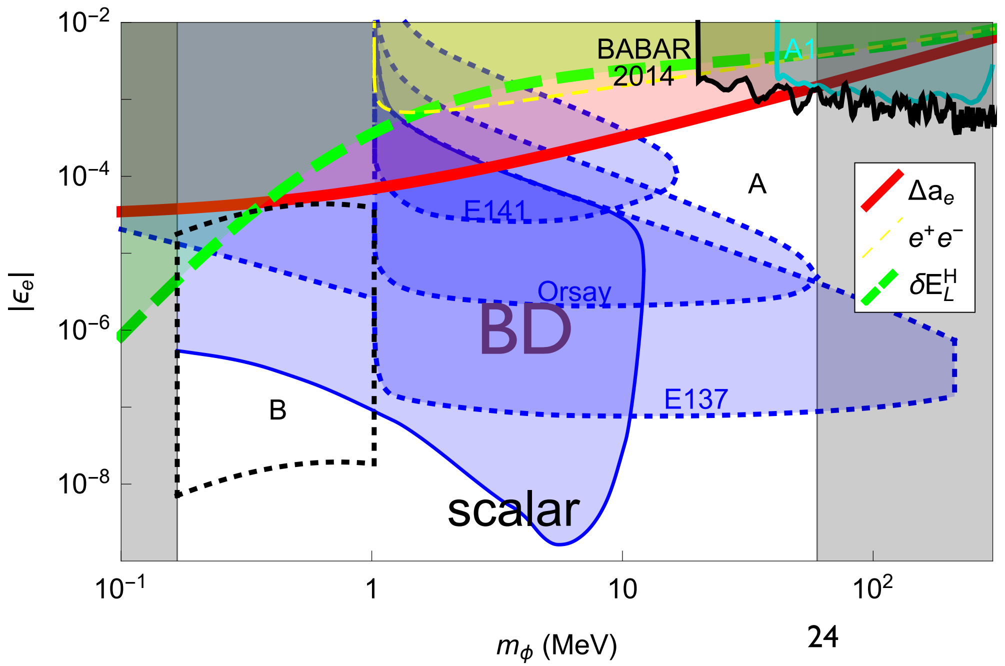
More constrains Coupling to electron

ϵ_e

Hydrogen atom Lamb shift $\delta E_L^H < 45 \text{ kHz}$

e^+e^- resonant (?) scattering not seen

electron anomalous magnetic moment



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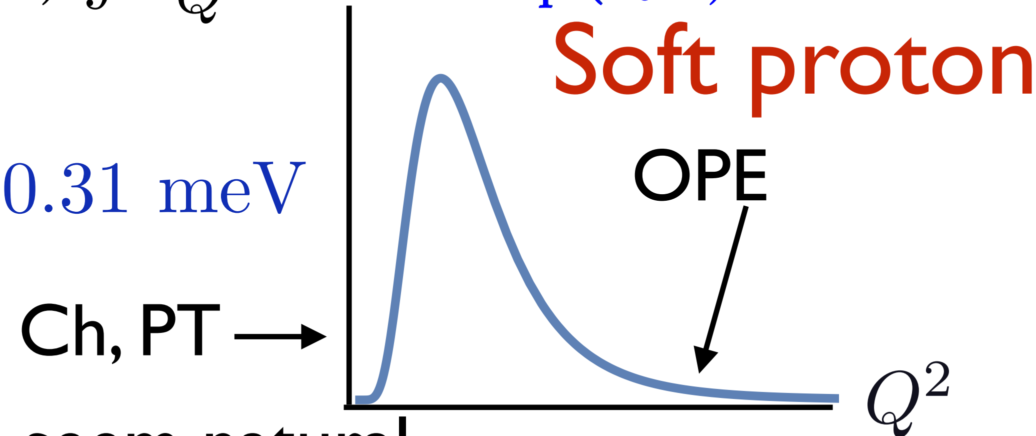
BD is beam dump discussed next

Electrophobic

Almost unknown $\bar{T}_1(0, Q^2)$ Miller PLB 2012

$$\Delta E^{\text{subt}} \propto \alpha^2 m \Psi_S^2(0) \int \frac{dQ^2}{Q^2} \cdots F_{\text{loop}}(Q^2)$$



number of
Infinite $F_{\text{loop}}(Q^2)$ give 0.31 meV
satisfy all constraints

Recast in EFT- parameters seem natural

So far size of this term cannot be determined from theory-experiment is needed

lattice QCD may disprove above sentence

Science 12 Aug 2016:
Vol. 353, Issue 6300, pp. 669-673

Deuteron is smaller too

Electron (D-H) isotope shift (2S-1S) 2 photon spectroscopy PRL 104, 233001

$r_d^2 - r_p^2 = 3.82007(65)$

$\mu - D$ Lamb shift $r_d = 2.12562(78)$ fm Science 353 (2016) 669

CODATA (2010)$r_d = 2.1424(26)$ fm - mainly electron scattering

Use $r_p = 0.84087$ in $r_d^2 - r_p^2 = 3.82007(65)$ gives $r_d = 2.12769$ fm

μD and Electron (D-H) isotope shift are consistent$\rightarrow$ redo eD scattering?

If NO proton radius puzzle, there still is a missing Lamb shift

OR:

CODATA deuteron radius is too large 2.1424 vs 2.12769

remeasure deuteron ?

