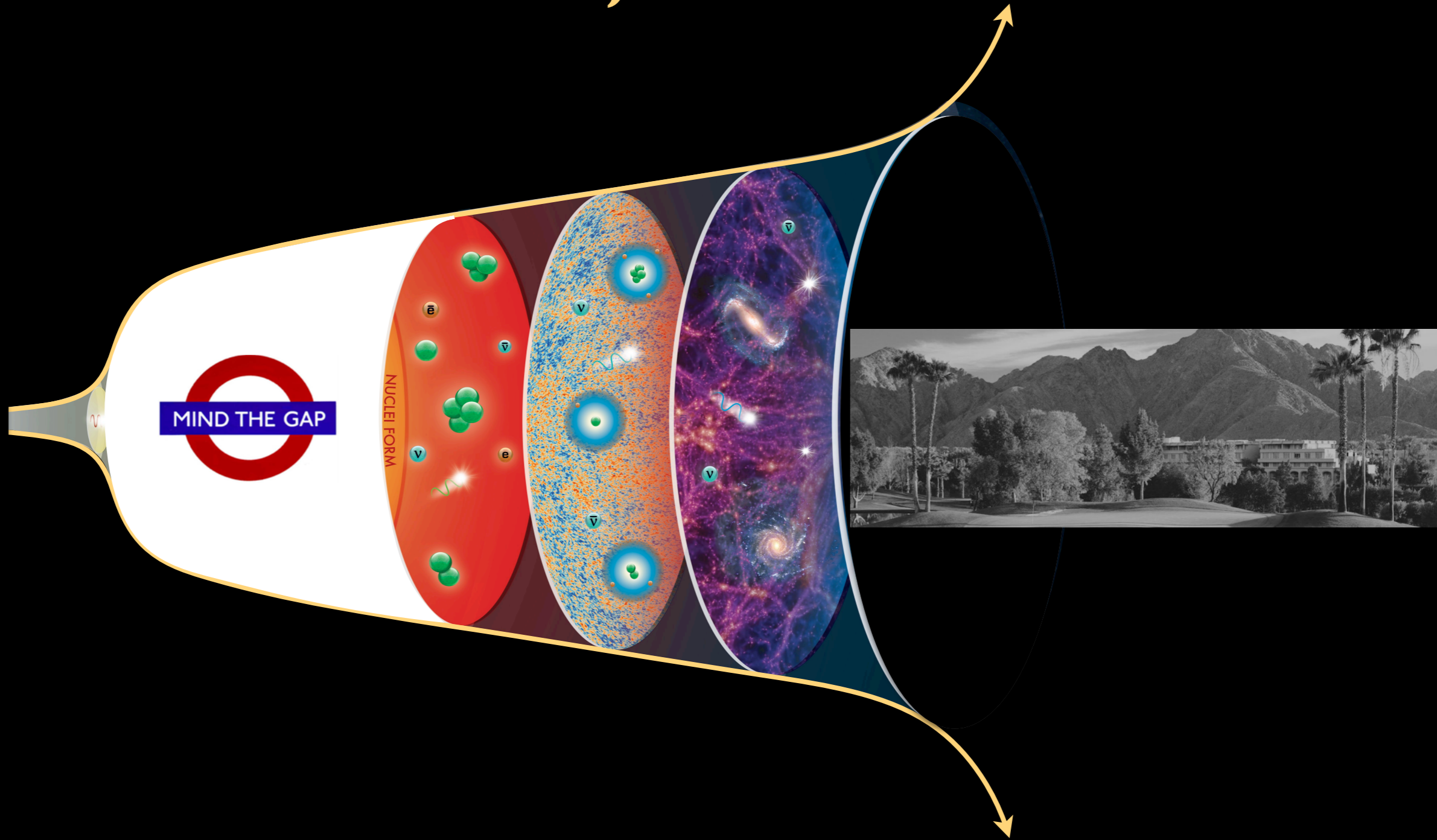


# Inflation Ends, What's Next ?



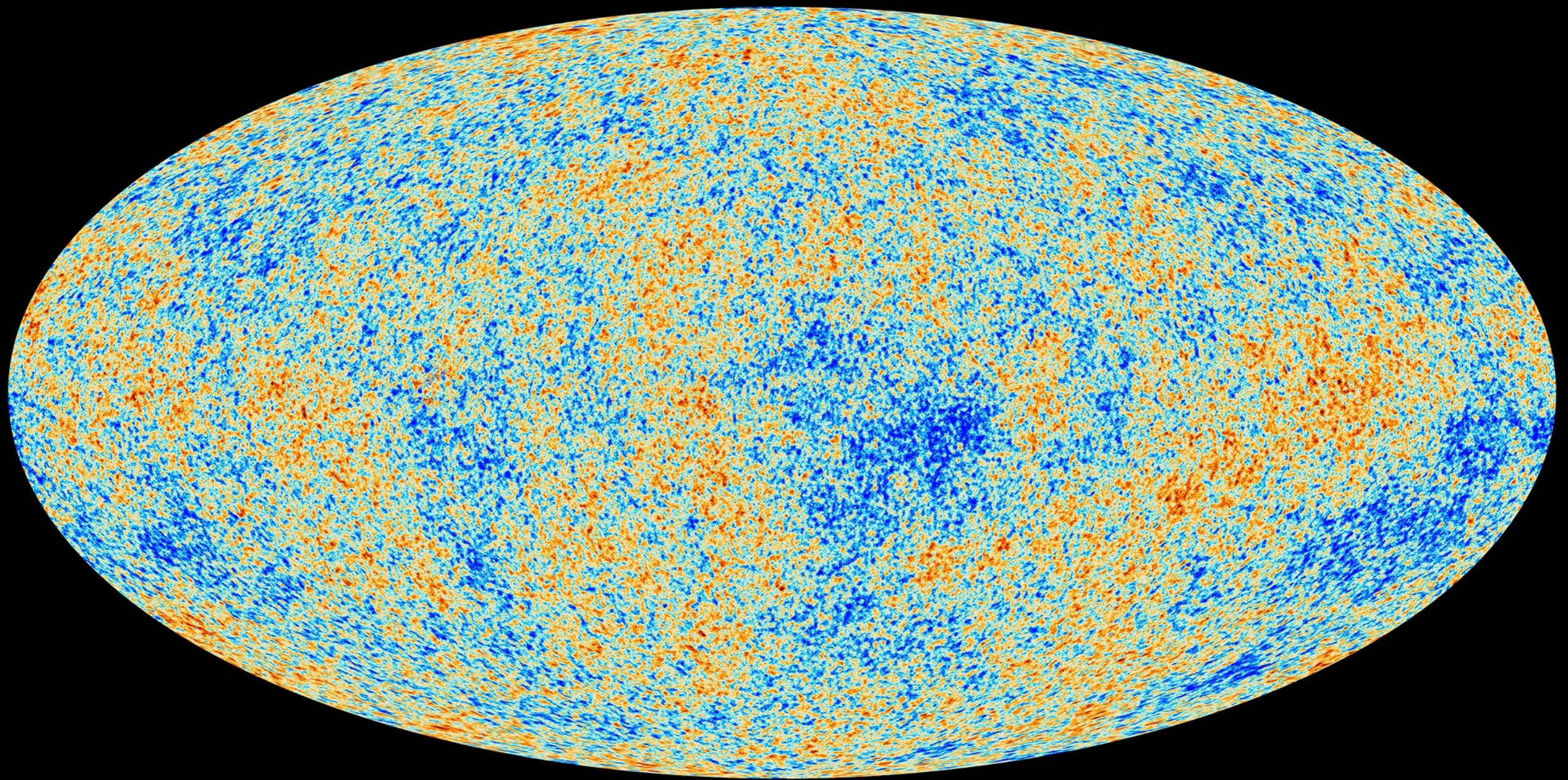
Mustafa A. Amin



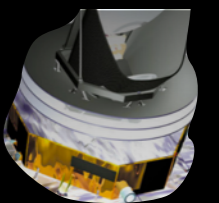
**motivation**



# anisotropies: cosmic microwave background



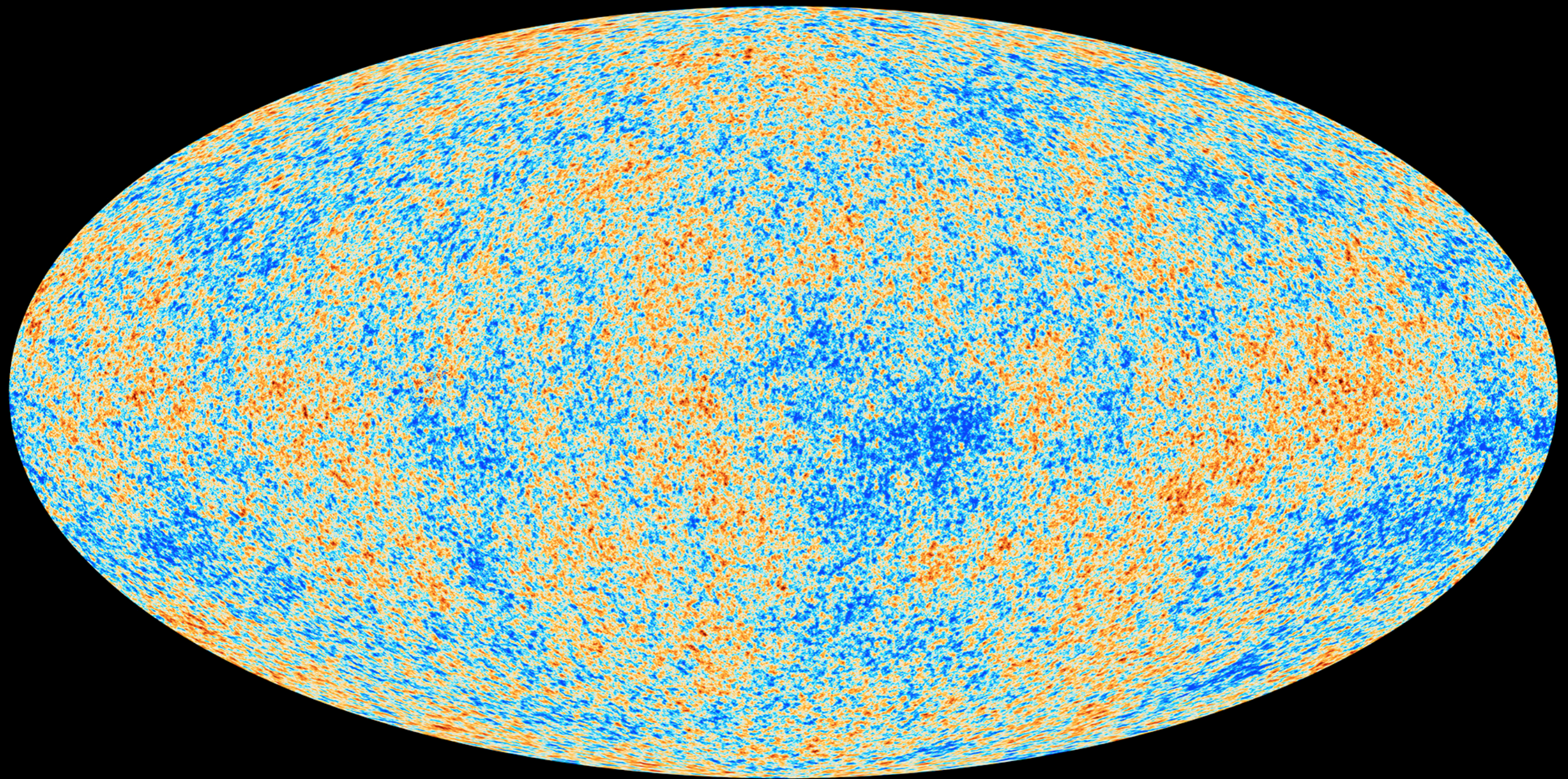
$$\delta T/T \sim 10^{-5}$$





~ gaussian

almost scale invariant



seemingly "acausal"

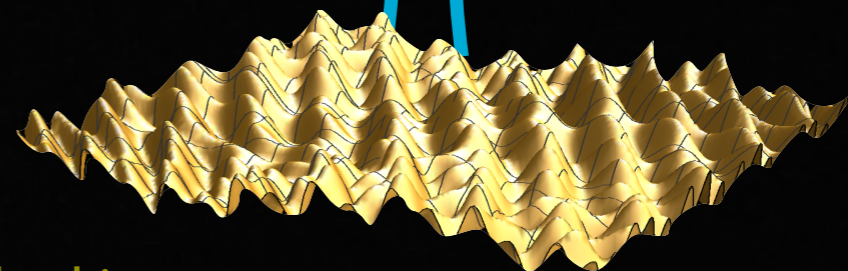
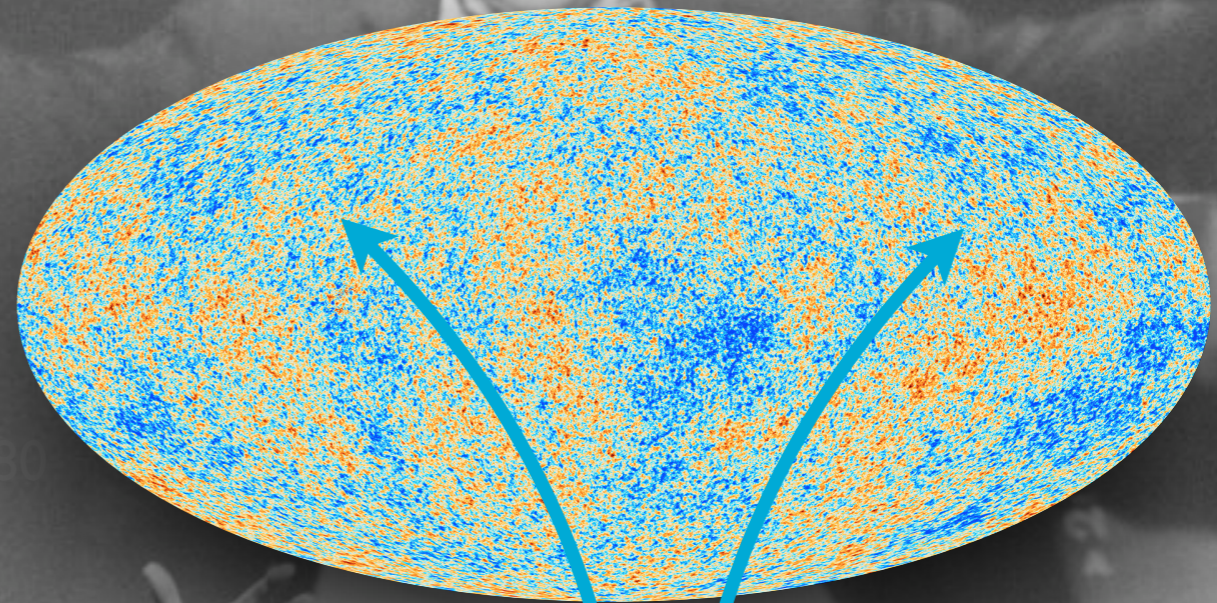
~ adiabatic



# inflation: a “simple” explanation?

$$a \sim e^{Ht}$$

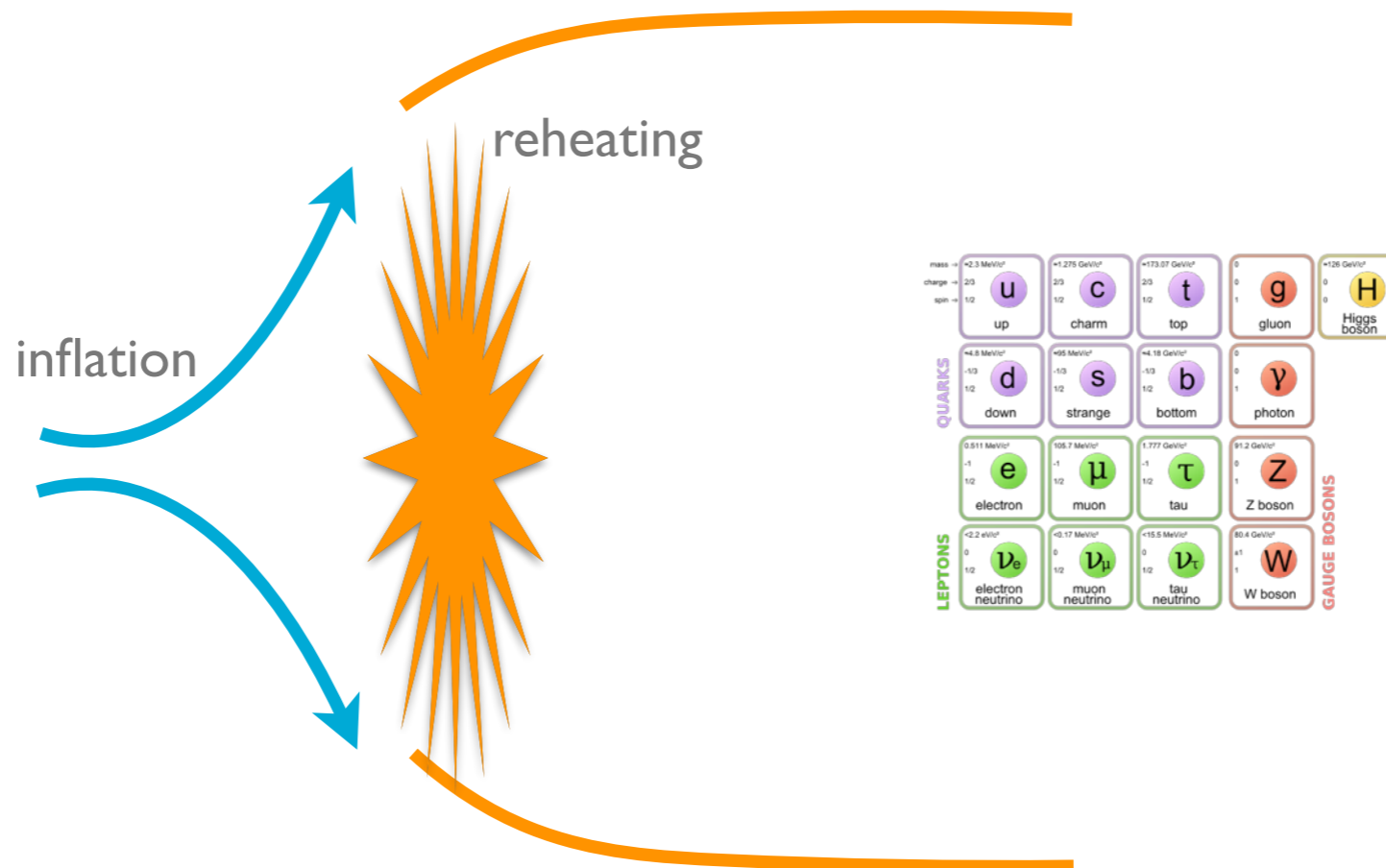
- “acausal”
- almost gaussian
- scale invariant
- adiabatic



quantum  
fluctuations

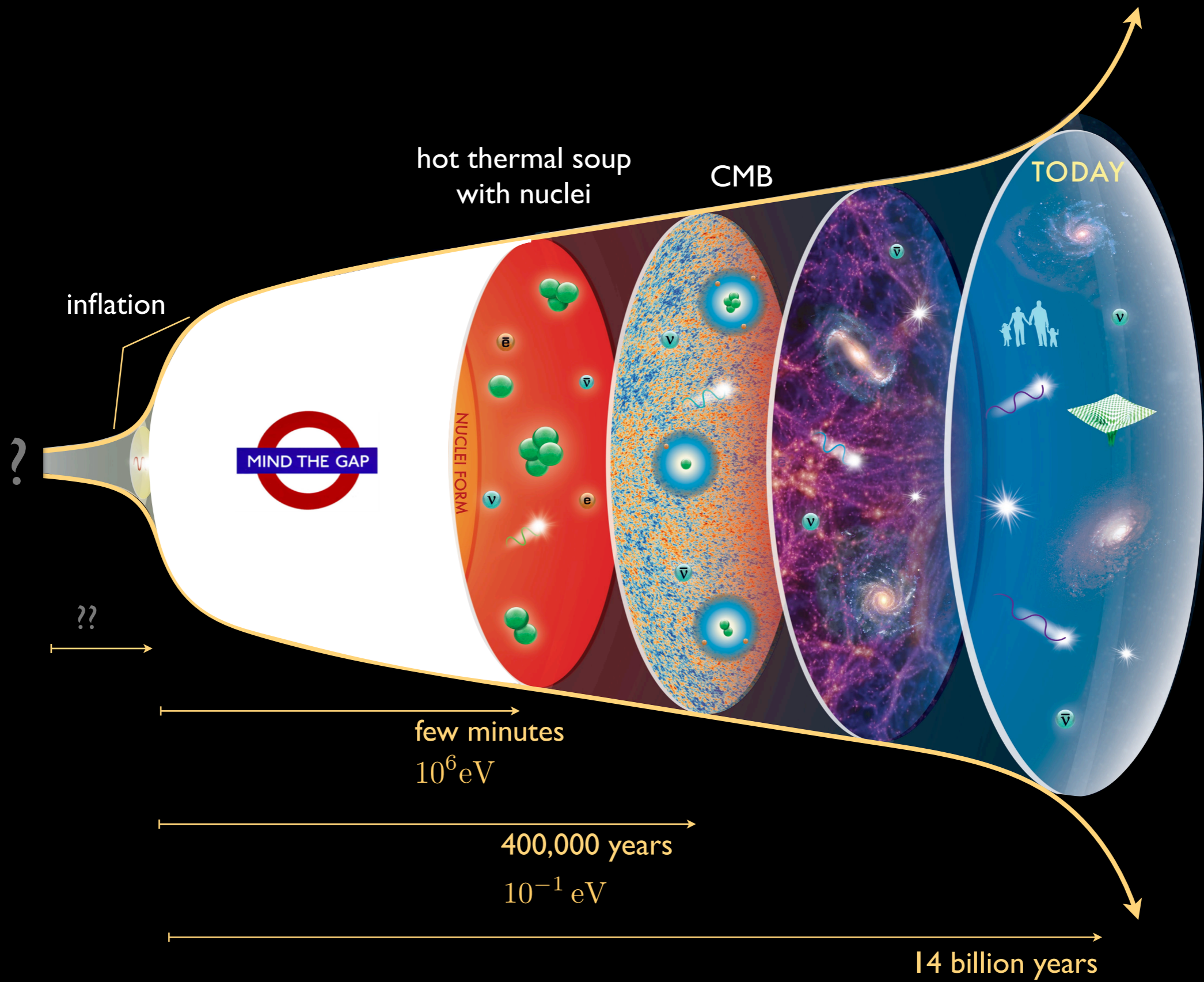
# physics of inflation and its end?

