The Weak Charge: From Low Energy to the Z-pole

Misha Gorshteyn - Mainz University

In collaboration with

Chuck Horowitz,
Michael Ramsey-Musolf,
Hubert Spiesberger,
…

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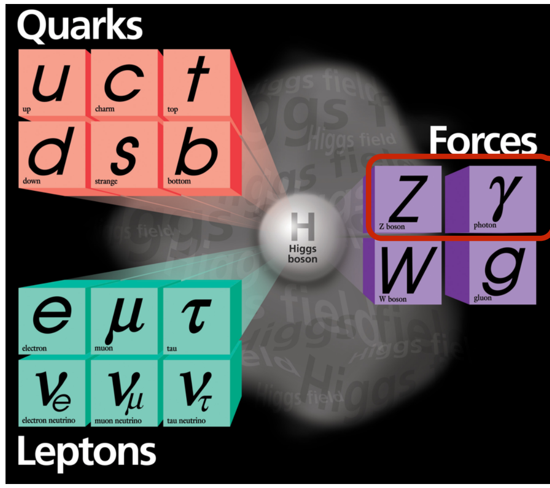
Outline

- Running $\sin^2\theta_W$ in Standard Model
- Sensitivity to New Physics
- $\sin^2\theta_W$ at low energy: weak charges
- Importance of electroweak boxes

Weak Mixing Angle



$$\begin{pmatrix} \gamma \\ Z^0 \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} B^0 \\ W^0 \end{pmatrix}$$

WMA determines the relative strength of the weak NC vs. e.-m. interaction

Tree-level: $\sin^2\theta_W = 1 - \frac{M_W^2}{M_Z^2} = \frac{g'^2}{g^2 + g'^2}$

Modified by radiative corrections

Universal radiative corrections can be absorbed into running $\sin^2\theta_W$

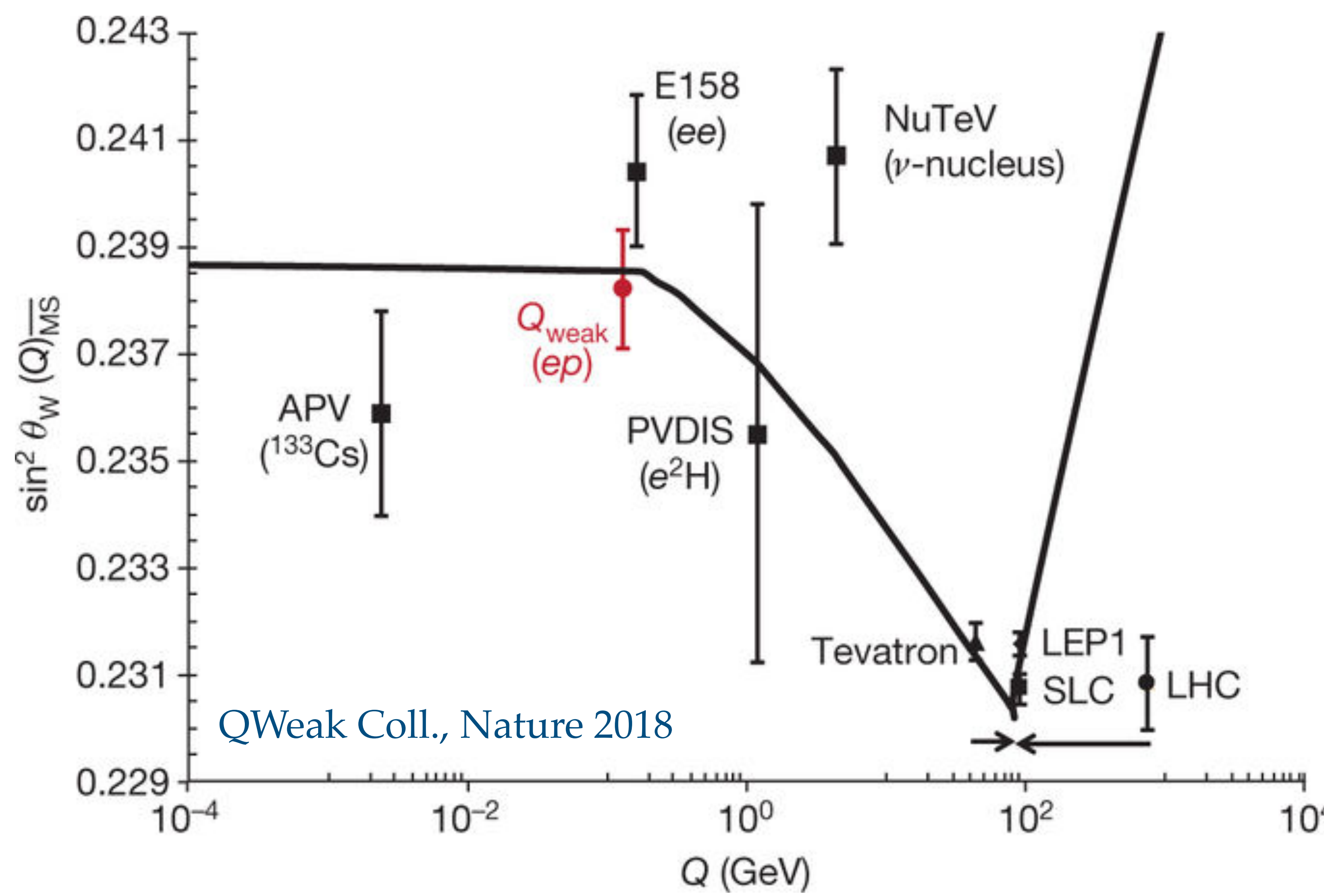
MS-bar scheme $\sin^2\theta_W = \frac{g'^2}{g^2 + g'^2}$

Most precise measurements to date:

QWeak (see Kent's talk Wed.) and LEP1/SLC (Z-pole) 3% apart

SM prediction: 2 x 10^{-5} precision

Erler, Ramsey-Musolf, hep-ph/0409169
Erler, Ferro Hernandez, arXiv:1712.09146



Precision running of $\sin^2\theta_W(\mu)$

Main idea:
running of WMA with respect to running of α

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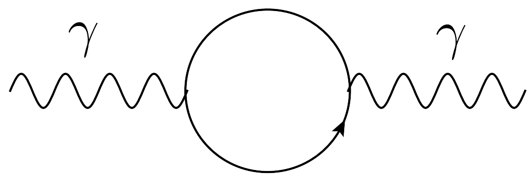

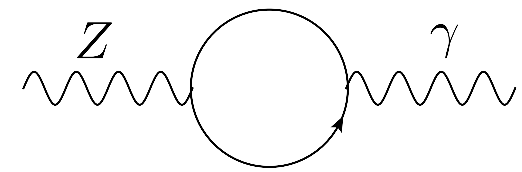

RG equation for em and weak vector coupling very similar

$$\mu^2 \frac{d\hat{\alpha}}{d\mu^2} = \frac{\hat{\alpha}^2}{\pi} \left[\frac{1}{24} \sum_i K_i \gamma_i Q_i^2 + \sigma \left(\sum_q Q_q \right)^2 \right]$$

$$\hat{v}_f = T_f - 2Q_f \sin^2 \hat{\theta}_W$$

$$\mu^2 \frac{d\hat{v}_f}{d\mu^2} = \frac{\hat{\alpha} Q_f}{24\pi} \left[\sum_i K_i \gamma_i \hat{v}_i Q_i + 12\sigma \left(\sum_q Q_q \right) \left(\sum_q \hat{v}_q \right) \right]$$

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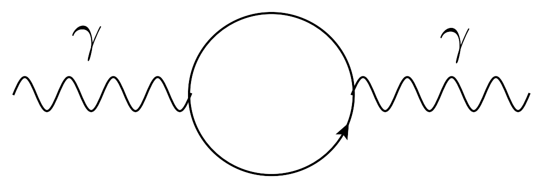

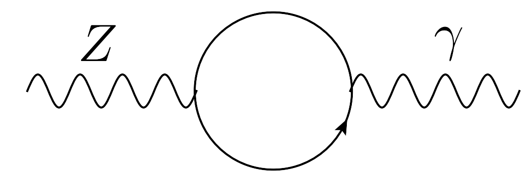

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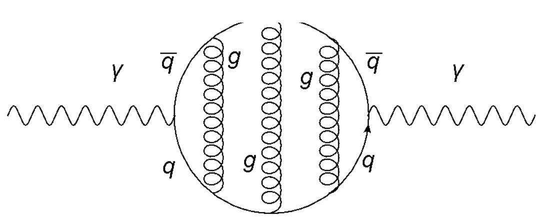
Connected contributions
Q_i, v_i - el. and weak charges
γ_i - field-dependent constants
K_i - h.o. coefficients



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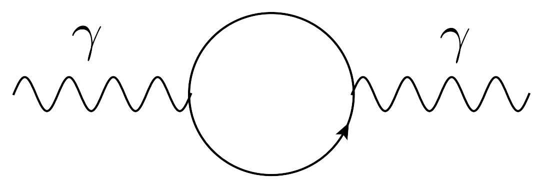

