Interactions between Decuplet Baryons from Lattice QCD

Shinya Gongyo (RIKEN)

HAL(Hadrons to Atomic nuclei from Lattice) QCD Collaboration

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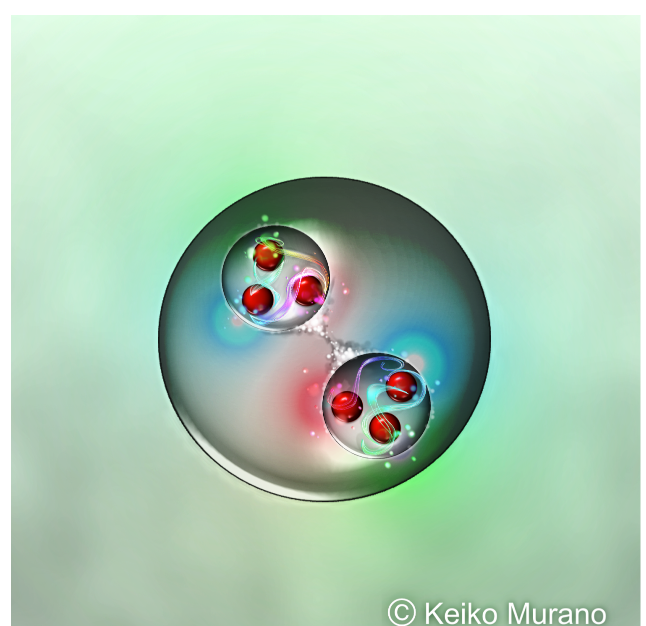
SG, K.Sasaki + (HAL QCD Coll.), Phys.Rev.Lett. 120 (2018) 212001

ΩΩ interaction with J=0
at almost physical point

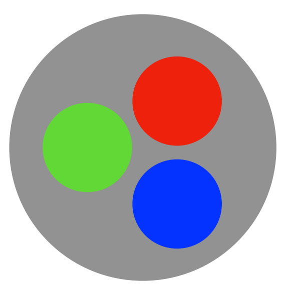


“di-Omega”
Most Strange dibaryon

May 29, 2018@CIPANP2018

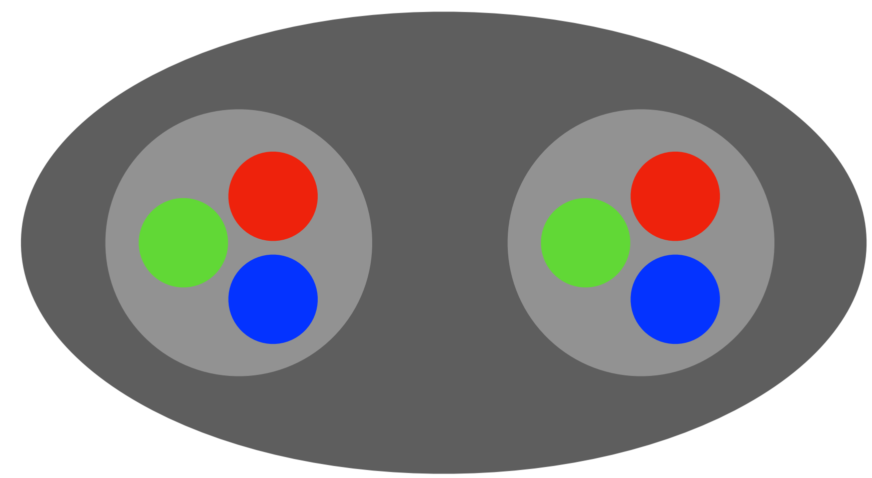
Introduction

Baryon (B=1)



Proton, Neutron,
Lambda,…

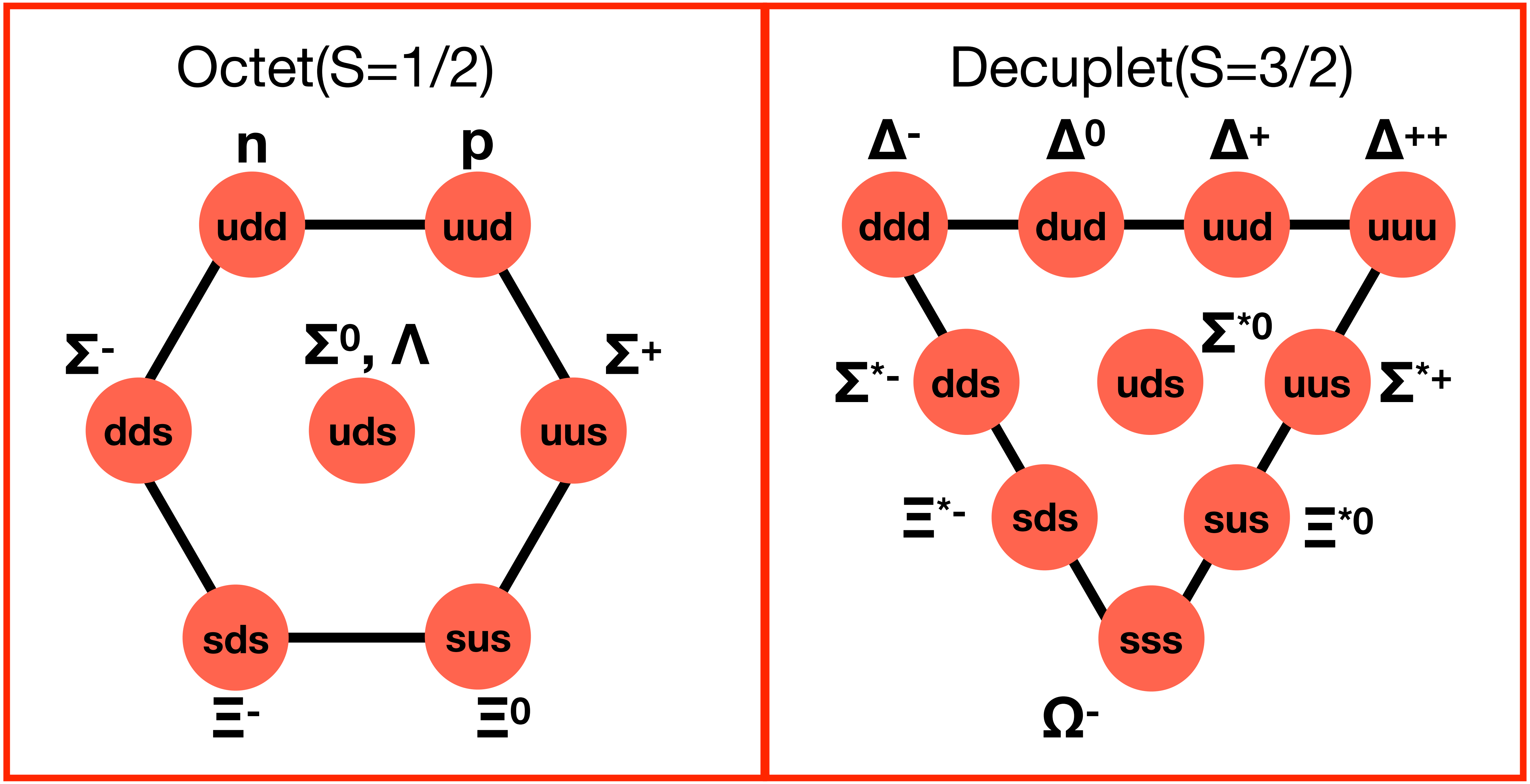
Dibaryon (B=2)



Deuteron(1930s)

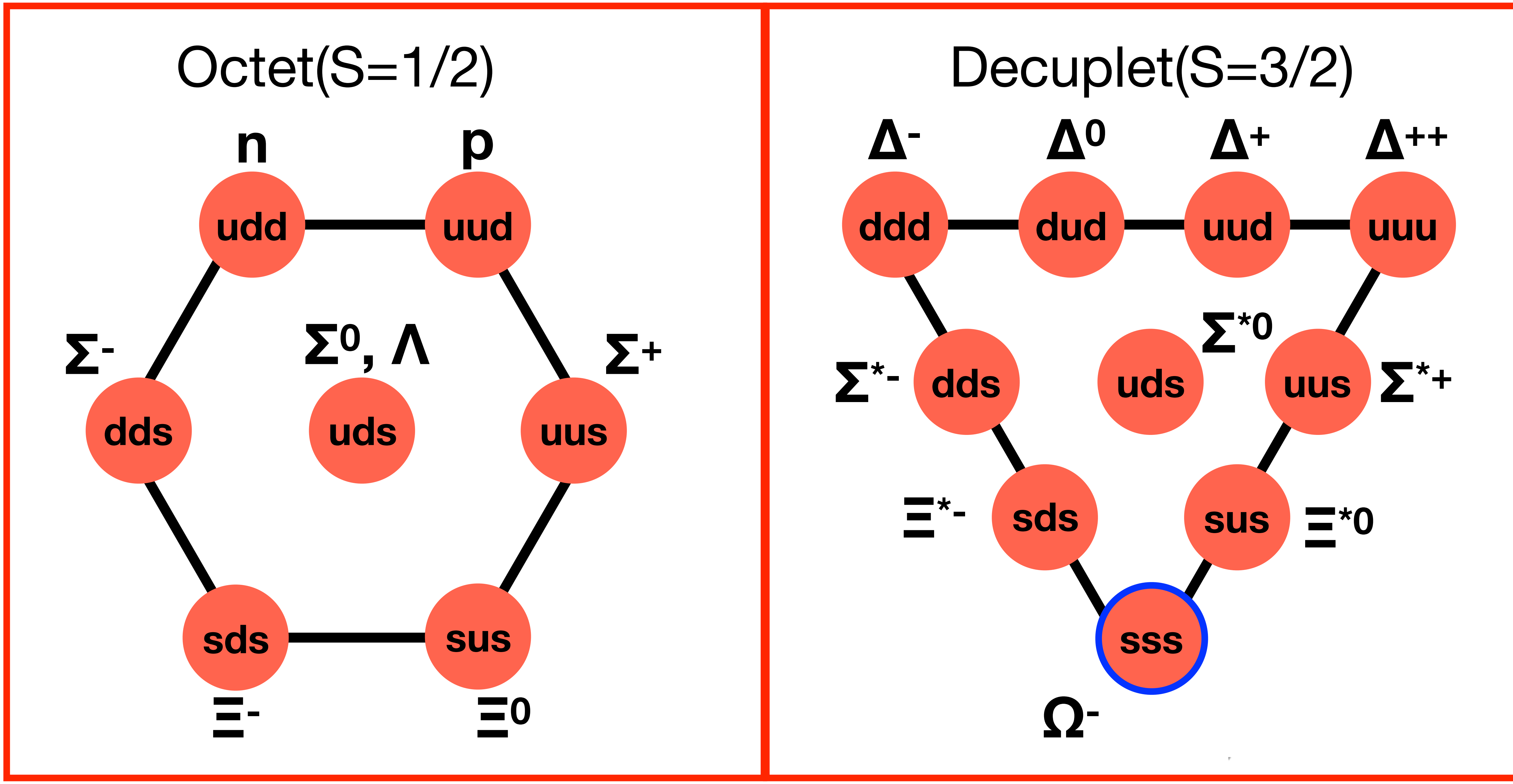
Dibaryon = Bound (or resonance) two baryon states