Nuclear Physics constraints

$$\epsilon_n/\epsilon_p$$

- Low energy scattering of neutrons on ^{208}Pb using ϕ-nucleon coupling g_N.
  $\frac{g_N^2}{e^2} \to \frac{A-Z}{A}\epsilon_n^2 + \frac{Z}{A}\epsilon_p\epsilon_n$ cancellation evades previous limits

- NN charge-independence breaking scattering length

  $\Delta a = (a_{pp} + a_{nn})/2 - a_{np}$, measured: 5.64(60) fm, theory: 5.6(5)

  Scalar boson exchange: $\Delta a_\phi \propto \int_0^\infty \Delta V \bar{u} u_{np} dr \leq 1.6$ fm (2 S.D.)

- Change in binding energy/A infinite nuclear matter: less than 1 MeV

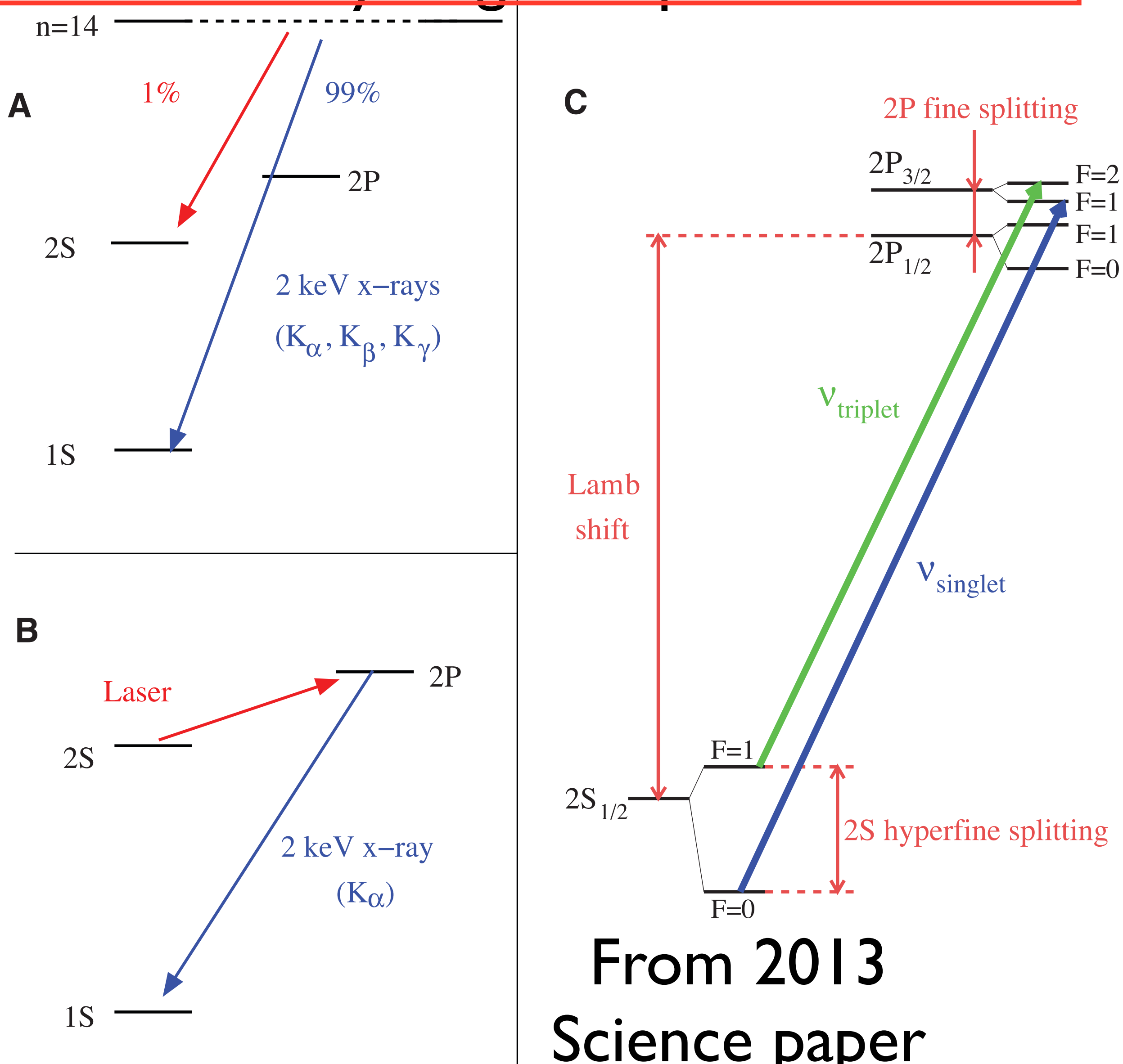
- binding energy B(^3He) $- B(^3H)$= 763.76 keV due to Coulomb (693 keV) + strong force charge symmetry breaking (68 keV) ϕ exchange < 30 keV

MUSE and scalar

$$V_\phi(r) = -1.7 \times 10^{-6}\, \alpha \frac{e^{-m_\phi\, r}}{r}$$

- No scattering experiment can detect a coupling this weak
- If this scalar exists (and other e H large radius experiments correct) MUSE will find electrons/positrons see the same large radius and
- muons and anti-muons will see the same large radius
- If no puzzle: all leptons see small radius
- In any case MUSE limits two photon (biggest uncertainty)

Muonic Hydrogen Experiment



From 2013 Science paper

Fig. 1. (**A**) Formation of μp in highly excited states and subsequent cascade with emission of "prompt" $K_{\alpha,\ \beta,\ \gamma}$. (**B**) Laser excitation of the 2S-2P transition with subsequent decay to the ground state with K_α emission. (**C**) 2S and 2P energy levels. The measured transitions ν_s and ν_t are indicated together with the Lamb shift, 2S-HFS, and 2P-fine and hyperfine splitting.

