Lattice QCD for Hadronic Parity Violation

CIPANP 2018
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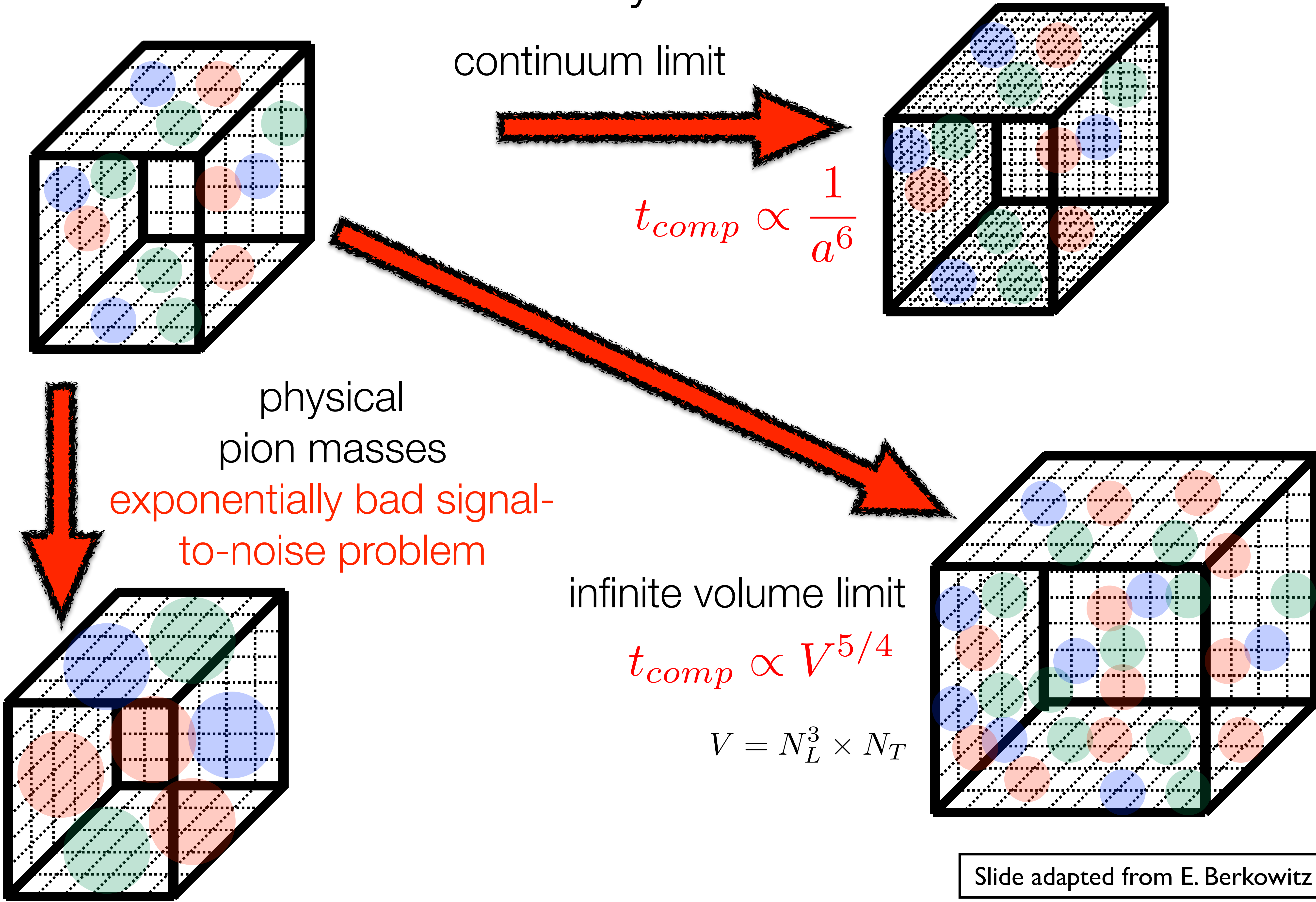
Palm Springs

André Walker-Loud
LBNL

Lattice QCD for Hadronic Parity Nonconservation

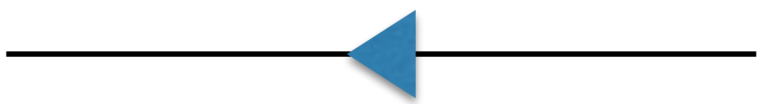
- Why haven't we updated Joe Wasem's result? Why haven't we published our I=2 PNC result?
  - Lattice QCD Challenges for Nuclear Physics
  - Lattice QCD Challenges for Parity Nonconservation (PNC)
- Inspiration for Lattice QCD Calculations of I=2 PNC
  - New method for two-nucleon calculations

LQCD Systematics



LQCD Challenges for NP

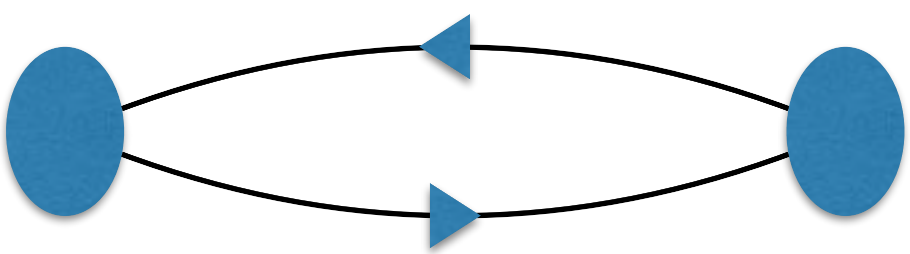
- The most difficult challenge in applying LQCD to NP is an exponentially bad signal-to-noise problem for nucleons



$$\sim e^{-\frac{1}{2}m_\pi t} + e^{-\frac{1}{3}m_N t} + \cdots$$

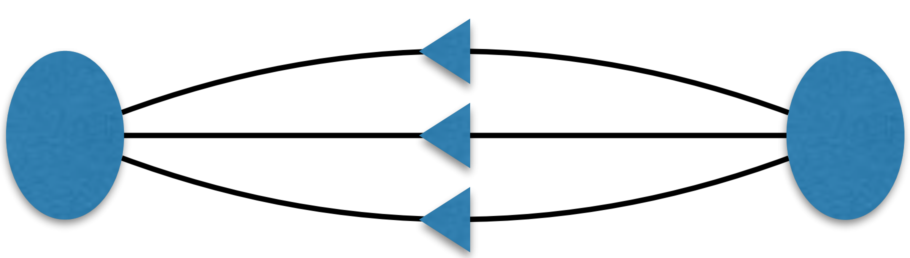
each **quark propagator** carries information about pions and nucleons (conversations with David Kaplan)

$$\lambda_\pi(t) \gg \lambda_N(t)$$



$$\bar{d}\gamma_5 u : C(t) = A_\pi e^{-m_\pi t} + \cdots$$

For the nucleon - the large pion eigenvalues must cancel to expose the small nucleon eigenvalues



$$(u^T C\gamma_5 d)u : C(t) = A_N e^{-m_N t} + \cdots$$

$$\frac{\text{Signal}}{\text{Noise}} \sim \sqrt{N}\exp\left[-A\left(m_N - \frac{3}{2}m_\pi\right)t\right]$$