Introduction: SU(3) classification for Baryon (B=1)



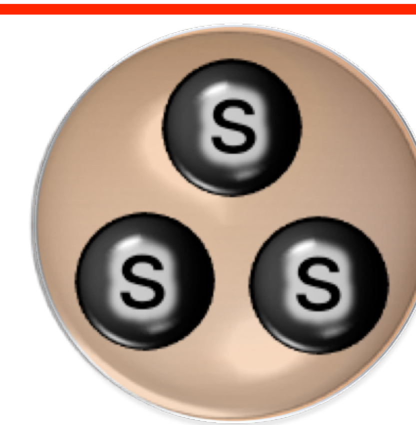
All octet baryons are stable under strong decay

Introduction: SU(3) classification for Baryon (B=1)

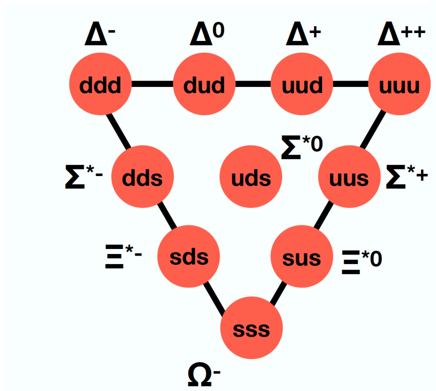


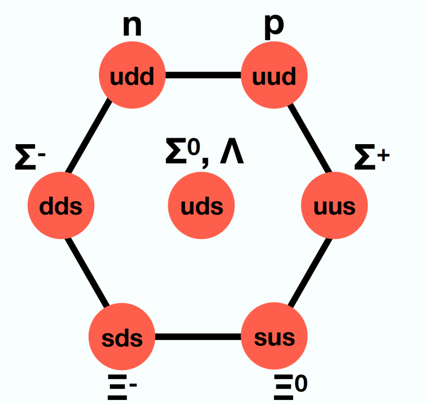
All octet baryons are stable under strong decay

In Decuplet baryons, only **Ω-baryon is stable**



Introduction: SU(3) classification for Dibaryon candidates (B=2)





1) octet-octet system

H-dibaryon(J=0) Jaffe (1977)

$$8 \otimes 8 = 27 \oplus 8_s \oplus \boxed{1} \oplus \boxed{\overline{10}} \oplus 10 \oplus 8_a$$

Deuteron(J=1)

2) decuplet-octet system

NΩ system (J=2)

$$10 \otimes 8 = 35 \oplus \boxed{8} \oplus 10 \oplus 27$$

Goldman et al (1987)

3) decuplet-decuplet system

found as a "resonance" by CELSIUS/WASA, 2009

$$10 \otimes 10 = \boxed{28} \oplus 27 \oplus 35 \oplus \boxed{\overline{10}}$$

ΩΩ system (J=0)

Zhang et al(1997)

ΔΔ system (J=3)

Dyson, Xuong (1964)
Oka, Yazaki(1980)

Previous model works on ΩΩ in J=0

$$\Delta M_{\Omega\Omega} \equiv E_{\Omega\Omega} - 2M_{\Omega}$$

SU(3) chiral quark model
Z.Y. Zhang et al(1997)

$$\Delta M_{\Omega\Omega} = -166\mathrm{MeV}$$

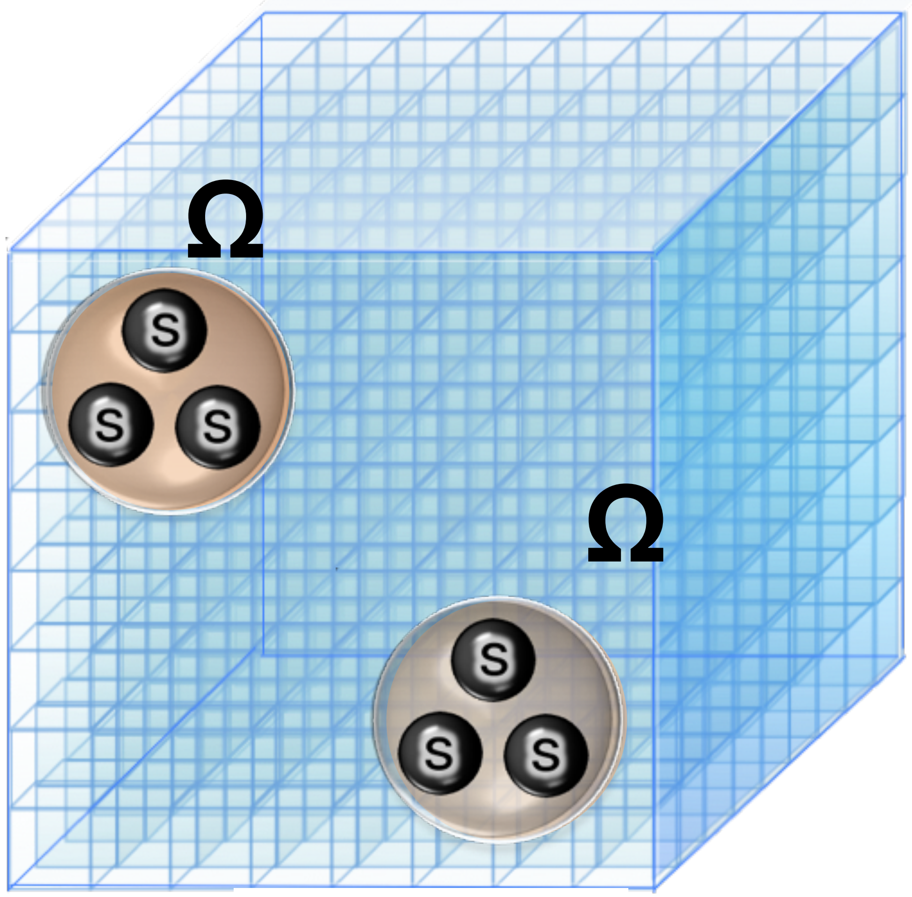
Quark Disloc/Color-screen model
F. Wang et al(1992)

$$\Delta M_{\Omega\Omega} = 43 \pm 18\mathrm{MeV}$$

- **Bound/unbound problem highly depends on models and their parameters.**
- **To clarify ΩΩ interaction in our world, first-principle calculations are needed.**

Baryon-Baryon interaction from lattice QCD
-HAL method-

Aoki, Hatsuda, Ishii, PTP123, 89 (2010)



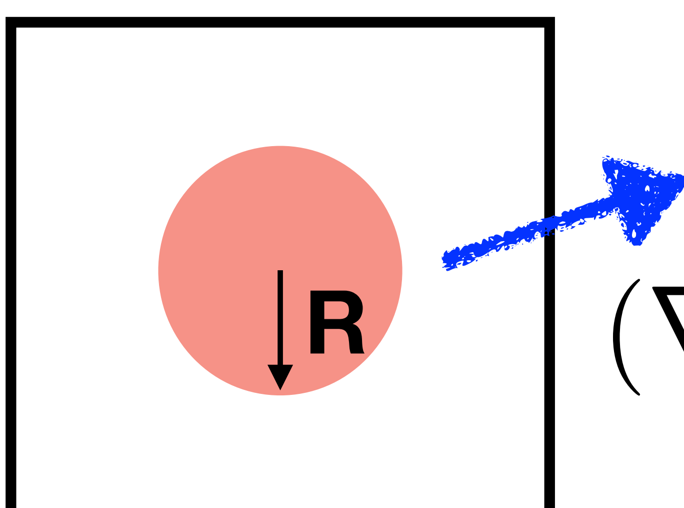
c.f. anothor method: Luscher's direct method

Nambu-Bethe-Salpeter (NBS) w.f.

$$\Psi_n(\vec{r})\, e^{-E_n t} = \sum_{\vec{x}} \langle 0|\, B_1(t, \vec{r}+\vec{x}) B_2(t, \vec{x})\, |E_n\rangle$$

Local operators B_1 and B_2 for Ω baryon

$$\Omega_{\alpha,k}(x) = \epsilon^{abc}\left[s_a^T(x) C\gamma_k s_b(x)\right] s_{c,\alpha}(x)$$



In asymptotic region (r>>R)
Helmholtz eq. is satisfied:

$$(\nabla^2 + k^2)\Psi(\vec{r}) = 0$$

$$\Psi(\vec{r}) \simeq A\frac{\sin(kr - l\pi/2 + \delta(k))}{kr}$$

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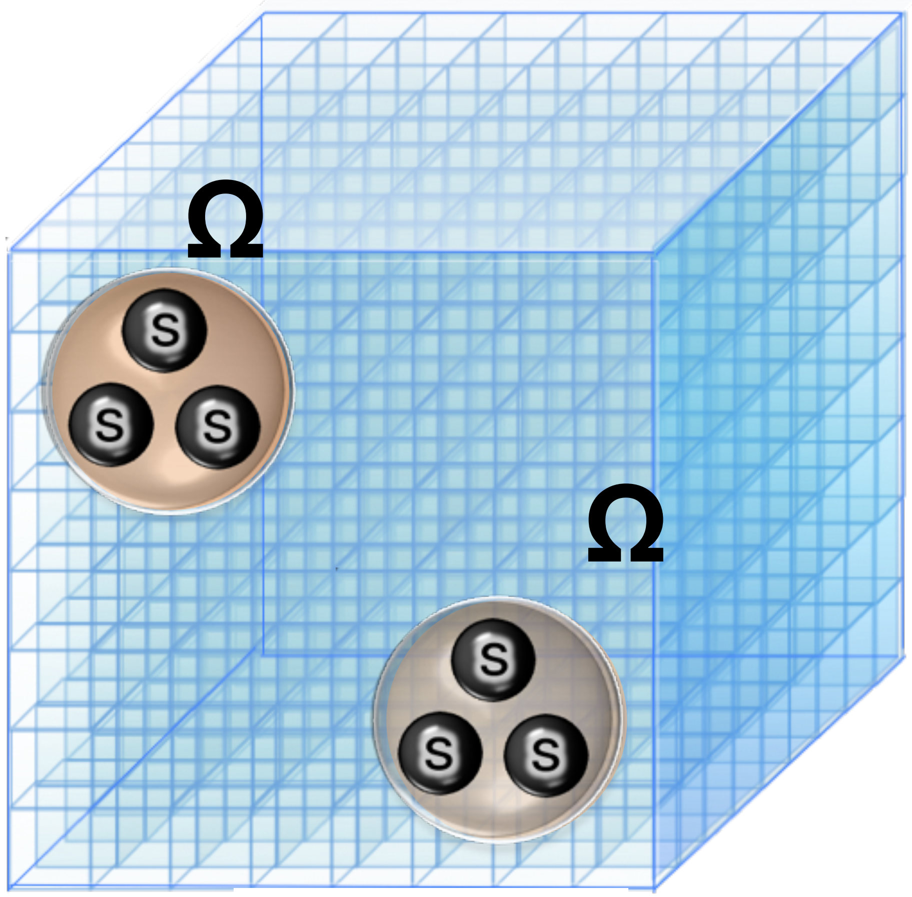