Randolf Pohl

Aldo Antognini

PHYSICAL REVIEW D **95,** 036010 (2017)

Validity of the Weizsäcker-Williams approximation and the analysis of beam dump experiments: Production of a new scalar boson

Yu-Sheng Liu,* David McKeen,† and Gerald A. Miller‡

- previous cross sections obtained w. WW approximation
- cross sections not accurate
- exclusion plots changed substantially
- if discovery, WW gives wrong parameters
- not necessary to assume mass of new particle is much much greater than mass of electron

Lepton-universality violating one boson exchange

- Tucker-Smith & Yavin PRD83, 101702 new particle scalar or vector coupling
- Brax & Burrage scalar particles PRD 83, 035020 &'14
- Batell, McKeen & Pospelov PRL 107, 011803 new gauge boson kinetically mixing with $F^{\mu\nu}$ plus scalar for muon mag. mom. 1401.6154 W decays enhanced
- Carlson Rislow PRD 86, 035013 fine tune scalar pseudoscalar or polar and axial vector couplings
- Barger et al PRL106, 153001 - new particles ruled out but assumes universal coupling
- Kaon decays provide constraints

Looking for new scalars is not new
Low mass Higgs searches

$$p(1.88\,\mathrm{MeV}) +^{19} F \rightarrow \alpha +^{16} O^*(6.05)$$
$$^{16}O^*(6.05) \rightarrow^{16} O(GS) + \phi$$

Kohler et al PRL 33, 1628 (1974)

Freedman et al. PRL 52, 240 (1984)

$$p +^3 H \rightarrow^4 He(20.1) \rightarrow 4He(GS) + \phi$$

No Scalars found, but assumed coupling constants were much larger than what we will use

Nuclear dependence of short-ranged mu-p effects

- Energy shift is proportional to square of muon wave function at the origin
- Suppose you have effect that gives energy shifts Ep (on proton) E_n (on neutron)

GAM
1501.01036

$$E_A = \left(\frac{1+\frac{m_\mu}{m_p}}{1+\frac{m_\mu}{Am_p}}\right)^3 Z^3(ZE_p+NE_n)\left(1-\mathcal{O}\left(\frac{R_A^2}{a_\mu^2}\right)\right) \approx \left(\frac{1+\frac{m_\mu}{m_p}}{1+\frac{m_\mu}{Am_p}}\right)^3 Z^3(ZE_p+NE_n),$$

Nuclear shift

Square of wave fun

Counting

Size of nucleus

My model: ~0.3 meV (1+0.3)(8)(2) =-6.3 meV about 6 st. dev off **RIP any short range idea**

Almost unknown $\overline{T}_1(0,Q^2)$ Miller PLB 2012

$$\Delta E^{\rm subt} = \frac{\alpha^2}{m}\Psi_S^2(0)\int_0^\infty dQ^2\,\frac{h(Q^2)}{Q^2}\overline{T}_1(0,Q^2)$$

Soft proton

$$\lim_{Q^2\to\infty} h(Q^2)\sim\frac{2m^2}{Q^2},\ \text{chiral PT}:\ \overline{T}_1(0,Q^2)=\frac{\beta_M}{\alpha}Q^2+\cdots$$

$\rightarrow$ Logarithmic divergence

Typo bleow

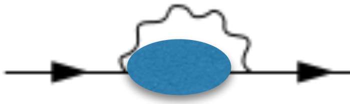
$\overline{T}_1(0,Q^2)\rightarrow\frac{\beta_M}{\alpha}Q^2F_{\rm loop}(Q^2)$ Cuts off integral

Birse & McGovern assume dipole : $\Delta E^{\rm subt} = 0.004\,\text{meV}$ very small

Miller $F_{\rm loop}(Q^2)=\left(\frac{Q^2}{M_0^2}\right)^n\frac{1}{(1+aQ^2)^N},\ n\geq 2,\ N\geq$ n+3

Infinite parameter set gets needed 0.31meV, NO constraint on neutron

Choose parameters so shift in proton mass <0.5 MeV (current uncertainty)



Recast in EFT- parameters seem natural

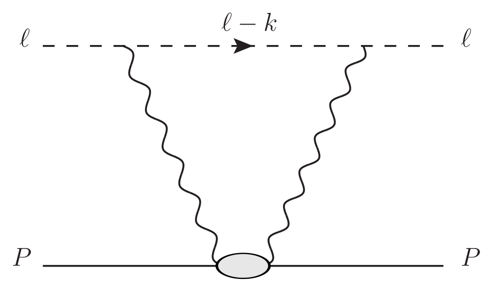
Two photon exchange

Measured =206.2949(32)= 206.0573(45)-5.2262 r_p^2+0.0347 r_p^3 meV

computed

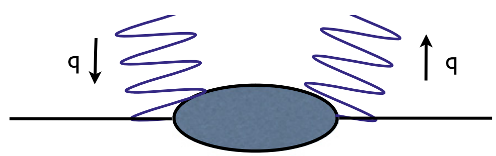
Explain puzzle with radius as in H atom increase 206.0573 meV by 0.31 meV-attractive effect on 2S state needed