→ **exponential noise power-law statistics**

LQCD Challenges for NP

2-point correlation function

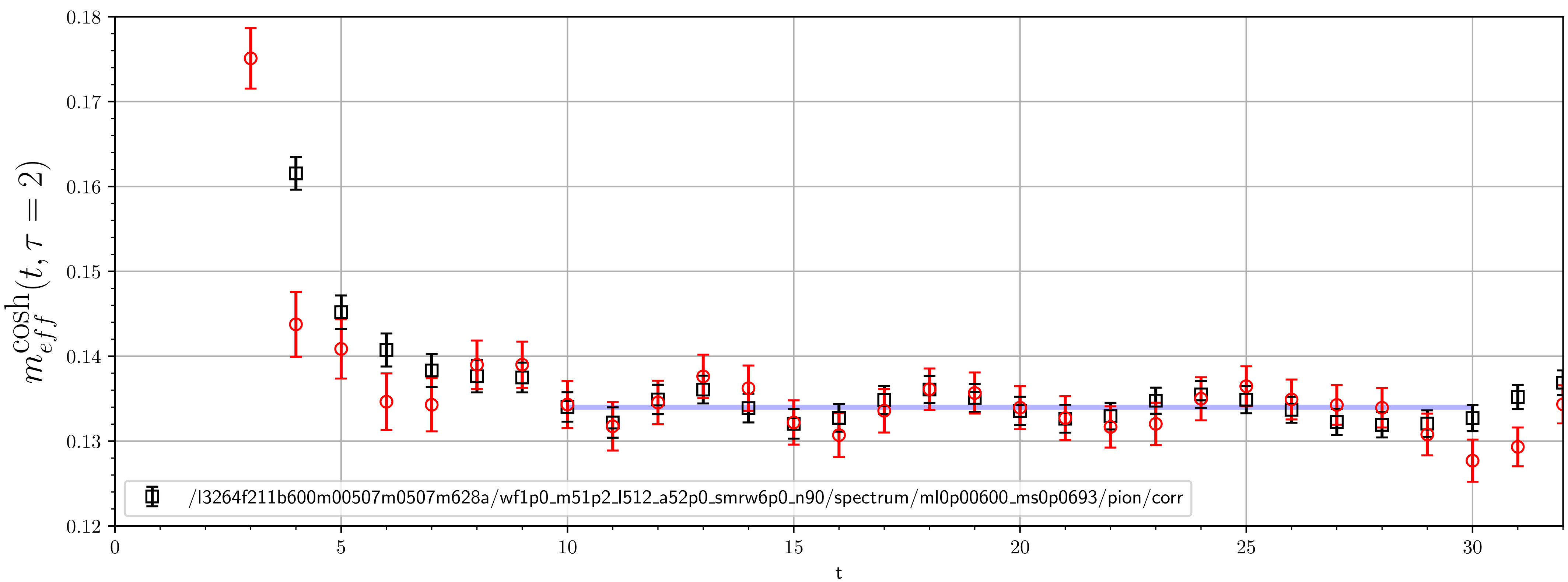
$$C(t) = \sum_n z_n z_n^\dagger e^{-E_n t}$$

$$m_{eff}(t) = \frac{1}{\tau} \ln \left(\frac{C(t)}{C(t+\tau)} \right)$$

For pions, need to consider leading finite temperature effects

$$C(t) = \sum_n z_n z_n^\dagger \left(e^{-E_n t} + e^{-E_n (T-t)} \right)$$

$$m_{eff}^{\cosh}(t,\tau) = \frac{1}{\tau} \cosh^{-1} \left(\frac{C(t+\tau) + C(t-\tau)}{2C(t)} \right)$$



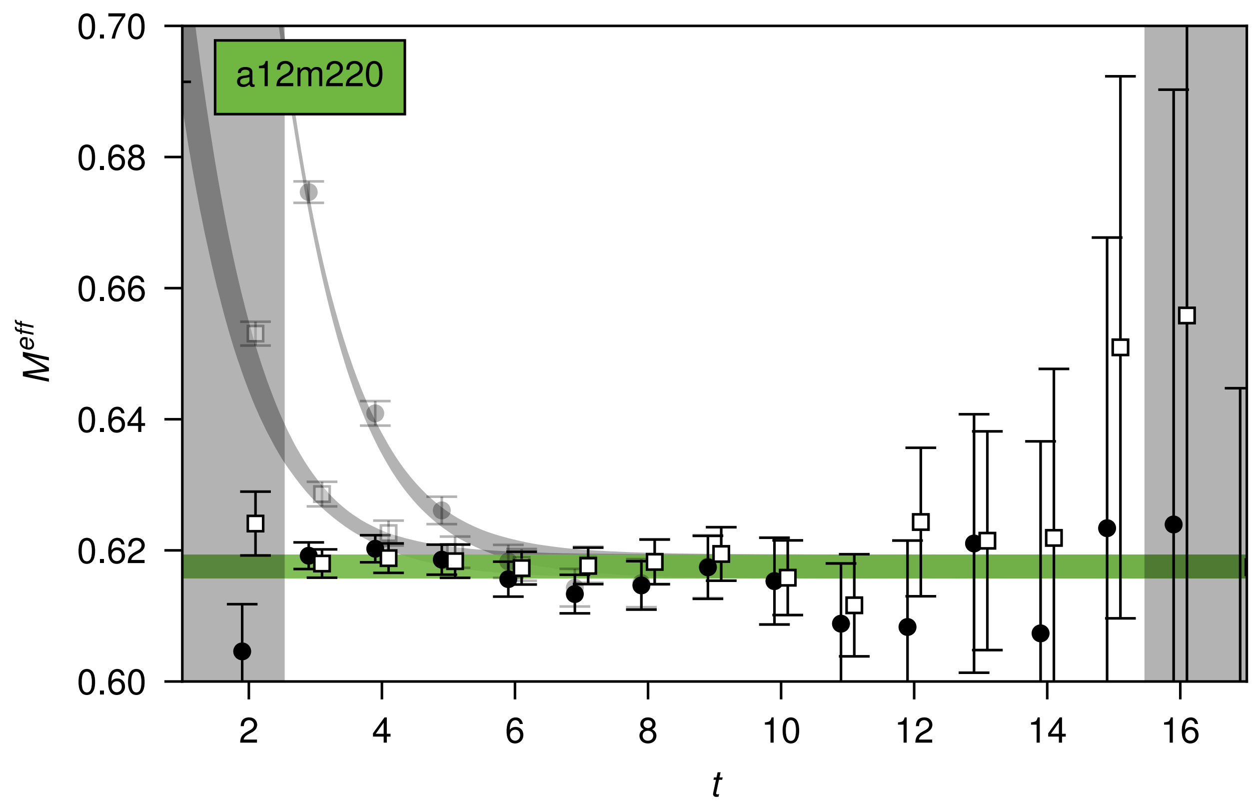
Effective mass of Pion 2-point correlation function