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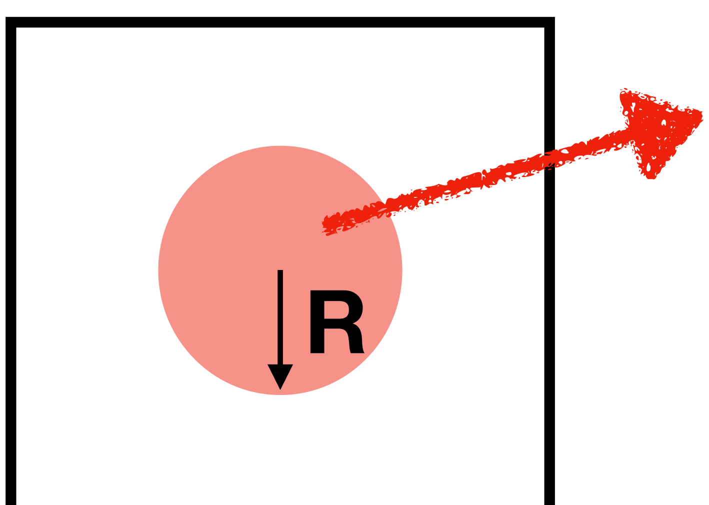


In interacting region,

Schroedinger type equation is satisfied

$$\left(\vec{p}_n^2 + \nabla^2\right)\Psi_n(\vec{r}) = 2\mu \int d\vec{r'}\, U(\vec{r}, \vec{r'})\Psi_n(\vec{r'})$$

Aoki, Hatsuda, Ishii, PTP123, 89 (2010)

Nonlocal potential U(r,r')

$$\left(\vec{p}_n^2 + \nabla^2\right) \Psi_n\left(\vec{r}\right) = 2\mu \int d\vec{r'} U(\vec{r}, \vec{r'}) \Psi_n(\vec{r'})$$

- The potential is energy-independent but non-local.
- The local leading potential can be obtained by its derivative expansion (c.f. Okubo-Marshak expansion):

$$\begin{aligned} U(\vec{r}, \vec{r'}) &= \underline{V_c(r) + V_\sigma(r)(\vec{S}_1 \cdot \vec{S}_2) + S_{12} V_{T_1}(r)} \\ &\quad + O(\nabla^2) \\ &= \underline{V_C^{eff}(r)} + O(\nabla^2) \end{aligned}$$

- The convergence of the expansion can be checked.
- The NLO term is explicitly determined by utilizing two source functions (Iritani et. al, arXiv:1805.02365)

N. Ishii et al, PLB712, 437 (2010)

Time-dependent HAL method

- original (t-indep) HAL method=> applicable for each NBS w.f.

$$G_{BB}(\vec{r},t) = \langle 0|\, B(\vec{y},t)B(\vec{x},t)\bar{\mathcal{J}}(t_0;J^P)\,|0\rangle$$

$$\mathcal{R}(\vec{r},t;J^P) = G_{BB}(\vec{r},t)/G_B(t)G_B(t) = \sum_n A_n \psi_n(\vec{r}) e^{-(W_i-2m_B)t}$$

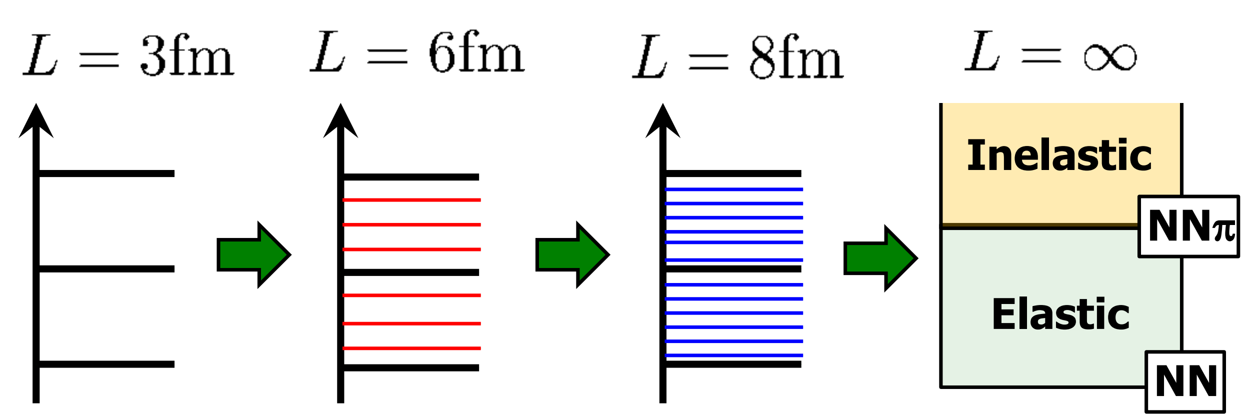
$$\int d\boldsymbol{r}'\, U(\boldsymbol{r},\boldsymbol{r}')\underline{\psi_{W_0}(\boldsymbol{r}')} = (\underline{E_{W_0}} - H_0)\underline{\psi_{W_0}(\boldsymbol{r})}$$

$$\int d\boldsymbol{r}'\, U(\boldsymbol{r},\boldsymbol{r}')\underline{\psi_{W_1}(\boldsymbol{r}')} = (\underline{E_{W_1}} - H_0)\underline{\psi_{W_1}(\boldsymbol{r})}$$

$$\cdots$$

- Many states contribute to the R-correlator
- As lattice size increases, the extraction of g.s. becomes difficult

$$E_n \sim 2\sqrt{m_B^2 + (2\pi n/L)^2} \sim E_0 \ (L \gg 1)$$



The same problem appears for the direct method (Iritani et al. JHEP(2016), PRD(2017))

N. Ishii et al, PLB712, 437 (2010)

Time-dependent HAL method

- new (t-dep) HAL method=> directly applicable for R-correlator

$$\int d\boldsymbol{r}' U(\boldsymbol{r},\boldsymbol{r}') \underline{\psi_{W_0}(\boldsymbol{r}')} = (\underline{E_{W_0}} - H_0)\underline{\psi_{W_0}(\boldsymbol{r})}$$
$$\int d\boldsymbol{r}' U(\boldsymbol{r},\boldsymbol{r}') \underline{\psi_{W_1}(\boldsymbol{r}')} = (\underline{E_{W_1}} - H_0)\underline{\psi_{W_1}(\boldsymbol{r})}$$
$$\cdots$$

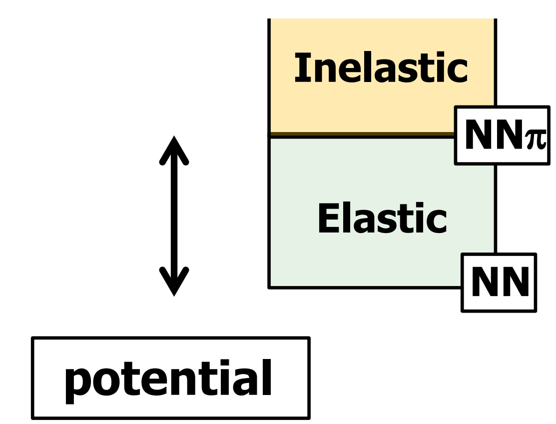
$$\Delta E_n = \frac{k_n^2}{m_B} - \frac{\Delta E_n^2}{4m_N}$$

All equations are combined as

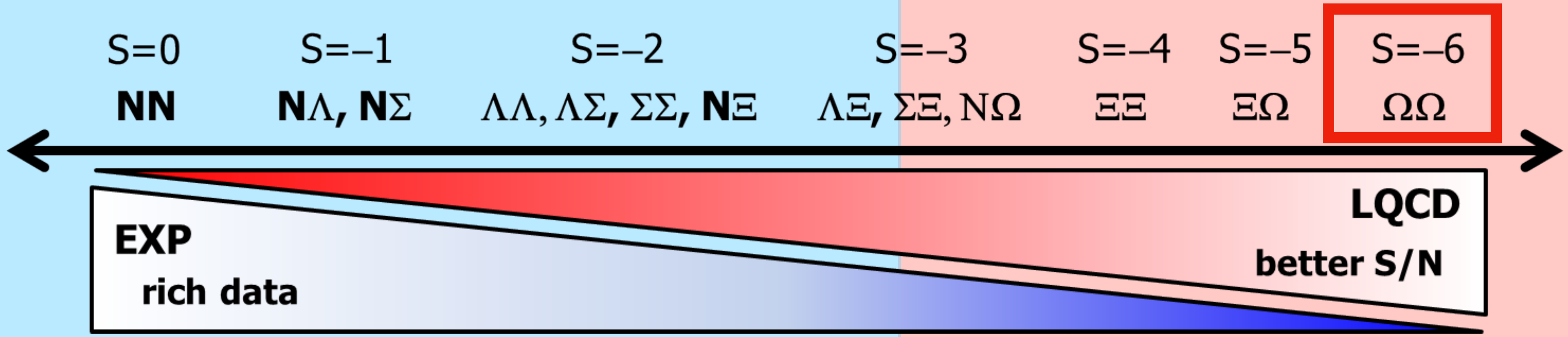
$$\left(\frac{1}{4m_B}\frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} + \frac{\nabla^2}{m_B}\right)\mathcal{R} = \int U(\vec{r},\vec{r'})\mathcal{R} d^3 r'$$