Our idea



energy shift proportional to lepton mass[4]

$T^{\mu\nu} =$



$$= -(g^{\mu\nu} - \cdots)T_1 + (P^{\mu} - \cdots)(P^{\mu} - \cdots)T_2$$

Dispersion relation: $Im[T_1] \propto W_1$ measured

Large virtual photon energy ν, $W_1 \sim \nu$ integral over energy diverges

Subtraction function needed: $\bar{T}_1(0, Q^2)$ zero energy

Hill & Paz- big uncertainty in dispersion approach

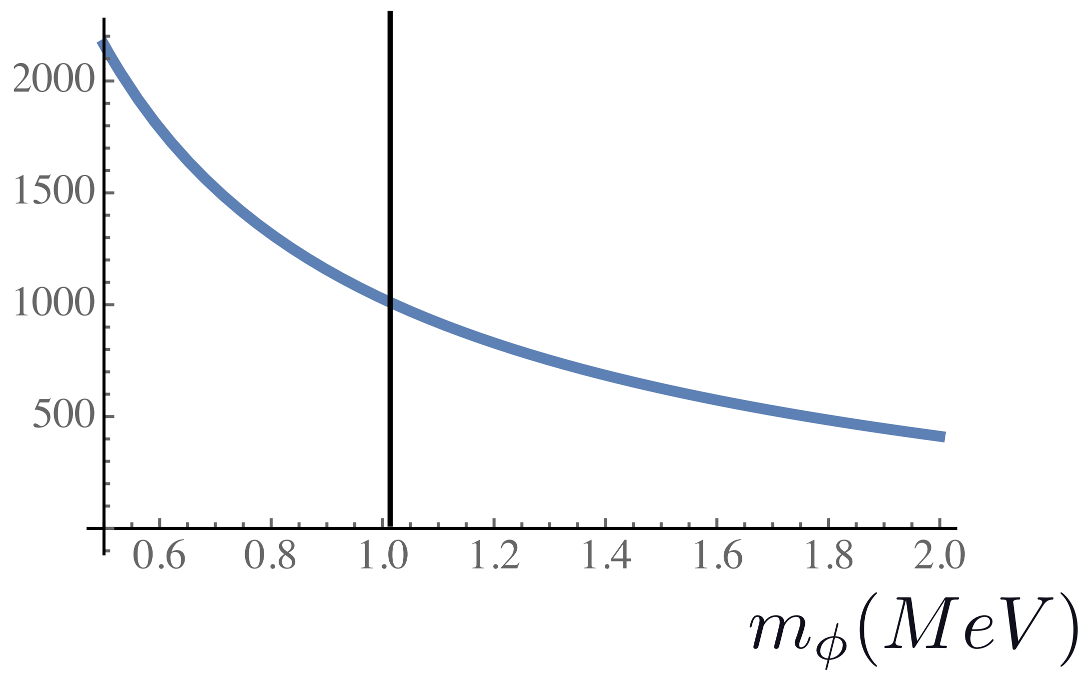
$$^{16}O(6.05, 0^+) \rightarrow ^{16} O(\mathrm{GS}, 0^+) + \phi, \quad \text{No single photon decay}$$

From electron g-2

$$\frac{\tau(A^* \rightarrow A + e^+ e^-)}{\tau(A^* \rightarrow A + \phi} = 3.3 \times 10^3 \frac{g^2_{\phi e \bar{e}}}{e^2} \left(1 - \left(\frac{m_\phi}{6\mathrm{MeV}}\right)^2\right)^{5/2}$$

$$\tau(A^* \rightarrow A + e^+ e^-): \quad \text{lifetime is } 10^{-10}\mathrm{s}$$

Decay length (m)
nuclear emission of
scalar boson



Several new electron spectroscopy experiments

- Independent measurement of Rydberg constant. This would change only extracted r_p nothing else
- 2S-6S UK, 2S-4P Germany, 1S-3S France
- 2S-2P classic, Canada
- Highly charged single electron ions NIST

2S-4P has reported preliminary results- small radius not yet published

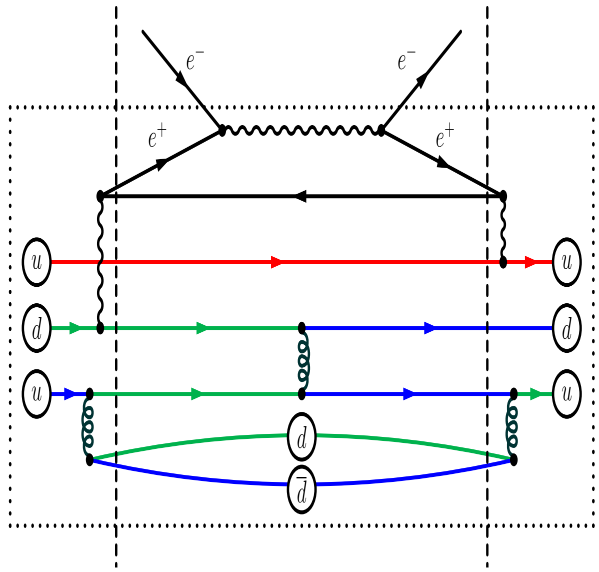
Yes it really is G_E

- Non-relativistic reduction of one-photon exchange leads to the spin independent interaction being $G_E(Q^2)/Q^2$
- All recoil effects properly accounted for:Breit-Pauli Hamiltonian computed for non-zero lepton and proton momentum

Light Sea Fermions in Electron–Proton and Muon–Proton Interactions

U. D. Jentschura Phys.Rev.A88 (2013) 062514

If we assume an average of roughly 0.7×10−7 light sea positrons per valence quark, then we can show that virtual electron-positron annihilation processes lead to an extra term in the electron-proton versus muon-proton interaction, which has the right sign and magnitude to explain the proton radius discrepancy.