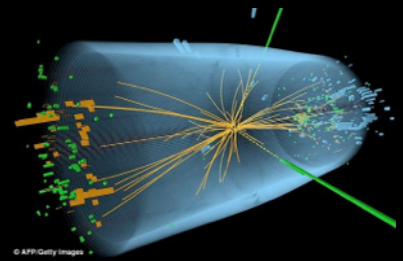
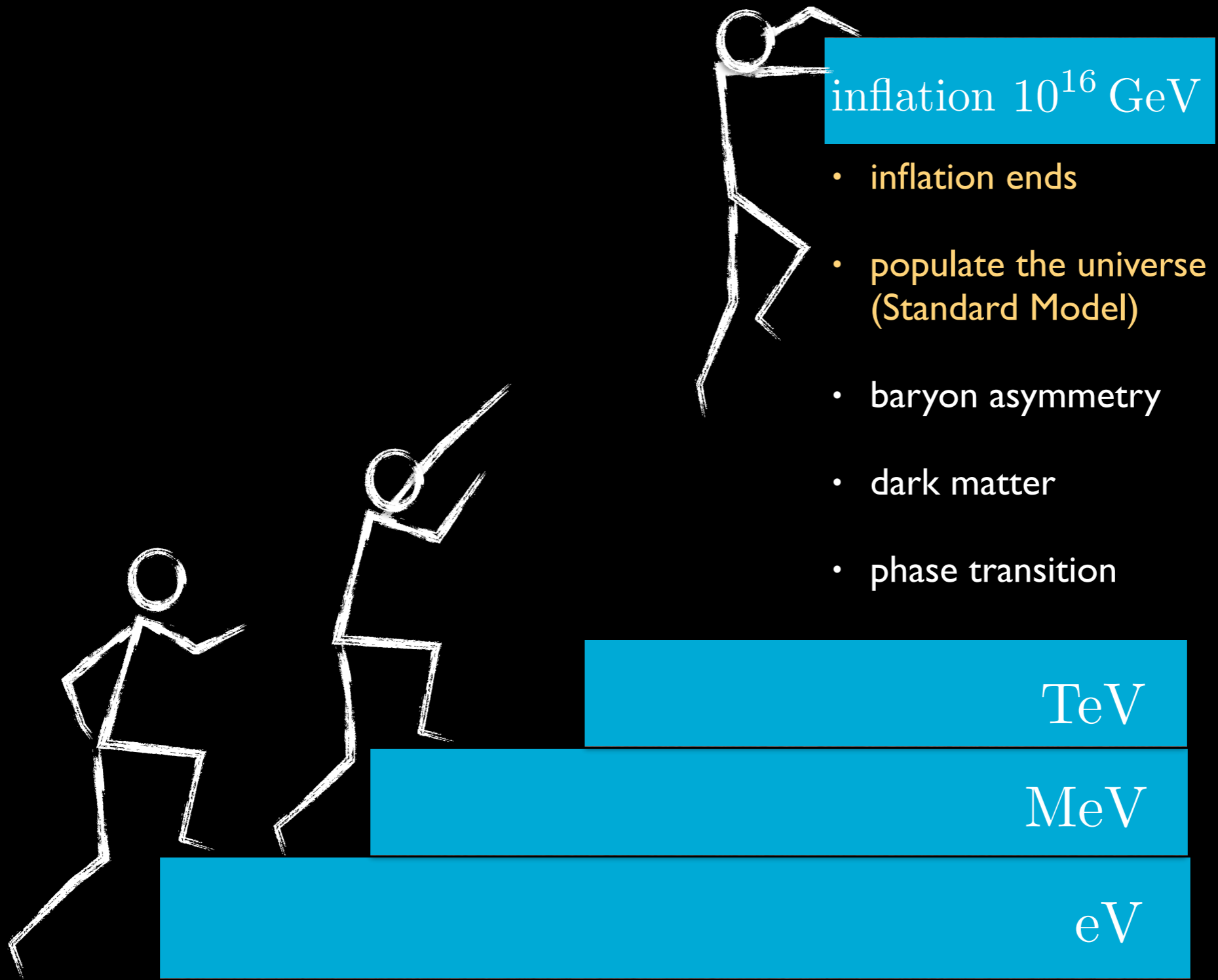
- what is the physics of **inflation** ?
- how did inflation end ?
- how did the universe get populated with particles after inflation ? (**reheating**)





# aftermath of inflation: a GAP in our cosmic history





# modeling inflation & its' aftermath

**SIMPLE**

**problem oriented**

**COMPLEX**

# modeling inflation & its' aftermath

**SIMPLE**

**problem oriented**

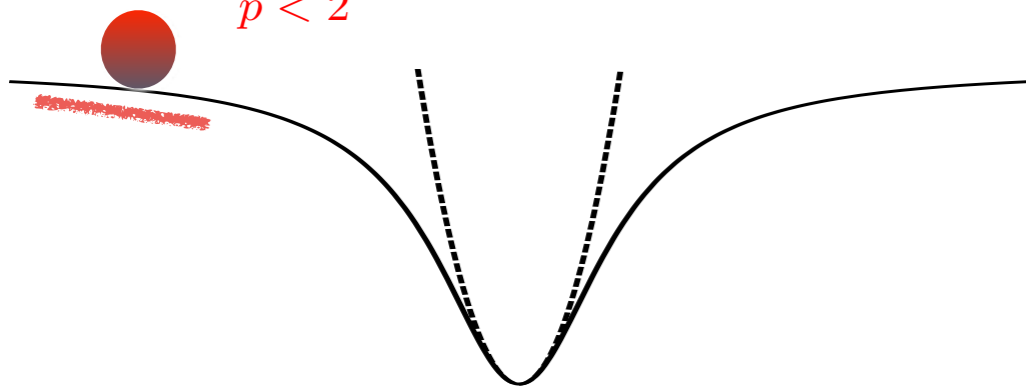
**COMPLEX**



# simplest models: single scalar field driven inflation constraints from observations

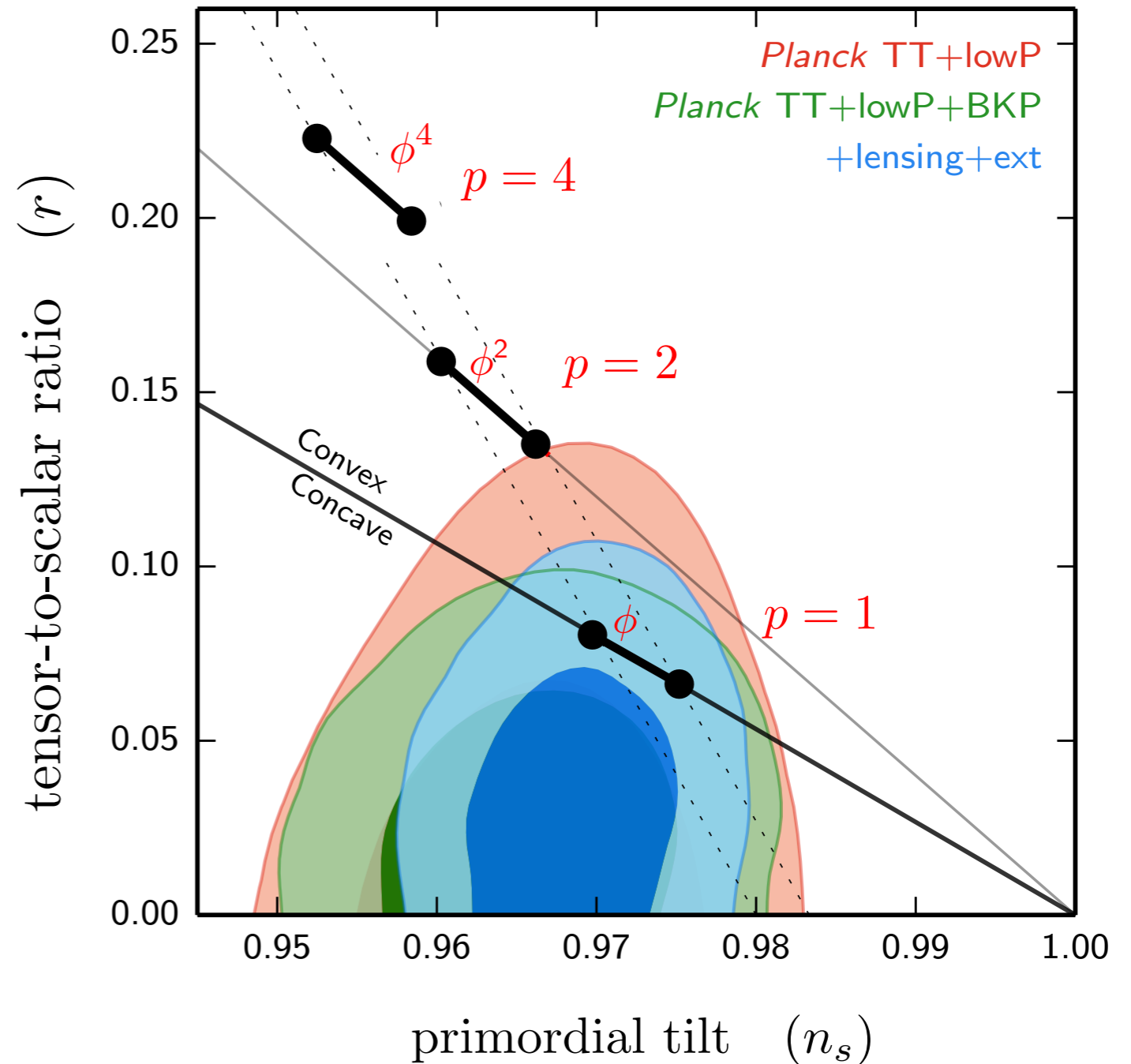
$$V(\phi) \propto \phi^p$$

$$p < 2$$



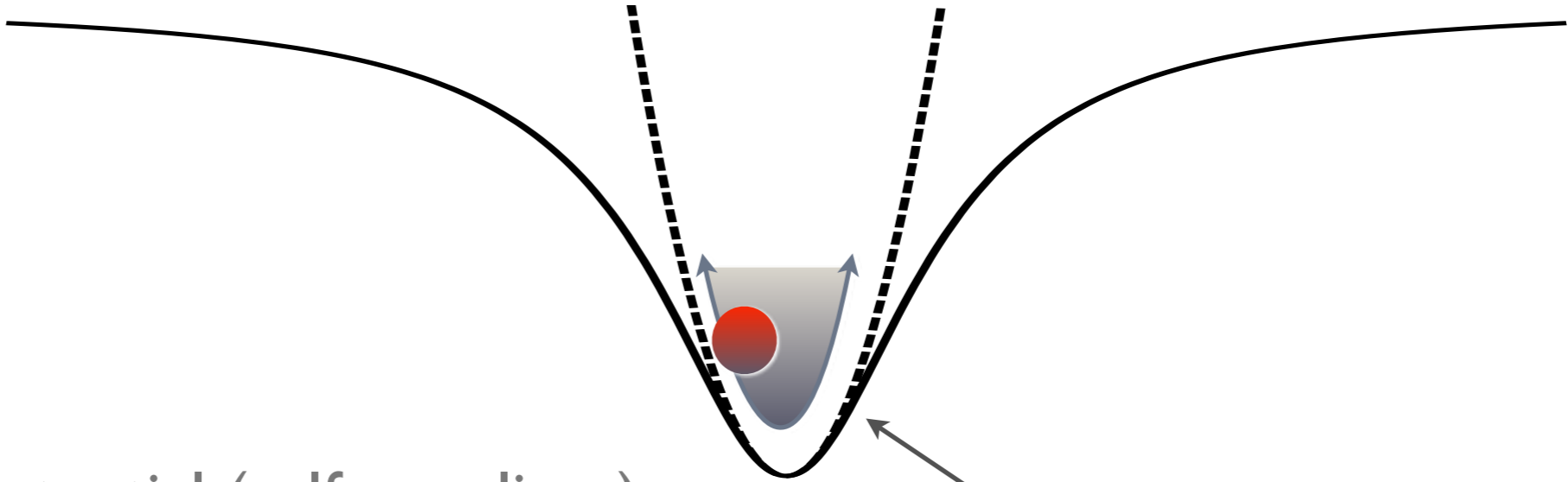
for example:

Starobinsky Inflation (1979)  
Silverstein & Westphal (2008)  
Kallosh & Linde (2013)



# detailed dynamics after inflation ?

- shape of the potential (self couplings)
- couplings to other fields

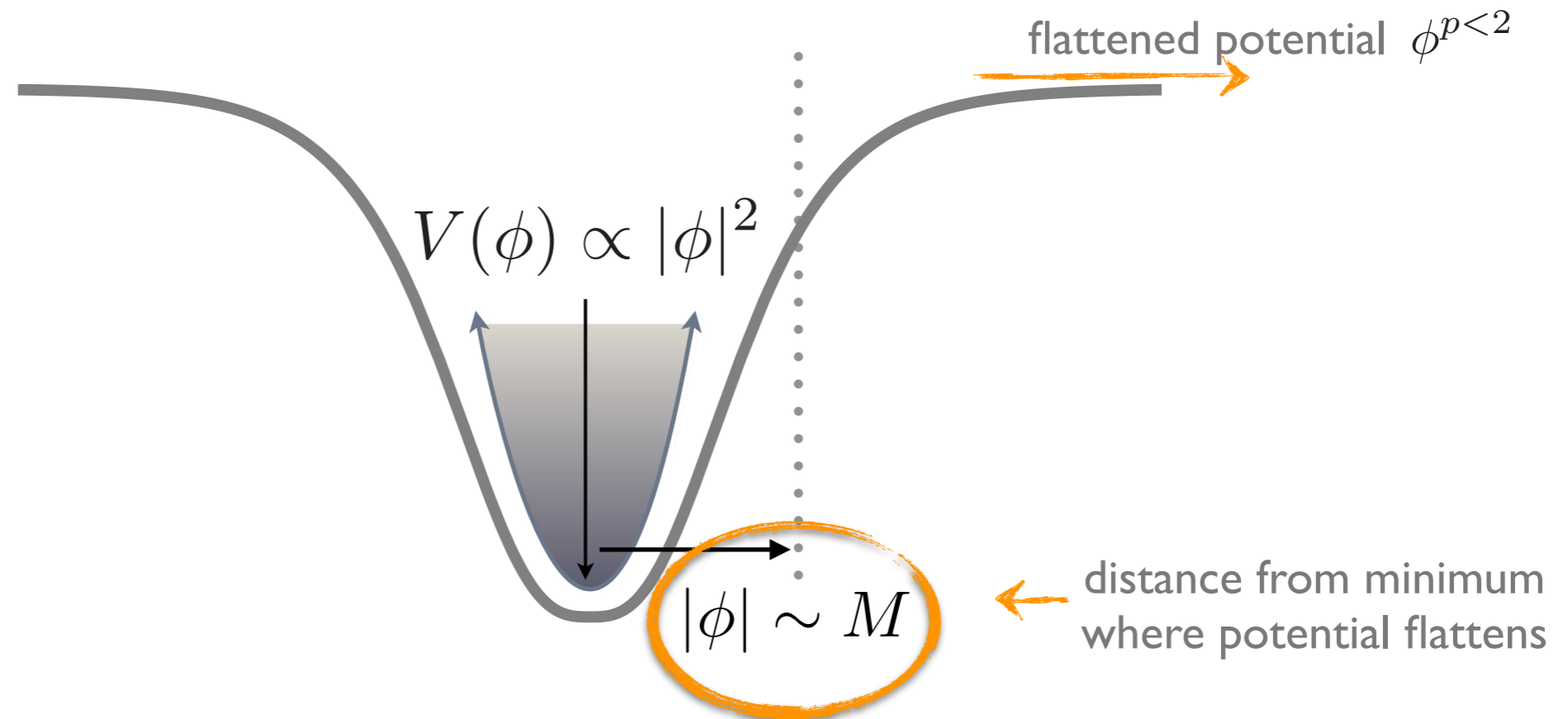


$\chi, \psi$

|          |   |                                       |                                      |                         |                         |
|----------|---|---------------------------------------|--------------------------------------|-------------------------|-------------------------|
| mass →   | ~2.3 MeV/c <sup>2</sup>                   | ~1.275 GeV/c <sup>2</sup>             | ~173.07 GeV/c <sup>2</sup>           | 0                       | ~126 GeV/c <sup>2</sup> |
| charge → | 2/3                                       | 2/3                                   | 2/3                                  | 0                       | 0                       |
| spin →   | 1/2                                       | 1/2                                   | 1/2                                  | 1                       | 0                       |
|          | <b>u</b><br>up                            | <b>c</b><br>charm                     | <b>t</b><br>top                      | <b>g</b><br>gluon       | <b>H</b><br>Higgs boson |
|          | <b>d</b><br>down                          | <b>s</b><br>strange                   | <b>b</b><br>bottom                   | <b>γ</b><br>photon      |                         |
|          | <b>e</b><br>electron                      | <b>μ</b><br>muon                      | <b>τ</b><br>tau                      | <b>Z</b><br>Z boson     |                         |
|          | <b>ν<sub>e</sub></b><br>electron neutrino | <b>ν<sub>μ</sub></b><br>muon neutrino | <b>ν<sub>τ</sub></b><br>tau neutrino | <b>W</b><br>W boson     |                         |
|          | 0.511 MeV/c <sup>2</sup>                  | 105.7 MeV/c <sup>2</sup>              | 1.777 GeV/c <sup>2</sup>             | 91.2 GeV/c <sup>2</sup> |                         |
|          | -1  | -1                                    | -1                                   | 0                       |                         |
|          | 1/2                                       | 1/2                                   | 1/2                                  | 1                       |                         |
|          | 0   | 0                                     | 0                                    | ±1                      |                         |
|          | 1/2                                       | 1/2                                   | 1/2                                  | 1                       |                         |
|          | <2.2 eV/c <sup>2</sup>                    | <0.17 MeV/c <sup>2</sup>              | <15.5 MeV/c <sup>2</sup>             | 80.4 GeV/c <sup>2</sup> |                         |
|          | 0   | 0                                     | 0                                    | ±1                      |                         |
|          | 1/2                                       | 1/2                                   | 1/2                                  | 1                       |                         |

Traschen & Brandenburger (1990)  
 Kofman, Linde & Starobinsky (1994, 97)  
 Shtanov, Traschen & Brandenberger (1995)  
 Kofman, Linde & Starobinsky (1997)

# end of inflation in “simple” models



- shape of the potential (self couplings)

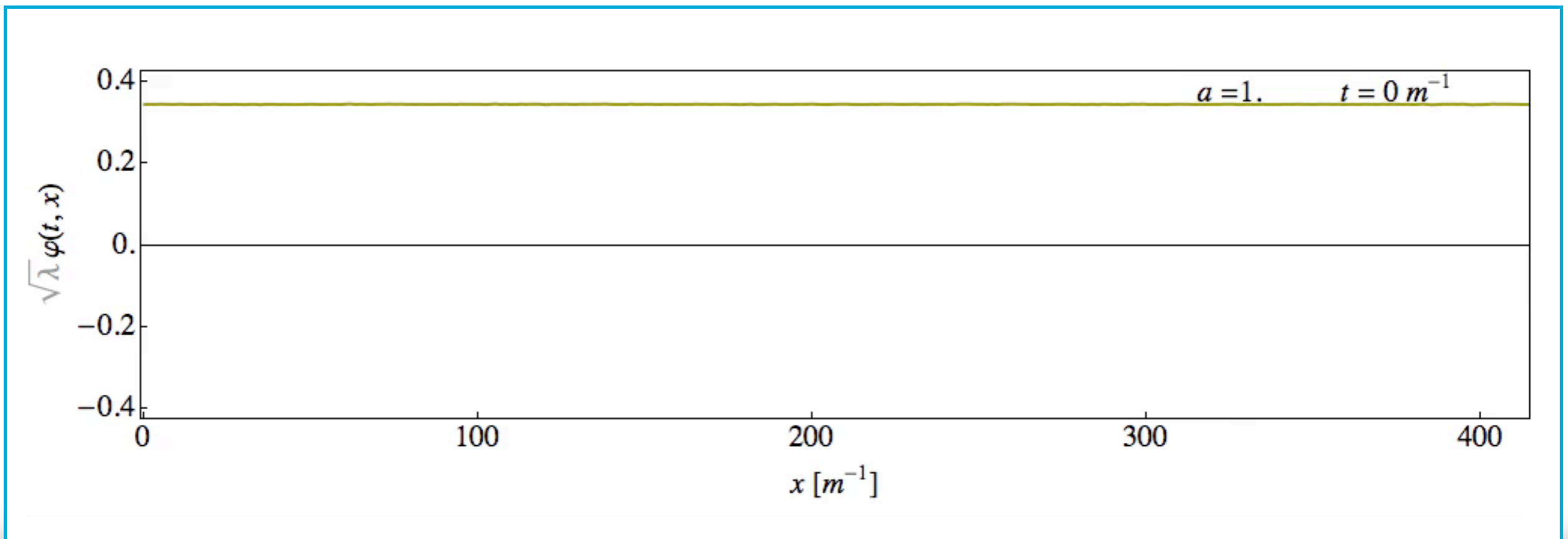
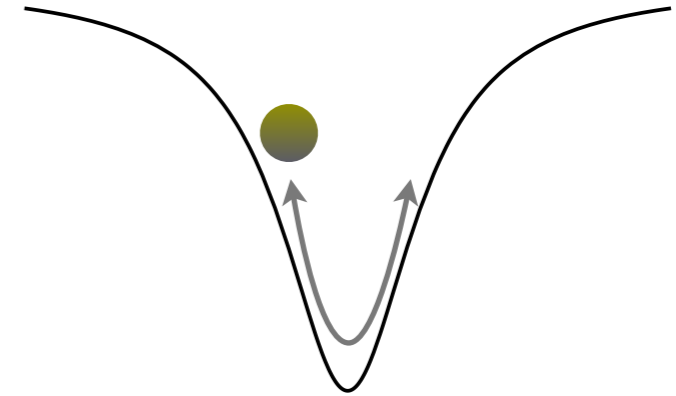
- ~~couplings to other fields~~  
 $\sim$  gravitational strength

|         |               |               |               |                              |                            |                            |              |                  |
|---------|---------------|---------------|---------------|------------------------------|----------------------------|----------------------------|--------------|------------------|
| QUARKS  | UP<br>u       | DOWN<br>d     | CHARM<br>c    | STRANGE<br>s                 | TOP<br>t                   | BOTTOM<br>b                | GLUONS<br>g  | HIGGS BOSON<br>H |
| LEPTONS | ELECTRON<br>e | MUON<br>$\mu$ | TAU<br>$\tau$ | ELECTRON NEUTRINO<br>$\nu_e$ | MUON NEUTRINO<br>$\nu_\mu$ | TAU NEUTRINO<br>$\nu_\tau$ | Z BOSON<br>Z | W BOSON<br>W     |

~~$\chi, \psi$~~

# dynamics after inflation

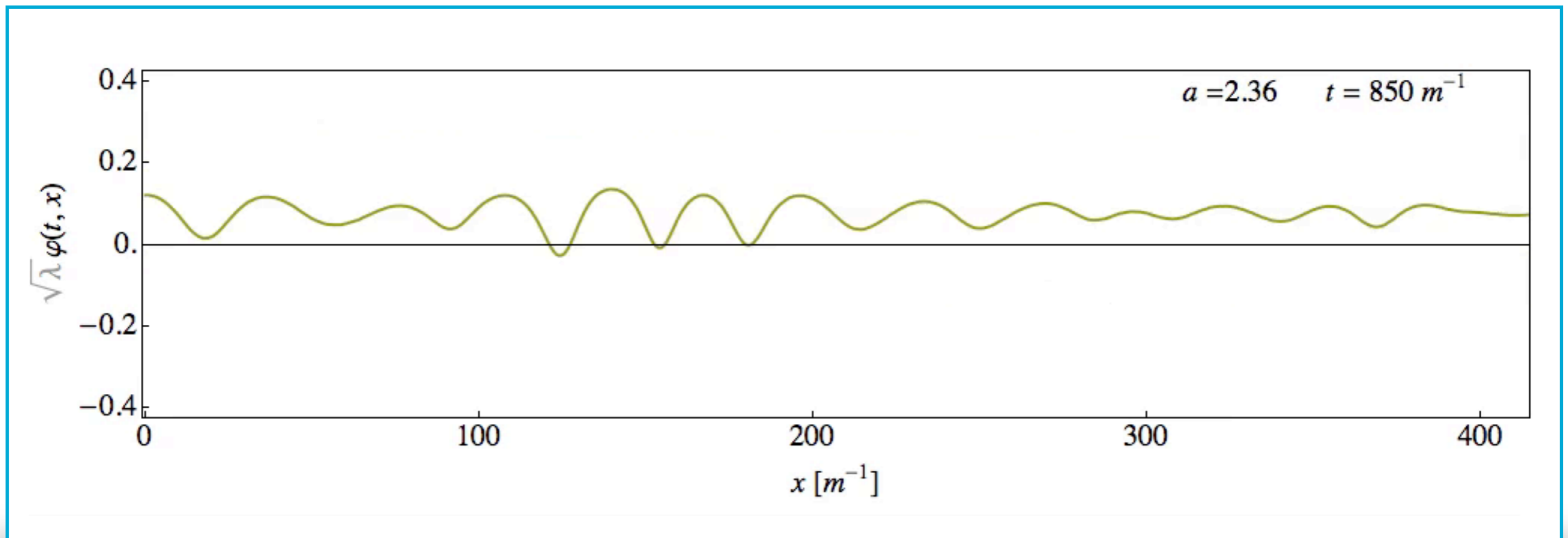
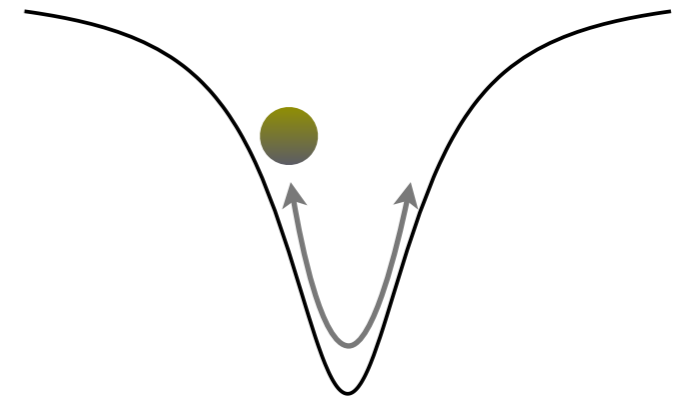
$$\square\phi = V'(\phi)$$