G.S. saturation is not required.
=>"**Elastic state saturation**" is required

Weaker condition as L → ∞



Experiment **Lattice QCD**



- rich data for less strange quarks
- More strange quarks,
  more difficult experiment
  due to short life time

- better S/N for more strange quarks
- Less strange quarks,
  more difficult numerical simulation
  due to increasing statistical noise

ΩΩ system is the best S/N ratio calculation on lattice

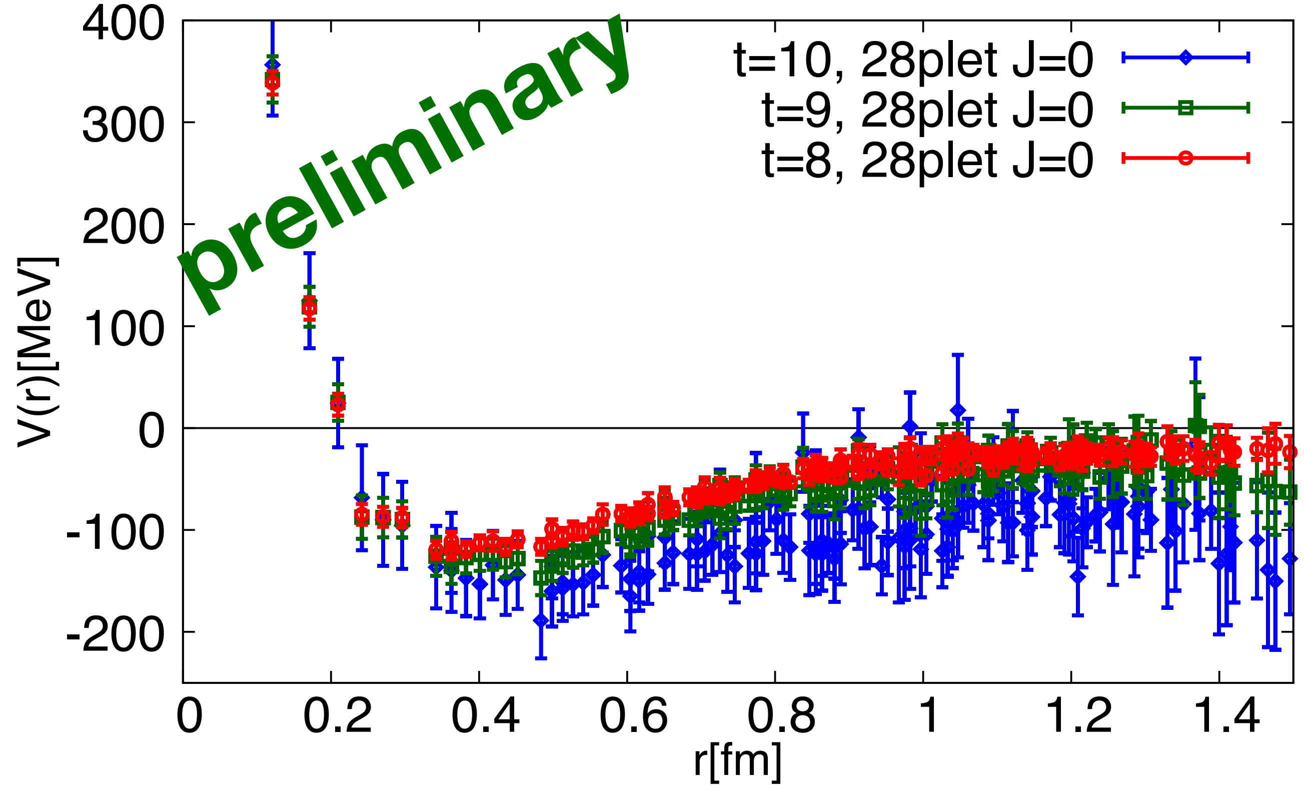
Interactions in ΩΩ (J=0) system

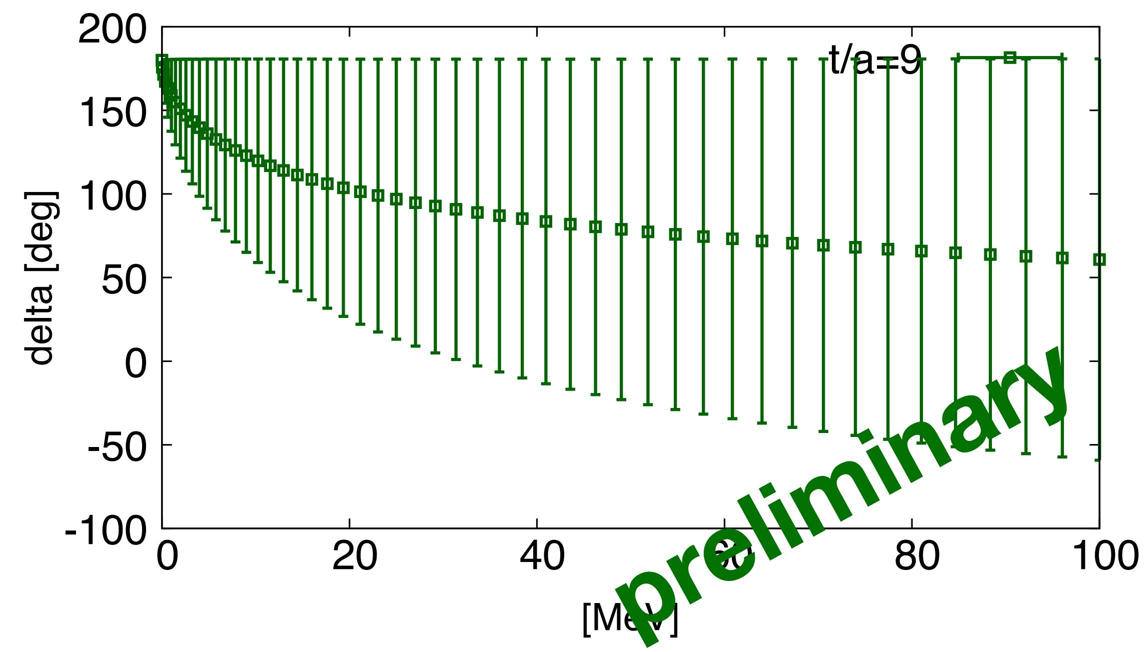
1) Nf = 2+1, L = 1.93fm, m_π=1015MeV, SU(3) limit
2) Nf=2+1, L = 3fm, m_π=700MeV, SU(3) breaking
3) Nf=2+1, L = 8.1fm, m_π=146MeV, almost physical mass

SG and K. Sasaki (HAL QCD)

1)Nf=2+1 full QCD with L = 1.93fm m_π= 1015MeV, SU(3) limit
ΩΩ in *J* =0

m_Ω = 2220 MeV





- Short range repulsive core and attractive pocket are found
- Phase shift shows the system is in the unitary limit

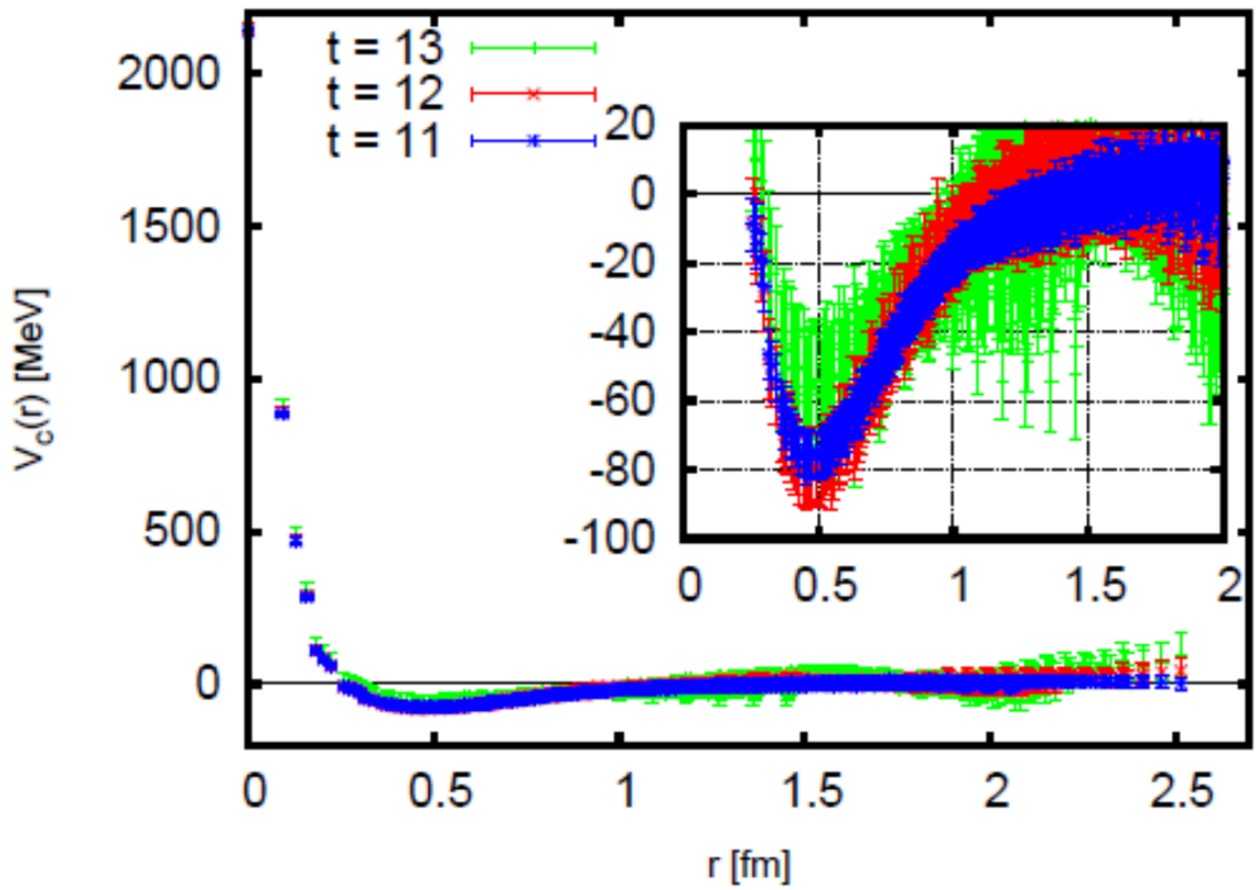
let's consider the lighter quark masses with SU(3) breaking