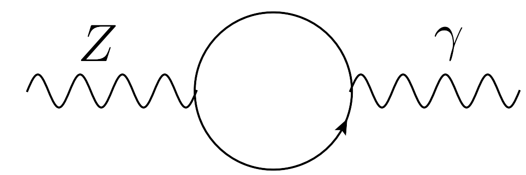

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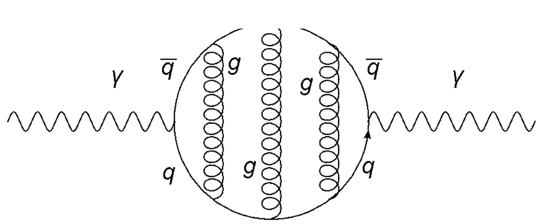
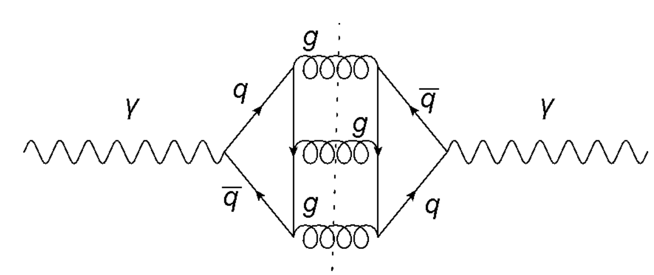
Disconnected contributions
σ - h.o. coefficients





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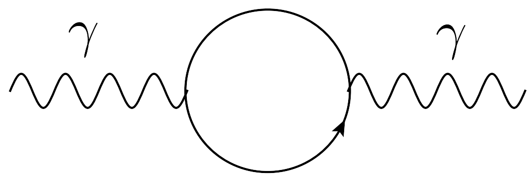

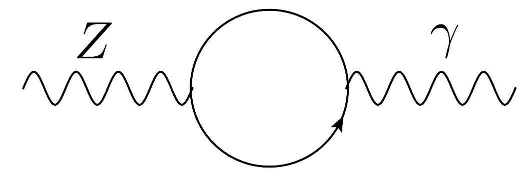

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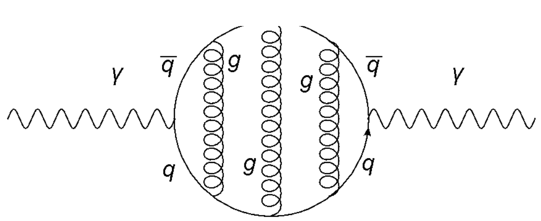
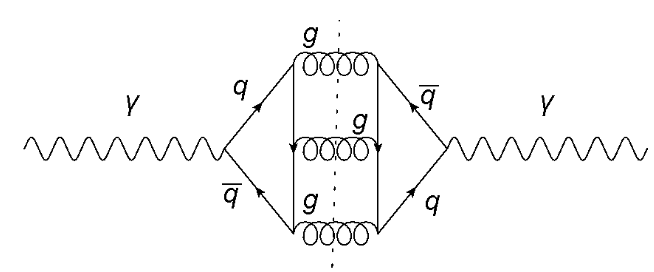


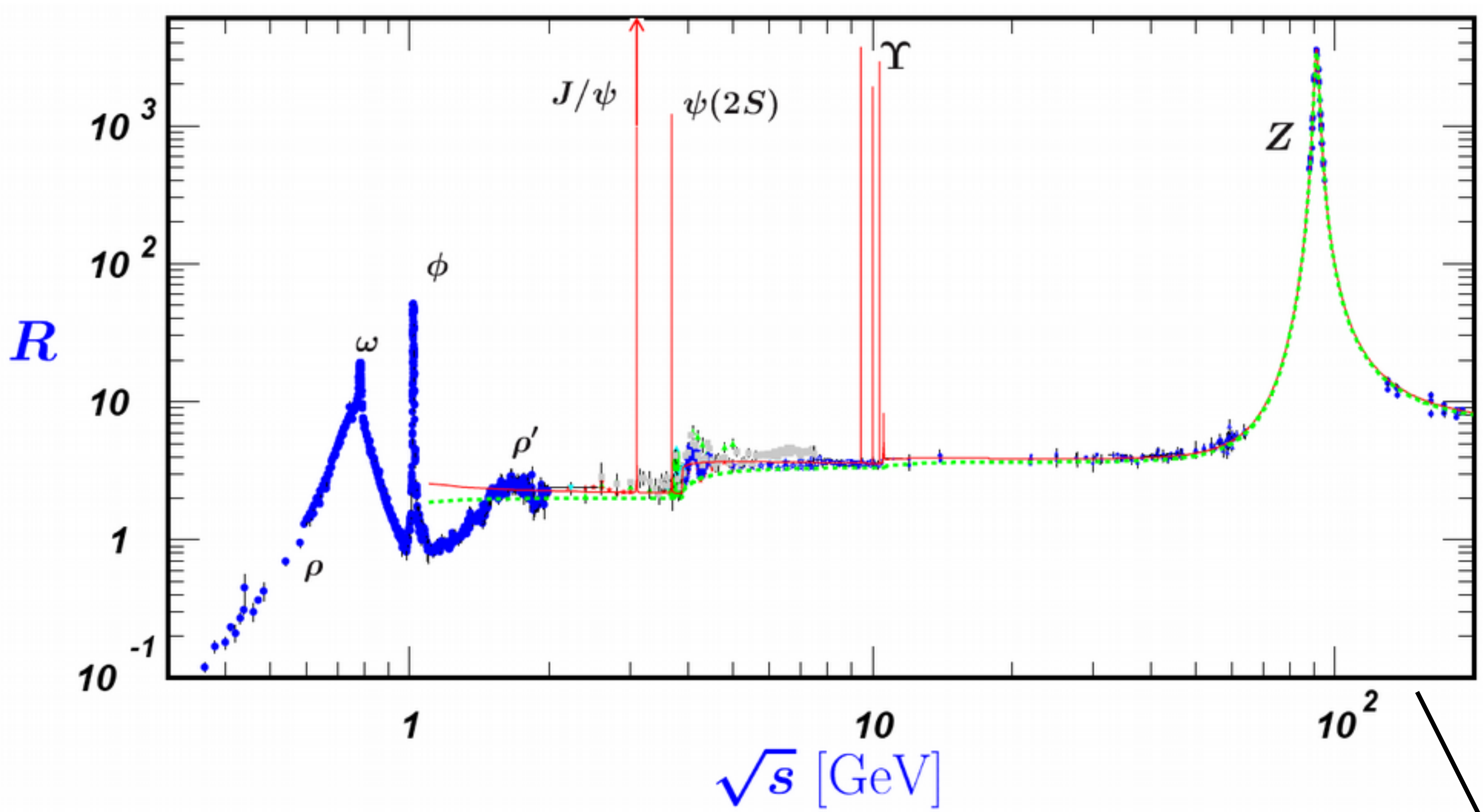


Run from Z-pole down: integrate heavy d.o.f. step by step, match at threshold

Precision running of $\sin^2\theta_W(\mu)$

By the time one gets down to low scale QCD is non-perturbative - use experimental input + dispersion relation

Use exp. known R(s)= $\sigma(e^+e^- \to \text{ hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$



At scale μ_0 = 2GeV: use pQCD input

$$\Delta\hat{\alpha}^{(3)}(\mu_0) = \frac{\alpha}{3\pi}\int_{4m_\pi^2}^{\mu_0^2} ds \frac{R(s)}{s - i\epsilon} + 4\pi I^{(3)}$$