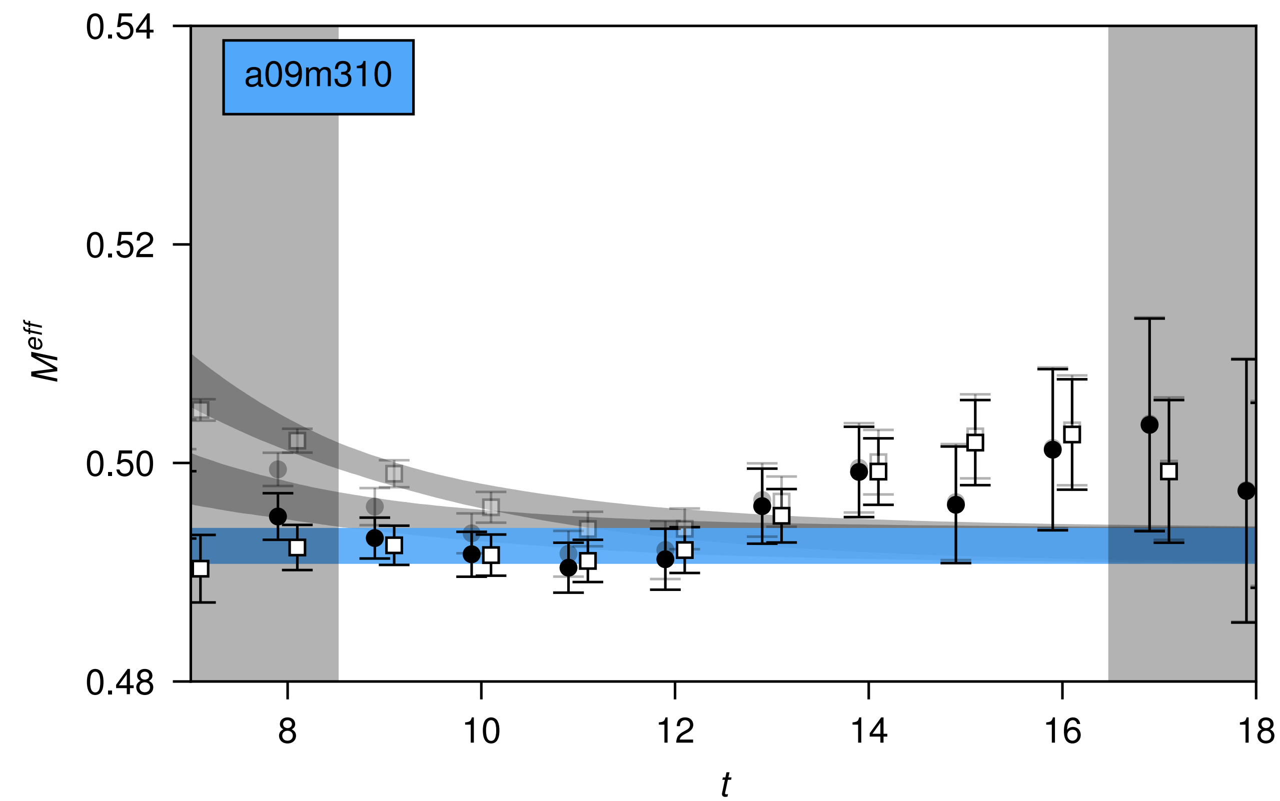
red and black “data” are from different choices of overlap operators

Noise is constant in time - can determine very clean ground state (blue band)

LQCD Challenges for NP

2-point correlation function





Two examples of nucleon effective mass

Noise is growing in time - can not simply go to the long-time limit without exponentially increasing the amount of statistics needed

$$\frac{Signal}{Noise} \rightarrow \sqrt{N_{stat}} e^{-(m_N - \frac{3}{2}m_\pi)t}$$

Correlated late-time fluctuations… what is the ground state?

Need sophisticated analysis to ensure you are not susceptible to correlated fluctuations

This problem is exacerbated with 2+ nucleons and form-factor calculations (g_A)