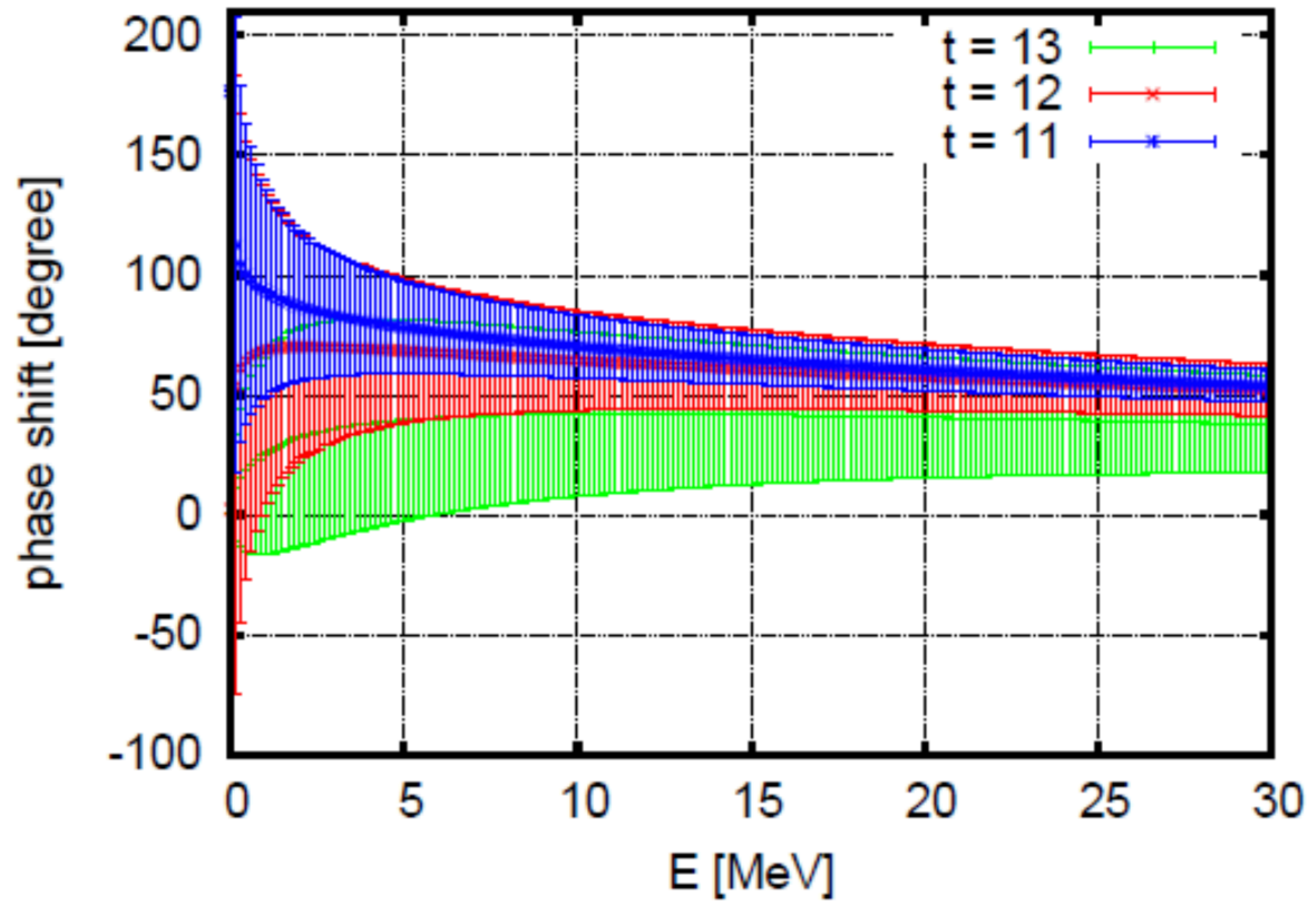
M. Yamada et. al.(HAL QCD), PTEP2015

2)Nf=2+1 full QCD with L = 3fm, m_π= 700MeV w. SU(3) breaking

ΩΩ in *J* =0

m_Ω= 1970MeV





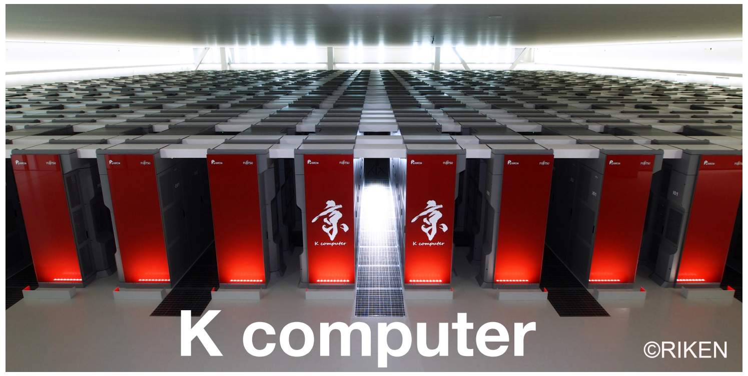
- Short range repulsive core and attractive pocket are found
- Potential is nearly independent on “t” within error
- Phase shift shows rapid changes depending on “t”
- *The system may appear close to the unitary limit*

c.f. Direct method by Buchoff et al., PRD(2012): L=4fm, m_π= 390MeV
a=0.16 ± 0.22 fm <= unitary limit

Numerical Setup at (almost) physical mass

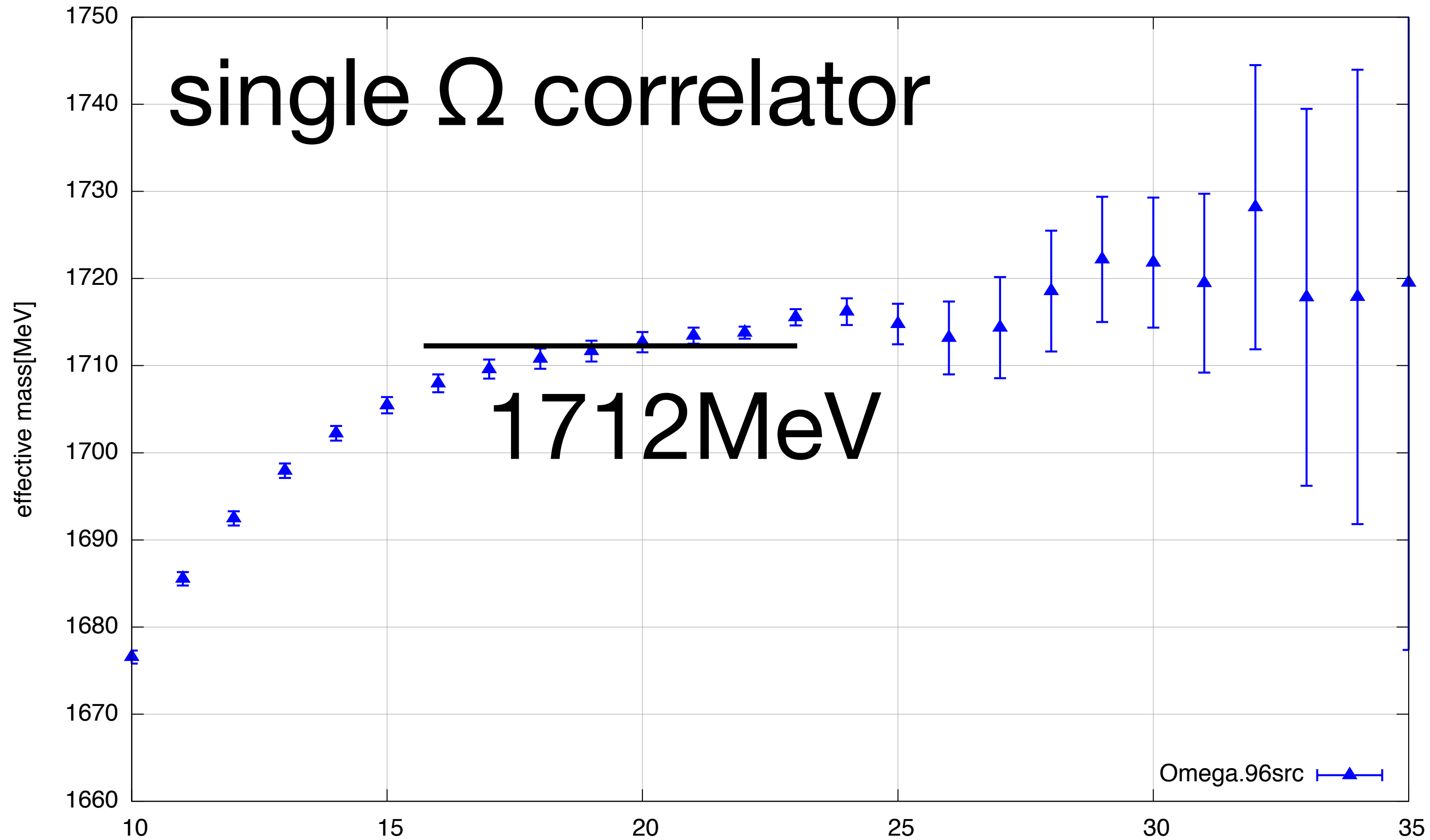
2+1 flavor gauge configurations

- Iwasaki gauge action & O(a) improved Wilson quark action
- a= 0.0846 [fm], a^{-1} = 2333 [MeV]
- 96^3x96 lattice, L = 8.1[fm]
- 400 confs x 48 source positions x 4 rotations



Wall source is employed. only S-wave state is produced.