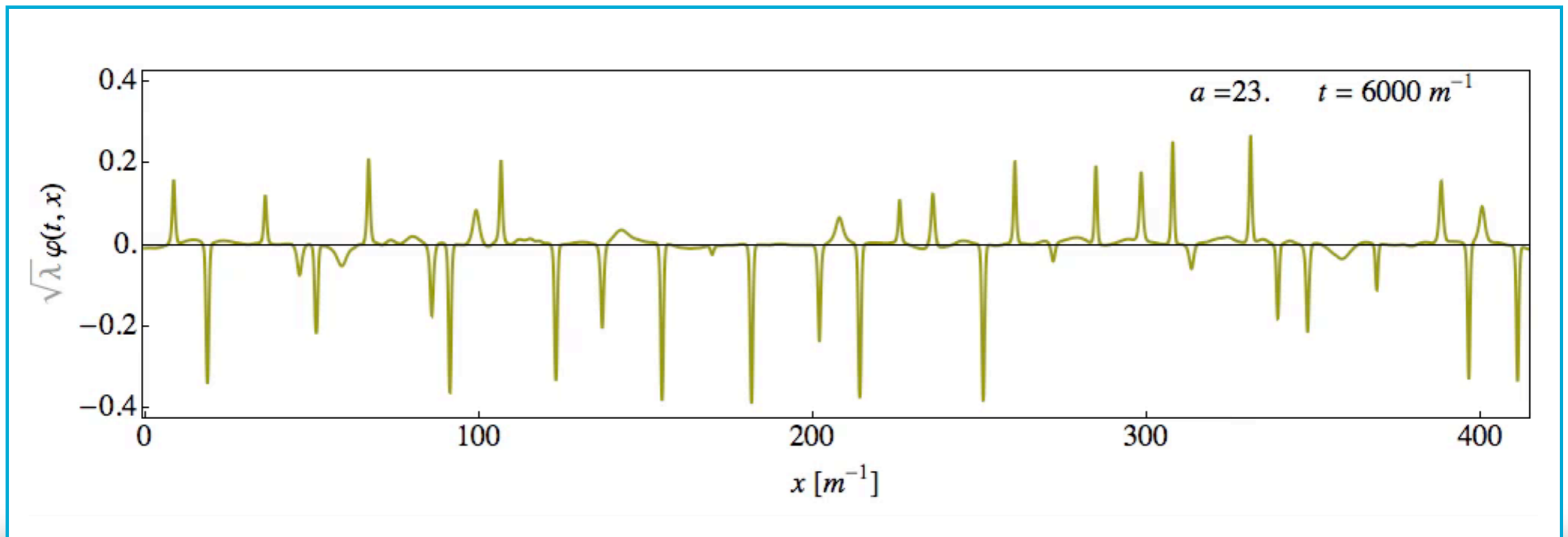
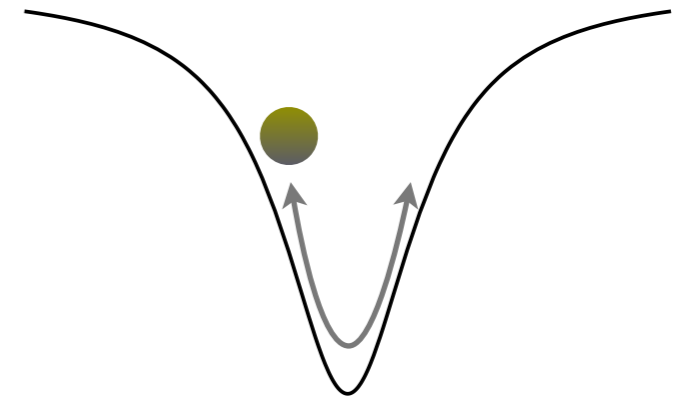
# dynamics after inflation

$$\square\phi = V'(\phi)$$



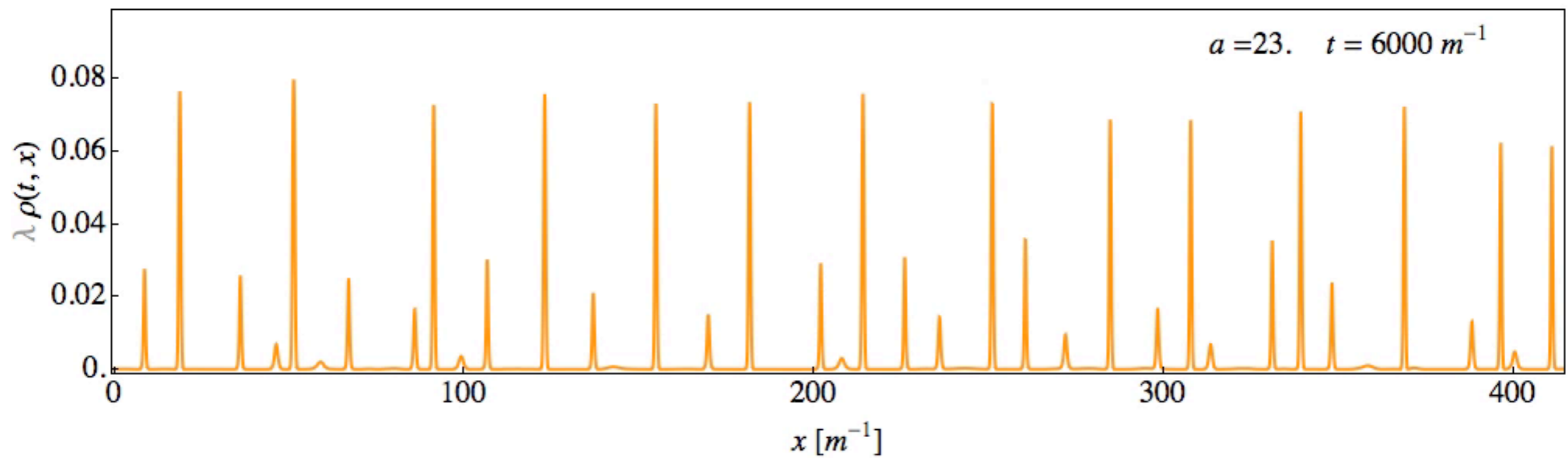
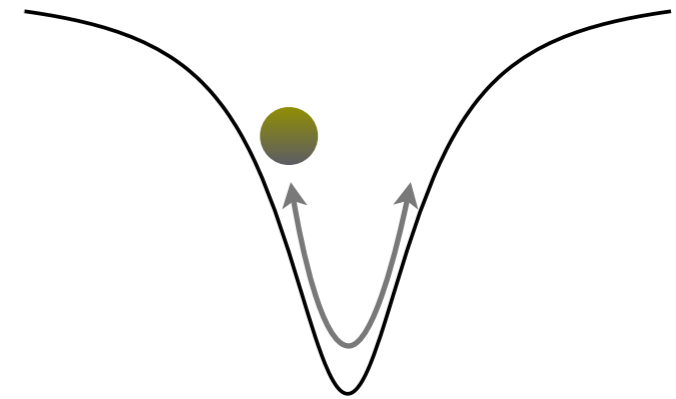
# dynamics after inflation

$$\square \phi = V'(\phi)$$

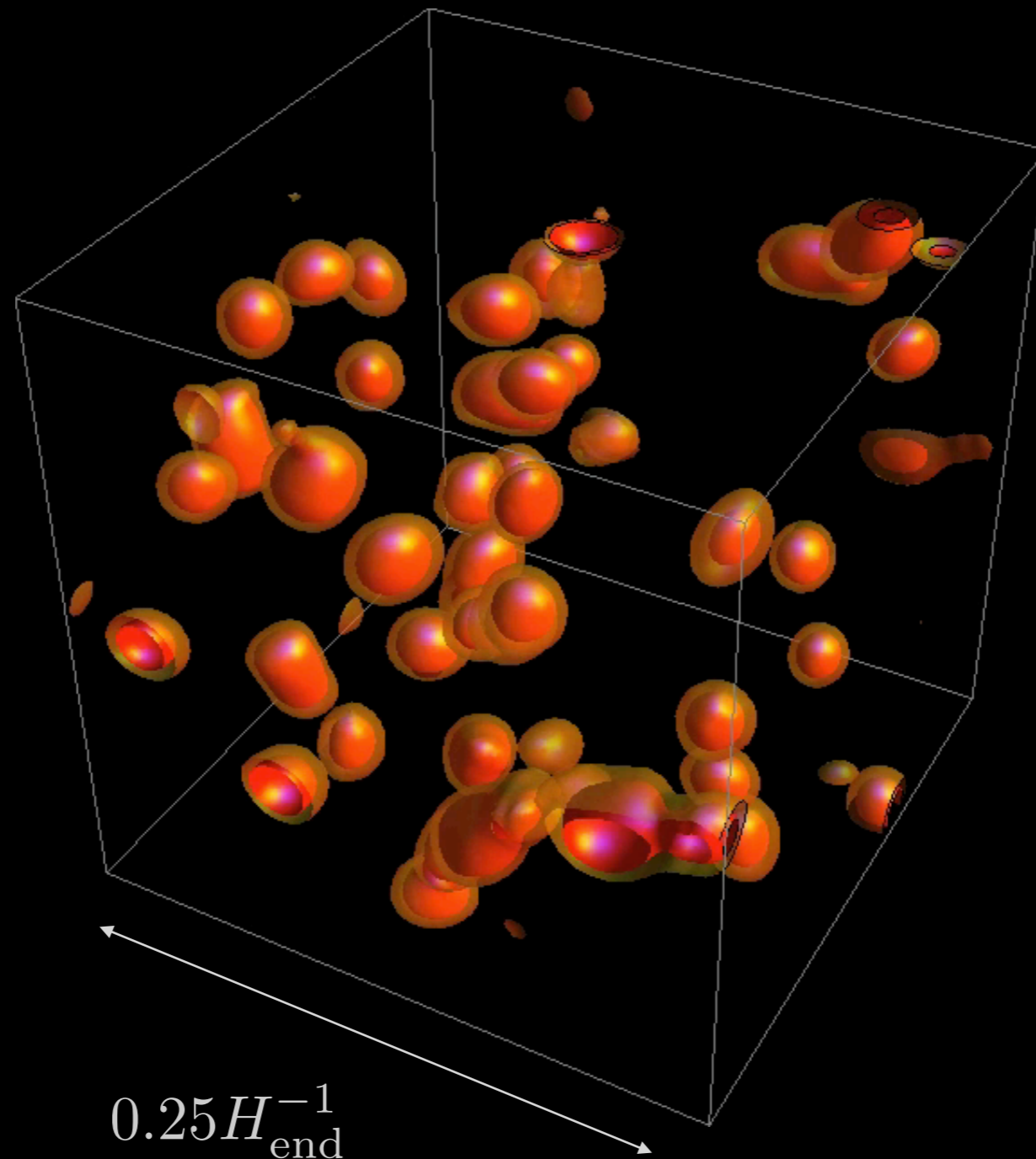


# dynamics after inflation

$$\square\phi = V'(\phi)$$



now in 3D:  
(iso-density surfaces)

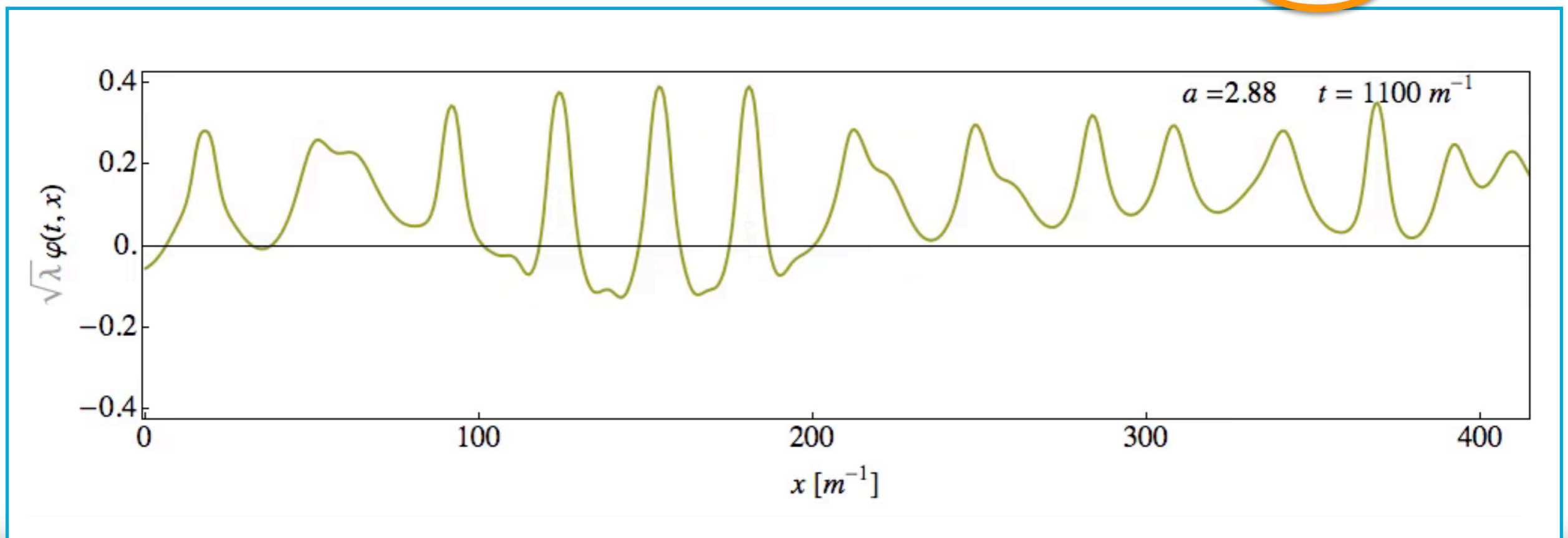
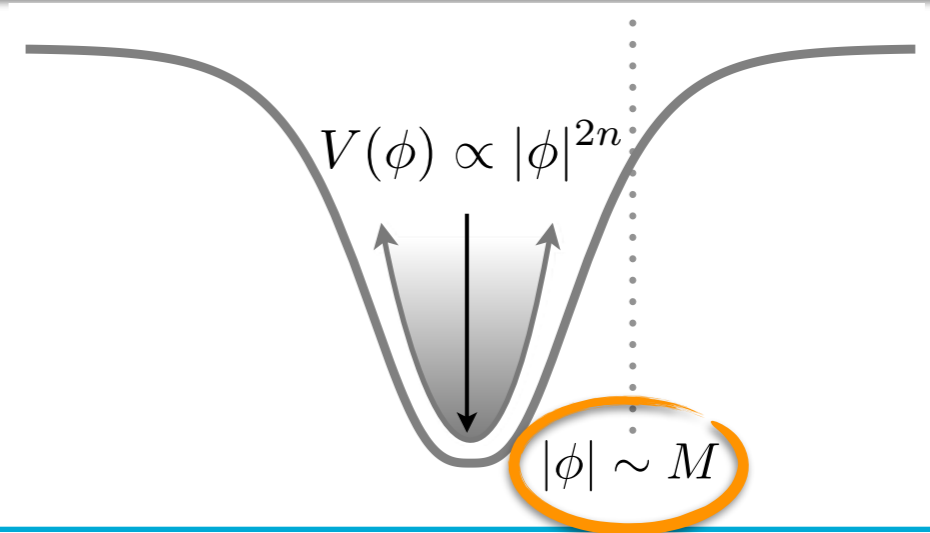


MA, Easter, Finkel, Flaugher & Hertzberg (2011)

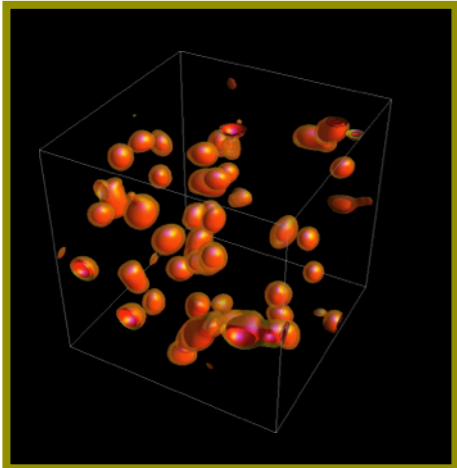


# condition for rapid fragmentation ?

$$\frac{\text{growth-rate of fluctuations}}{\text{expansion rate}} \sim \frac{m_{\text{pl}}}{M} \gg 1$$

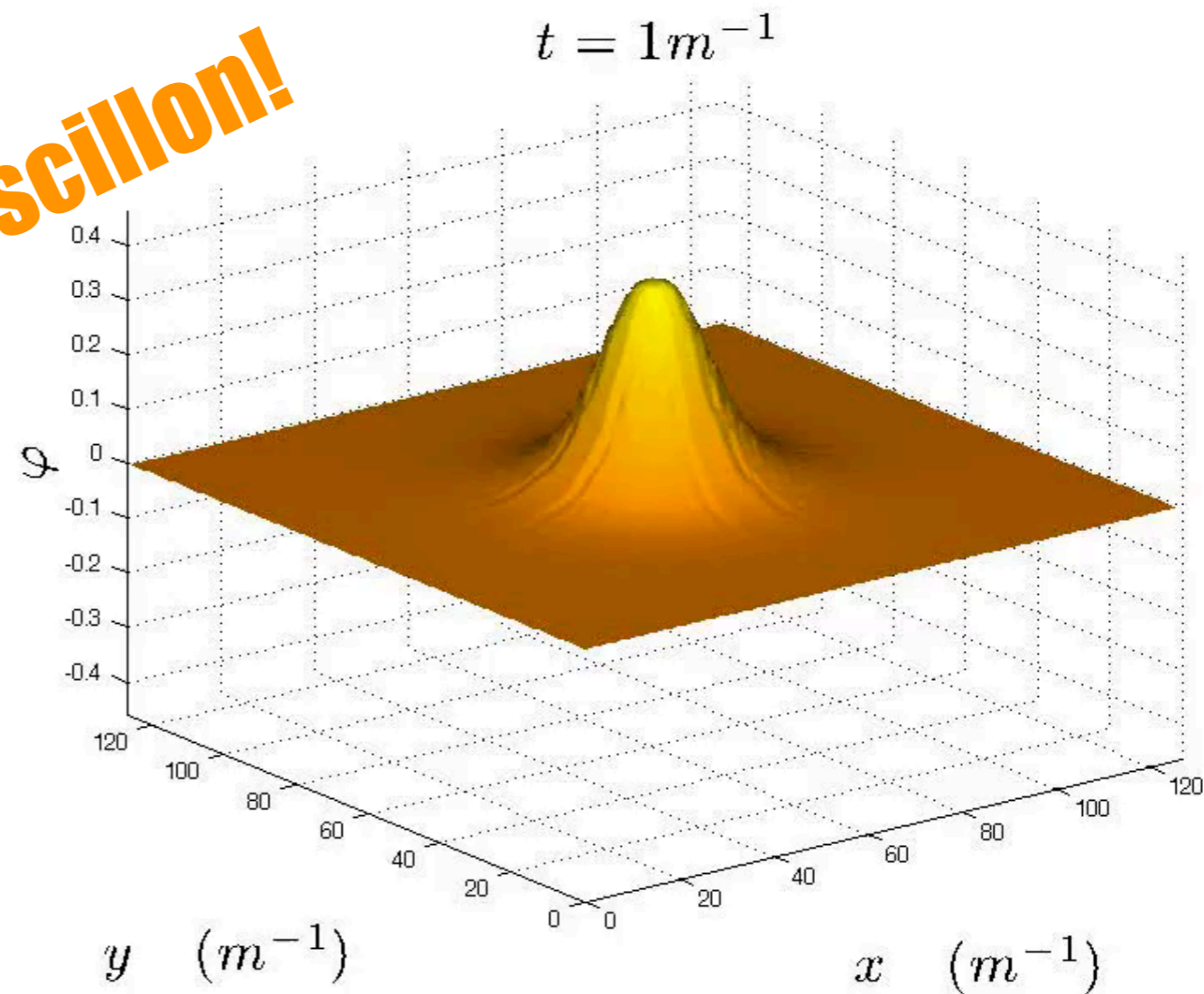


# lumps ?



(1) oscillatory (2) spatially localized (3) **very long lived**

**oscillon!**



**existence and stability:**

Segur & Kruskal (1987)

MA & Shirokoff (2010)

MA (2013)

Hertzberg (2011)

Mukaido et. al (2016,17)

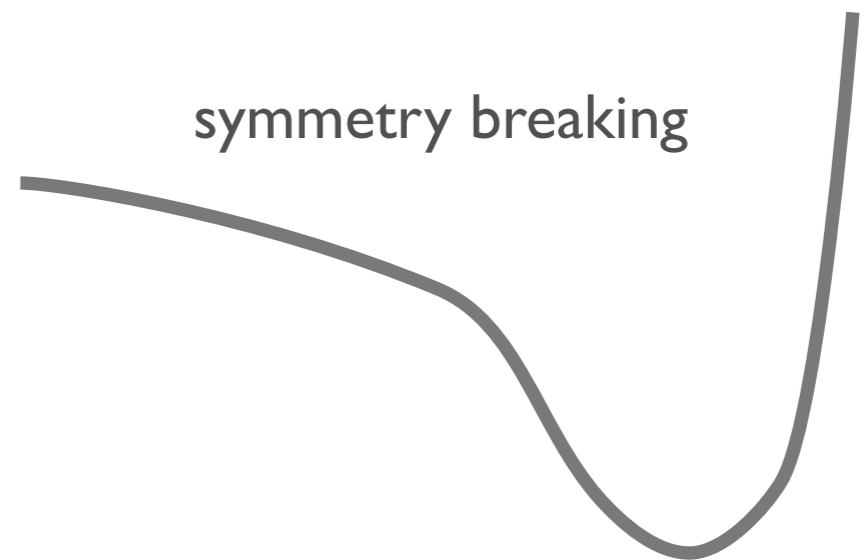
Bogolubsky & Makhankov (1976), Gleiser (1994), Copeland et. al (1995)

# existence conditions

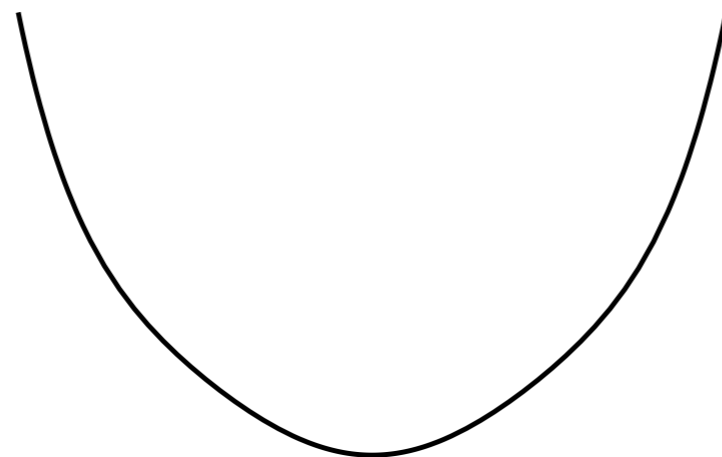
$$V(\varphi) = \frac{1}{2}\varphi^2 + \frac{1}{3}\lambda_3\varphi^3 + \frac{1}{4}\lambda_4\varphi^4 + \dots$$

$$\Delta \equiv -\lambda_4 + \frac{10}{9}\lambda_3^2 > 0$$

symmetry breaking



axions, axion monodromy



# existence conditions

$$\mathcal{L} = T(X, \varphi) - V(\varphi)$$

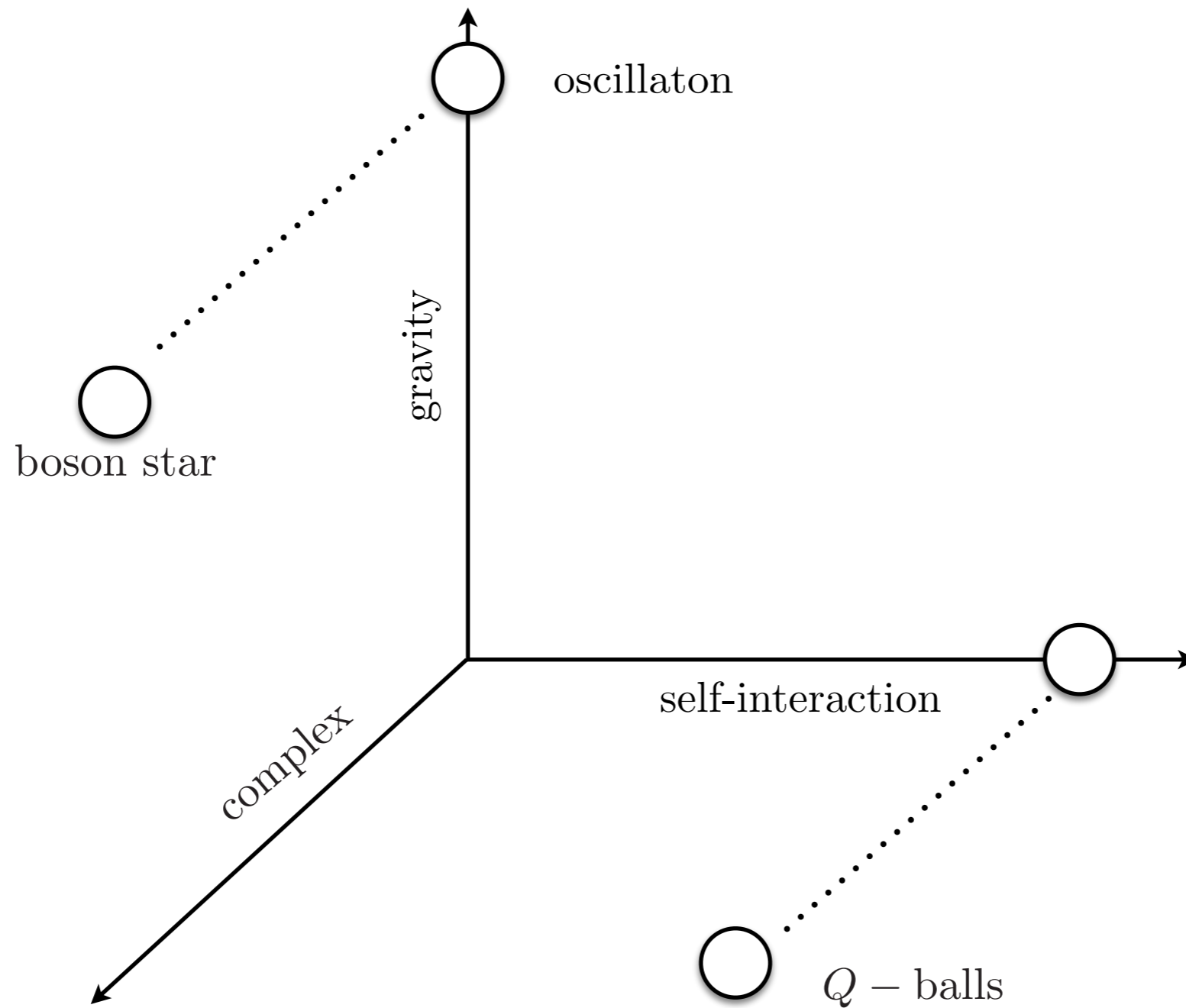
$$X = \frac{1}{2}(\partial\varphi)^2$$

$$T(X, \varphi) = X + \xi_2 X^2 + \xi_3 \varphi X^2 + \dots$$

$$V(\varphi) = \frac{1}{2}\varphi^2 + \frac{\lambda_3}{3}\varphi^3 + \frac{\lambda_4}{4}\varphi^4 + \frac{\lambda_5}{5}\varphi^5 + \dots$$

$$\Delta = \xi_2 - \lambda_4 + \frac{10}{9}\lambda_3^2 > 0.$$

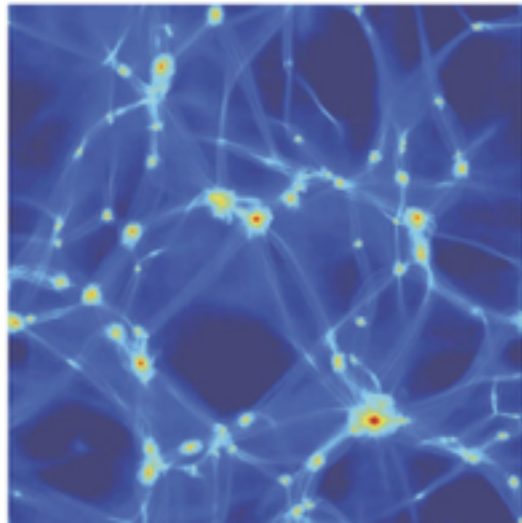
# family of related solitons



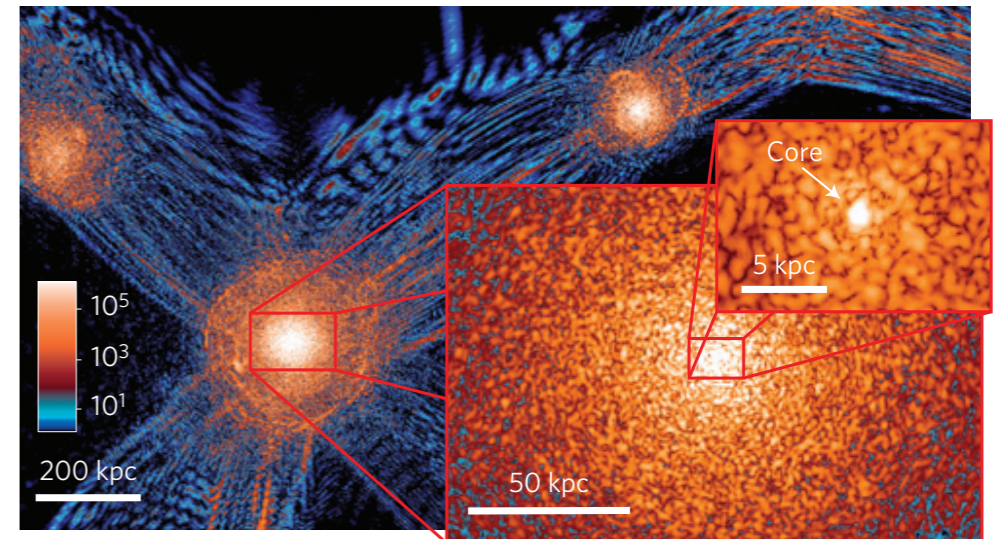
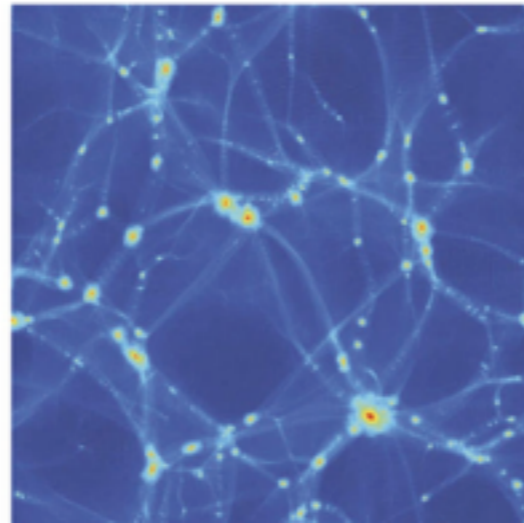