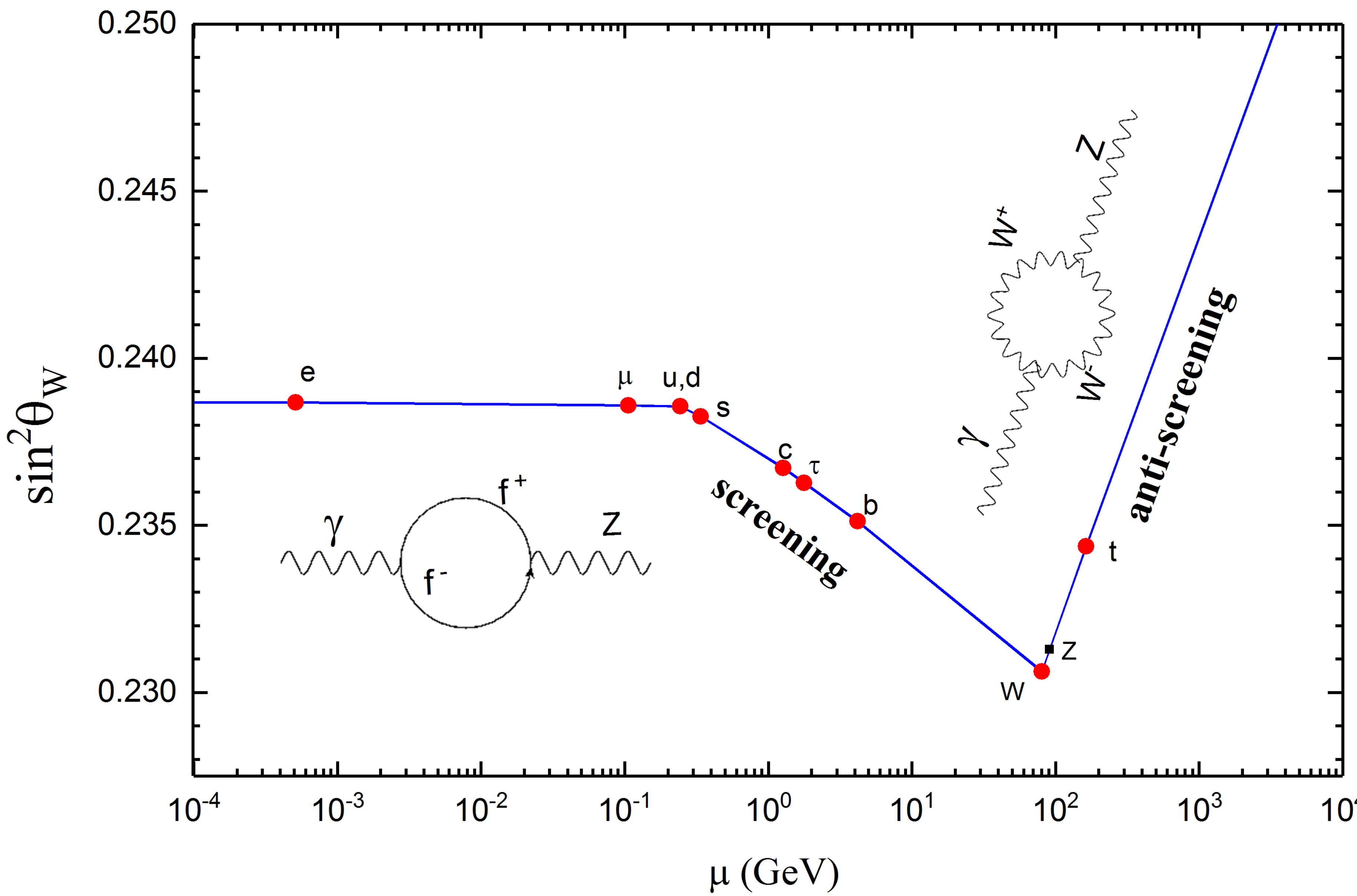
Final step - flavor rotate R to get Z coupling from e.-m. coupling

Precision running of $\sin^2\theta_W(\mu)$

SM prediction for low energy:

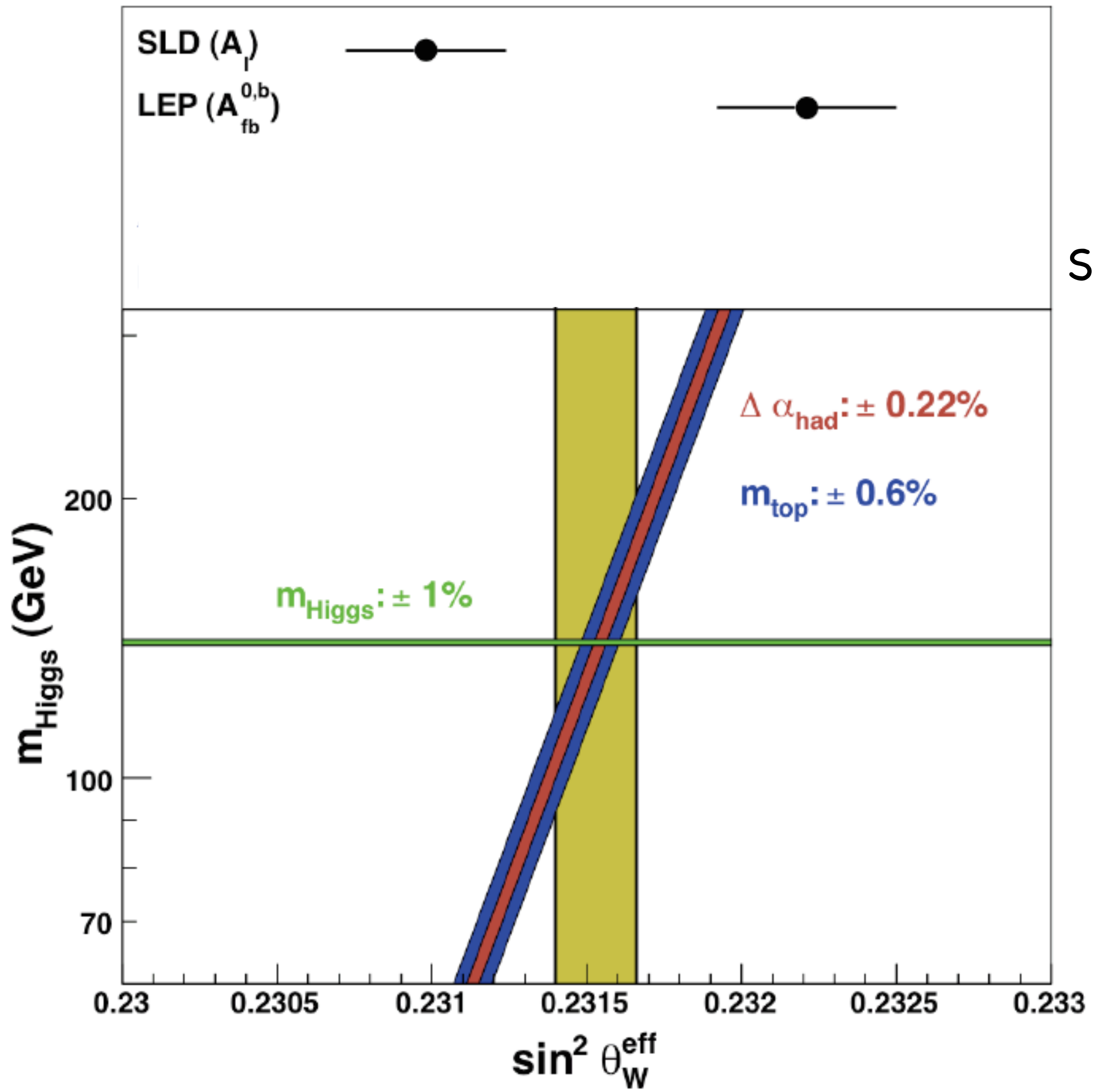
$$\sin^2 \hat{\theta}_W(0) = 0.23868 \pm 0.00005 \pm 0.00002$$

Erler, Ferro Hernandez, arXiv:1712.09146



Precision running of $\sin^2\theta_W(\mu)$

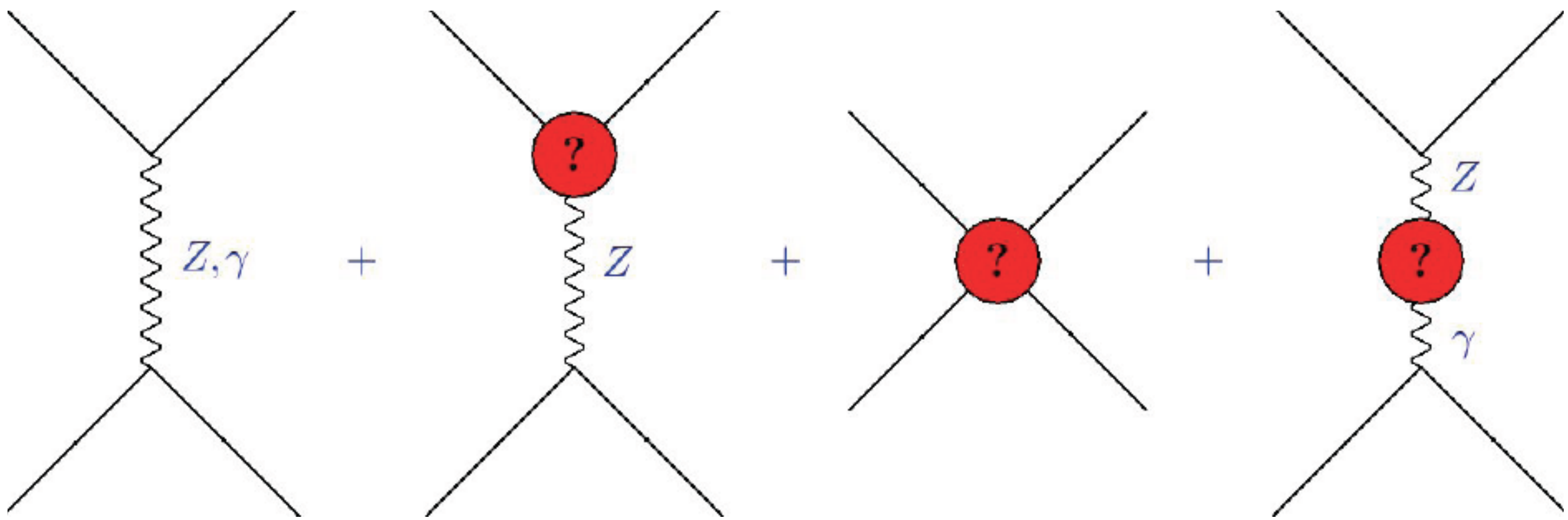
SM global fit at Z-pole: $\sin^2\hat{\theta}_W(M_Z) = 0.23129(5)$