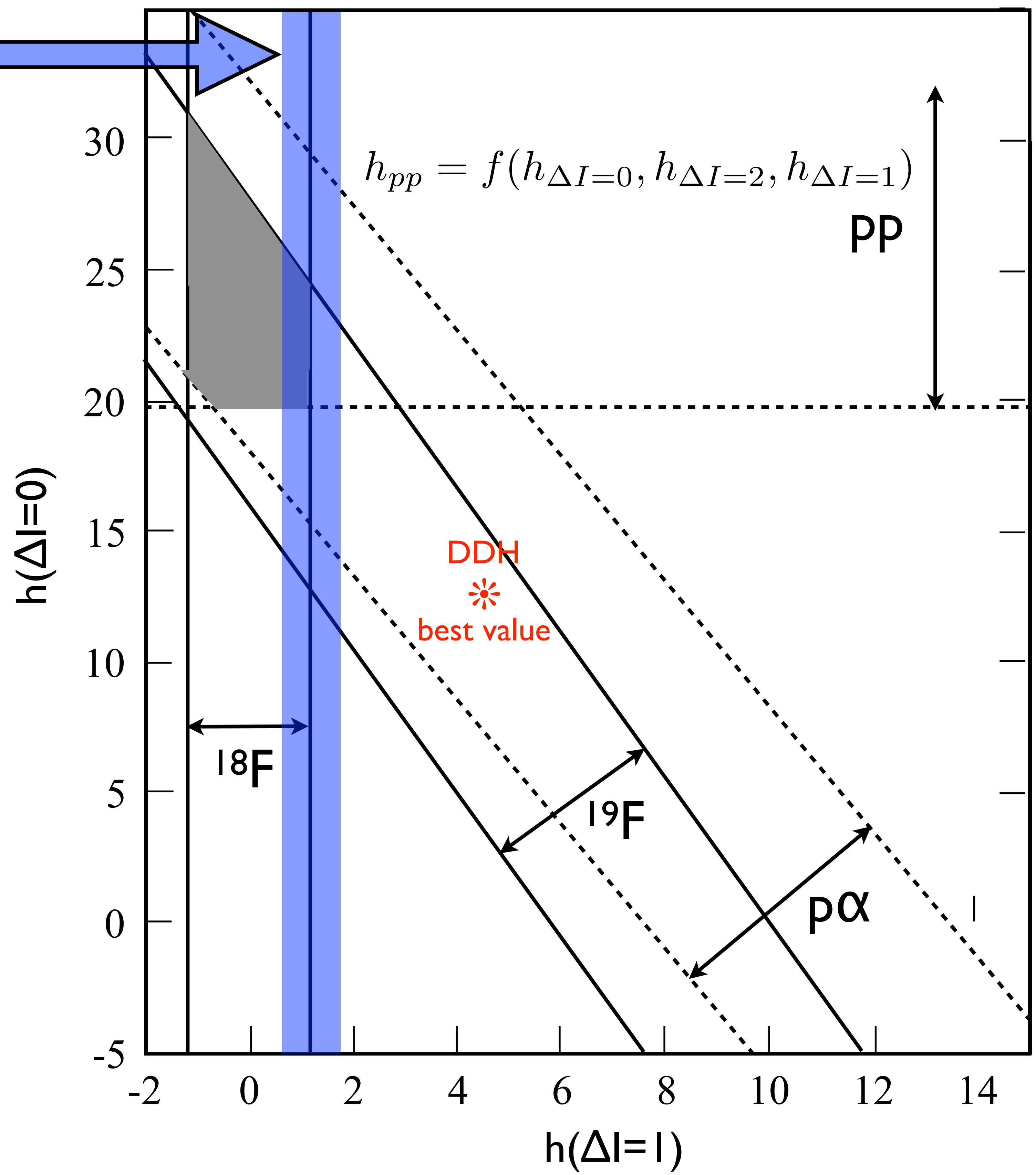
LQCD Challenges for Parity Nonconservation

first LQCD calculation of h_π^1 for
L=2.5 f a=0.123f m_π= 389 MeV
J. Wasem Phys. Rev. C85 (2012) 022501

Several unquantified approximations

- assumption about coupling of "wave function" used to Nπ state in LQCD calculations
- "disconnected" quark loops neglected
- single lattice spacing
- single pion mass
- single volume
- no renormalization

- This was a tour-de-force calculation carried out single handedly by Joe Wasem



LQCD Challenges for Parity Nonconservation

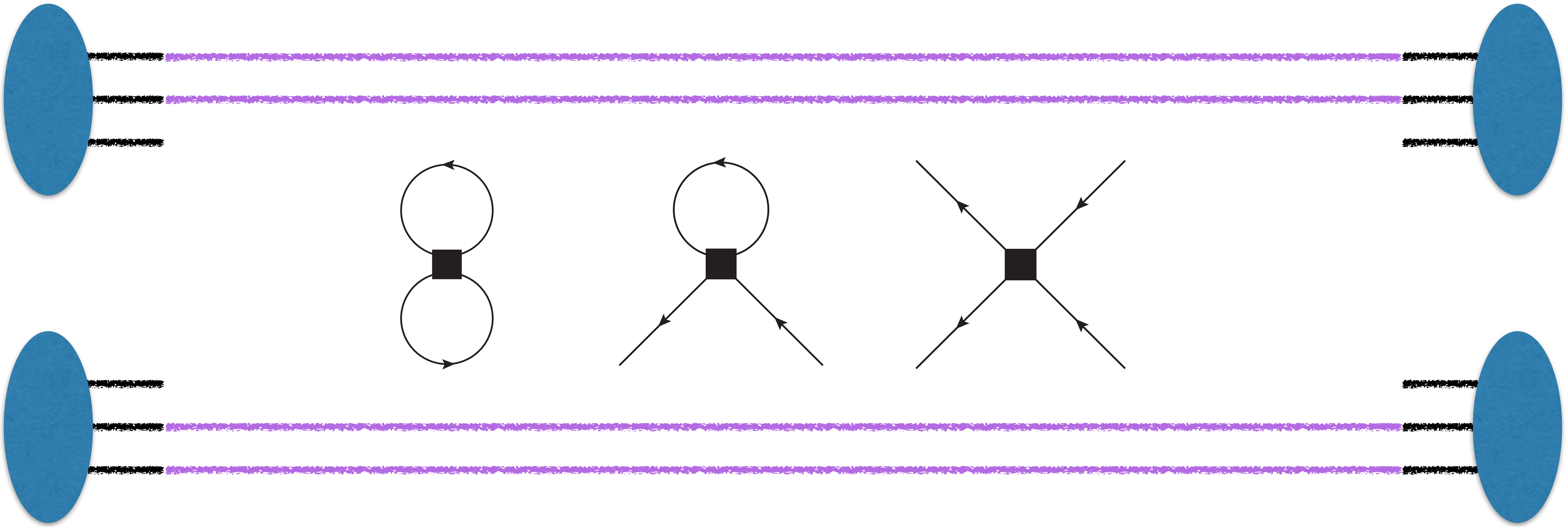
- Signal-to-noise is exponentially worse than single nucleon

$$\frac{S_{NN}}{\sigma_{NN}} \sim \left(\frac{S_N}{\sigma_N}\right)^2 = \left(\sqrt{N_{samples}}\; e^{(m_N - \frac{3}{2}m_\pi)t}\right)^2$$