# axions, moduli fields, BECs etc

(a)  $\psi$ DM

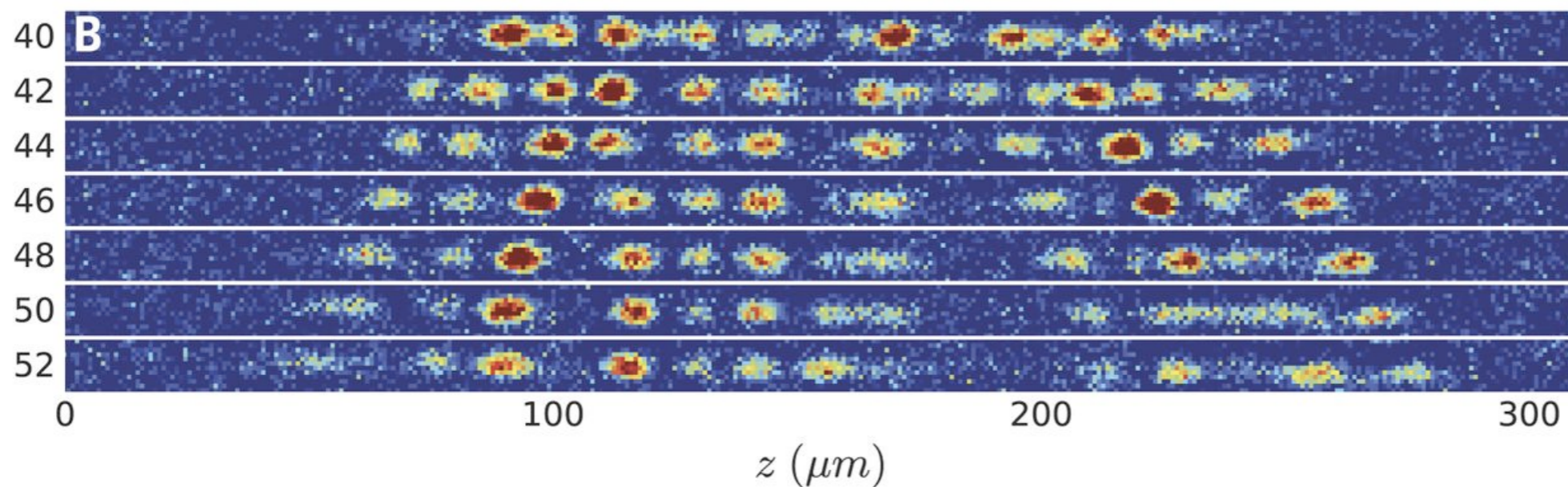


(b) CDM



Schive et. al (2014)

Nguyen, Luo & Hulet (2017)

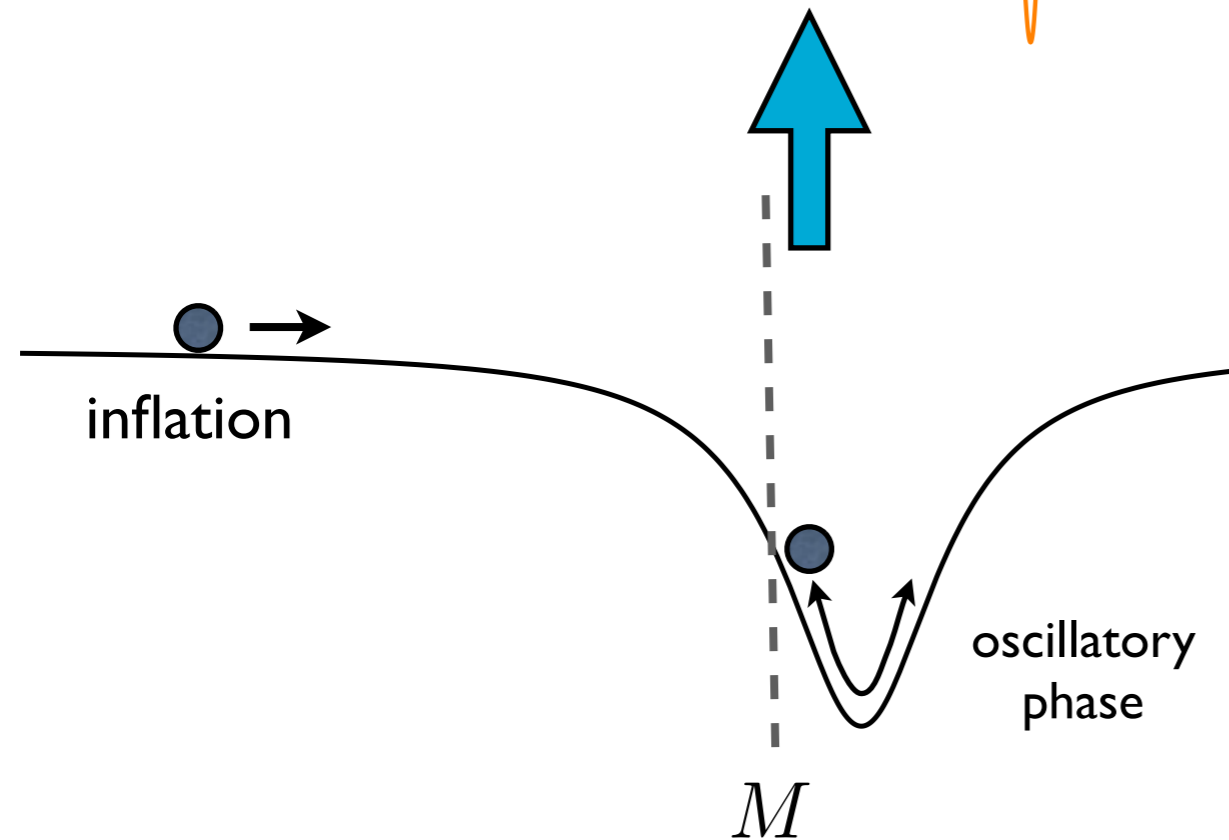
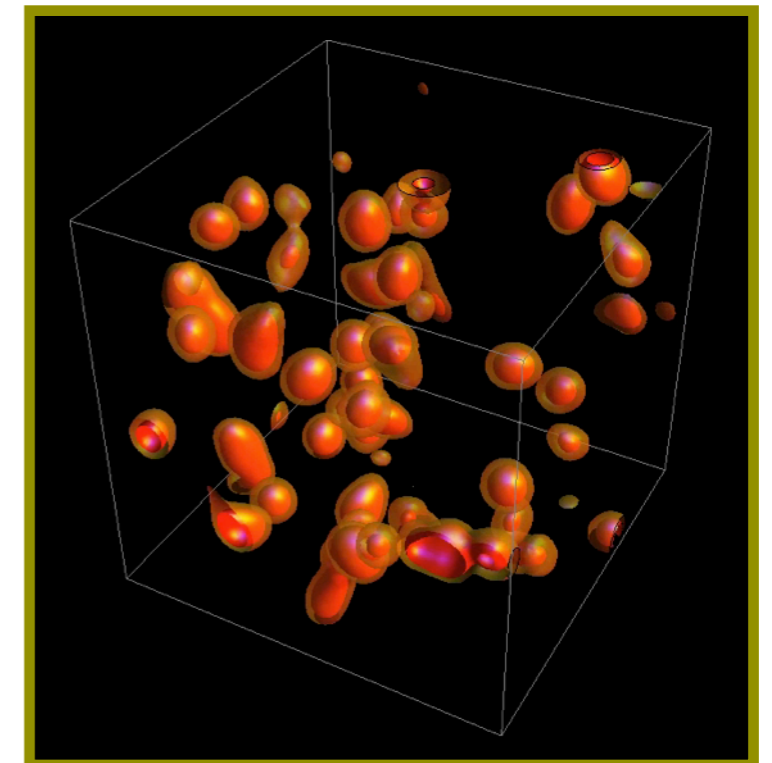
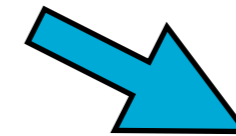
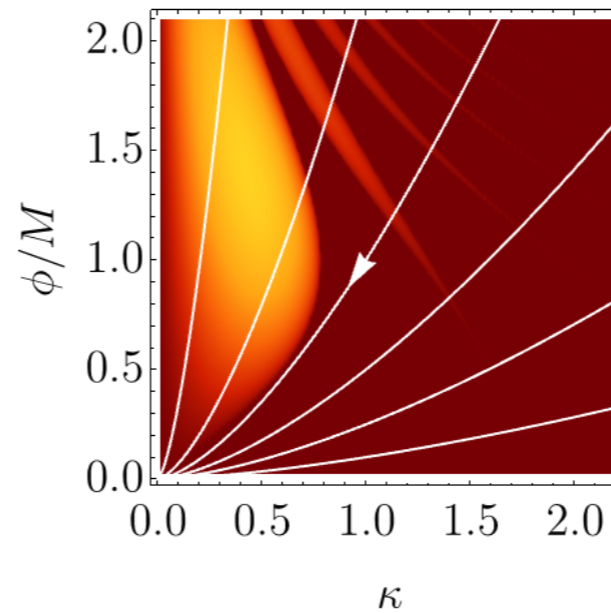


# so far : end of inflation

$$\delta\varphi_k(t) \propto e^{\mu_k t}$$

resonant growth

$$\frac{\mu_k}{H} \sim \frac{m_{\text{pl}}}{M} \gg 1$$

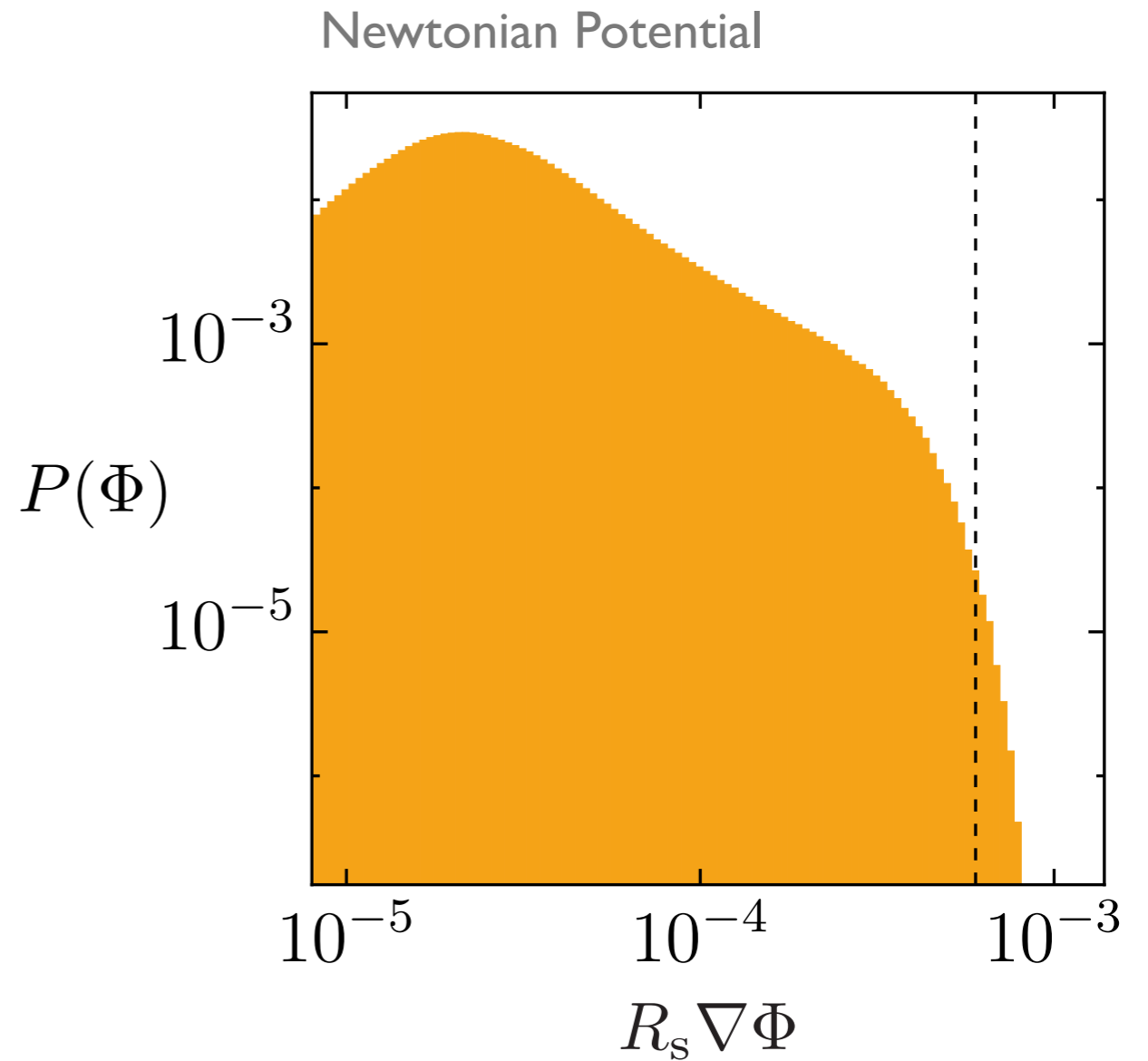
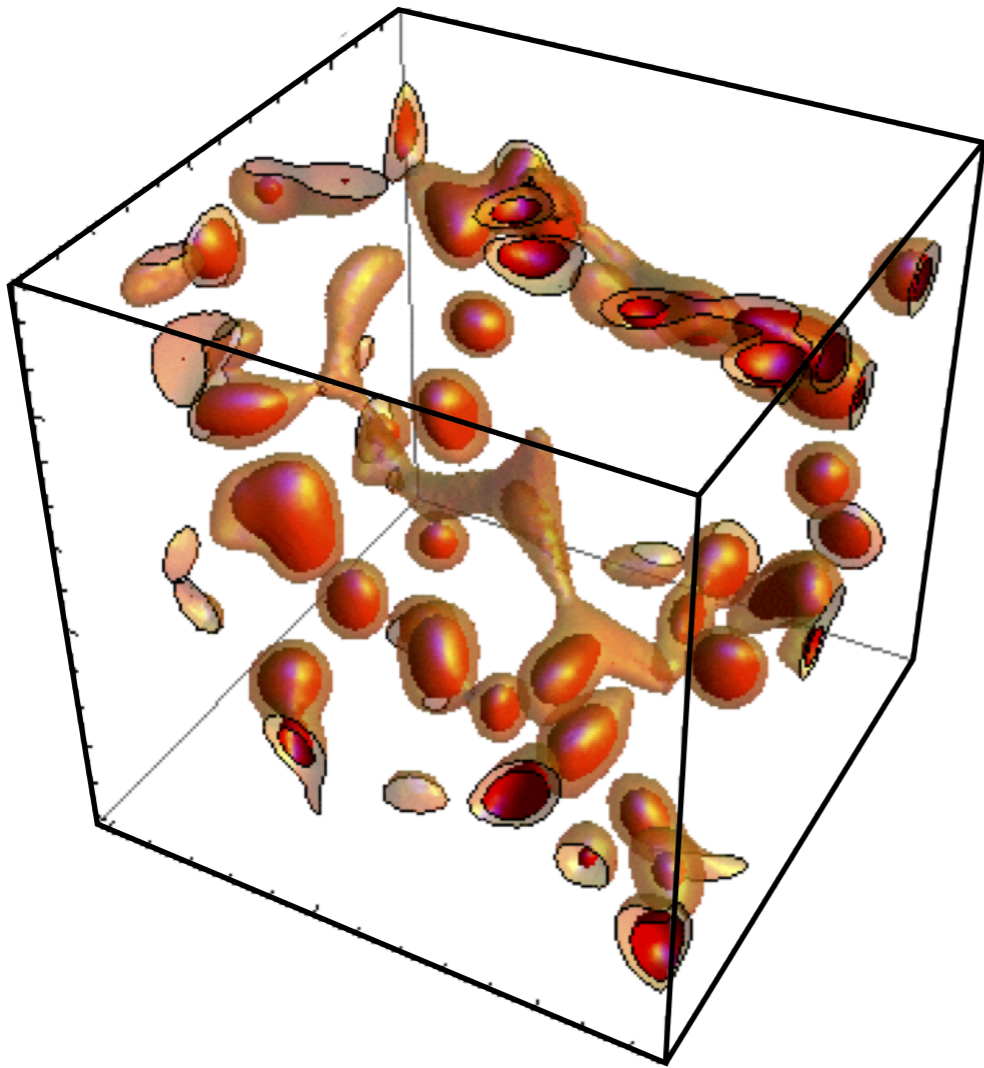




# consequences ?

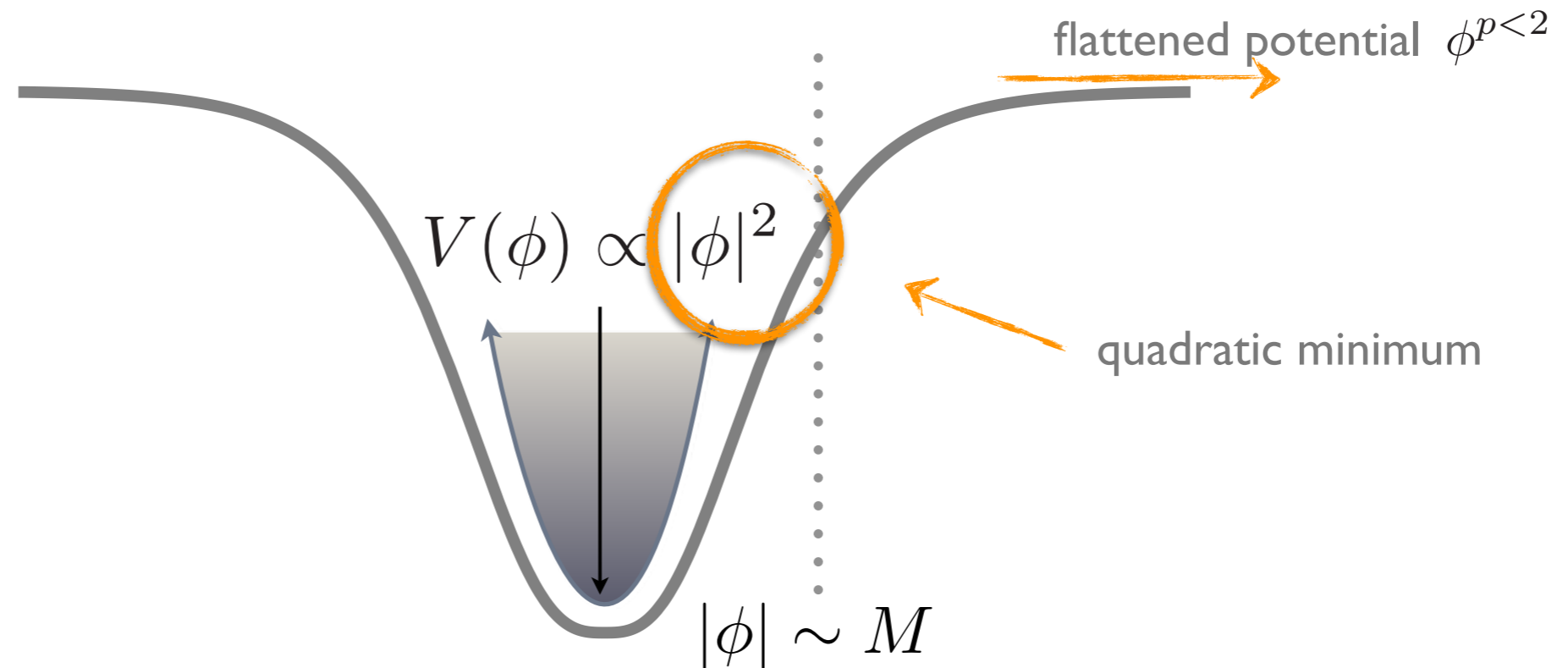
- equation of state/duration to radiation domination ?  
Lozanov & MA (2017, 2018)
- **black holes ?**
- gravitational waves ?  
Zhou et. al (2013), Antusch et. al (2015), MA et. al (2018)

# primordial black holes? gravitational clustering ?



Lozanov & MA (in progress)  
Mocz & MA (in progress)

# end of inflation in “simple” models



- shape of the potential (self couplings)

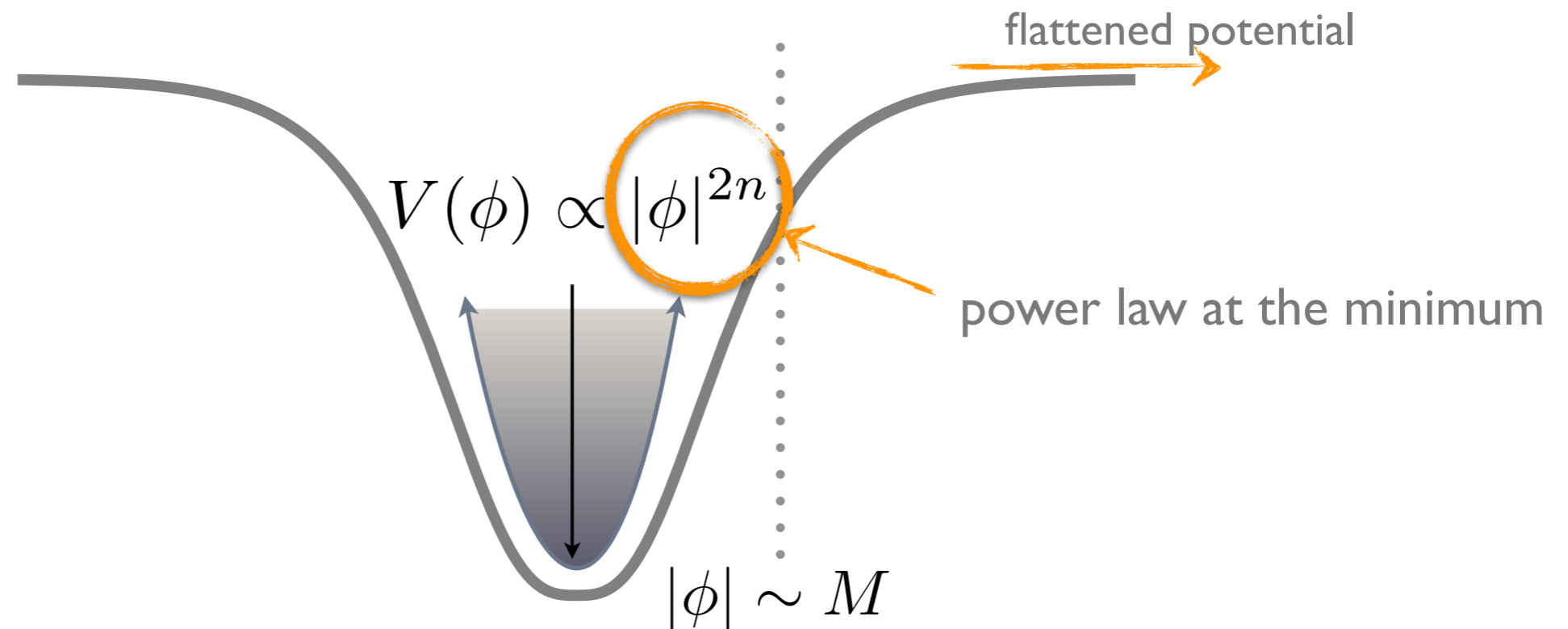
- ~~couplings to other fields~~

|                  |                    |                      |              |
|------------------|--------------------|----------------------|--------------|
| UP QUARKS        | DOWN QUARKS        | LEPTONS              | GAUGE BOSONS |
| u<br>up quark    | d<br>down quark    | e<br>electron        | g<br>gluon   |
| c<br>charm quark | s<br>strange quark | $\mu$<br>muon        | W<br>W boson |
| t<br>top quark   | b<br>bottom quark  | $\tau$<br>tau lepton | Z<br>Z boson |
| g<br>gluon       | photon             | electron neutrino    | photon       |
| H<br>Higgs boson |                    | muon neutrino        | W boson      |
|                  |                    | tau neutrino         |              |