Shown is $\sin^2\theta^{eff}$

Sensitivity to New Physics

The running is a unique prediction of the Standard Model
Deviation ANYWHERE = BSM signal



Dark Z

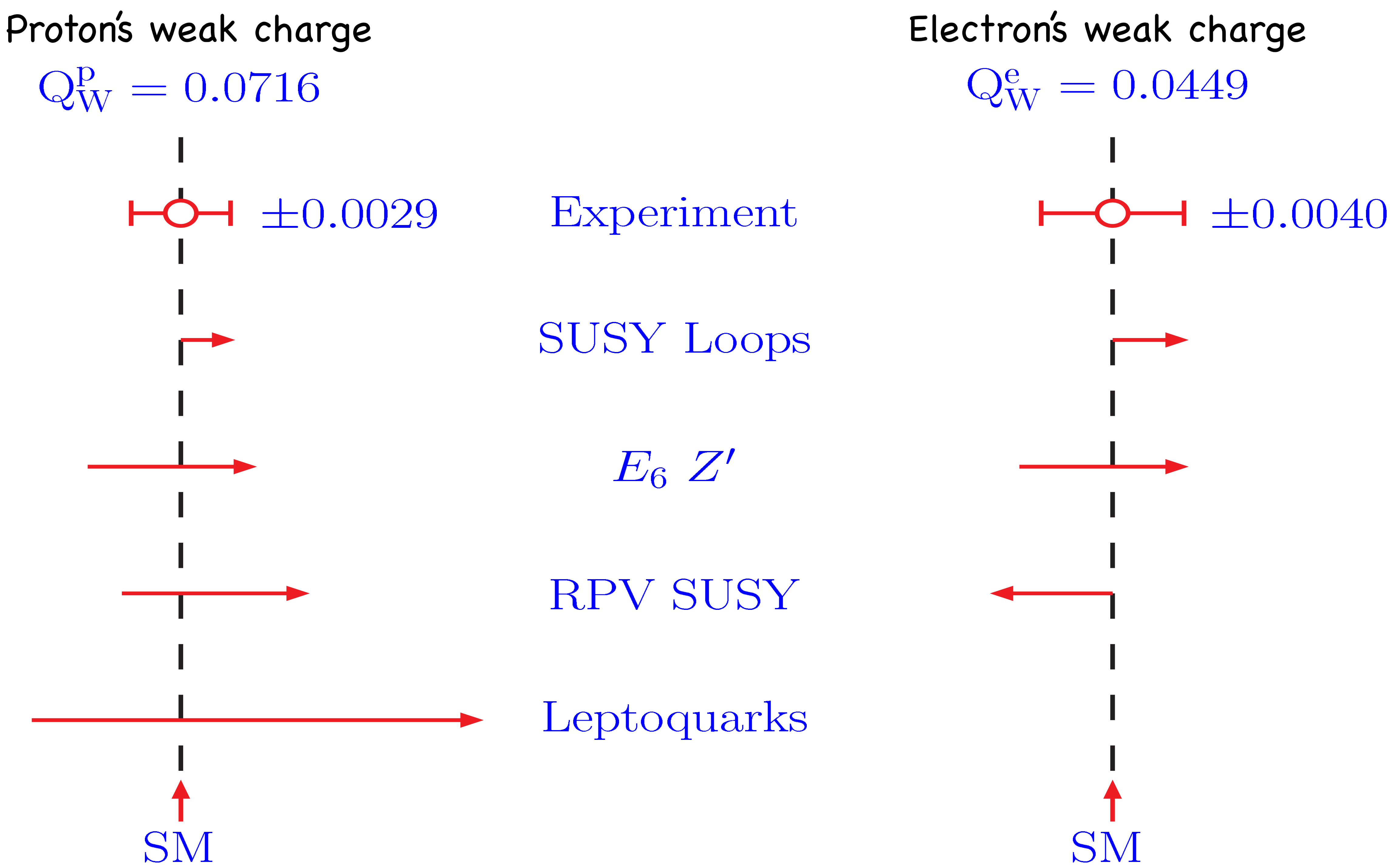
Mixing
w. dark Z or γ

New contact
interactions

New fermions

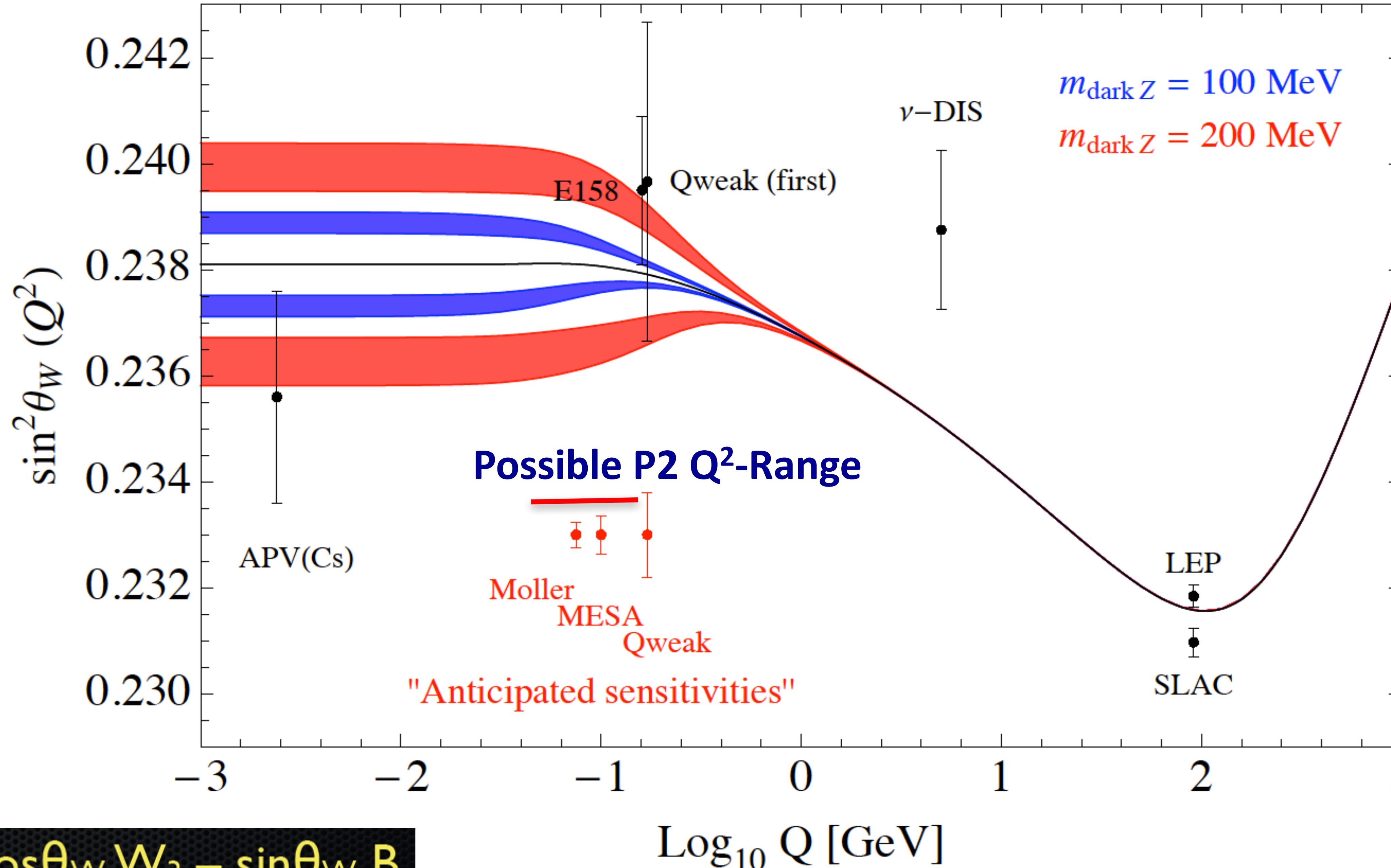
Sensitivity to New Physics

Complementary access to New Physics by PV e-p and e-e scattering



Erler, Kurylov, Ramsey-Musolf, hep-ph/0302149

Running $\sin^2\theta_W$ and Dark Parity Violation



$Z = \cos\theta_W W_3 - \sin\theta_W B$

$A = \sin\theta_W W_3 + \cos\theta_W B$

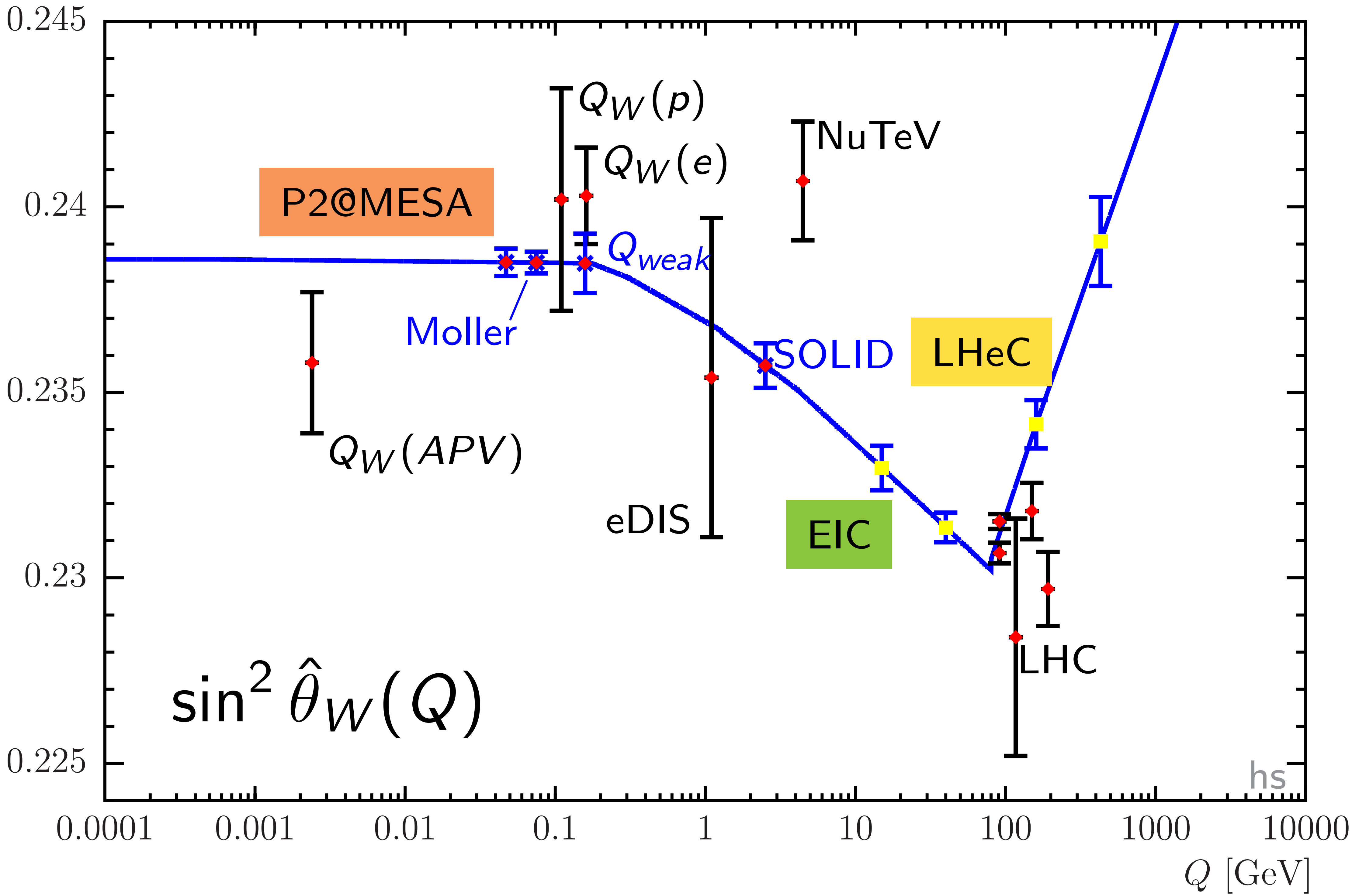
Bill Marciano

Heavy BSM reach of modern low-energy experiments: up to 49 TeV