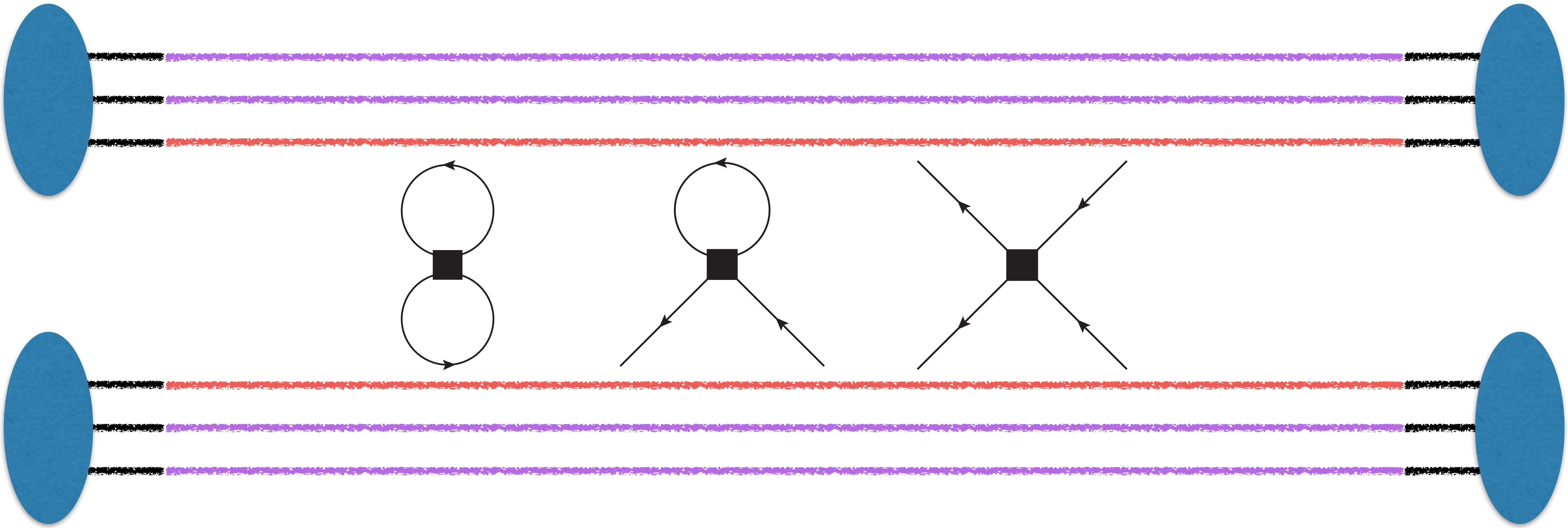
- Either need all-to-all quark propagators (1 or more orders of magnitude more expensive) or
  - can not do I=0 and I=1 PNC amplitudes
  - lose a Volume factor in statistics in I=2

- Wick contraction cost of connecting all quark lines is ~100 times more than for two-nucleons