~~$\chi, \psi$~~



# end of inflation in “simple” models



- shape of the potential (self couplings)

- ~~couplings to other fields~~

|                  |                    |                      |                           |
|------------------|--------------------|----------------------|---------------------------|
| UP QUARKS        | DOWN QUARKS        | LEPTONS              | GAUGE BOSONS              |
| u<br>up quark    | d<br>down quark    | e<br>electron        | g<br>gluon                |
| c<br>charm quark | s<br>strange quark | $\mu$<br>muon        | W <sup>±</sup><br>W boson |
| t<br>top quark   | b<br>bottom quark  | $\tau$<br>tau lepton | Z<br>Z boson              |
| g<br>gluon       | photon             | electron neutrino    | photon                    |
| H<br>Higgs boson |                    | muon neutrino        | W <sup>±</sup><br>W boson |
|                  |                    | tau neutrino         |                           |

~~$\chi, \psi$~~

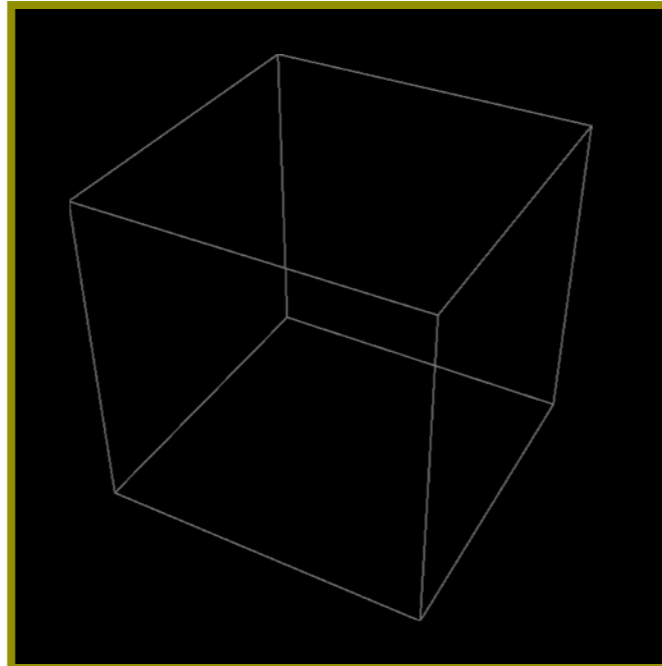
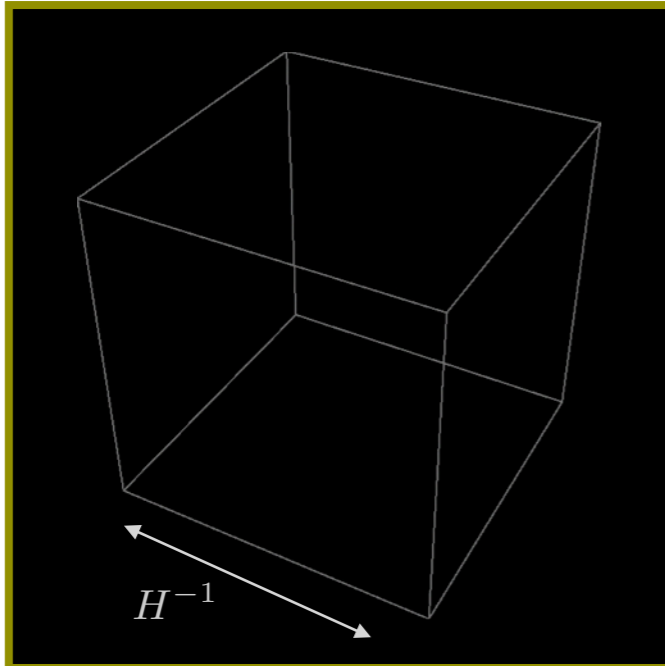
# result of fragmented dynamics

\* after sufficient time

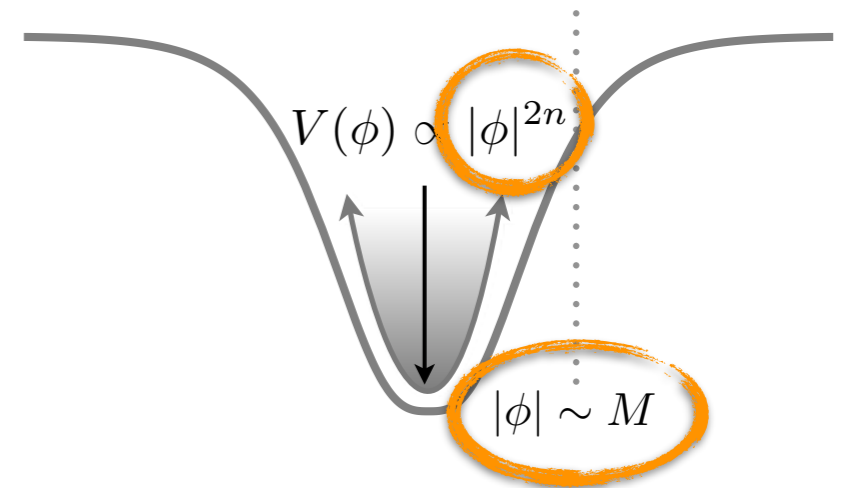
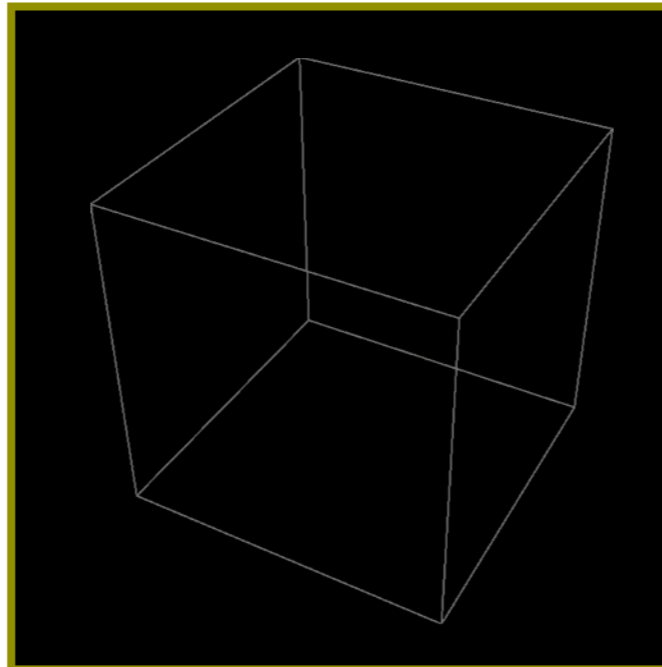
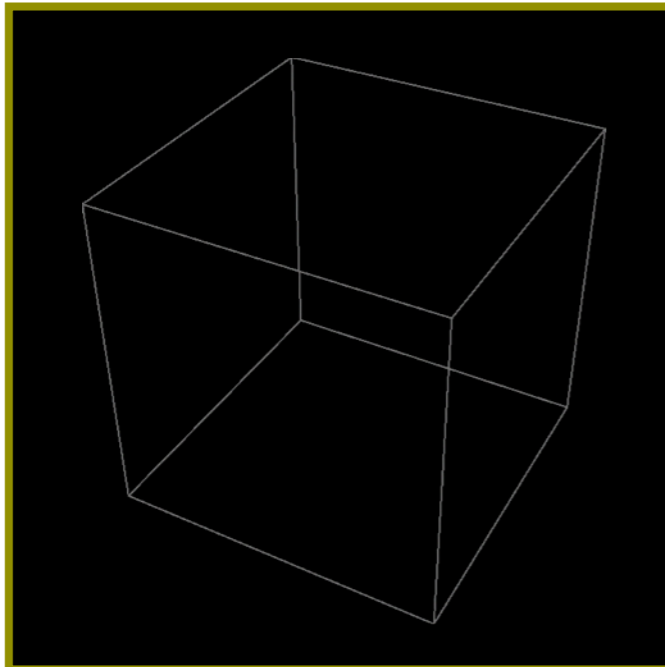
$n = 1$

$n > 1$

$M \sim m_{\text{pl}}$



$M \ll m_{\text{pl}}$



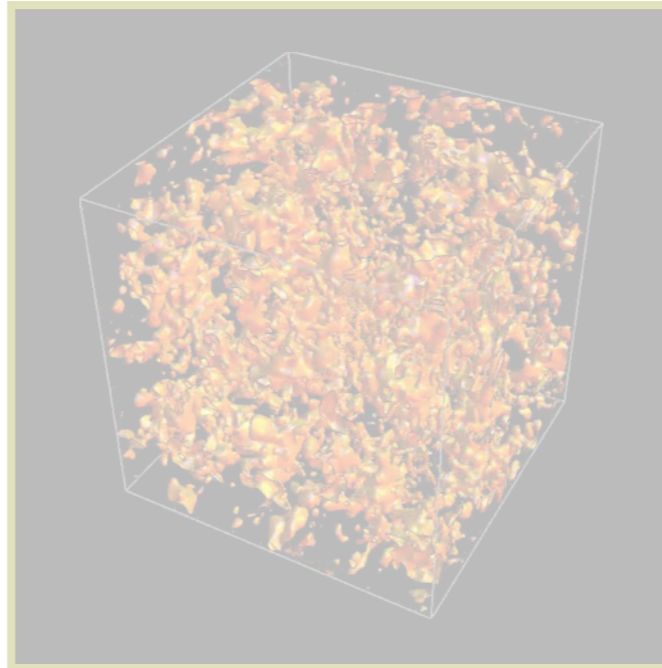
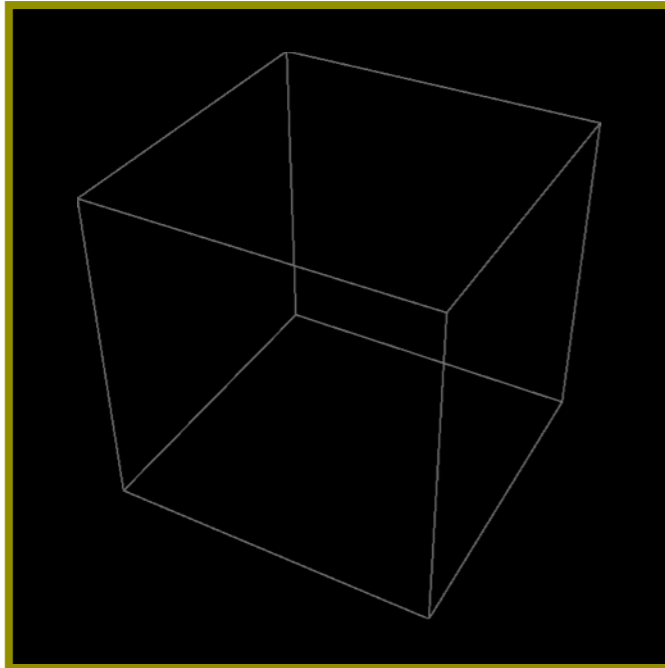
# result of fragmented dynamics

\* after sufficient time

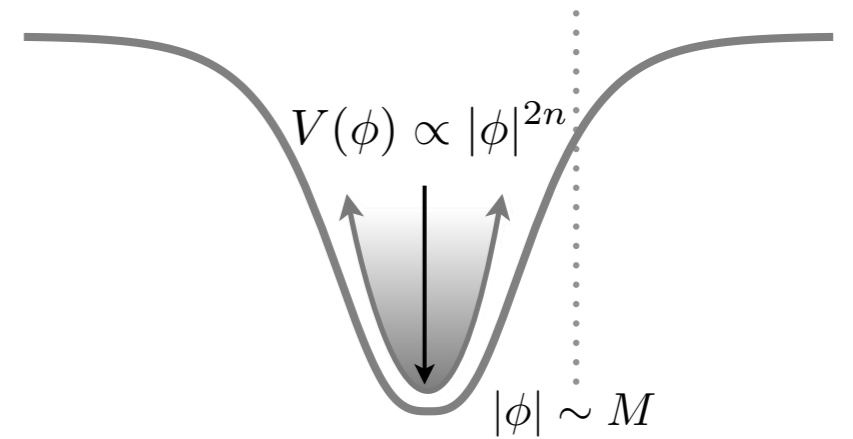
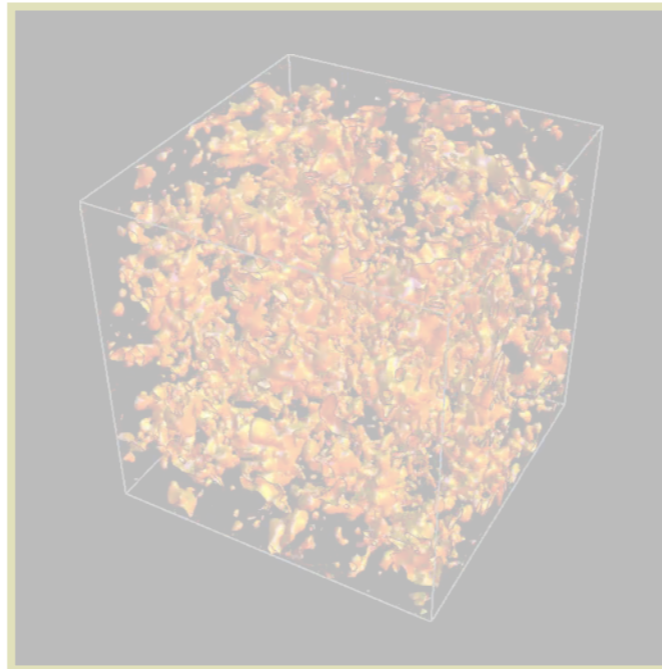
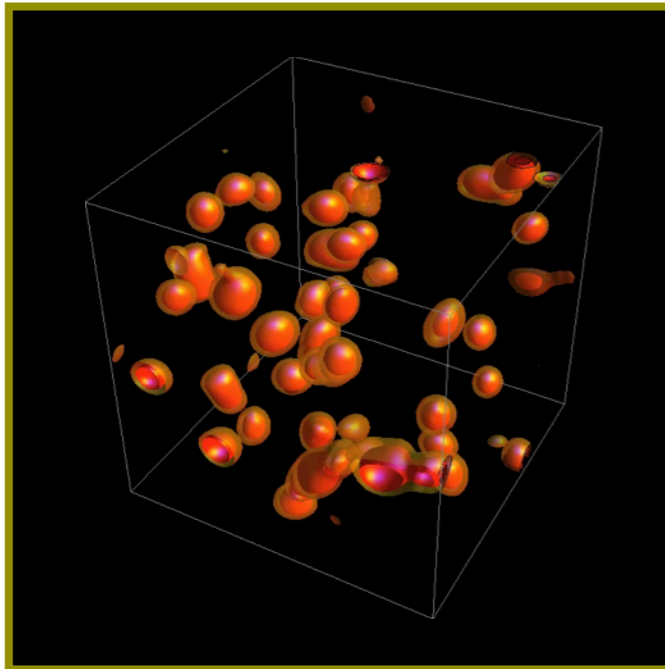
$n = 1$

$n > 1$

$M \sim m_{pl}$



$M \ll m_{pl}$





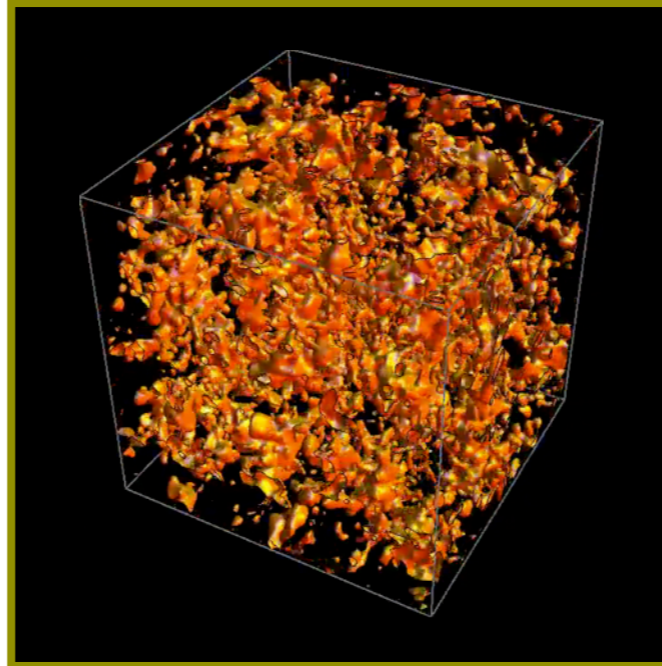
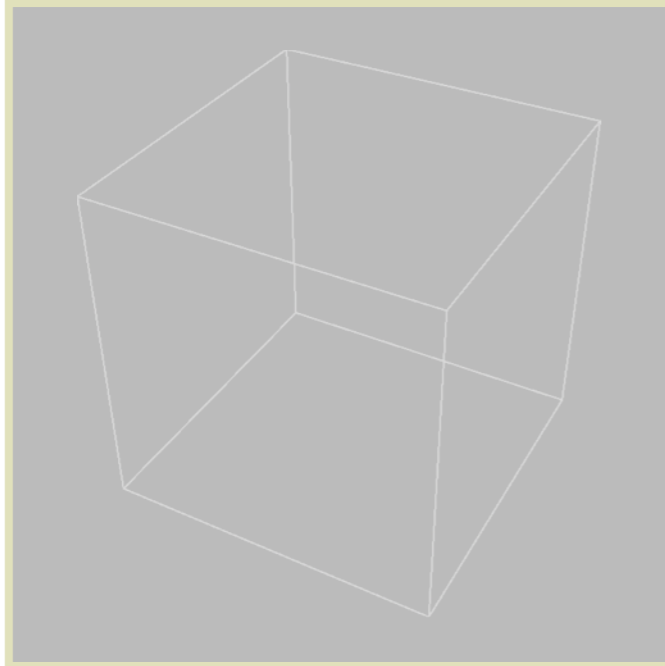
# result of fragmented dynamics

\* after sufficient time

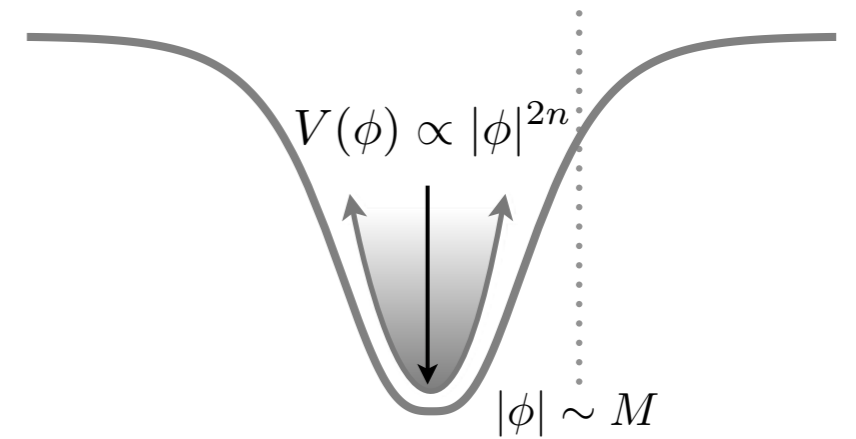
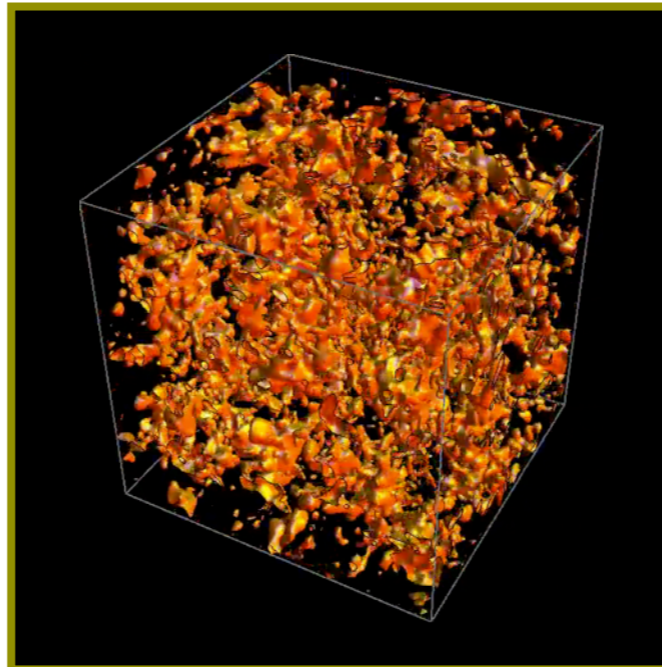
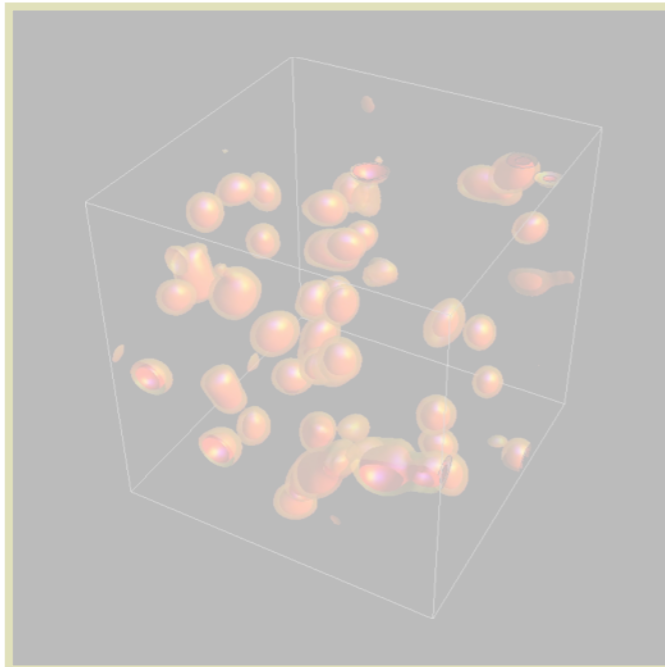
$n = 1$

$n > 1$

$M \sim m_{pl}$



$M \ll m_{pl}$



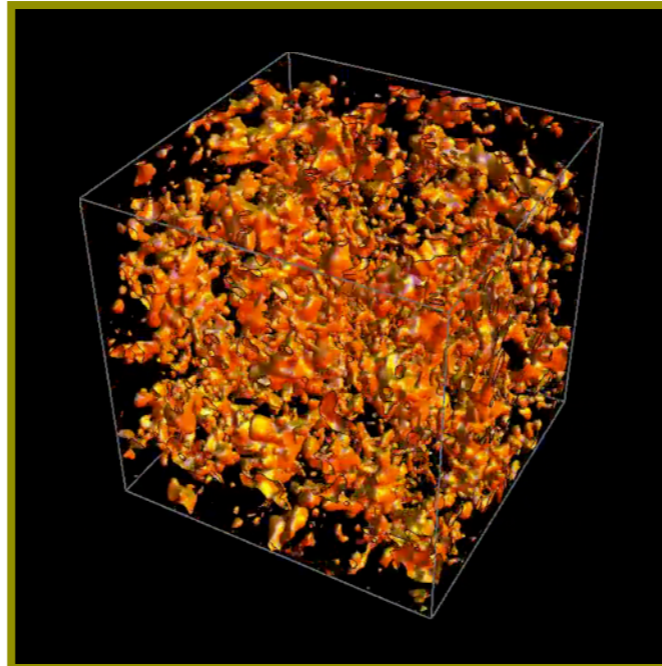
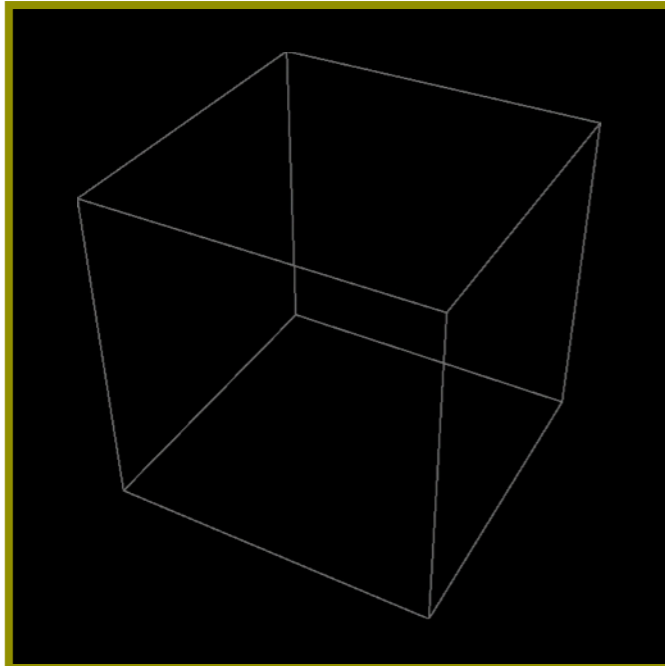
# result of fragmented dynamics