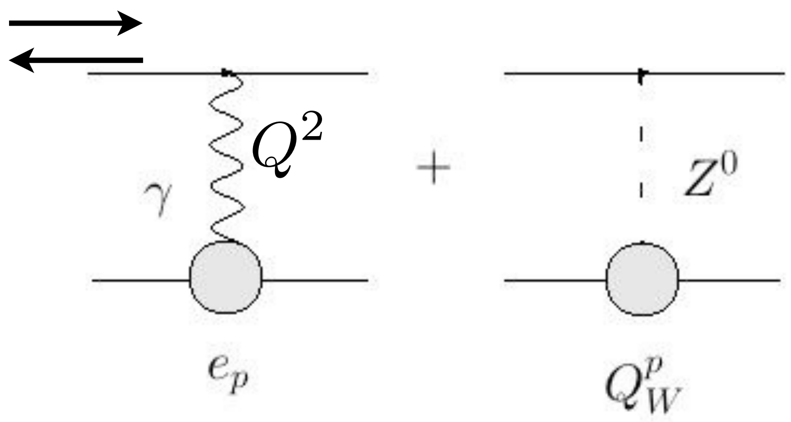
Sensitivity to light dark gauge sector

Complementary to colliders

Experimental tests of running $\sin^2\theta_W(\mu)$



Proton's weak charge with PVES



$$A^{\rm PV} = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L} = -\frac{G_F Q^2}{4\sqrt{2}\pi\alpha}\left[Q_W^p + Q^2 B(Q^2)\right]$$

Weak charge from PVES: $Q_W^p = \lim_{Q^2 \to 0}\left[-\frac{4\sqrt{2}\pi\alpha}{G_F Q^2} A^{exp}\right]$

$Q_W{}^p$ in SM at tree-level: accidentally suppressed

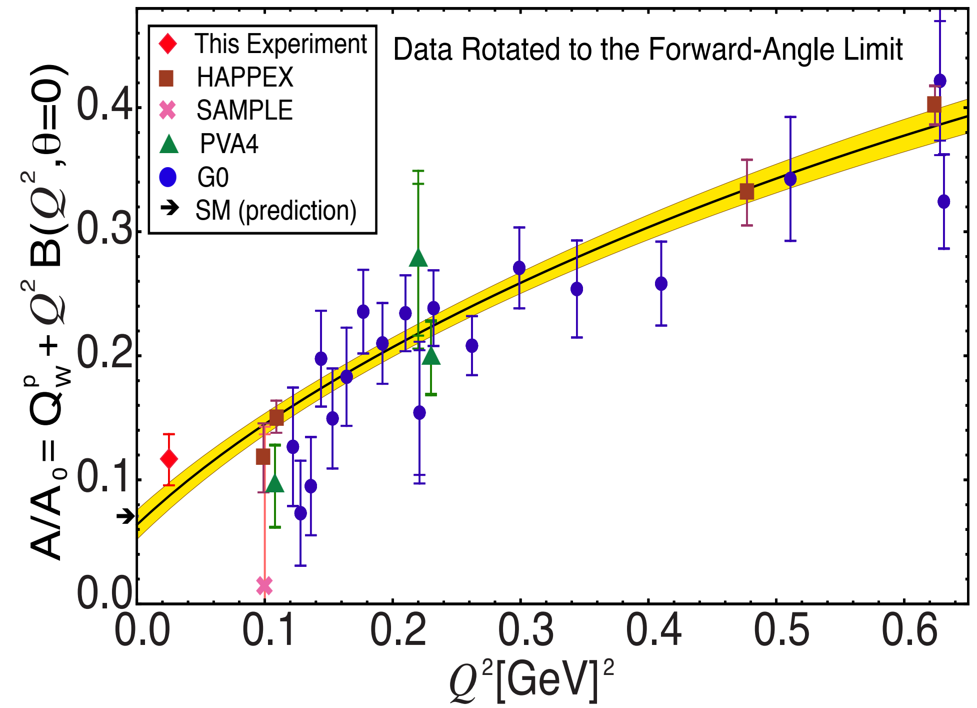
$$Q_W^{p,\,\rm tree} = 1 - 4\sin^2\theta_W \approx 0.07$$

A sensitive test of running of θ_W at low energy:

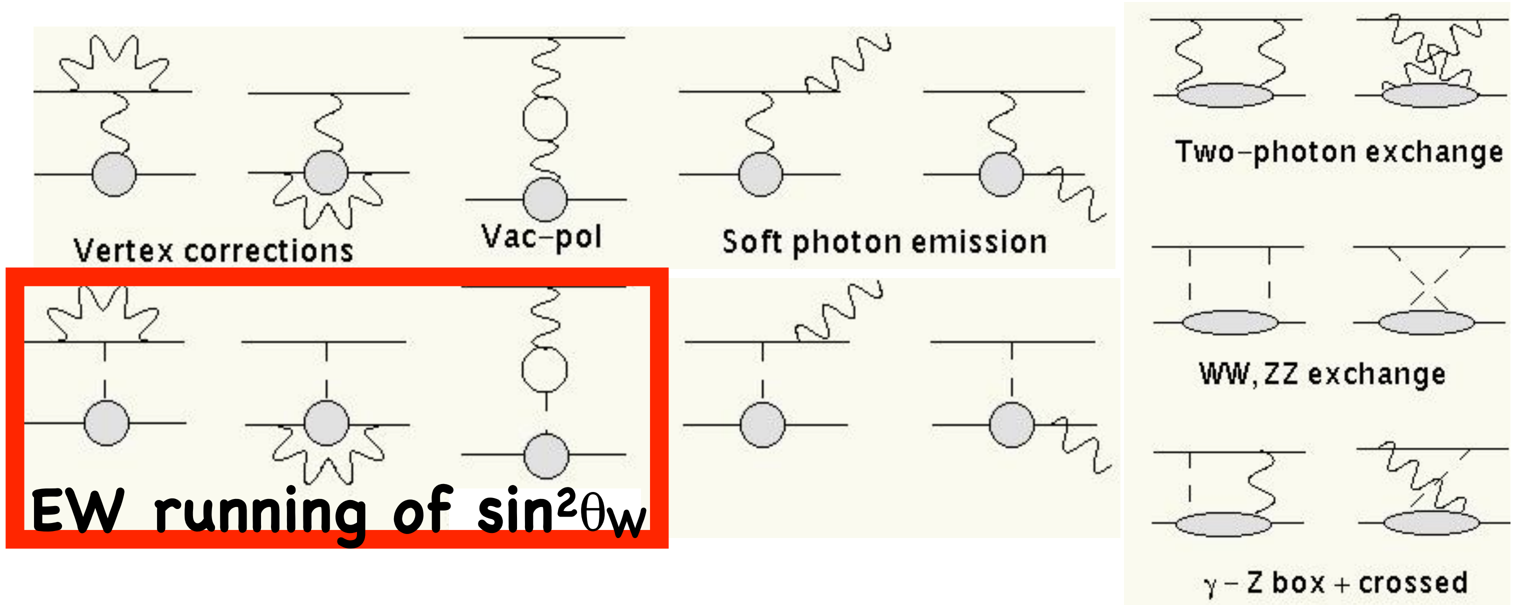
2% measurement of Q_W -> 0.14% on $\sin^2\theta_W$

$B(Q^2)$ - from non-forward PVES data

Young et al. '07;
Androic et al. [Qweak Coll.], '13



Weak charge with radiative corrections: EW boxes



Hadronic effects under control

$$Q_W^{p,\,1-\text{loop}} = (1+\Delta_\rho+\Delta_e)(1-4\sin^2\hat{\theta}_W+\Delta_e') + \Box_{WW} + \Box_{ZZ} + \Box_{\gamma Z}$$

Marciano, Sirlin '83,84; Erler, Musolf '05

Non-universal correction - depends on kinematics and hadronic structure

Marciano and Sirlin '84:

γZ-box mainly universal (large log)

same for PV in atoms and e-scattering

Residual dependence on hadronic scale Λ

No energy dependence assumed

$$\Box_{\gamma Z} = \frac{5\hat{\alpha}}{2\pi}(1-4\hat{s}^2)\left[\ln\left(\frac{M_Z^2}{\Lambda^2}\right) + C_{\gamma Z}(\Lambda)\right]$$

0.0052±0.0005 (7.3±0.7% of Q_W)

γZ-Box from Dispersion Relations

γZ-box from forward dispersion relation

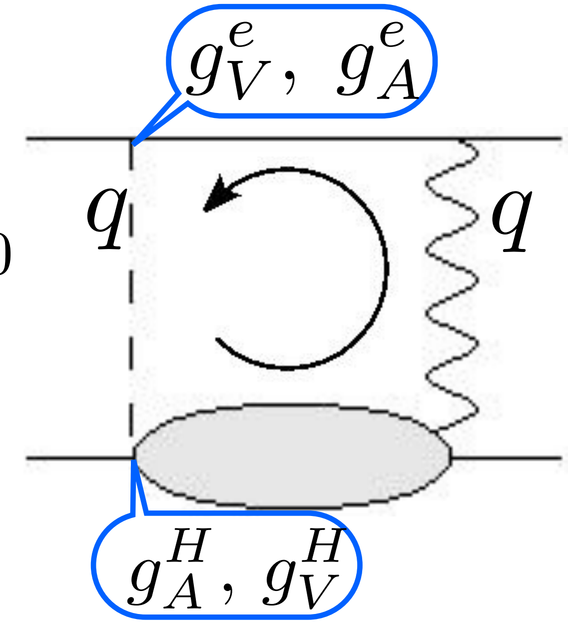
Imaginary part = on-shell states = data
Real part: from unitarity + analyticity + symmetries

MG, Horowitz '09; MG, Horowitz, Ramsey-Musolf '11

$$k^\mu = (E, \vec{k})$$

$$Q^2 = -q_\mu q^\mu \geq 0$$
$$W^2 = (p+q)^2$$

$$p^\mu = (M, \vec{0})$$



Lower blob: forward interference Compton tensor

$$\mathrm{Im} W^{\mu\nu} = -\hat{g}^{\mu\nu} F_1^{\gamma Z} + \frac{\hat{p}^\mu \hat{p}^\nu}{(p \cdot q)} F_2^{\gamma Z} + \frac{i\epsilon^{\mu\nu\alpha\beta} p_\alpha q_\beta}{2(p \cdot q)} F_3^{\gamma Z}$$

Forward dispersion relation for $\Box_{\gamma Z} = g_V^e \Box_{\gamma Z_A} + g_A^e \Box_{\gamma Z_V}$

$$\mathrm{Re}\Box_{\gamma Z_V}(E) = \frac{2E}{\pi} \int_0^\infty dQ^2 \int_{W_\pi^2}^\infty dW^2 \left[A F_1^{\gamma Z}(W^2, Q^2) + B F_2^{\gamma Z}(W^2, Q^2) \right]$$

$$\mathrm{Re}\Box_{\gamma Z_A}(E) = \frac{2}{\pi} \int_0^\infty dQ^2 \int_{(M+m_\pi)^2}^\infty dW^2 \, C F_3^{\gamma Z}(W^2, Q^2)$$

Inclusive PV data
– little available

Energy dependence of γZ-Box from Dispersion Relations

Vector box $$\mathrm{Re}\Box_{\gamma Z_V}(E) = \frac{2E}{\pi}\int_0^\infty dQ^2 \int_{W_\pi^2}^\infty dW^2 \left[AF_1^{\gamma Z}(W^2,Q^2) + BF_2^{\gamma Z}(W^2,Q^2)\right]$$

Not much data on $F^{\gamma Z}_{1,2}$ available – relate to e.-m. $F_{1,2}$

As for running $\sin^2\theta_W$ from α: need flavor separation + rotation

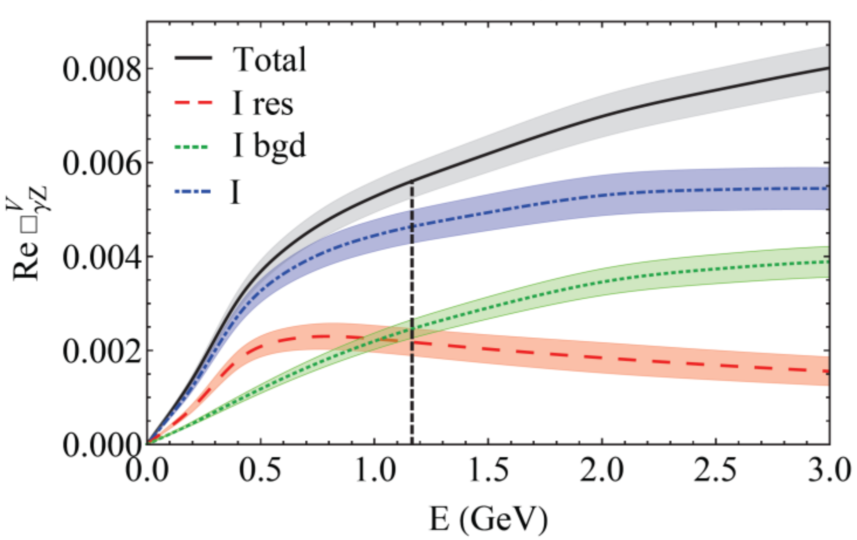
Unfortunately, unlike for e^+e^- -> hadrons this flavor separation can only be done in very limited kinematical range