LQCD Challenges for Parity Nonconservation

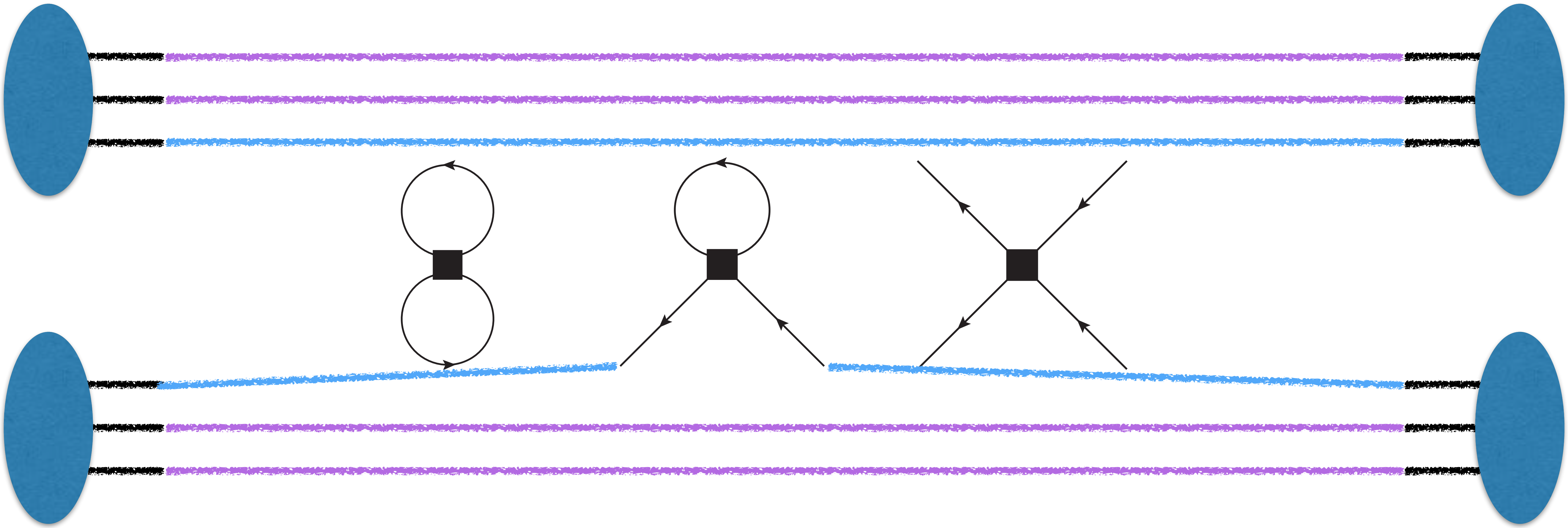


LQCD Challenges for Parity Nonconservation



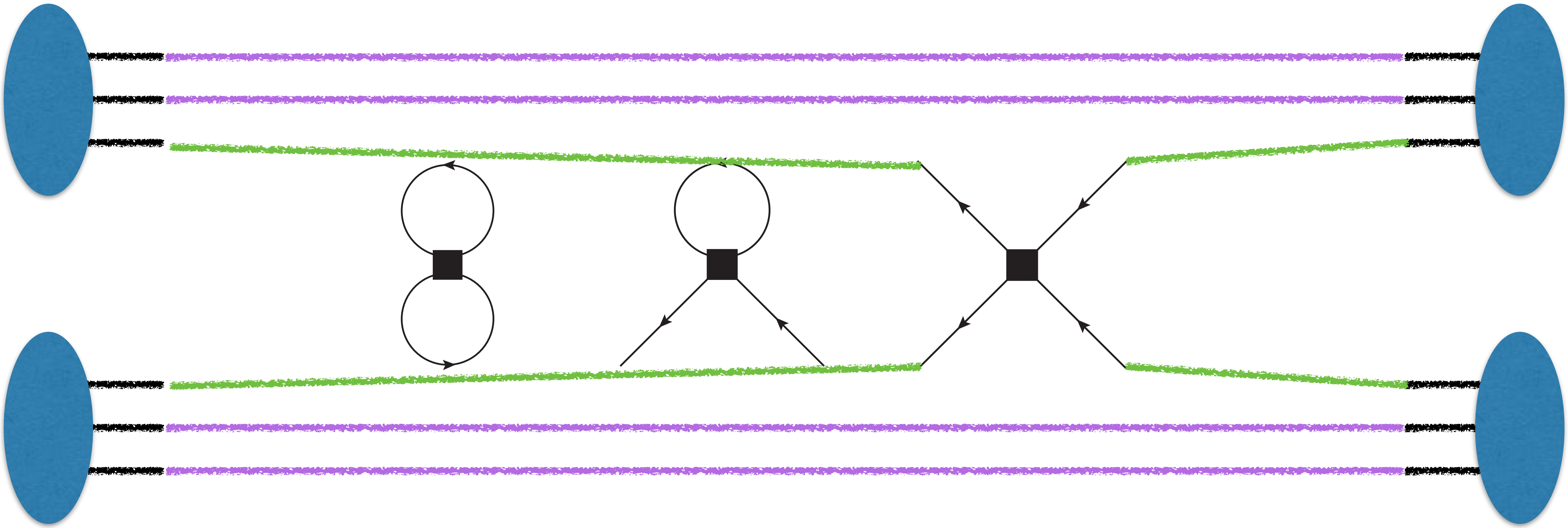
ΔI=0

LQCD Challenges for Parity Nonconservation



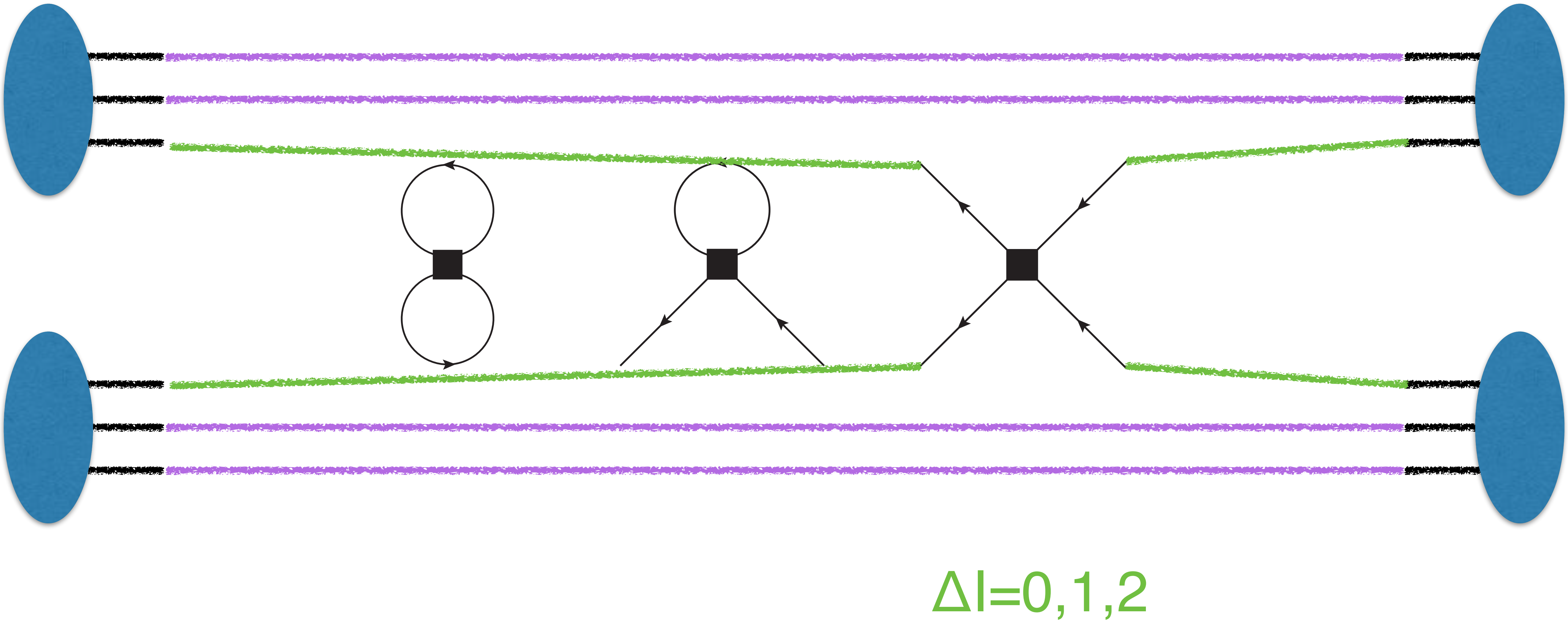
$\Delta I=0,1$

LQCD Challenges for Parity Nonconservation



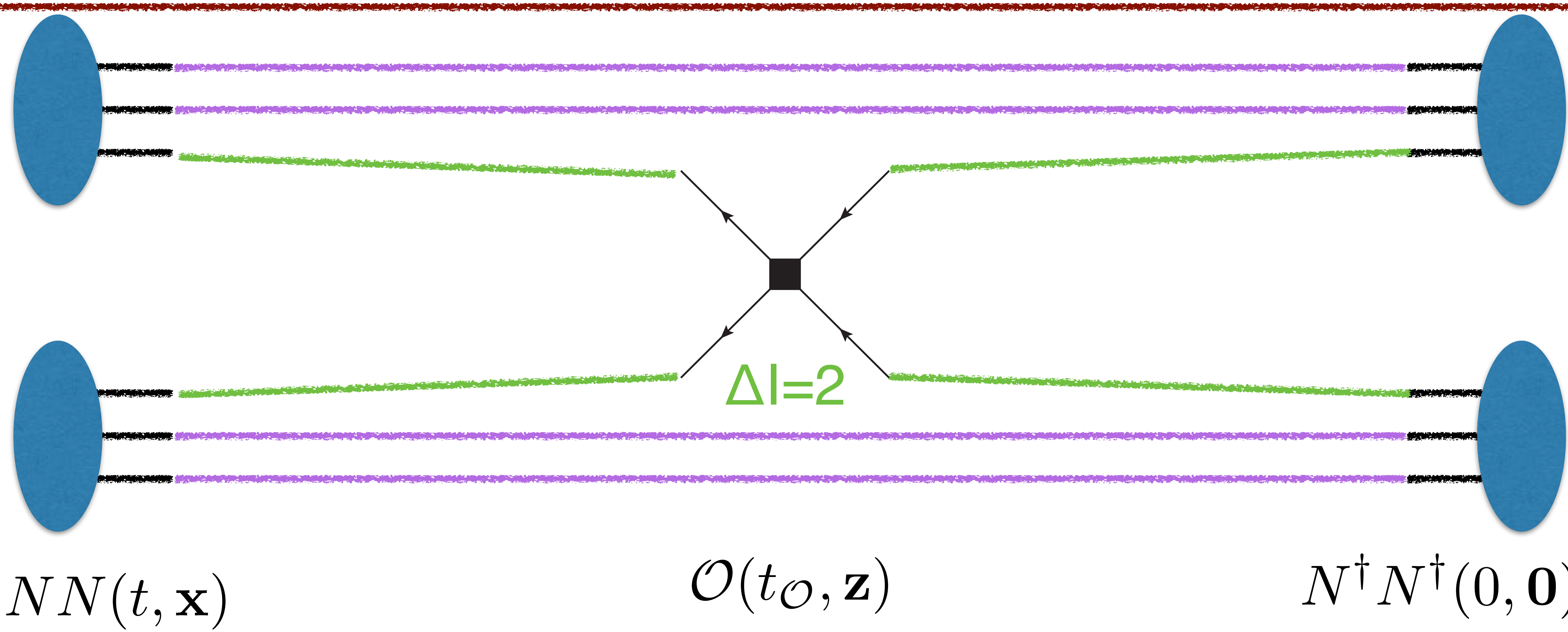
ΔI=0,1,2

LQCD Challenges for Parity Nonconservation



- The “disconnected” quark loops are numerically more expensive, and stochastically noisier
- LQCD calculations can project onto definite ΔI

LQCD Challenges for Parity Nonconservation