	[MeV]	phys.
π	146	8%
K	525	6%
N	964	3%
Ω	1712	2%

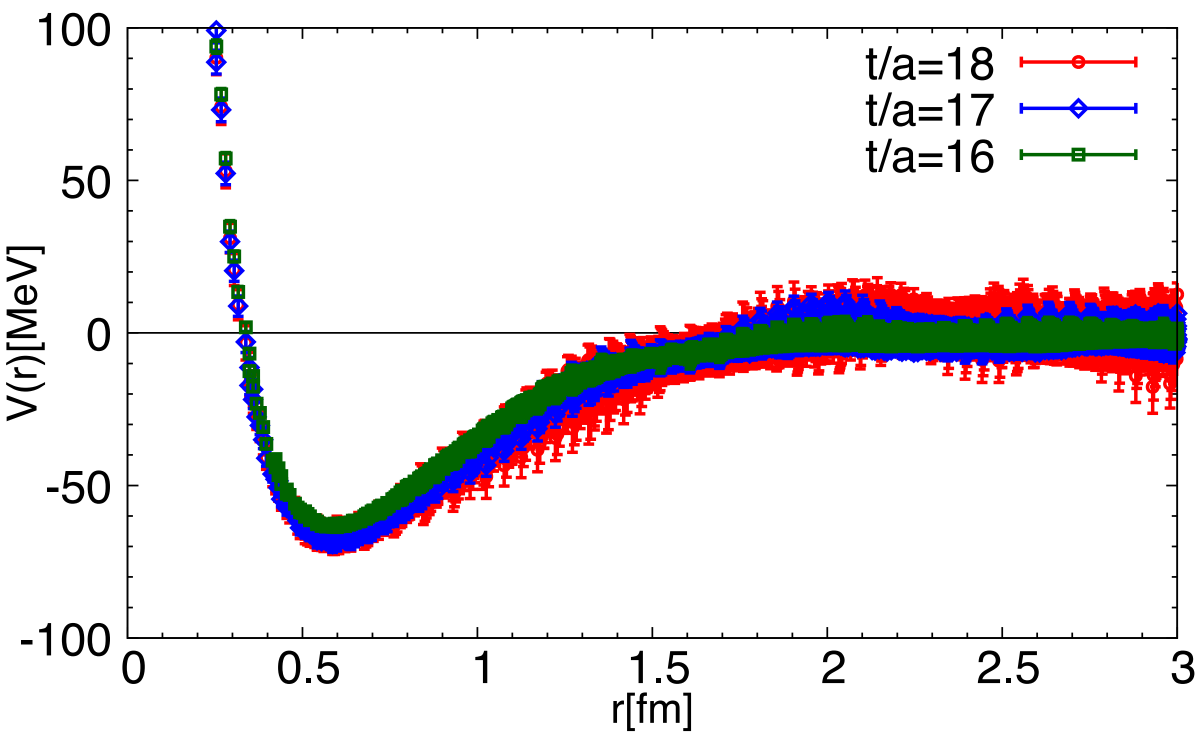


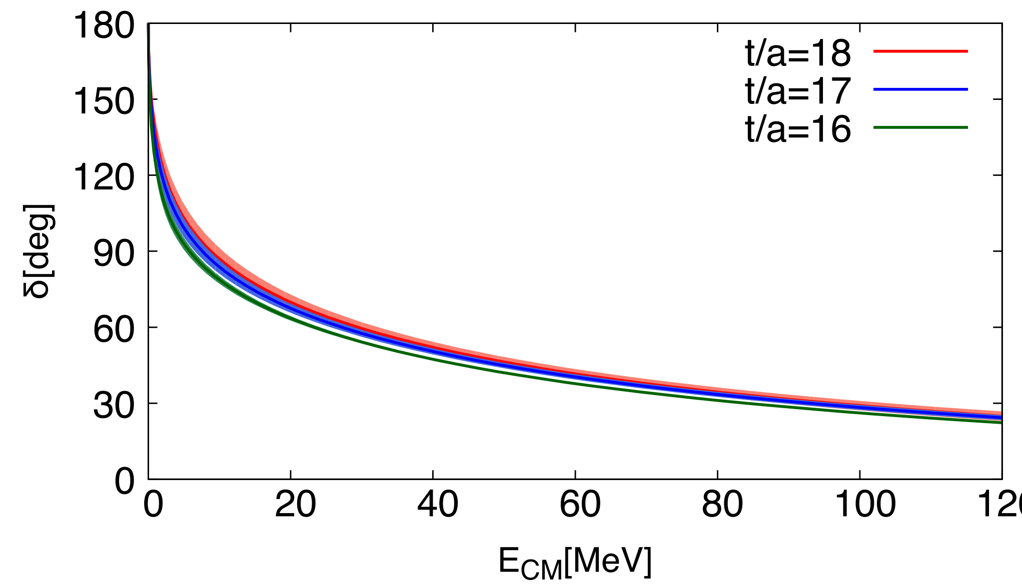
SG and K. Sasaki et.al.(HAL), PRL(2018)

ΩΩ in *J* =0

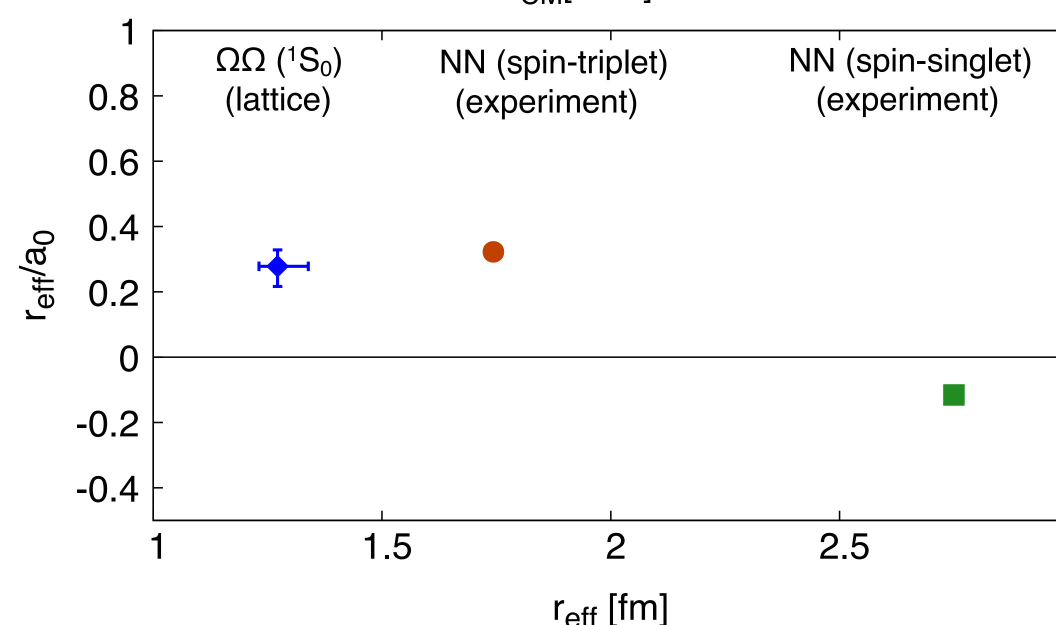
"most strange dibaryon"

Nf=2+1 full QCD with L = 8.1fm, m_π= 146MeV





$$a_0^{(\Omega\Omega)} = 4.6(6)(^{+1.2}_{-0.5}) \quad \mathrm{fm},$$
$$r_{\mathrm{eff}}^{(\Omega\Omega)} = 1.27(3)(^{+0.06}_{-0.03}) \quad \mathrm{fm}.$$



- Short range repulsive core and attractive pocket are found
- Phase shift shows the presence of a bound state
- The state is very close to the unitary region (r/a<1)

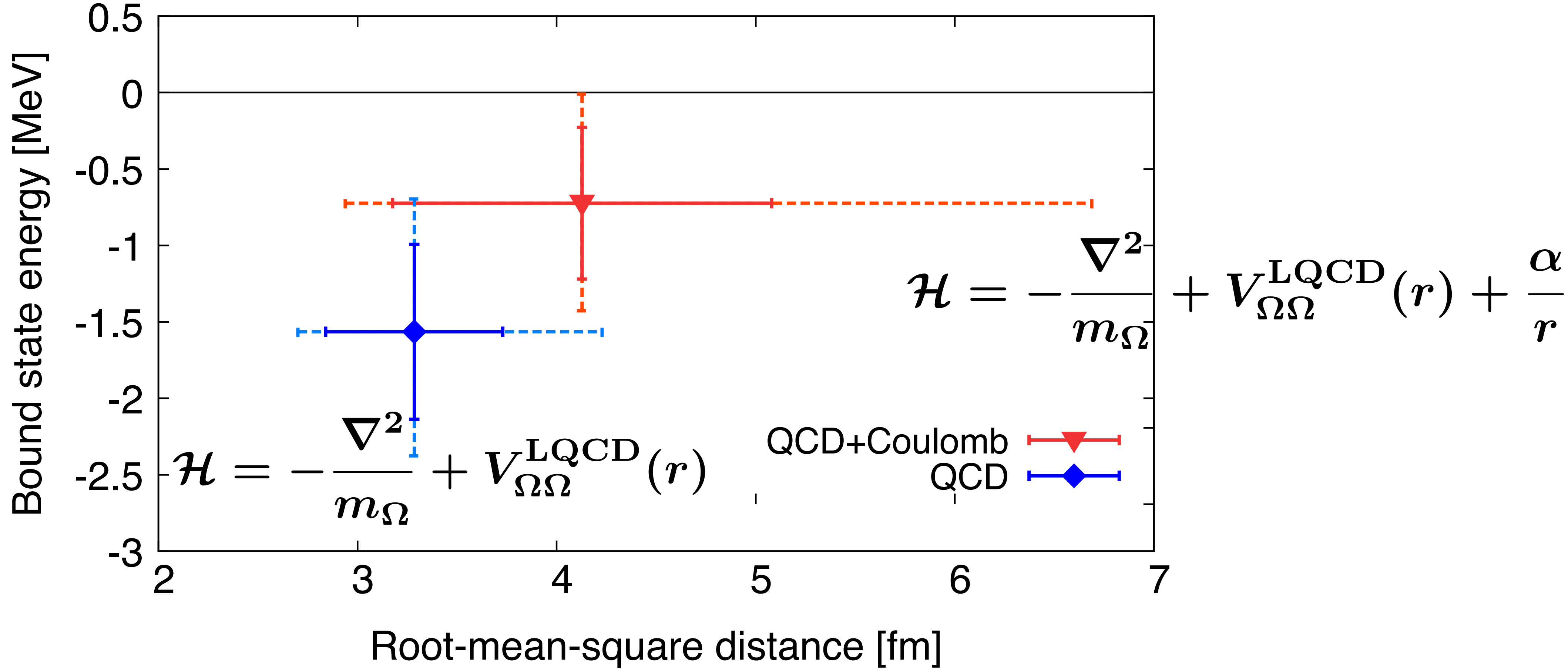
SG and K. Sasaki et.al.(HAL), PRL(2018)