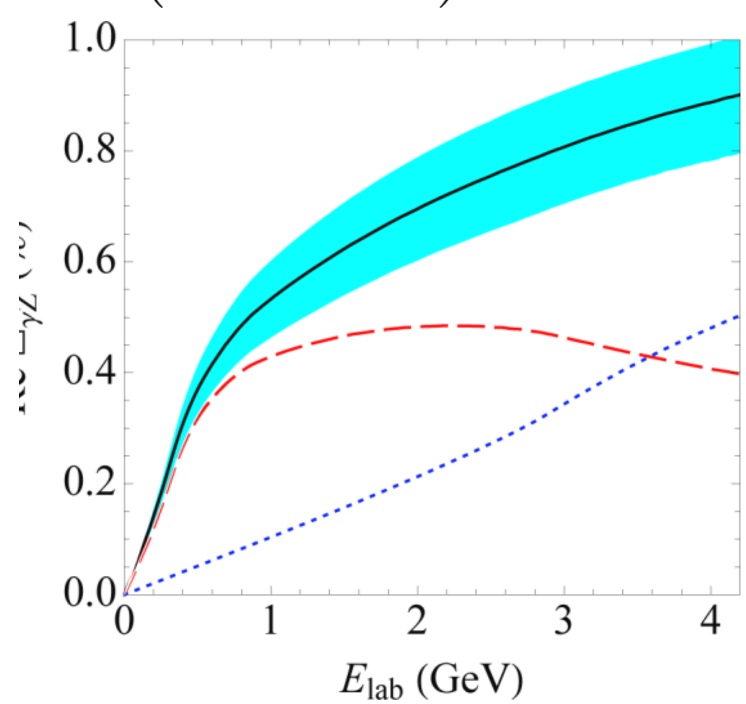
-> larger model dependence

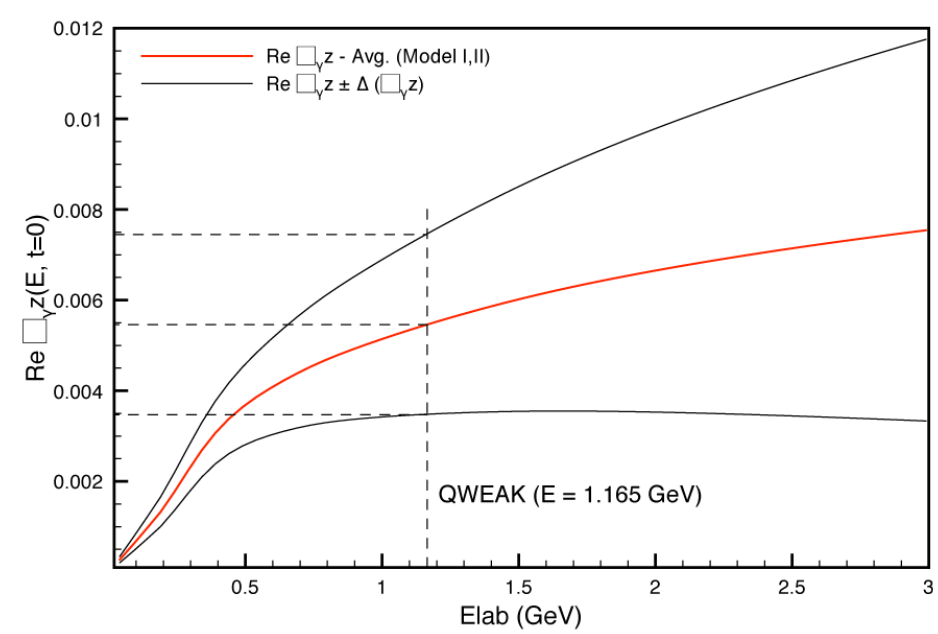
Three groups estimated γZ^V box for QWeak energy E = 1.165 GeV:

Central values agree, some discussion on the errors

	Hall et al '13	Rislow, Carlson '11	MG, Horowitz, Ramsey-Musolf '11
$\mathrm{Re}\Box_{\gamma Z_V}(E) =$	$(5.6 \pm 0.36) \times 10^{-3}$	$(5.7 \pm 0.9) \times 10^{-3}$	$(5.4 \pm 2.0) \times 10^{-3}$



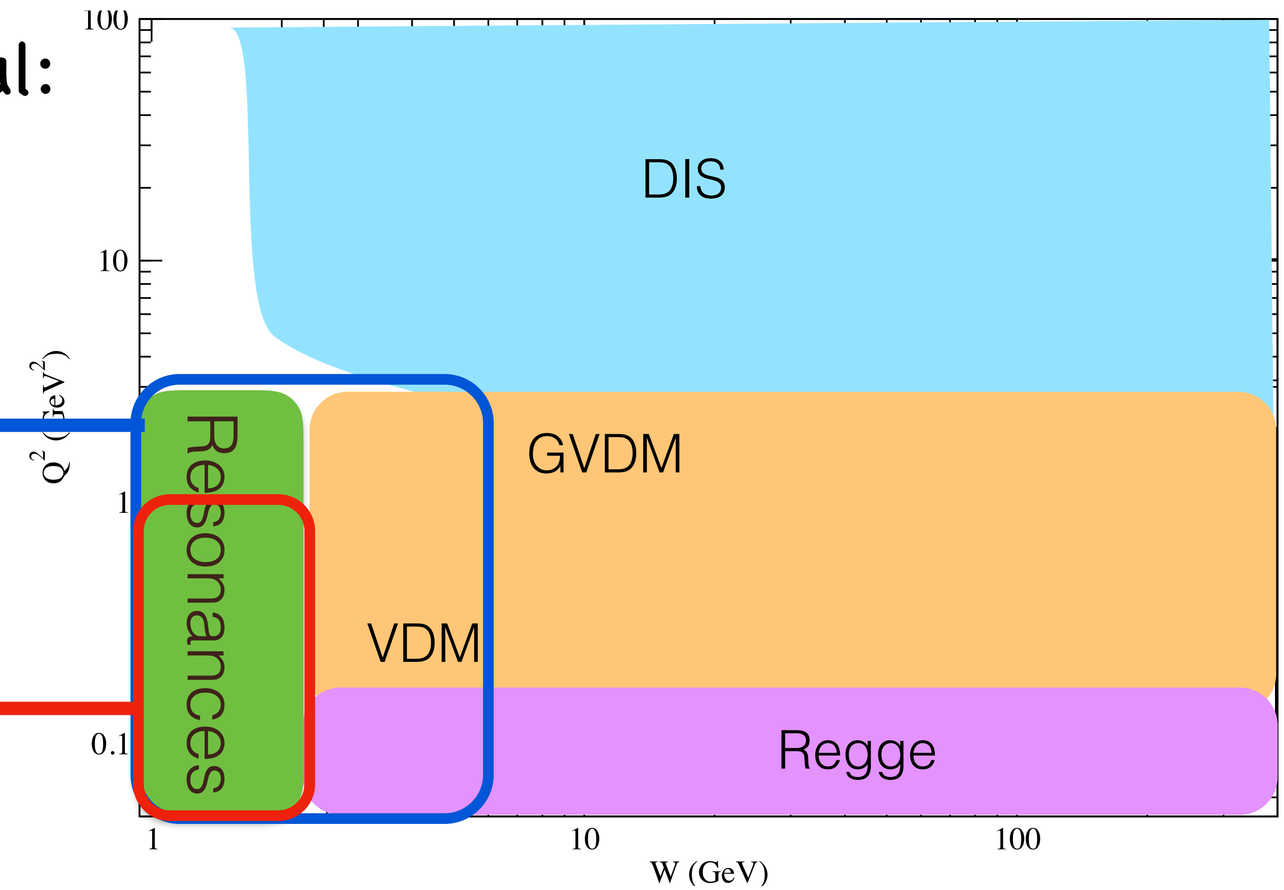




Energy dependence of γZ-Box from Dispersion Relations

Model dependence stems from that in the flavor separation/rotation

Importance of the input for the integral:



For QWeak energy E = 1.165 GeV

Main contribution: W < 5 GeV, Q^2 < 2 GeV^2

For P2 energy E = 155 MeV

Main contribution: W < 2.5 GeV, Q^2 < 1 GeV^2

Energy dependence required

a formal redefinition of the weak charge

$$Q_W^p = \lim_{E,\,Q^2\to 0}\left[-\frac{4\sqrt{2}\pi\alpha}{G_F Q^2}A^{exp}\right]$$

Steep energy dependence of the γZ^V box (much smaller for smaller energy)

Flavor (isospin) structure in the resonance region well understood

- Furnished a strong motivation for the P2 experiment in Mainz at E=155 MeV

$$\mathrm{Re}\,\Box^V_{\gamma Z}(E=155\,\mathrm{MeV}) = (1.1\pm 0.18)\times 10^{-3}$$

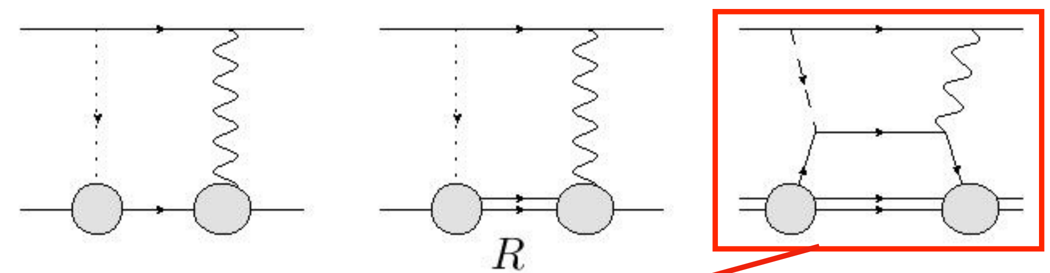
Frank's talk today

γZ-Box from Dispersion Relations

Axial box - additional energy dependence

$$\mathrm{Re}\,\Box^{A}_{\gamma Z}(E) = \frac{2\alpha}{\pi}(1-4s^2\hat{\theta}_W)\int_0^\infty dQ^2 \int_0^\infty dW^2 C(E,W,Q^2) F_3^{\gamma Z}(W,Q^2)$$

To evaluate the integral need F_3 for $M<W<\infty$, $0<Q^2<\infty$



The part of the Q^2 integral above $\Lambda^2 \gg M^2$ is model-independent: scattering on perturbative quarks