- To project the operator, *O*, onto definite momentum, and to project the final NN state onto definite momentum, we need all-to-all propagators (expensive): $\sum_{\mathbf{x}}, \sum_{\mathbf{z}}$
- For now - fix source-sink separation (t) and do NOT sum over **x**, loss of spatial volume in statistics

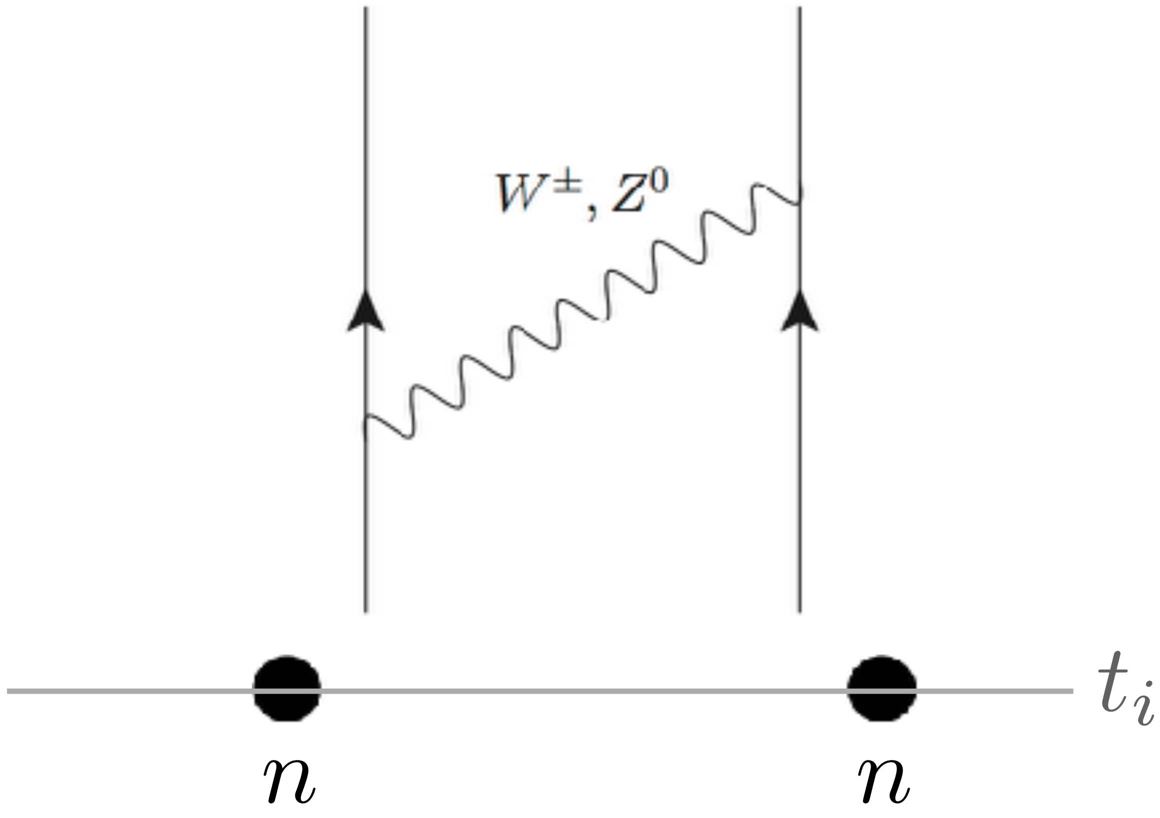
Hadronic Parity Violation

- 2 Baryon „s-wave“ source

Slide adapted from T. Kurth

Hadronic Parity Violation

- 2 Baryon „s-wave" source
- EW vertices ⇒ 4-quark operator insertion



Slide adapted from T. Kurth

Hadronic Parity Violation

- 2 Baryon „s-wave“ source

$$\mathcal{O}^{\Delta I=2} \equiv \sum_{\mathbf{x}} \left[(\bar{q}\tau^3 q)_A (\bar{q}\tau^3 q)_V - \frac{1}{3} (\bar{q}\vec{\tau} q)_A (\bar{q}\vec{\tau} q)_V \right] (\mathbf{x})$$