\* after sufficient time

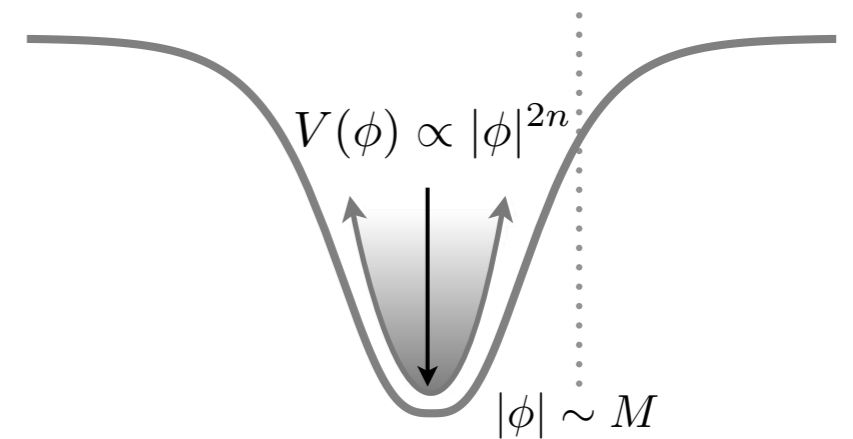
$n = 1$

$n > 1$

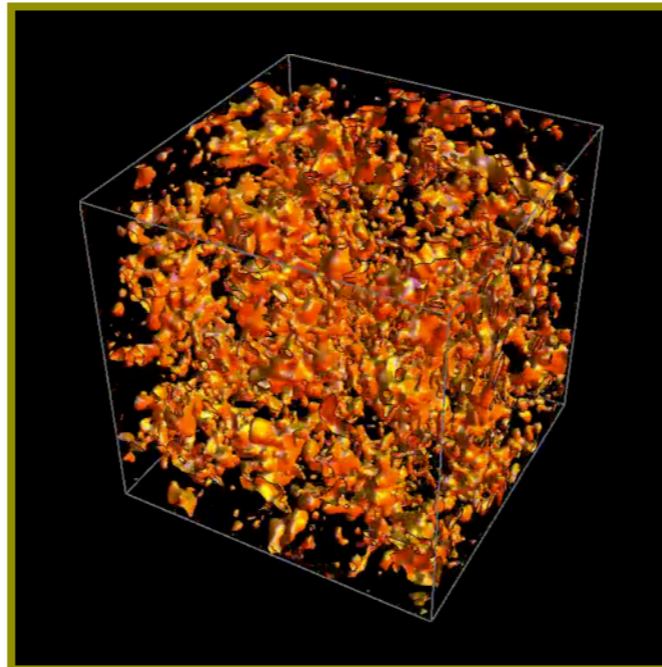
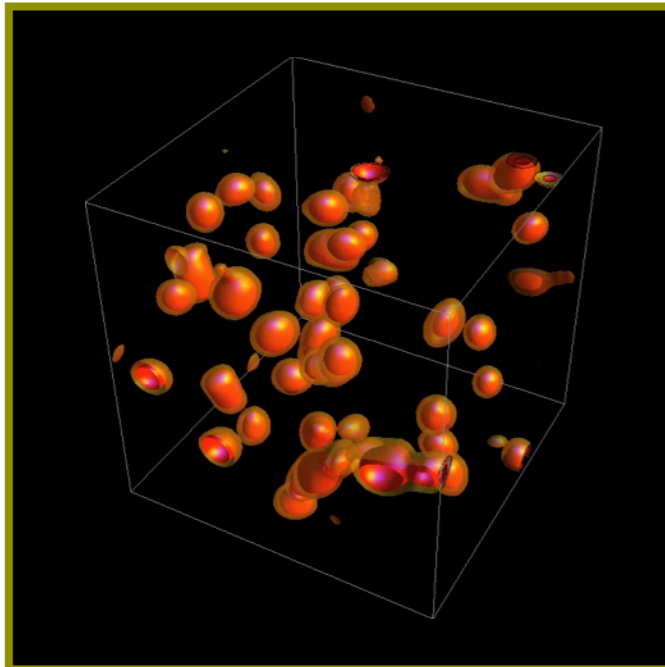
$M \sim m_{pl}$



slow



$M \ll m_{pl}$



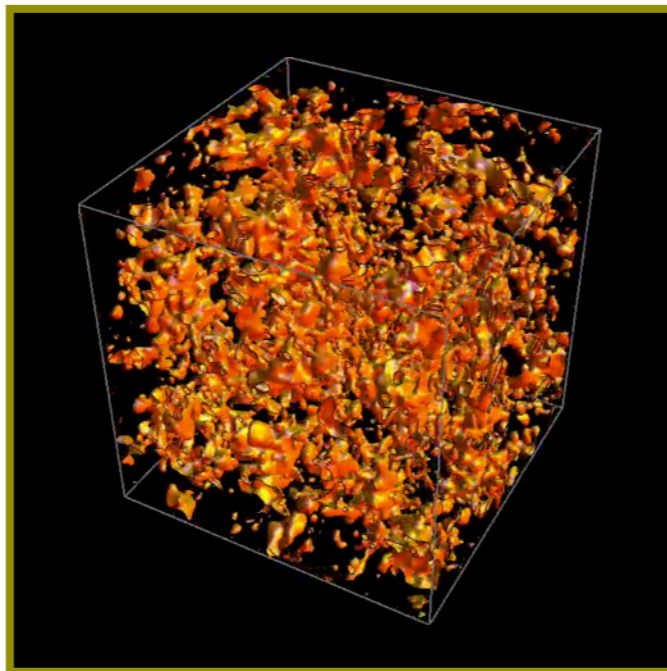
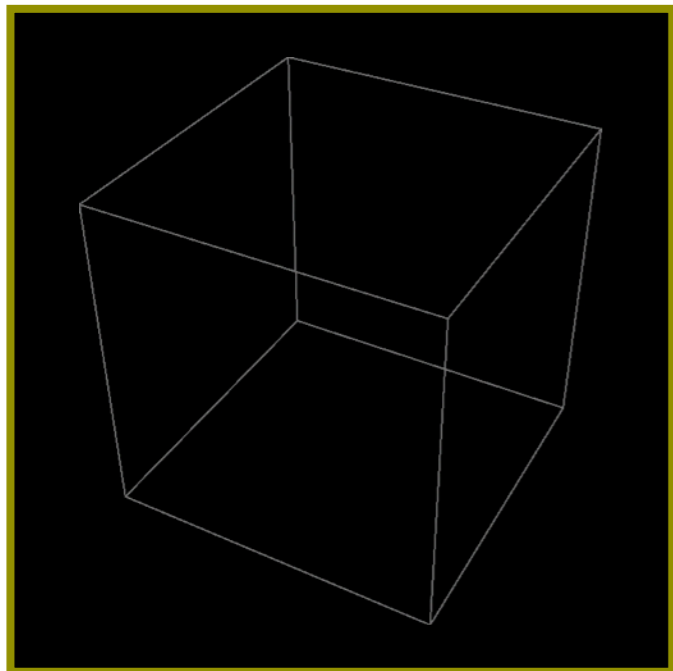
fast

eq. of state  $w = \text{pressure/density}$   
 \* after sufficient time

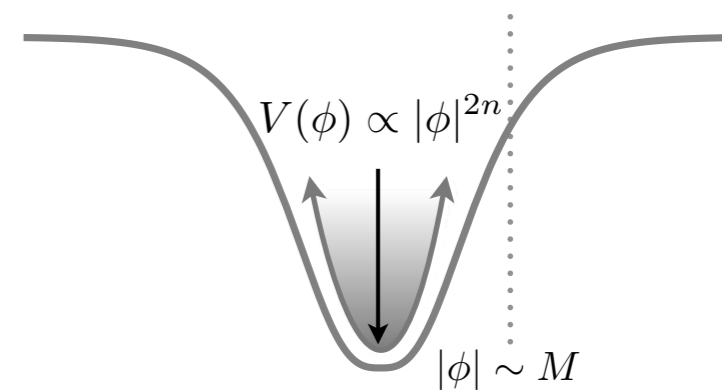
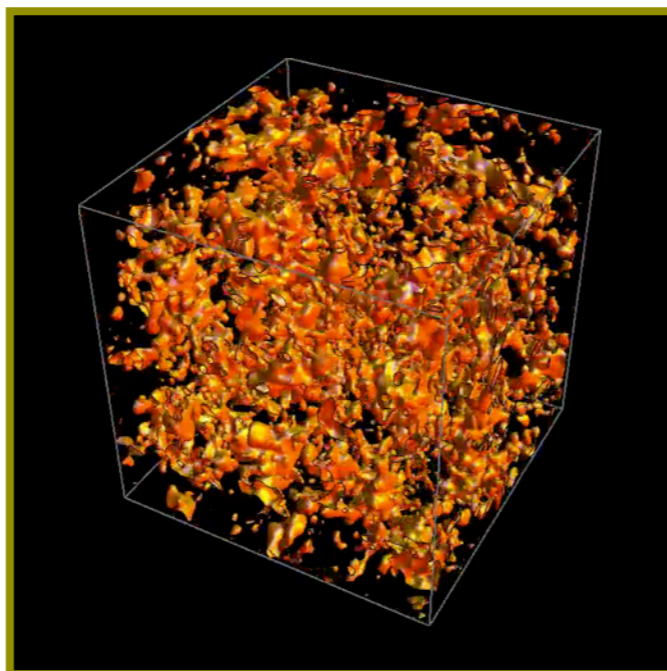
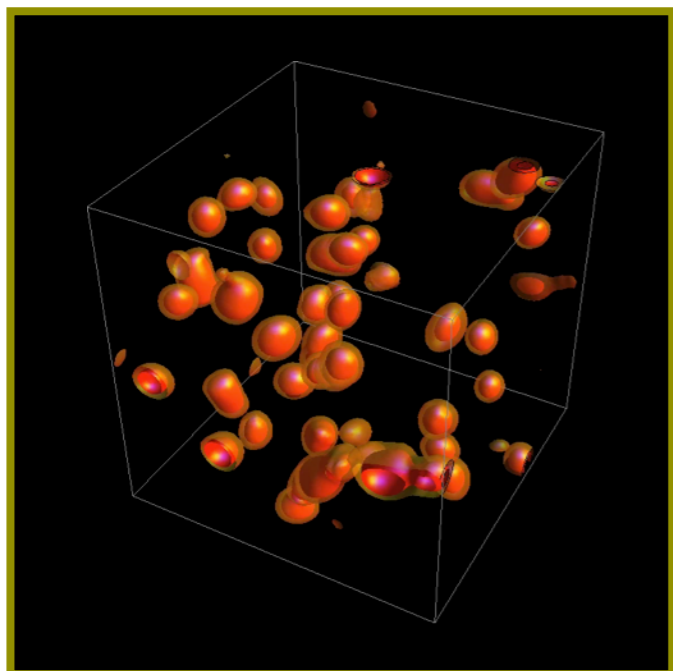
$n = 1$

$n > 1$

$M \sim m_{\text{pl}}$



$M \ll m_{\text{pl}}$



slow

$$w \rightarrow \begin{cases} 0 & \text{if } n = 1 \\ 1/3 & \text{if } n > 1 \end{cases}$$

independent of  $M$

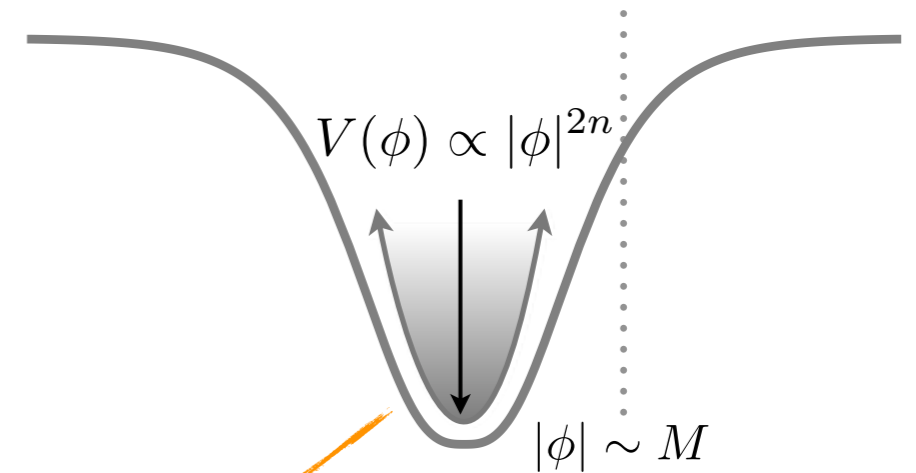
fast

$$w \neq \frac{n-1}{n+1}$$



# an upper bound on duration to radiation domination

$$\Delta N_{\text{rad}} \sim \begin{cases} 1 & M \lesssim 10^{-2} m_{\text{Pl}} \\ \frac{n+1}{3} \ln \left( \frac{\kappa}{\Delta\kappa} \frac{10M}{m_{\text{Pl}}} \right) & M \gtrsim 10^{-2} m_{\text{Pl}} \end{cases}$$

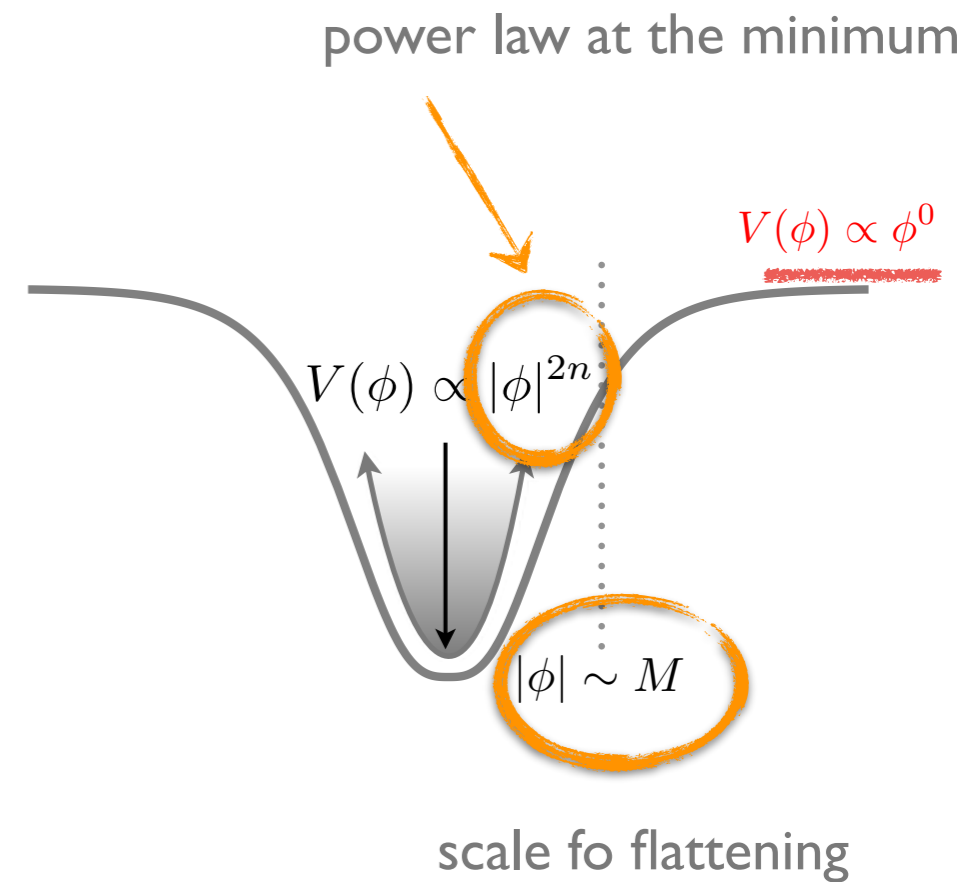
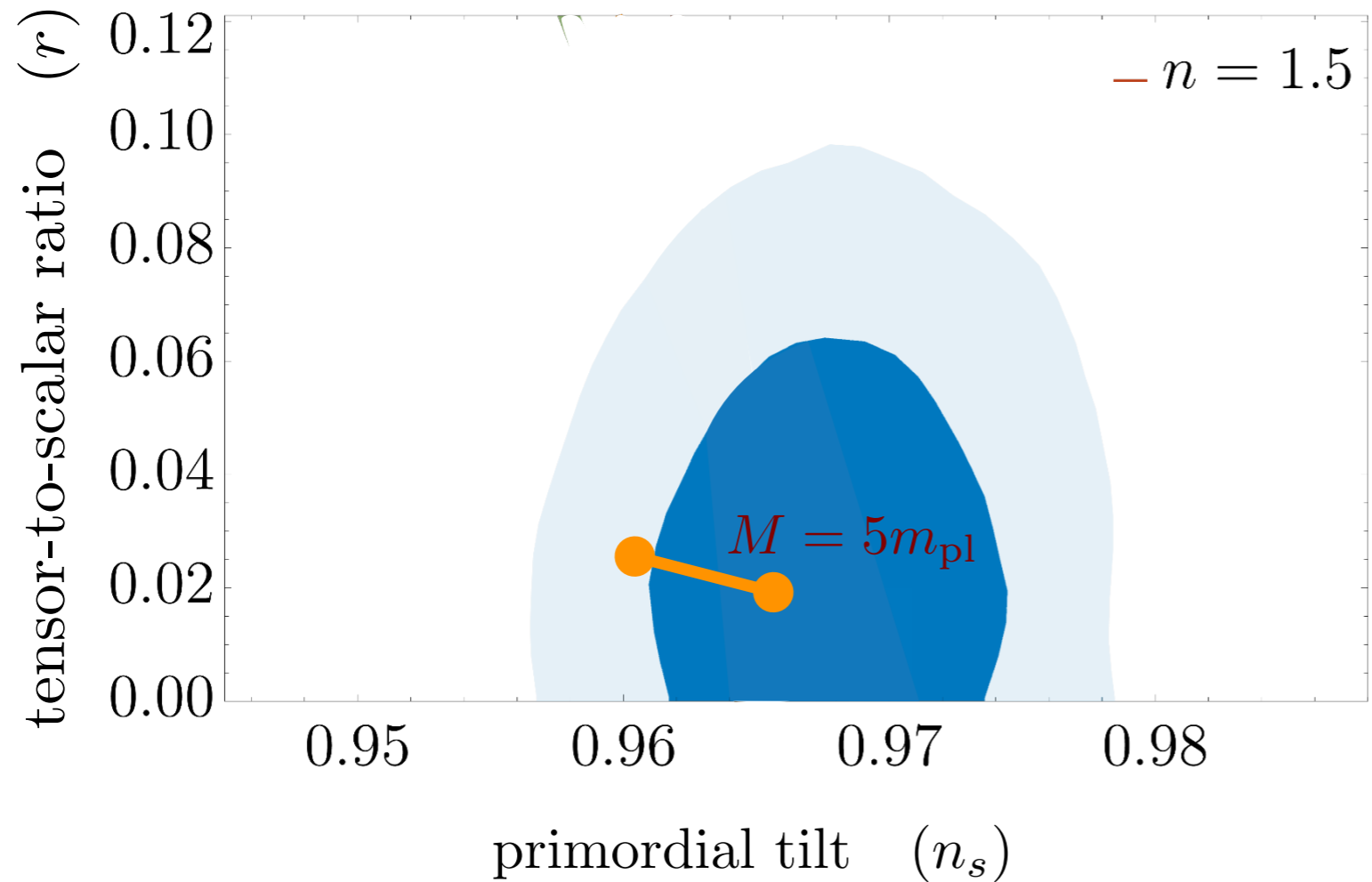


additional *light (massless) fields* can  
only decrease the duration!

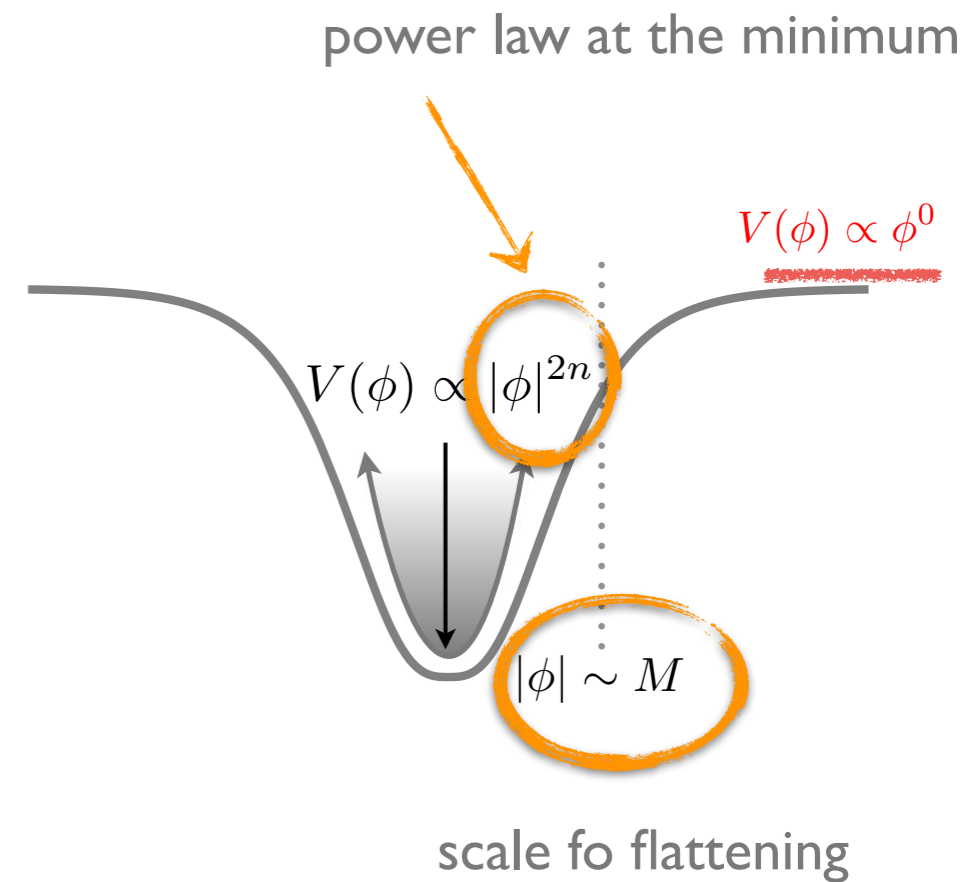
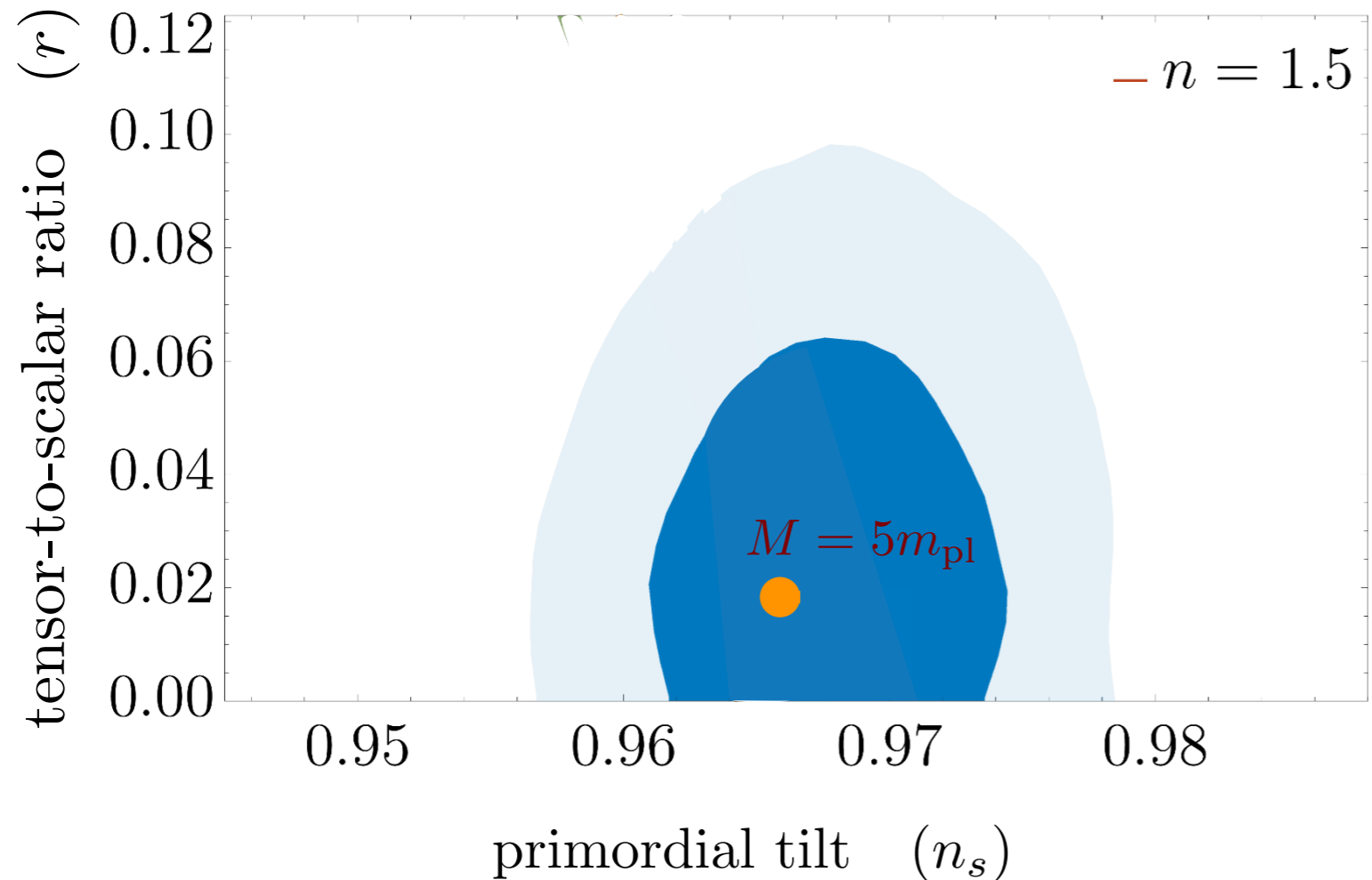
|   |                                       |                                      |                     |                                  |
|---|---------------------------------------|--------------------------------------|---------------------|----------------------------------|
| mass =<br>12 MeV/c <sup>2</sup>           | mass =<br>1.275 GeV/c <sup>2</sup>    | mass =<br>173.01 GeV/c <sup>2</sup>  | 0                   | mass =<br>125 GeV/c <sup>2</sup> |
| charge =<br>2/3                           | charge =<br>2/3                       | charge =<br>2/3                      | charge =<br>0       | charge =<br>0                    |
| spin =<br>1/2                             | spin =<br>1/2                         | spin =<br>1/2                        | spin =<br>1         | spin =<br>0                      |
| <b>u</b><br>up                            | <b>c</b><br>charm                     | <b>t</b><br>top                      | <b>g</b><br>gluon   | <b>H</b><br>Higgs boson          |
| <b>d</b><br>down                          | <b>s</b><br>strange                   | <b>b</b><br>bottom                   | <b>γ</b><br>photon  |                                  |
| <b>e</b><br>electron                      | <b>μ</b><br>muon                      | <b>τ</b><br>tau                      | <b>Z</b><br>Z boson |                                  |
| <b>ν<sub>e</sub></b><br>electron neutrino | <b>ν<sub>μ</sub></b><br>muon neutrino | <b>ν<sub>τ</sub></b><br>tau neutrino | <b>W</b><br>W boson |                                  |
| LEPTONS                                   |                                       |                                      | GAUGE BOSONS        |                                  |