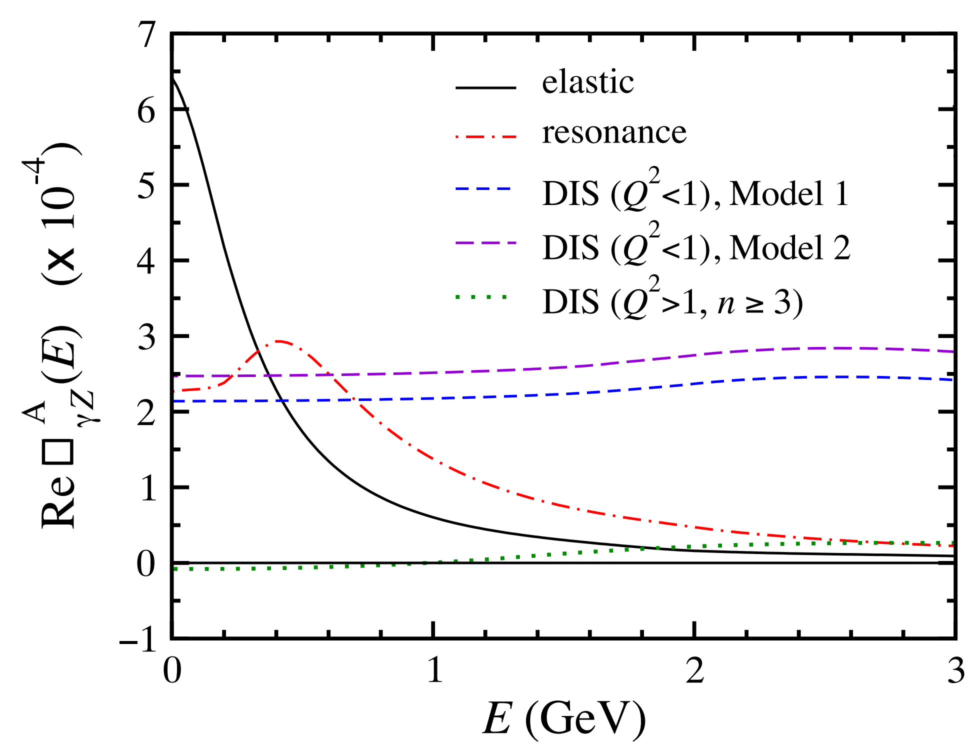
$$\Box_{\gamma Z} = \frac{5\hat{\alpha}}{2\pi}(1-4\hat{s}^2)\left[\ln\left(\frac{M_Z^2}{\Lambda^2}\right) + C_{\gamma Z}(\Lambda)\right]$$

Marciano, Sirlin '84

Can evaluate the low-Q^2 integral explicitly:

elastic form factors; resonances;
higher moments of quark PDFs

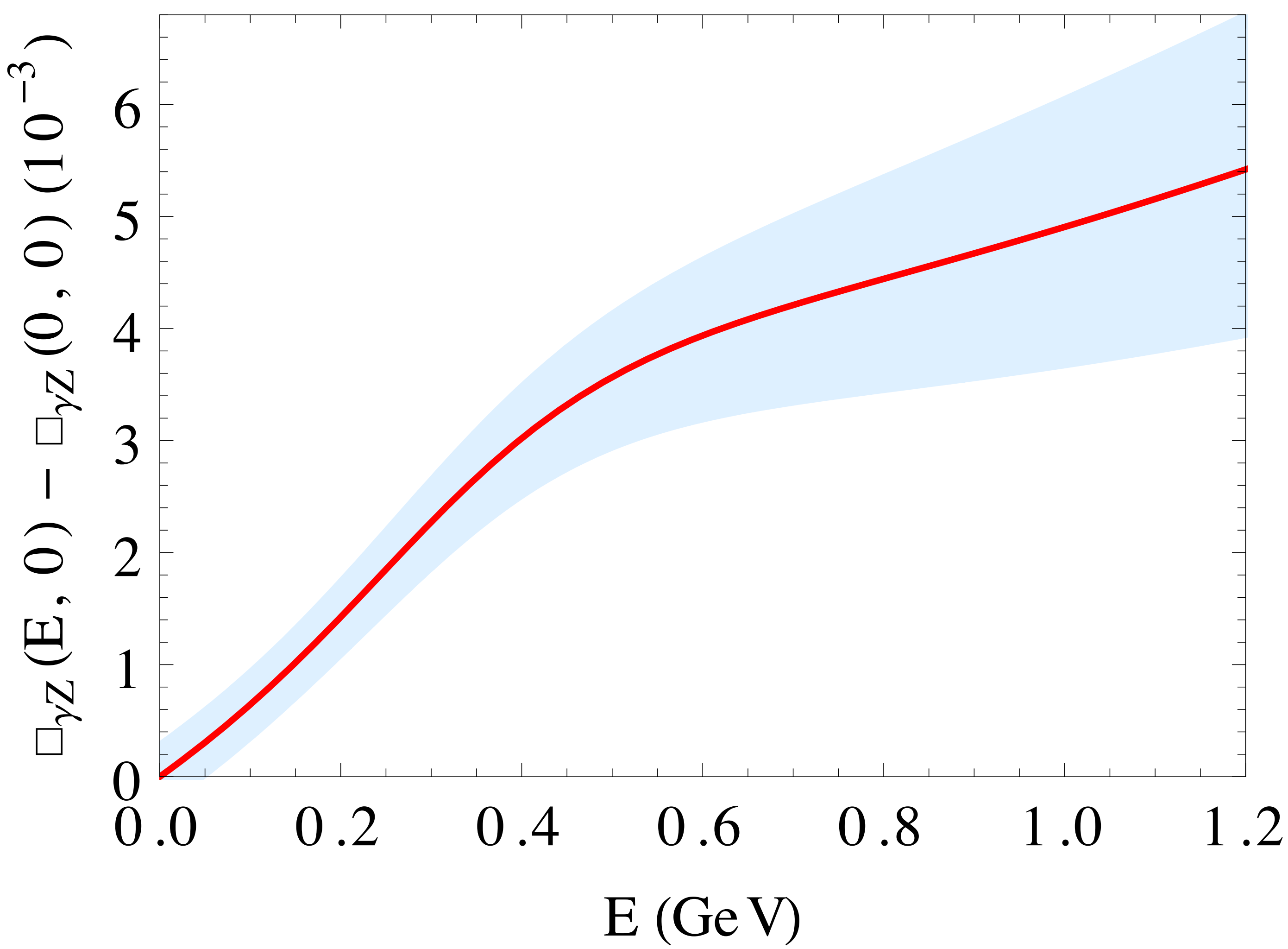
Blunden et al. '11



Update of the old result

$$\mathrm{Re}\,\Box^{A}_{\gamma Z}(E=0) = (5.2\pm 0.5)\times 10^{-3} \;\rightarrow\; \mathrm{Re}\,\Box^{A}_{\gamma Z}(E=1.165\,\mathrm{GeV}) = (3.7\pm 0.4)\times 10^{-3}$$

Status of the energy-dependent γZ-Box



MG, Horowitz, PRL 102 (2009) 091806;
Nagata, Yang, Kao, PRC 79 (2009) 062501;
Tjon, Blunden, Melnitchouk, PRC 79 (2009) 055201;
Zhou, Nagata, Yang, Kao, PRC 81 (2010) 035208;
Sibirtsev, Blunden, Melnitchouk, PRD 82 (2010) 013011;
Rislow, Carlson, PRD 83 (2011) 113007;
MG, Horowitz, Ramsey-Musolf, PRC 84 (2011) 015502;
Blunden, Melnitchouk, Thomas, PRL 107 (2011) 081801;
Rislow, Carlson PRD 85 (2012) 073002;
Blunden, Melnitchouk, Thomas, PRL 109 (2012) 262301;
Hall et al., PRD 88 (2013) 013011;
Rislow, Carlson, PRD 88 (2013) 013018;
Hall et al., PLB 731 (2014) 287;
MG, Zhang, PLB 747 (2015) 305;
Hall et al., PLB 753 (2016) 221;
MG, Spiesberger, Zhang, PLB 752 (2016) 135;

QWEAK energy: $\mathrm{Re}\,\Box_{\gamma Z}^{A+V}(E = 1.165\,\mathrm{GeV}) = (9.3 \pm 1.5) \times 10^{-3}$ (mostly vector box)

QWEAK final result: $Q^p_W = 0.0719 \pm 0.0045$ (error mostly experimental)

P2 energy: $\mathrm{Re}\,\Box_{\gamma Z}^{A+V}(E = 155\,\mathrm{MeV}) = (5.4 \pm 0.4) \times 10^{-3}$ (mostly axial box)

P2 expectation: $Q^p_W = 0.0713 \pm 0.0013$

Summary

- Running $\sin^2\theta_W$ in SM is known very precisely
- Bridges low and high energies, light and heavy BSM
- Proton's weak charge: enhanced sensitivity
- Electroweak boxes play a major role at low energies (same for V_{ud} extraction, $0\nu\beta$ decay, ...)