Contribution for electron not muon

Non-perturbative lepton-pair exists in proton wave function. UDJ: energy shift $\propto 1/m_l^2$, from annihilation at rest. GAM: Shift $\propto 1/(\text{constituent quark mass})^2$

Any effect is small and same for electron and muon atoms

arXiv:1501.01036

Arbitrary functions

$$\overline{T}_1(0,Q^2) = \frac{\beta_M}{\alpha} Q^2 F_{\rm loop}(Q^2)\,.$$

$$F_{\rm loop}(Q^2) = \left(\frac{Q^2}{M_0^2}\right)^n \frac{1}{(1+aQ^2)^N},\; n \geq 2,\; N \geq n+3,$$

$$\overline{T}_1(0,Q^2) \sim \frac{1}{Q^4} \text{ or faster, } \beta_M \to \beta$$

$$\Delta E^{\rm subt} \approx 3\alpha^2 m \Psi_S^2(0) \frac{\beta}{\alpha} \gamma^n B(N,n), \gamma \equiv \frac{1}{M_0^2 a}$$

3 parameters: n, N, a $\quad (M_0 = M_\beta)$

Choose parameters such that shift in proton mass < electromagnetic uncertainty of 0.5 MeV

Almost unknown $\overline{T}_1(0, Q^2)$

Miller PLB 2012

$$\Delta E^{\text{subt}} = \frac{\alpha^2}{m}\Psi_S^2(0)\int_0^\infty dQ^2\,\frac{h(Q^2)}{Q^2}\overline{T}_1(0,Q^2)$$

Soft proton

$$\lim_{Q^2\to\infty} h(Q^2) \sim \frac{2m^2}{Q^2},\ \text{chiral PT}:\ \overline{T}_1(0,Q^2) = \frac{\beta_M}{\alpha}Q^2 + \cdots$$

$\to$ Logarithmic divergence

Typo bleow

$\overline{T}_1(0,Q^2) \to \frac{\beta_M}{\alpha}Q^2 F_{\text{loop}}(Q^2)$ Cuts off integral

Birse & McGovern assume dipole : $\Delta E^{\text{subt}} = 0.004\,\text{meV}$ very small

$$\text{Miller}\quad F_{\text{loop}}(Q^2) = \left(\frac{Q^2}{M_0^2}\right)^n \frac{1}{(1+aQ^2)^N},\, n \geq 2, N \geq N+3$$

Infinite parameter set gets needed 0.31meV, NO constraint on neutron

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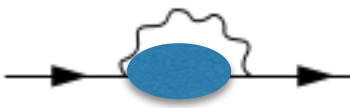
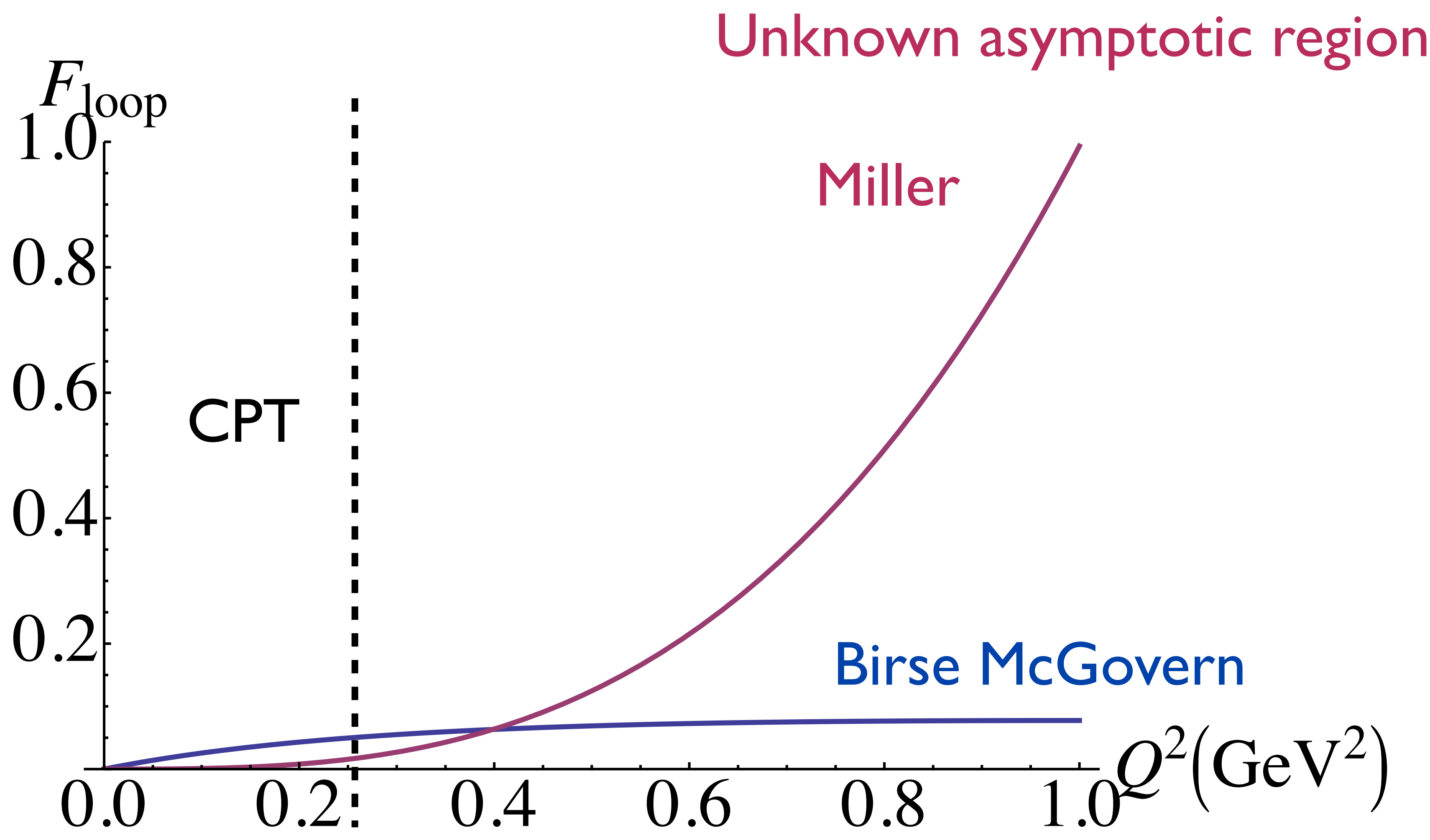