ΩΩ in *J* =*0*

Binding energy and the Coulomb effect

"most strange dibaryon"

Q=-1

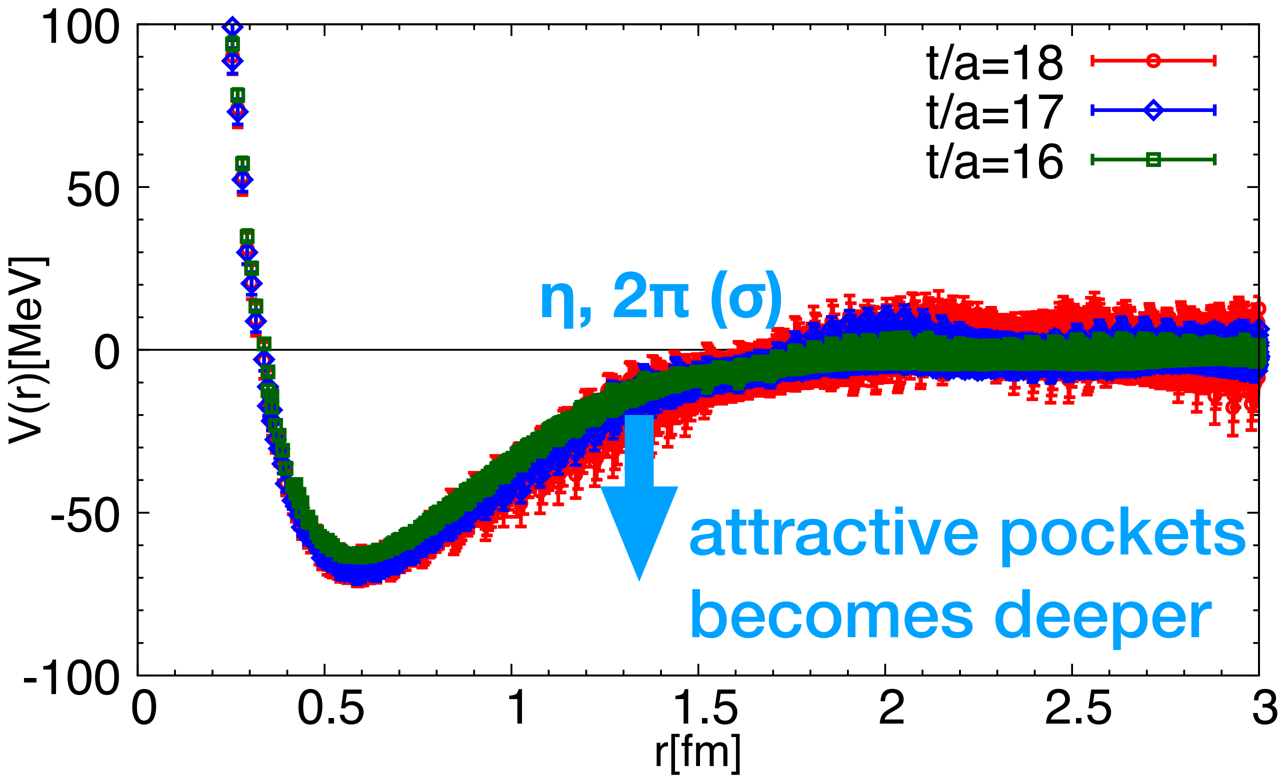


$$(B^{\rm (QCD)}_{\Omega\Omega}, B^{\rm (QCD+Coulomb)}_{\Omega\Omega}) = (1.6(6)\mathrm{MeV}, 0.7(5)\mathrm{MeV})$$

Conservative estimate at exact phys. pt.

m_π=146 MeV -> 135 MeV, m_Ω= 1712MeV -> 1672 MeV

B.E. ↑



V.S.

B.E. ↓

$$\mathcal{H} = \boxed{-\frac{\nabla^2}{m_\Omega}} + V_{\Omega\Omega}^{\mathrm{LQCD}}(r)$$

kinetic energy is increasing
-> B.E. is reduced

conservative estimate:
only change the mass of kinetic term

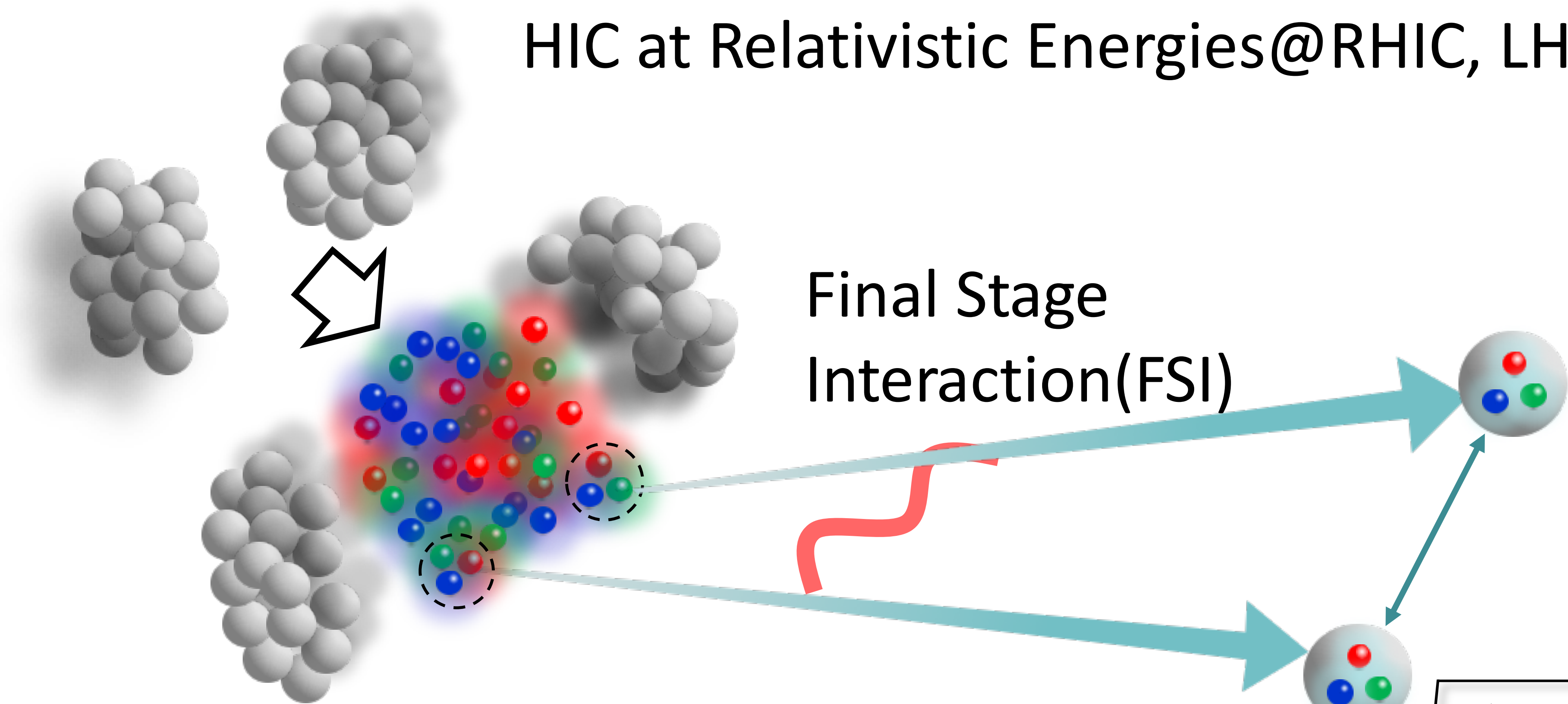
$$(B_{\Omega\Omega}^{(\mathrm{QCD})}, B_{\Omega\Omega}^{(\mathrm{QCD+Coulomb})}) = (1.6(6)\mathrm{MeV}, 0.7(5)\mathrm{MeV})$$
$$\to (1.3(5)\mathrm{MeV}, 0.5(5)\mathrm{MeV})$$

These changes are within errors

Kenji Morita (Wroclaw/RIKEN)

How HIC Can Tell Us Interaction?

HIC at Relativistic Energies@RHIC, LHC



$$Q = \sqrt{-\left(\frac{p_1 - p_2}{2} - \frac{(p_1 - p_2)\cdot P}{P^2}P\right)^2}$$

Measuring Pair Correlation

→Constrain Pairwise Interaction

Deviation from “1”,
tells us the behavior of the interaction

$$C_{AB}(Q) = \frac{N^{\mathrm{pair}}_{AB}(Q)}{N_A N_B(Q)} = \begin{cases} 1 & \textbf{No Correlation} \\ \text{others} & \textbf{Interaction, Interference etc} \end{cases}$$