\* decay to significantly massive fields can change this conclusion

# implications for CMB observables



# reduction in uncertainty!



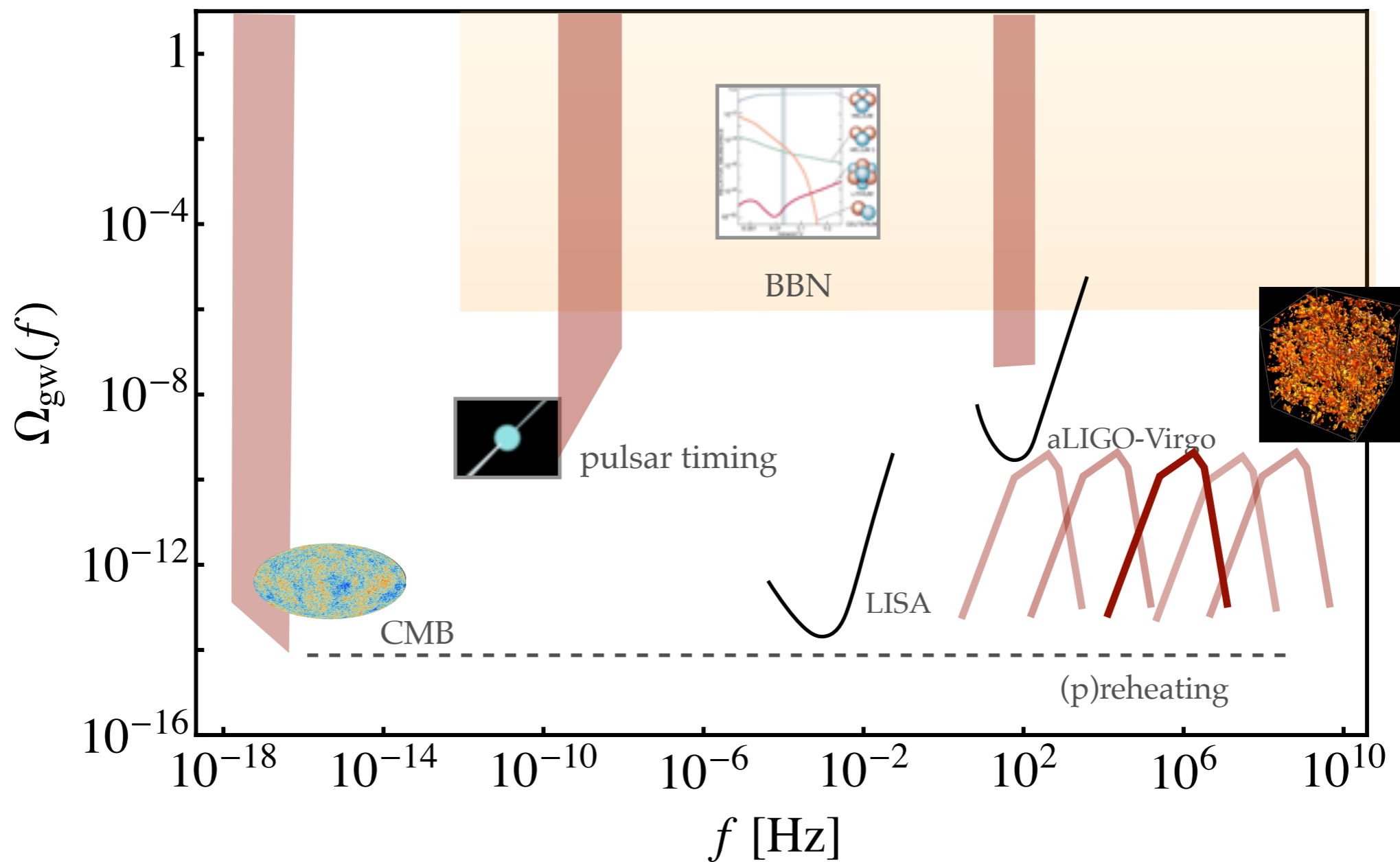
$$\Delta N_{\text{rad}} \sim \begin{cases} 1 & M \lesssim 10^{-2} m_{\text{Pl}}, \\ \frac{n+1}{3} \ln \left( \frac{\kappa}{\Delta\kappa} \frac{10M}{m_{\text{Pl}}} \right) & M \gtrsim 10^{-2} m_{\text{Pl}}. \end{cases}$$

\* non-quadratic minimum



# stochastic gravitational waves

$$\Omega_{\text{gw}}(f) = \frac{d \ln \rho_{\text{gw}}}{d \ln f} \sim \frac{\rho_{\text{gw}}}{\rho_{\text{crit}}}$$



# end of inflation



**“SIMPLE”**

- single field
- non-trivial dynamics
- eq. of state + gravitational waves

# after inflation

**SIMPLE**

**problem oriented**

conundrums in the present universe, with solutions/ implications in the early universe ...

examples: dark matter abundance/ distribution, matter anti-matter asymmetry etc. See upcoming talk by E. Erikcek

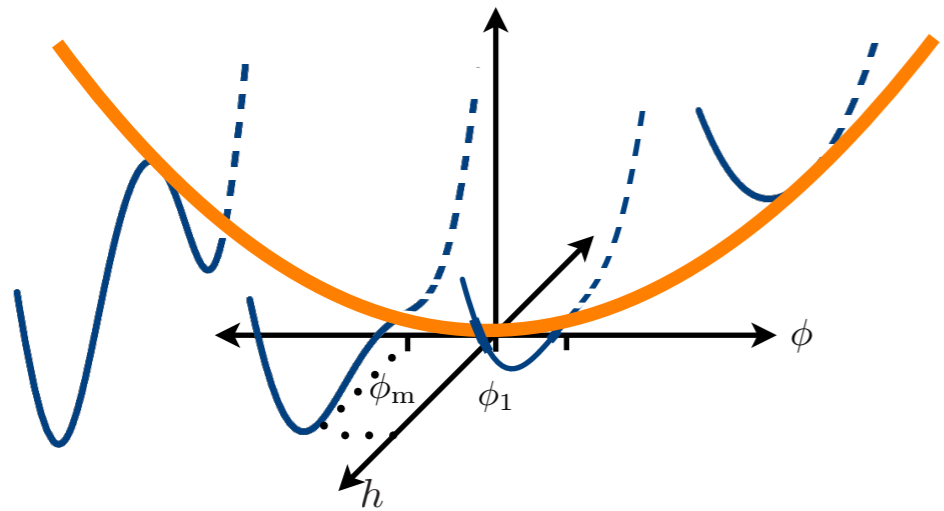


# Early Universe implications of Higgs Fine Tuning

LHC: Higgs, but no SUSY particles (so far). Is SUSY wrong or is the Higgs accidentally light within SUSY ?

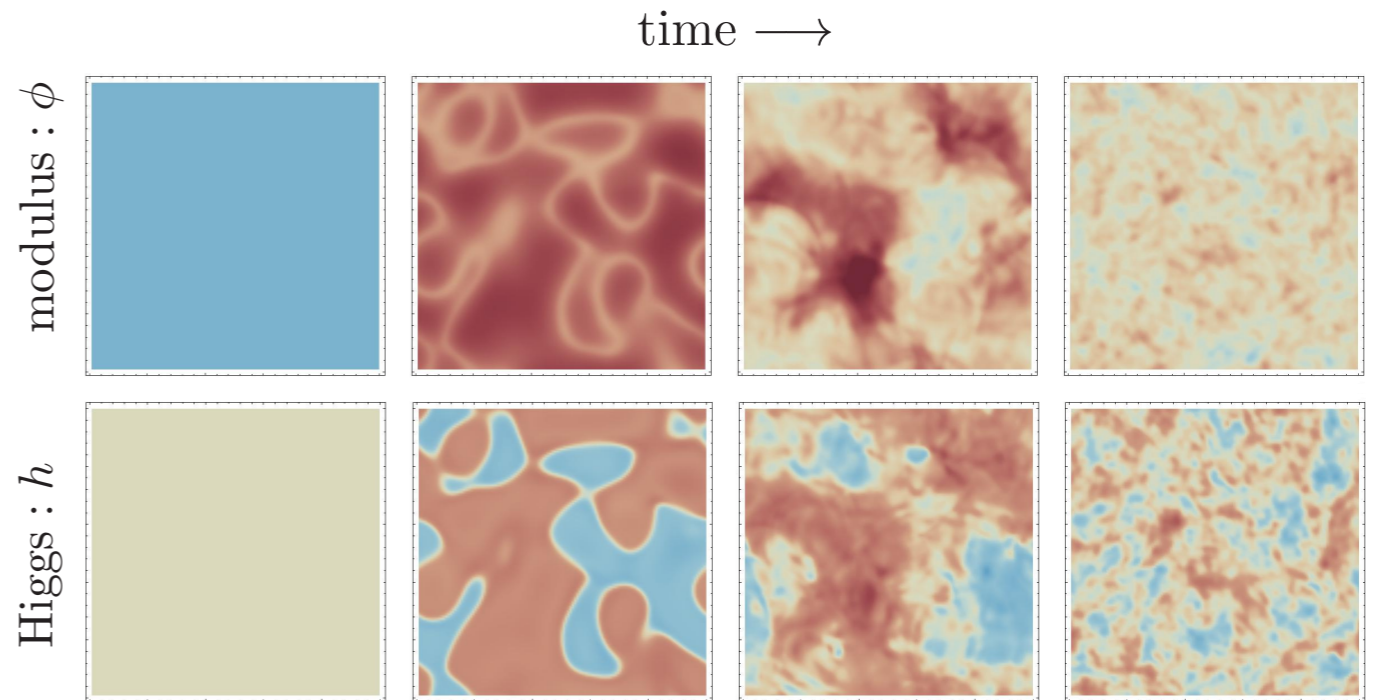
If the Higgs potential is **fine-tuned** (consistent with LHC so-far), are there **observable cosmological implications ?**

# Higgs fine tuning — implications from the early universe

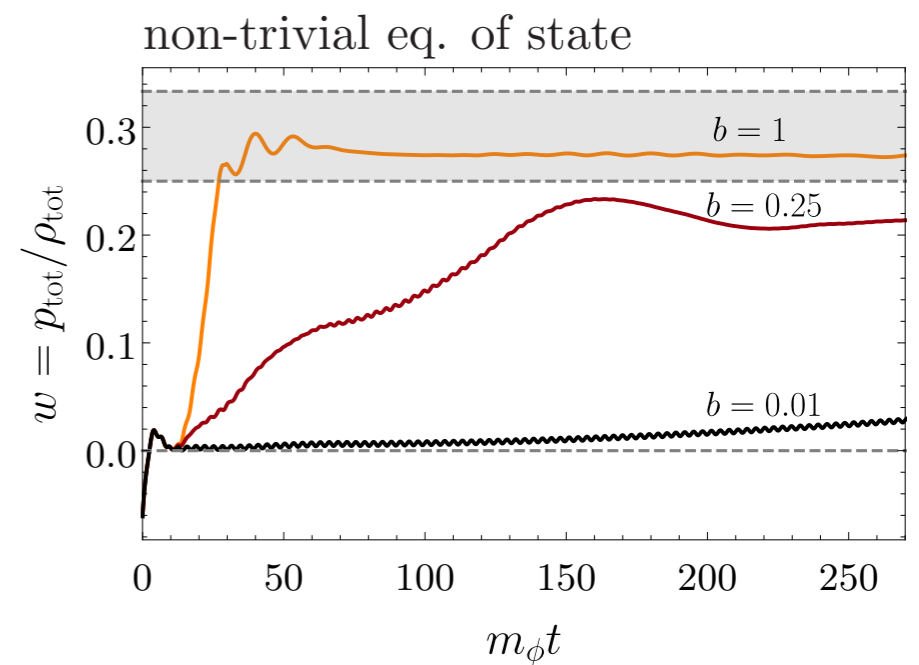
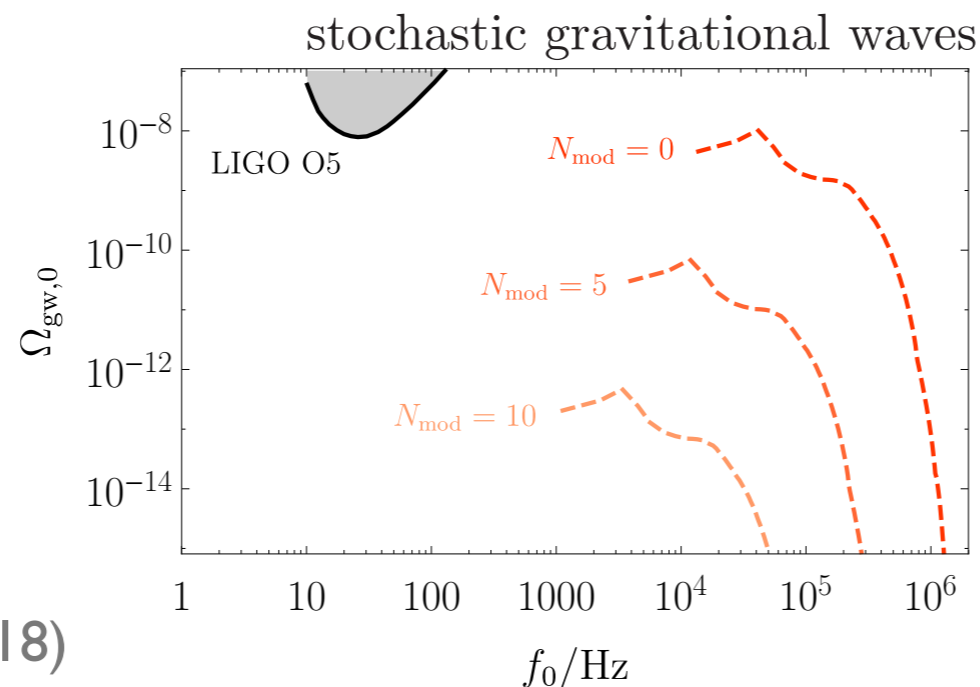


$$\frac{1}{2}m_\phi^2(\phi - \phi_1)^2 + M^2\frac{\phi - \phi_0}{f}h^\dagger h + \lambda(h^\dagger h)^2 + V_0$$

Necessary Fine Tuning  $\Leftrightarrow \frac{\phi_m - \phi_0}{f} \ll 1$



$$\frac{M^4}{2\lambda f^2 m_\phi^2} \rightarrow 1 \Leftrightarrow \text{rapid fragmentation}$$



# after inflation

**SIMPLE**

**problem oriented**

**COMPLEX**

# Early Universe implications of Higgs Fine Tuning

arXiv: 1802.00444

MA, Fan, Lozano & Reece

LHC: Higgs, but no SUSY particles (so far). Is SUSY wrong or is the Higgs accidentally light within SUSY ?

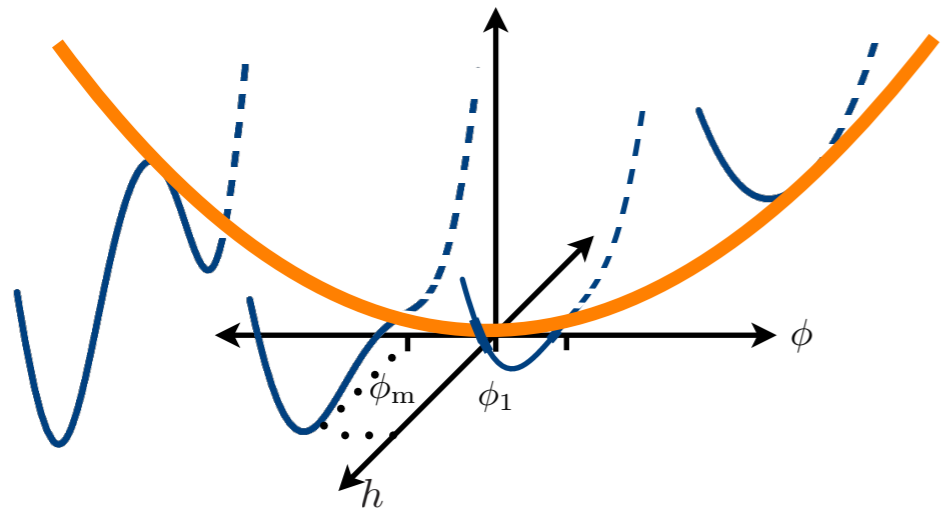
If the Higgs potential is **fine-tuned** (consistent with LHC so-far), are there **observable cosmological implications ?**



# Higgs fine tuning — implications from the early universe

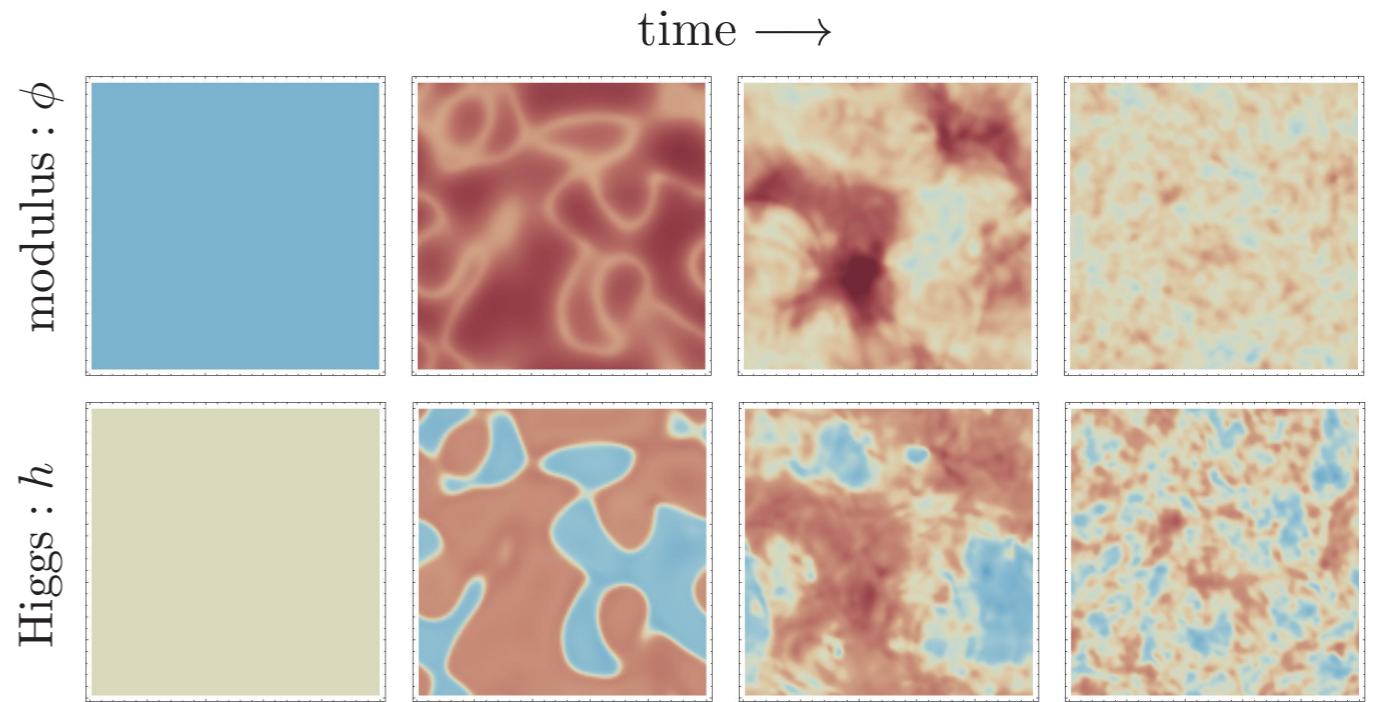
arXiv: 1802.00444

MA, Fan, Lozanov & Reece

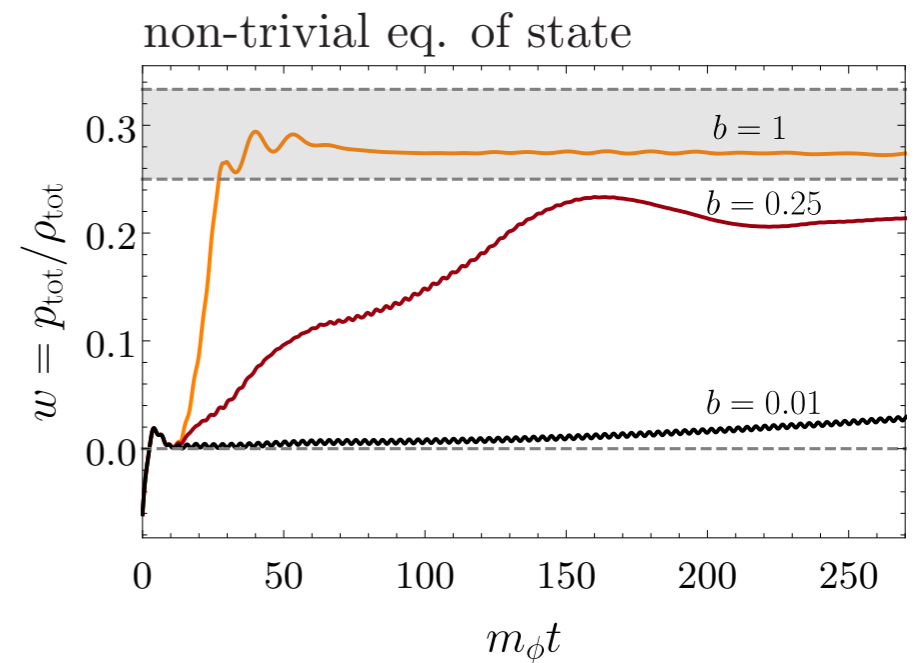
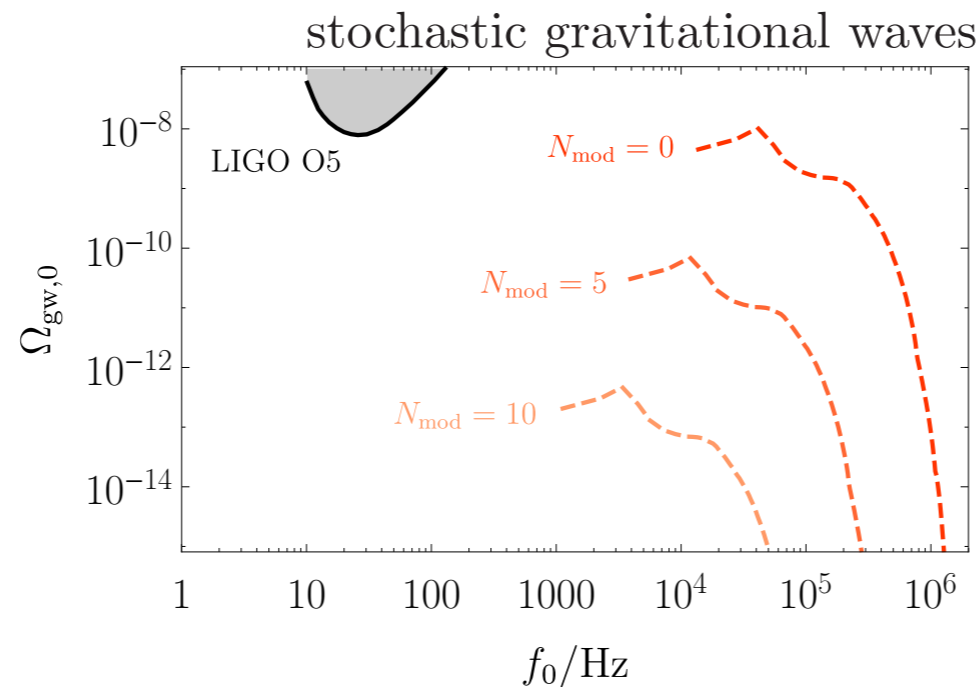


$$\frac{1}{2}m_\phi^2(\phi - \phi_1)^2 + M^2\frac{\phi - \phi_0}{f}h^\dagger h + \lambda(h^\dagger h)^2 + V_0$$

Necessary Fine Tuning  $\Leftrightarrow \frac{\phi_m - \phi_0}{f} \ll 1$



$$\frac{M^4}{2\lambda f^2 m_\phi^2} \rightarrow 1 \Leftrightarrow \text{rapid fragmentation}$$