Recast in EFT- parameters seem natural

Form factors

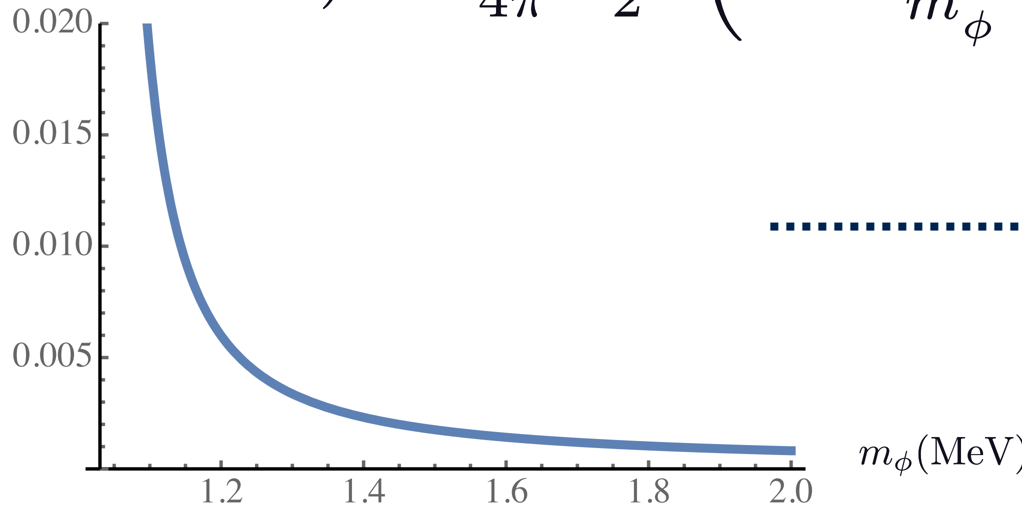


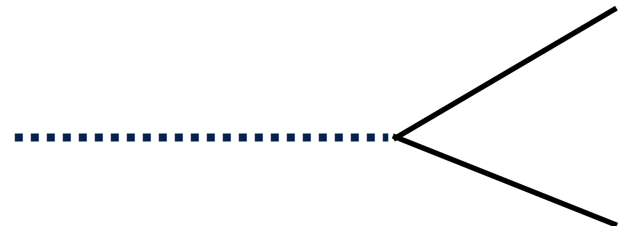
If recast into effective field theory strength seems natural

ϕ Decay modes

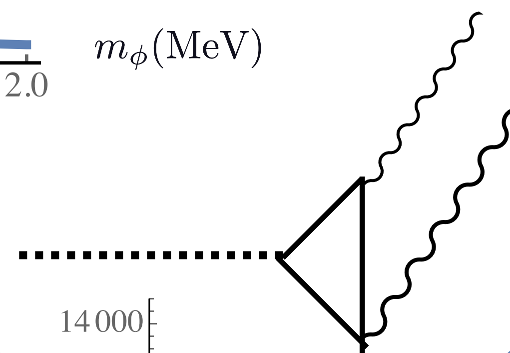
$$\Gamma(\phi \to e^+e^-) = \frac{g_{\phi e}^2}{4\pi}\frac{m_\phi}{2}\left(1 - \frac{4m_e^2}{m_\phi^2}\right)^{3/2}, m_\phi > 2m_e$$