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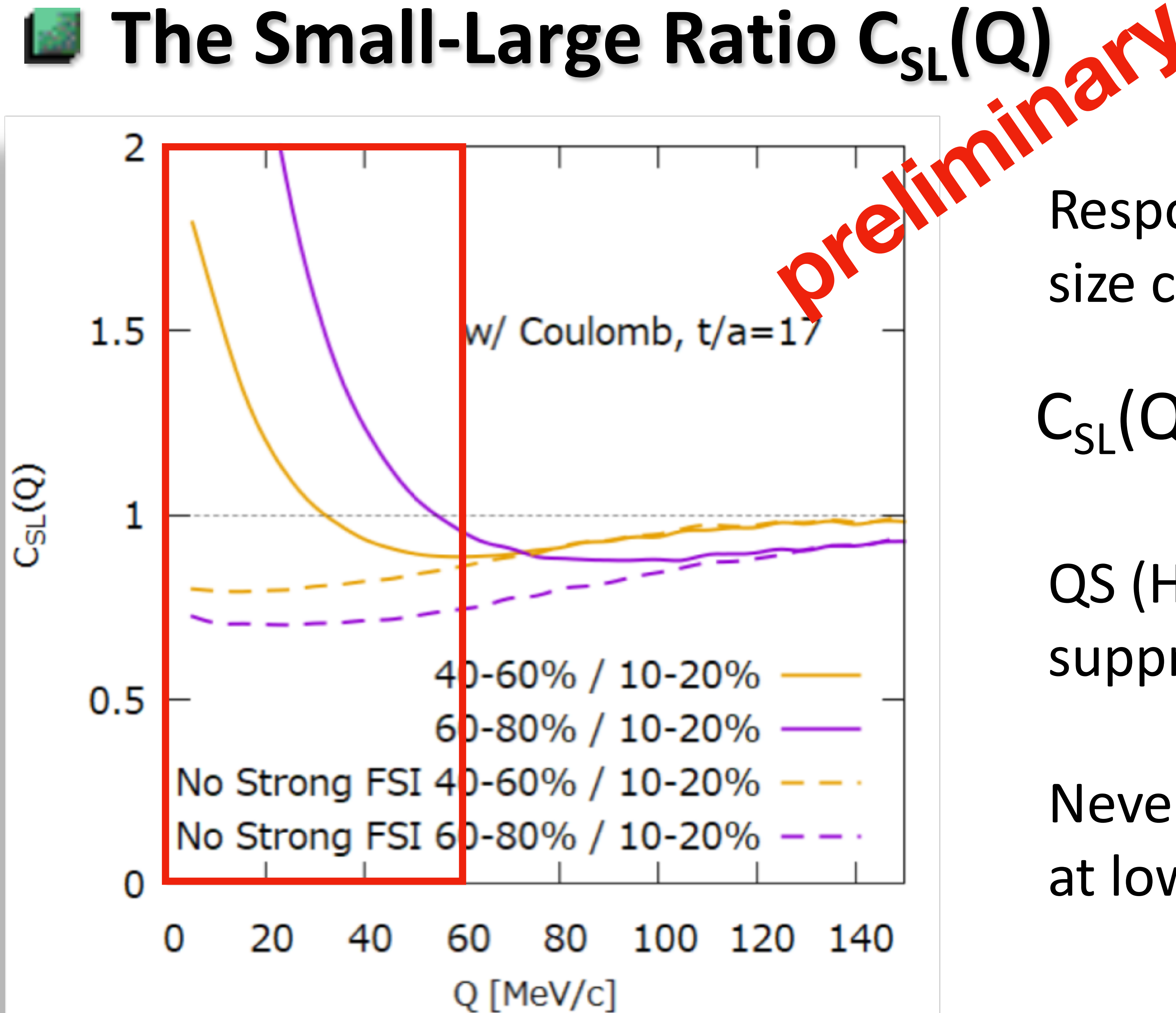
ΩΩ Correlation@LHC

The Small-Large Ratio $C_{SL}(Q)$



Response to system size change

$$C_{SL}(Q) = C_R(Q)/C_{R'}(Q)$$

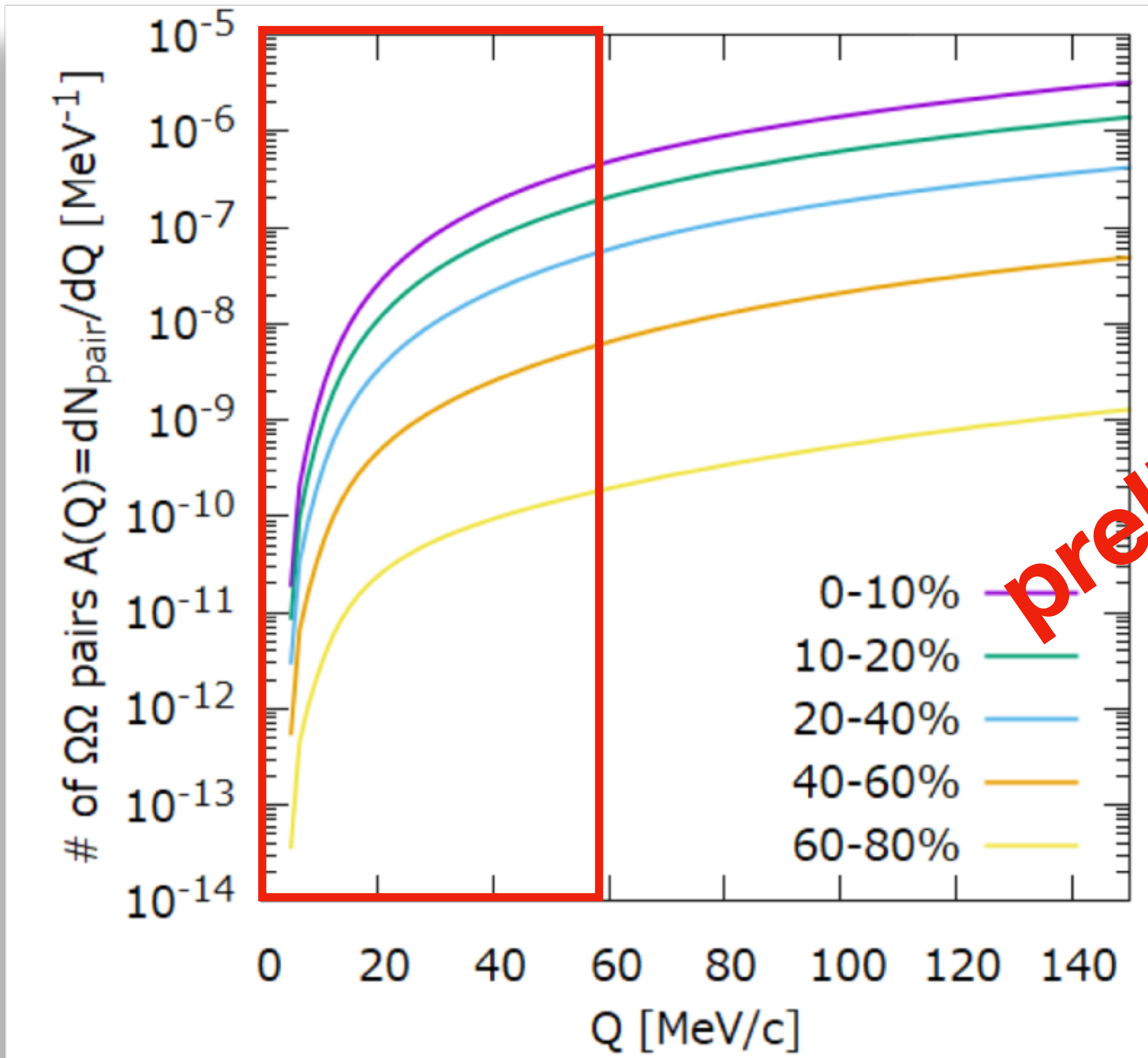
QS (HBT) Correlation suppresses the ratio

Nevertheless FSI dominates at low Q

ΩΩ Correlation: Statistics?

Kenji Morita (Wroclaw/RIKEN)

of pair A(Q)



To have 100 pairs at low Q:

Acceptance×Efficiency : 0.01

10^{11} events: unreachable at LHC

Not impossible in Future
J-PARC ?(int. rate 10^8 Hz)

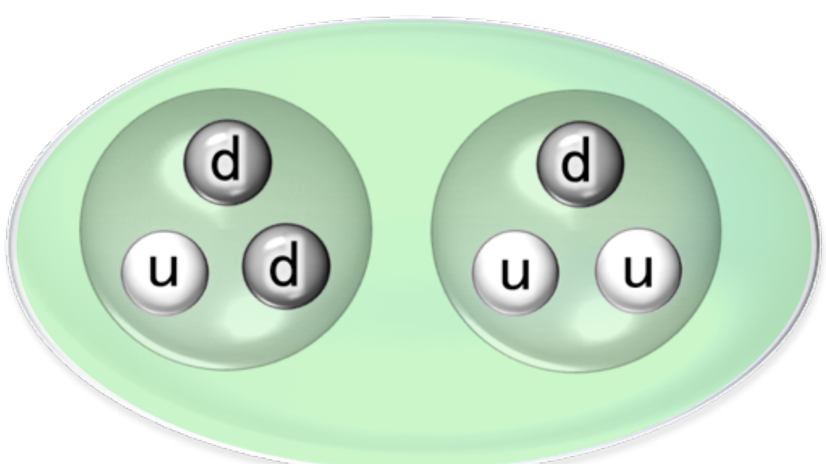
Summary

- We have investigated ΩΩ interaction (J=0) from lattice QCD

- (almost) physical pion masses:
  ΩΩ interaction in $^1S_0$
  - short range repulsive and attractive pocket
  - a very shallow bound state
  [Most strange dibaryon, di-Omega]

Dibaryon

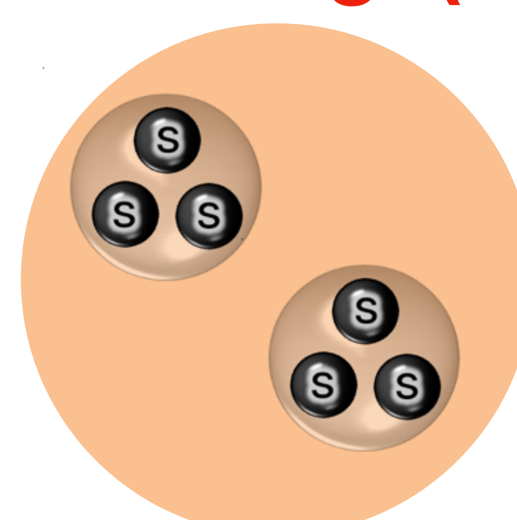
Deuteron



found in 1930s

+

di-Omega(ΩΩ)



?

will be found by J-PARC or FAIR?

Back Slides

Estimate NLO contribution for ΩΩ system at almost physical pt.

$$U(\boldsymbol{r}, \boldsymbol{r}') = V_0(r)\delta(\boldsymbol{r} - \boldsymbol{r}') + \sum_{n=1} V_{2n}(r)\nabla^{2n}\delta(\boldsymbol{r} - \boldsymbol{r}')$$

- Determining the higher order potentials explicitly by utilizing multiple quark sources is the best way to estimate their contributions.

c.f. Iritani et. al (HAL QCD), arXiv:1805.02365

$$V(r) = R^{-1}(\boldsymbol{r}, t)\left(\frac{\nabla^2}{m_\Omega} - \frac{\partial}{\partial t} + \frac{1}{4m_\Omega}\frac{\partial^2}{\partial t^2}\right)R(\boldsymbol{r}, t)$$
$$= V_0(r) + \boxed{\sum_{n=1} V_{2n}(r)R^{-1}(\boldsymbol{r}, t)\nabla^{2n}R(\boldsymbol{r}, t)}$$

- Instead, we have estimated in two alternative ways:
  1. their contributions are estimated from its t-dependence
  2. pertubative estimate on the binding energy
     $|V_2 / m_{2\pi}| \sim |V_0|$ + several functional forms such as square-well form…
     => B.E. changes less than 20% in all cases
     (within systematic errors from the t-dependence)