# end of inflation



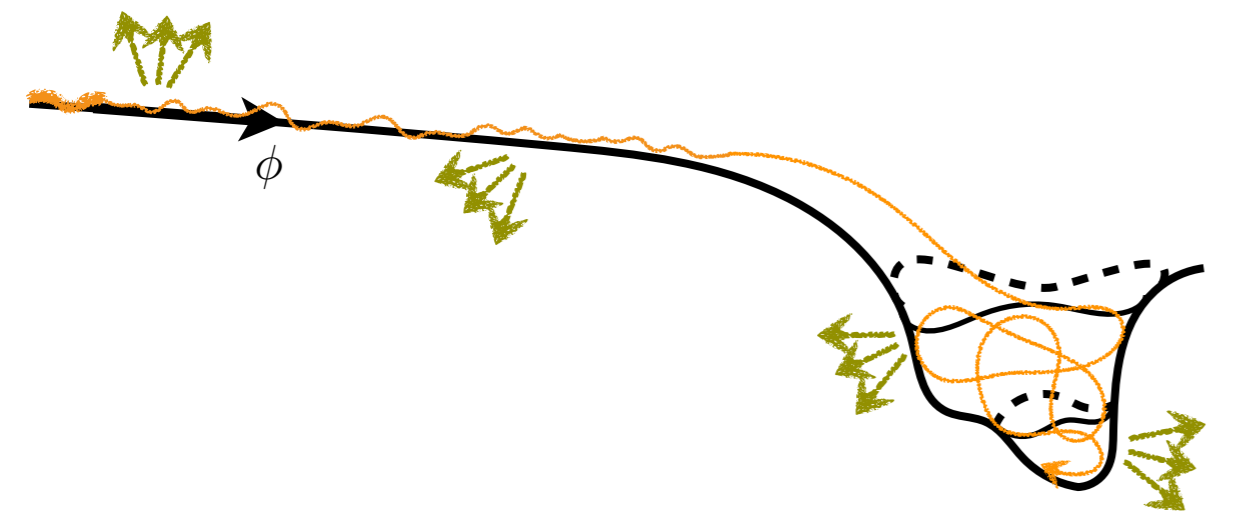
**SIMPLE**



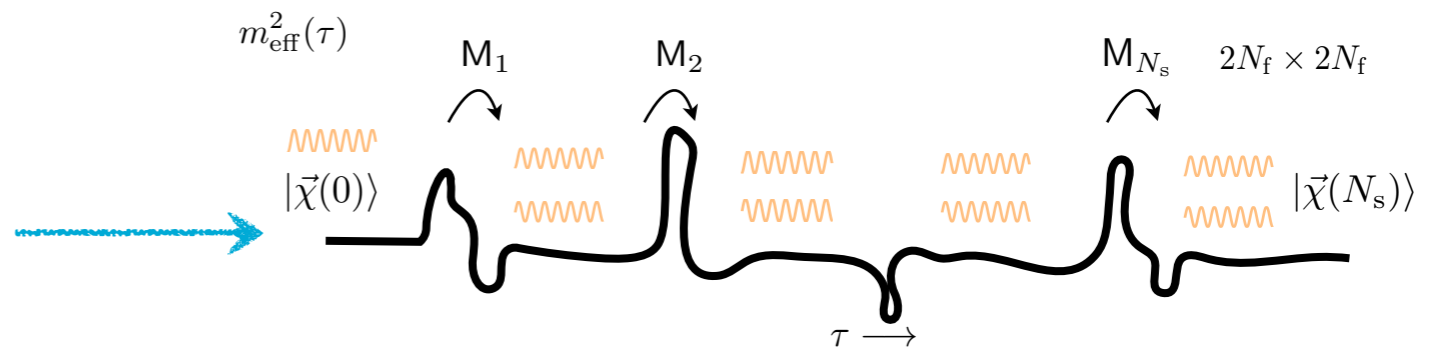
**problem oriented**

**COMPLEX**

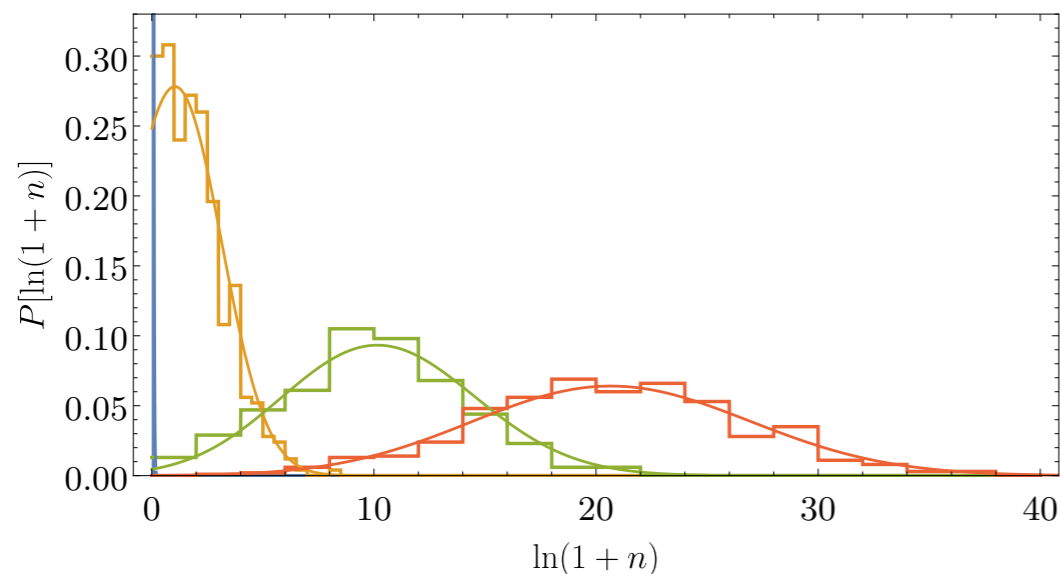
# complex enough: “universal” results



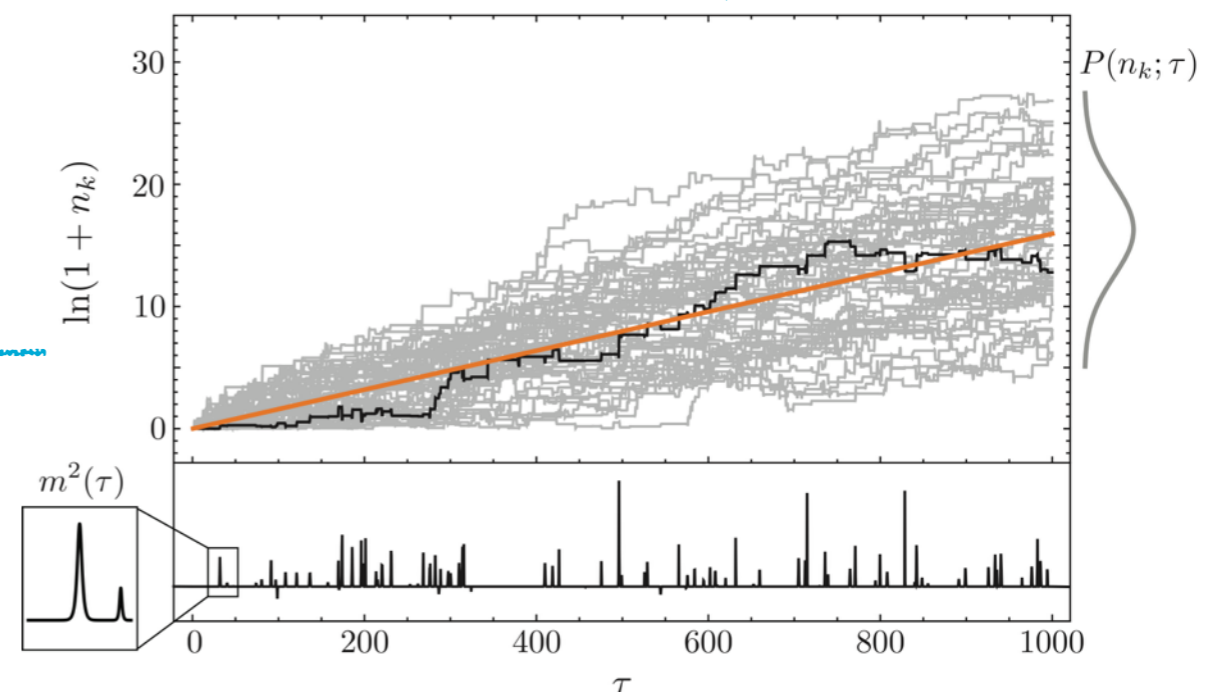
complex non-perturbative particle production



treat as scattering problems  
(similar to that when dealing with impurities in wires)



universally log-normal distributions



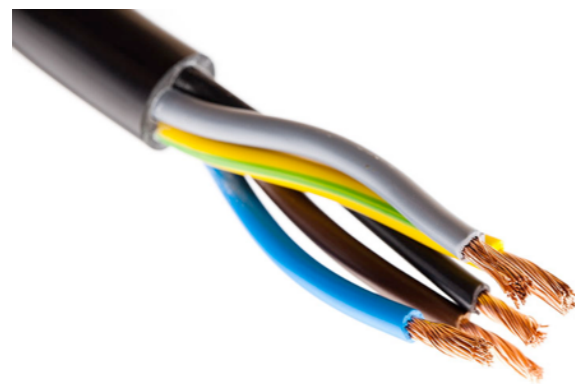
occupation numbers grow exponentially  
(universality similar to Anderson localization)

# From Wires to Cosmology

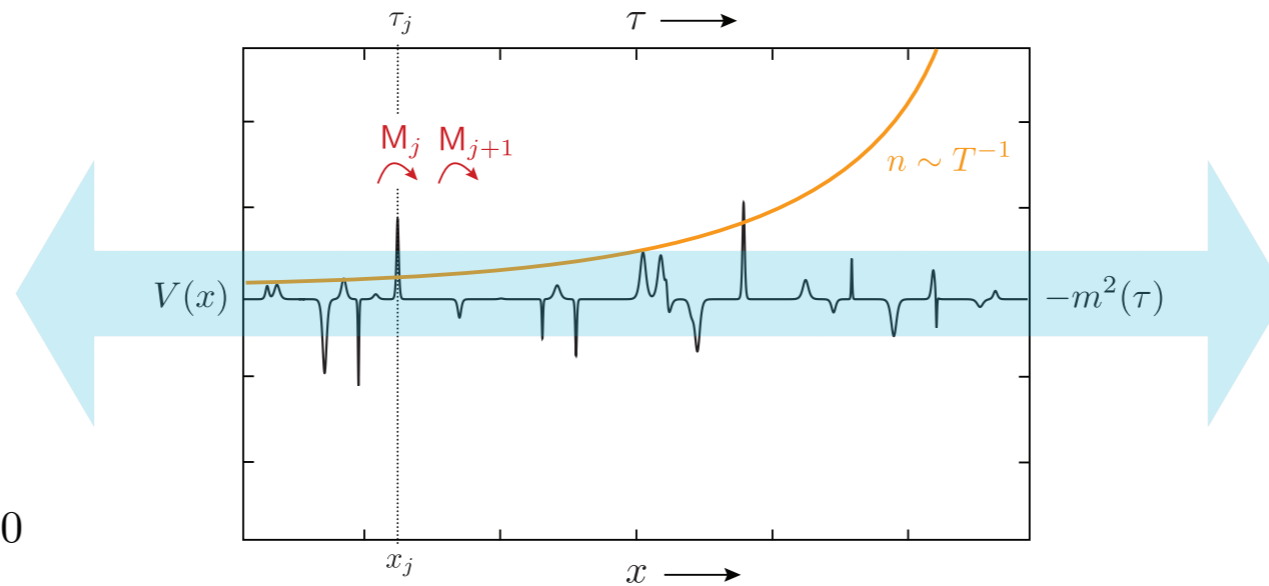
arXiv: 1512.02637

MA & Baumann

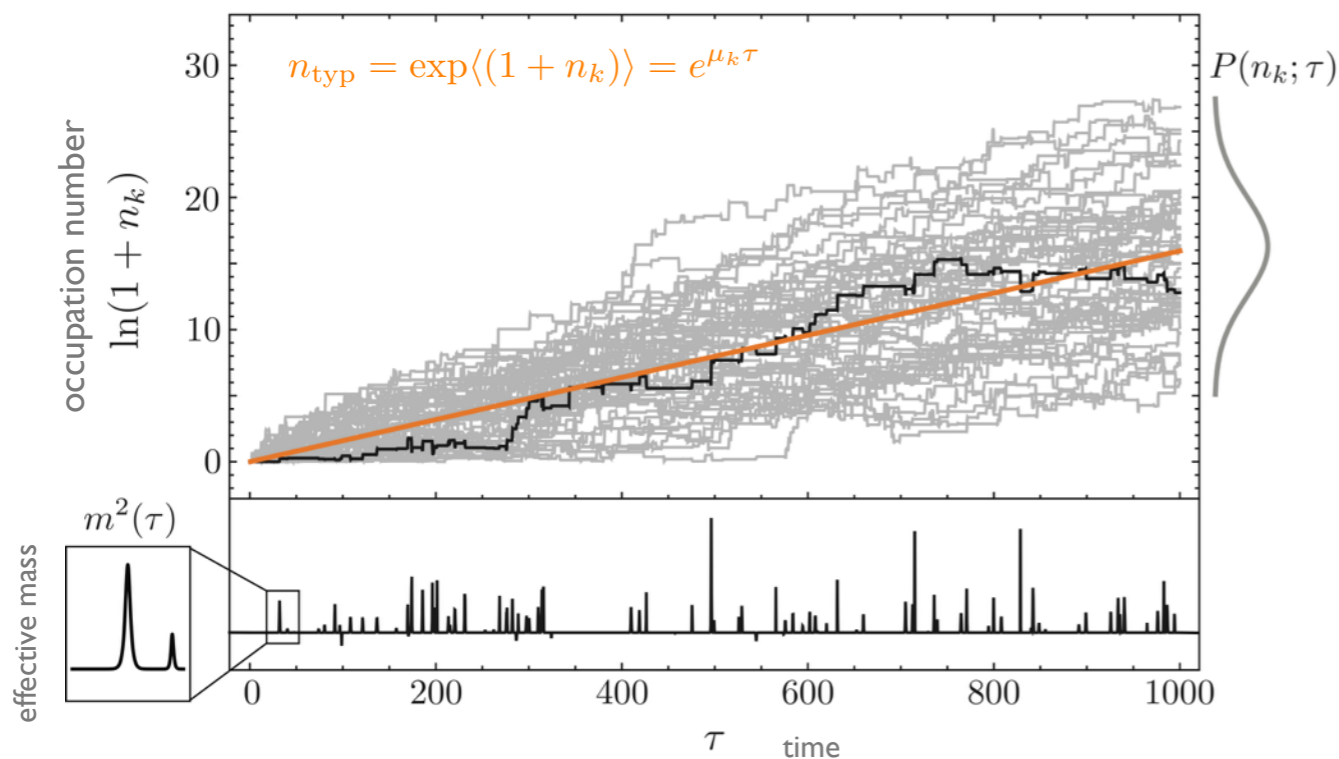
arXiv: 1706.02319 MA, Garcia, Xie & Wen



$$\frac{d^2\psi}{dx^2} + [k^2 - V(x)]\psi = 0$$



$$\frac{d^2\chi_k}{d\tau^2} + [k^2 + m^2(\tau)]\chi_k = 0$$



## What did we do?

We developed a statistical framework to calculate stochastic particle production in the early universe in complex scenarios with multiple interacting fields.

## Potential benefits:

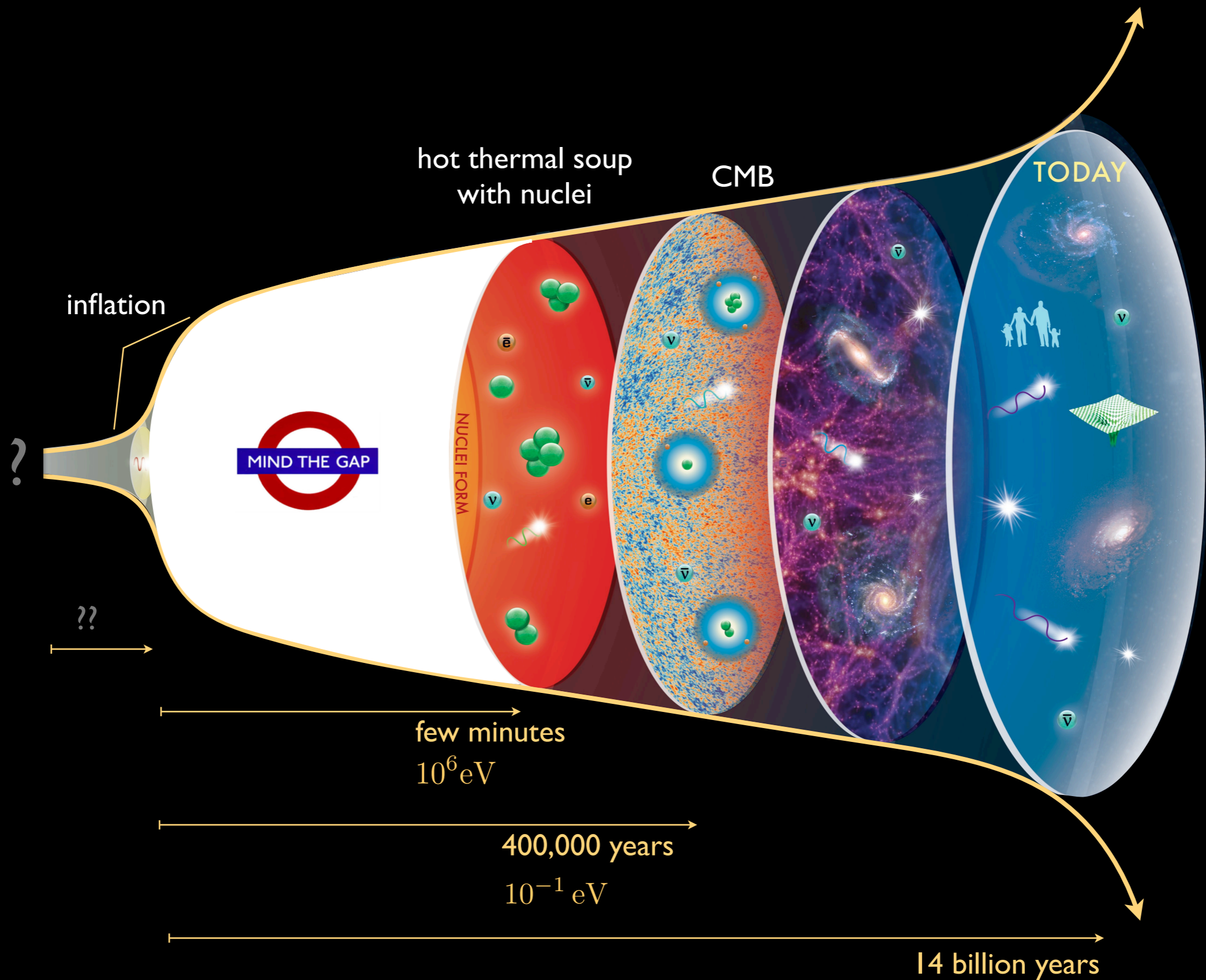
- useful when the microphysics is uncertain, dynamics are complex and only coarse grained predictions are needed.
- hints of universality in predictions when the no. of interactions and/or components is large.
- reduction in complexity to a few parameters

## The fun bit:

By establishing a mathematical map between current conduction in wires with impurities and stochastic particle production in cosmology, we benefited a lot from work on *Anderson Localization* since the 1950s.

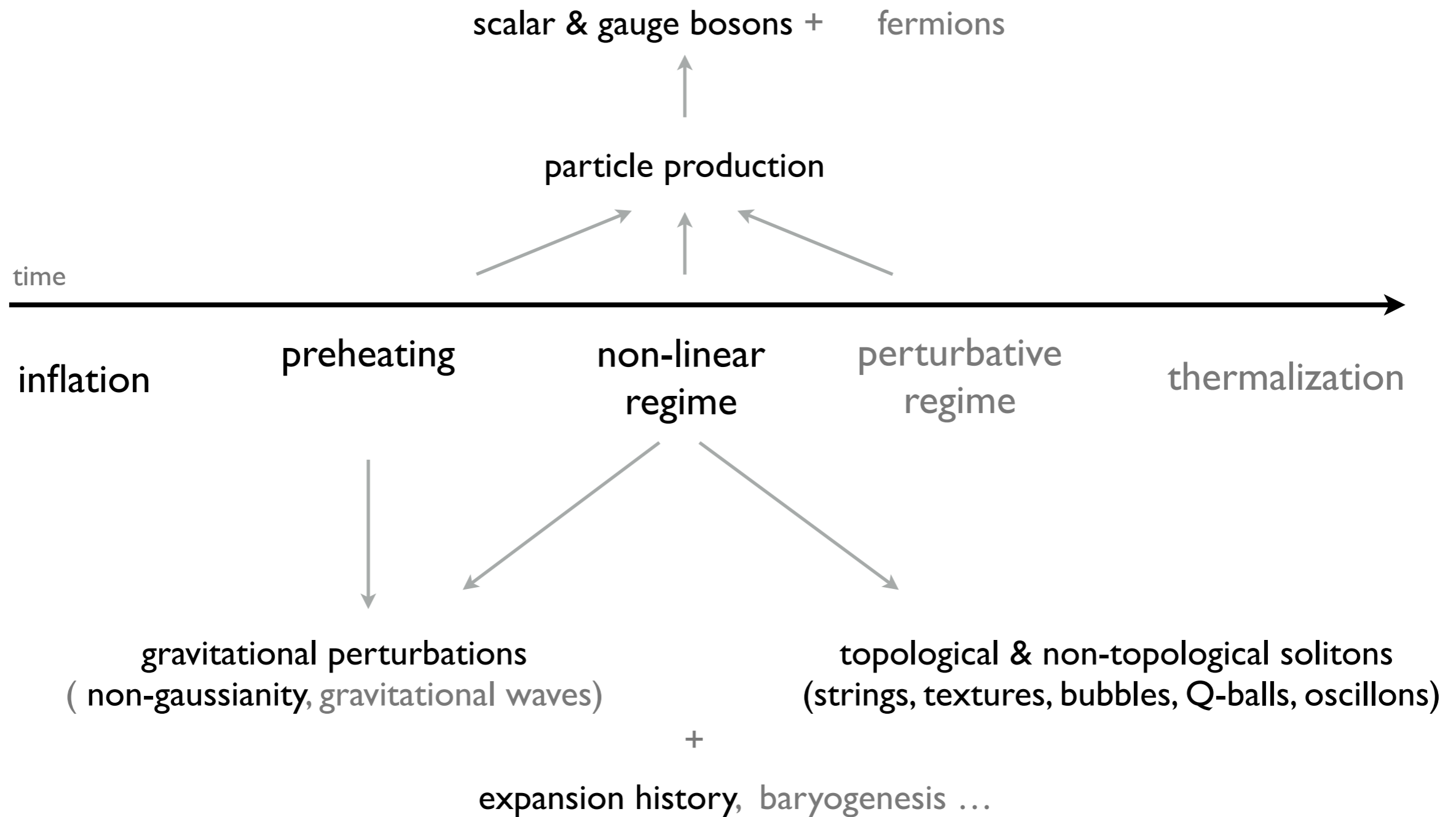


# aftermath of inflation: a GAP in our cosmic history

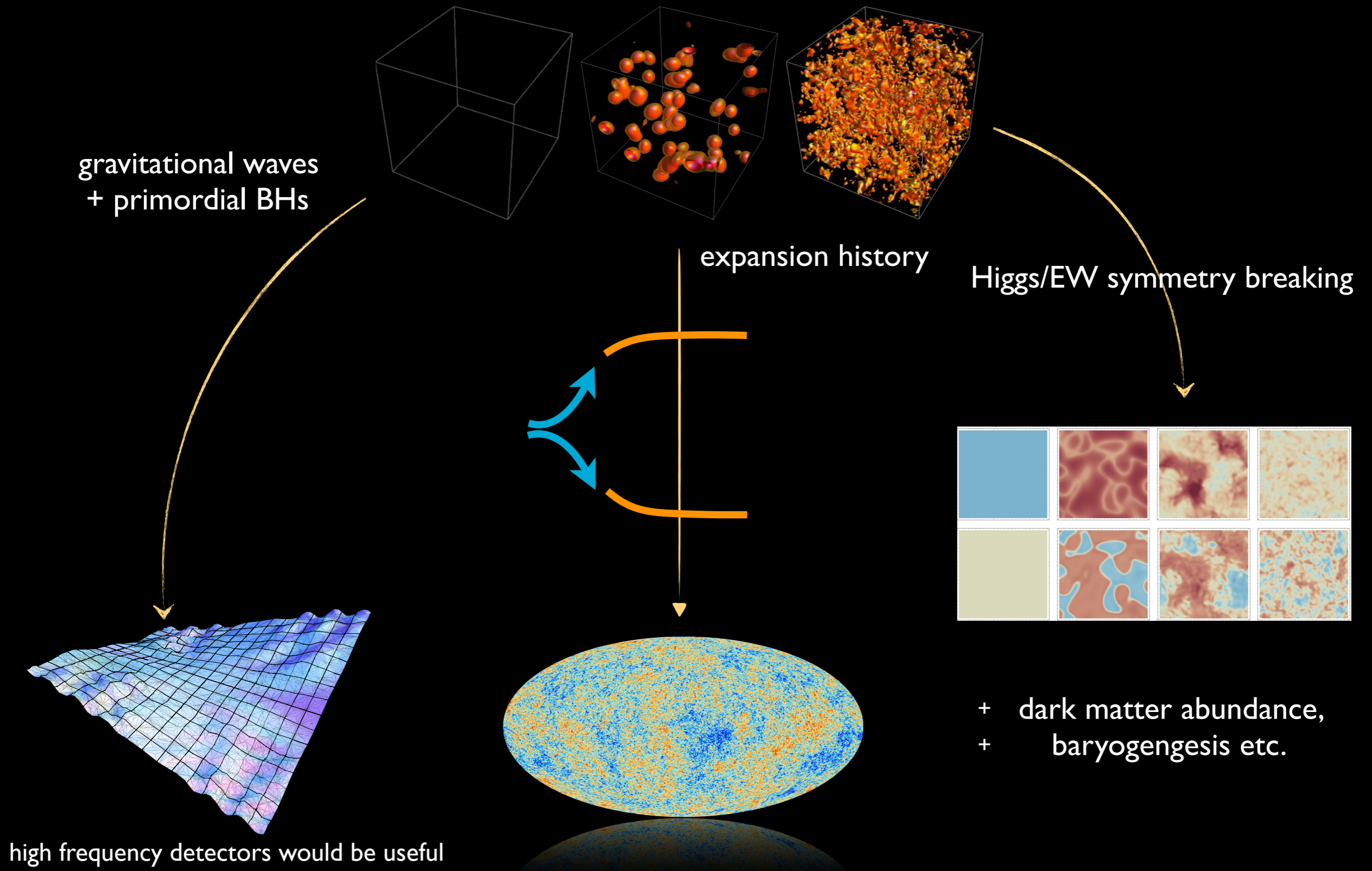


# inflation ends, what's next ?

inflaton dominated, "cold" universe



# sample probes/consequences





thanks!