$c\tau(\mathrm{m})$

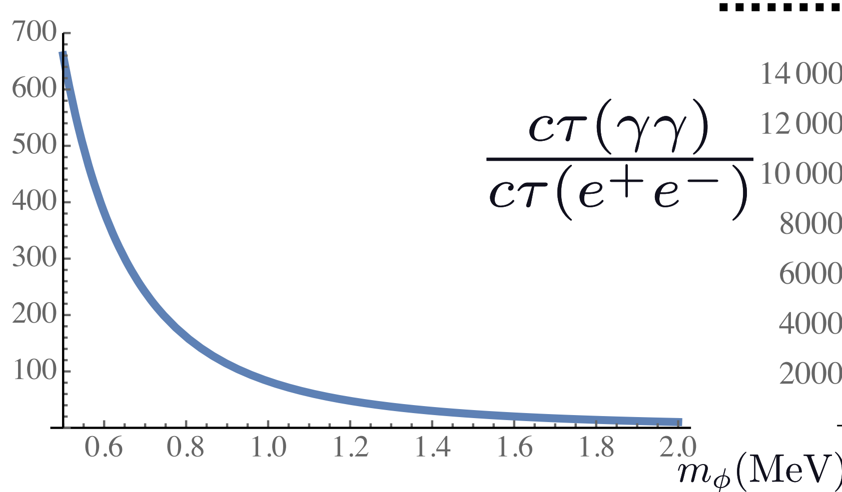




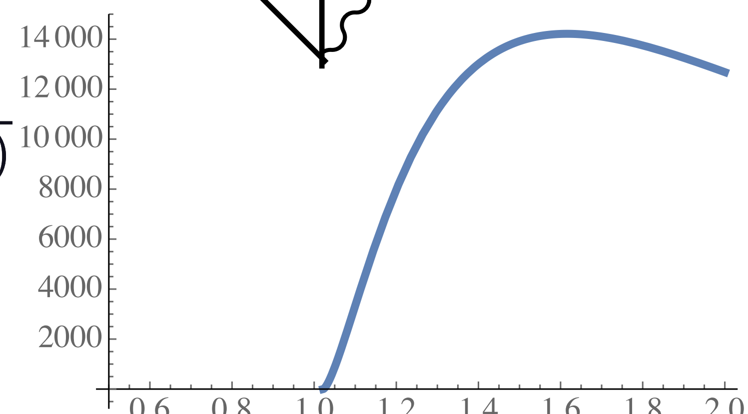
$$\Gamma(\phi \to \gamma\gamma) = g_{\phi e}^2 \frac{\alpha^2}{144\pi^3}\frac{m_\phi^3}{m_e^2}$$



$c\tau(\mathrm{m})$



$$\frac{c\tau(\gamma\gamma)}{c\tau(e^+e^-)}$$



Pohl et al. Table of calculations

Lamb shift: vacuum polarization many, many terms

#	Contribution	Ref.	Our selection Value	Unc.	Pachucki[1–3] Value	Unc.	Borie[5] Value	Unc.
1	NR One loop electron VP	1,2			205.0074			
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3	Relativistic one loop VP	5	205.0282				205.0282	
4	NR two-loop electron VP	5,14	1.5081		1.5079		1.5081	
5	Polarization insertion in two Coulomb lines	1,2,5	0.1509		0.1509		0.1510	
6	NR three-loop electron VP	11	0.00529					
7	Polarisation insertion in two and three Coulomb lines (corrected)	11,12	0.00223					
8	Three-loop VP (total, uncorrected)				0.0076		0.00761	
9	Wichmann-Kroll	5,15,16	−0.00103				−0.00103	
10	Light by light electron loop contribution (Virtual Delbrück scattering)	6	0.00135	0.00135			0.00135	0.00015
11	Radiative photon and electron polarization in the Coulomb line $\alpha^2(Z\alpha)^4$	1,2	−0.00500	0.0010	−0.006	0.001	−0.005	
12	Electron loop in the radiative photon of order $\alpha^2(Z\alpha)^4$	17–19	−0.00150					
13	Mixed electron and muon loops	20	0.00007				0.00007	
14	Hadronic polarization $\alpha(Z\alpha)^4 m_r$	21–23	0.01077	0.00038	0.0113	0.0003	0.011	0.002
15	Hadronic polarization $\alpha(Z\alpha)^5 m_r$	22,23	0.000047					
16	Hadronic polarization in the radiative photon $\alpha^2(Z\alpha)^4 m_r$	22,23	−0.000015					
17	Recoil contribution	24	0.05750		0.0575		0.0575	
18	Recoil finite size	5	0.01300	0.001			0.013	0.001
19	Recoil correction to VP	5	−0.00410				−0.0041	
20	Radiative corrections of order $\alpha^n(Z\alpha)^k m_r$	2,7	−0.66770		−0.6677		−0.66788	
21	Muon Lamb shift 4th order	5	−0.00169				−0.00169	
22	Recoil corrections of order $\alpha(Z\alpha)^5 \frac{m}{M} m_r$	2,5–7	−0.04497		−0.045		−0.04497	
23	Recoil of order α^6	2	0.00030		0.0003			
24	Radiative recoil corrections of order $\alpha(Z\alpha)^n \frac{m}{M} m_r$	1,2,7	−0.00960		−0.0099		−0.0096	
25	Nuclear structure correction of order $(Z\alpha)^5$ (Proton polarizability contribution)	2,5,22,25	0.015	0.004	0.012	0.002	0.015	0.004
26	Polarization operator induced correction to nuclear polarizability $\alpha(Z\alpha)^5 m_r$	23	0.00019					
27	Radiative photon induced correction to nuclear polarizability $\alpha(Z\alpha)^5 m_r$	23	−0.00001					
	Sum		206.0573	0.0045	206.0432	0.0023	206.05856	0.0046

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Total $< r_p^2 >$ contribution		−5.22619	$< r_p^2 >$	−5.2256	−5.2244
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Resolution 1- QED calcs not OK

α

Pohl et al. Table of calculations

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Mostly irrelevant- theory replaced by experiment

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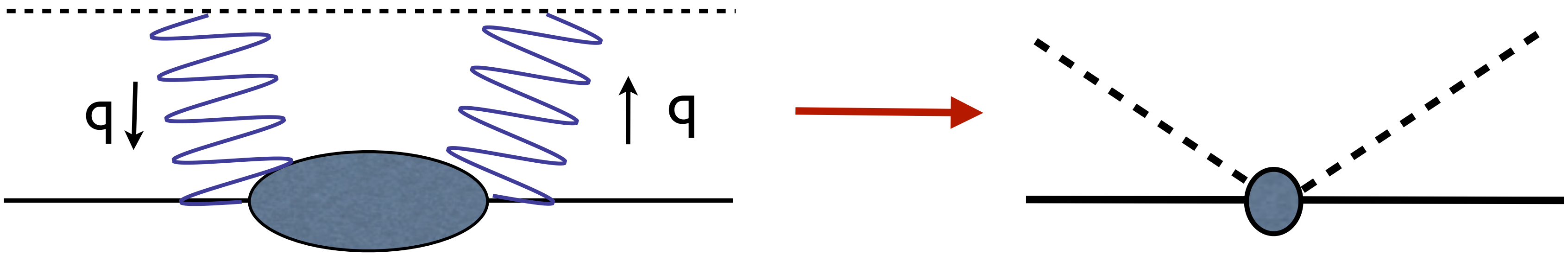
Resolution 1- QED calcs not OK

QED calcs expand in α

EFT of μp interaction

Caswell Lepage '86

- Compute Feynman diagram, remove log divergence using dimensional regularization
- include counter term in Lagrangian



$$\mathcal{M}_2^{DR} = \frac{3}{2} i\, \alpha^2 m \frac{\beta_M}{\alpha} \left[\frac{2}{\epsilon} + \log\frac{\mu^2}{m^2} + \frac{5}{6} - \gamma_E + \log 4\pi\right] \overline{u}_f u_i \overline{U}_f U_i,$$

$$= i\, \alpha^2 m \frac{\beta_M}{\alpha} (\lambda + 5/4)\, \overline{u}_f u_i \overline{U}_f U_i$$

Choose λ to get 0.31 meV shift

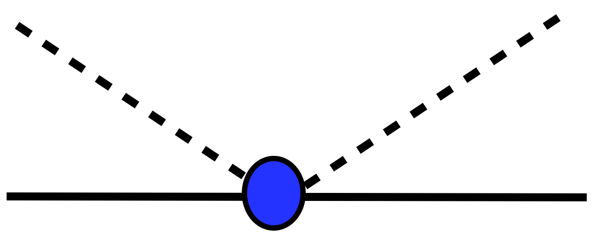
$$\Delta E^{\rm subt}(DR) = \alpha^2 m \frac{\beta_M}{\alpha} \Psi_S^2(0)(\lambda + 5/4)$$

$$\Delta E^{\rm subt}(DR) = 0.31 \text{ meV} \rightarrow \boxed{\lambda = 769}$$

$\beta_M(\text{magnetic polarizability}) = 3.1 \times 10^{-4}\text{fm}^3$ very small

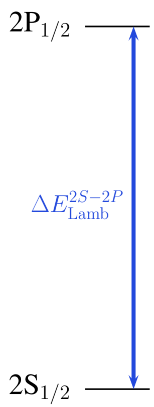
Natural units $\beta_M/\alpha \sim 4\pi/(4\pi f_\pi)^3$ Butler & Savage '92

$$\mathcal{M}_2^{DR} = i\, 3.95\, \alpha^2 m \frac{4\pi}{\Lambda_\chi^3} \overline{u}_f u_i \overline{U}_f U_i.$$



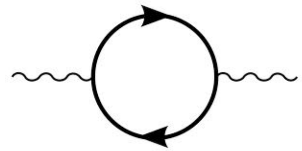
3.95 =natural

Muonic hydrogen experiment and r_p



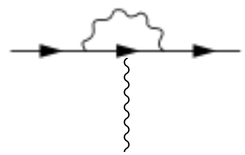
The Lamb shift is the splitting of the degenerate $2S_{1/2}$ and $2P_{1/2}$ eigenstates

Dominant in μH



vacuum polarization 205 of 206 meV

Dominant in eH



electron self-energy

Proton radius in Lamb shift

$$\Delta E = \langle \Psi_S | V_C - V_C^{\text{pt}} | \Psi_S \rangle = \frac{2}{3}\pi\alpha|\Psi_S(0)|^2 (-6G'_E(0))$$

$= r_p^2$

Muon/electron mass ratio 205! 8 million times larger for muon

Electronic Hydrogen -Pohl

- Need two levels to get Rydberg and Lamb shift-have ~ 20 available

$$E(nS) \approx \frac{R_\infty}{n^2} + \frac{L_{1S}}{n^3}$$