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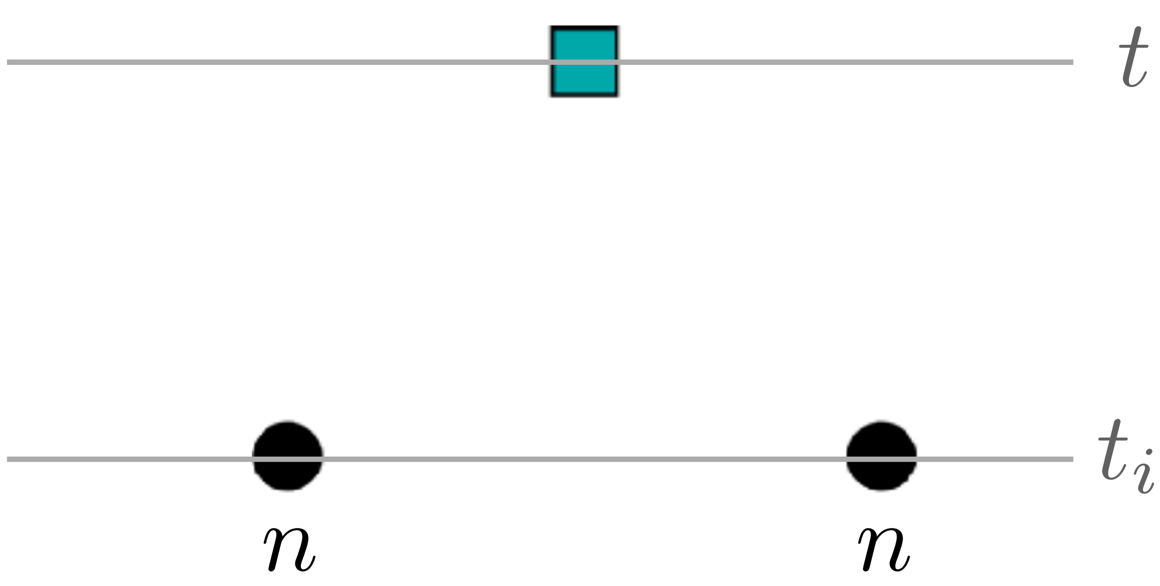

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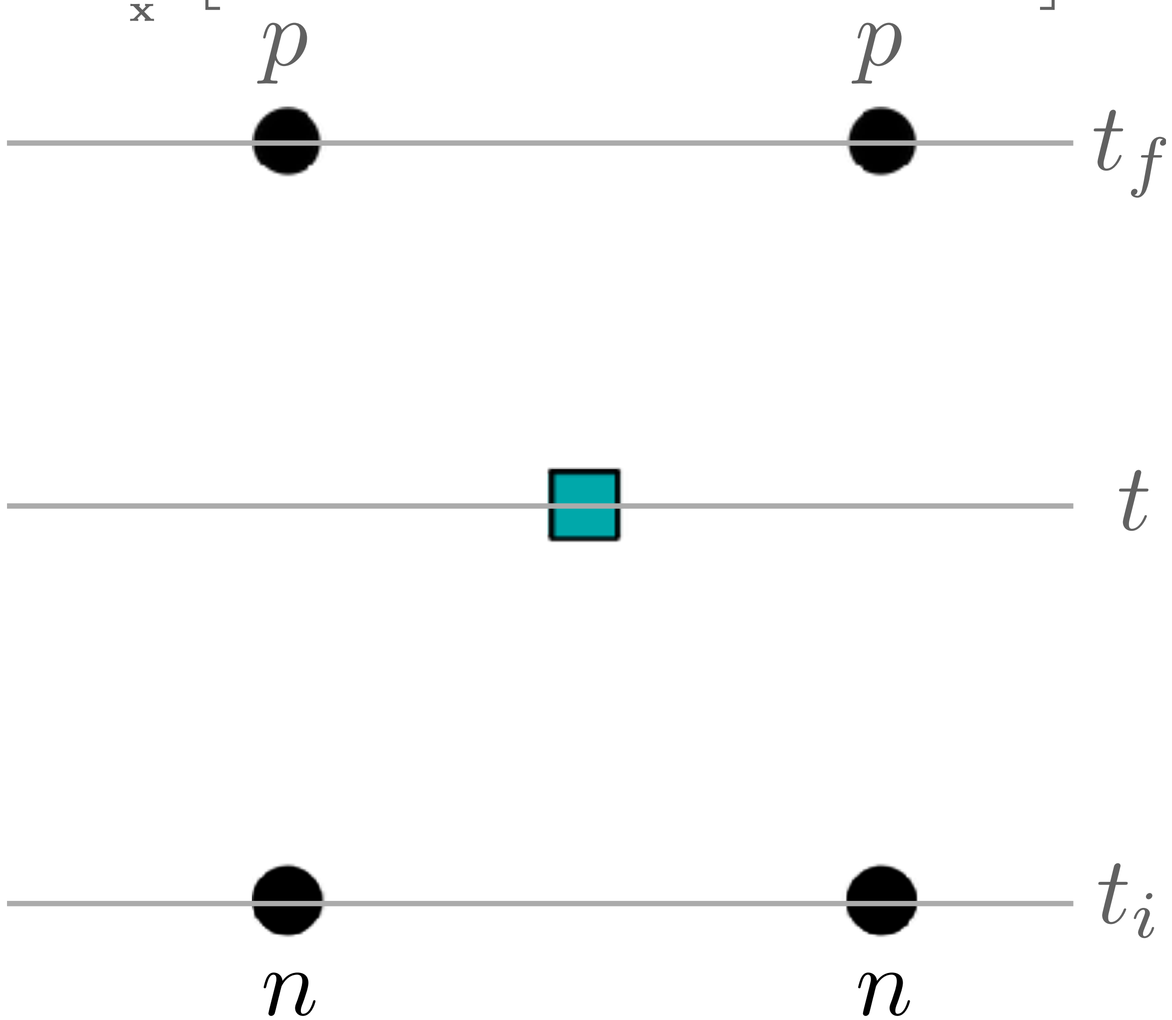
Hadronic Parity Violation

- 2 Baryon „s-wave“ source
- EW vertices ⇒ 4-quark operator insertion
- 2 Baryon „p-wave“ sink
- In total there are 4896 contractions
- isospin limit reduces this number to 2208

$$\mathcal{O}^{\Delta I=2} \equiv \sum_{\mathbf{x}} \left[(\bar{q}\tau^3 q)_A (\bar{q}\tau^3 q)_V - \frac{1}{3} (\bar{q}\vec{\tau} q)_A (\bar{q}\vec{\tau} q)_V \right] (\mathbf{x})$$



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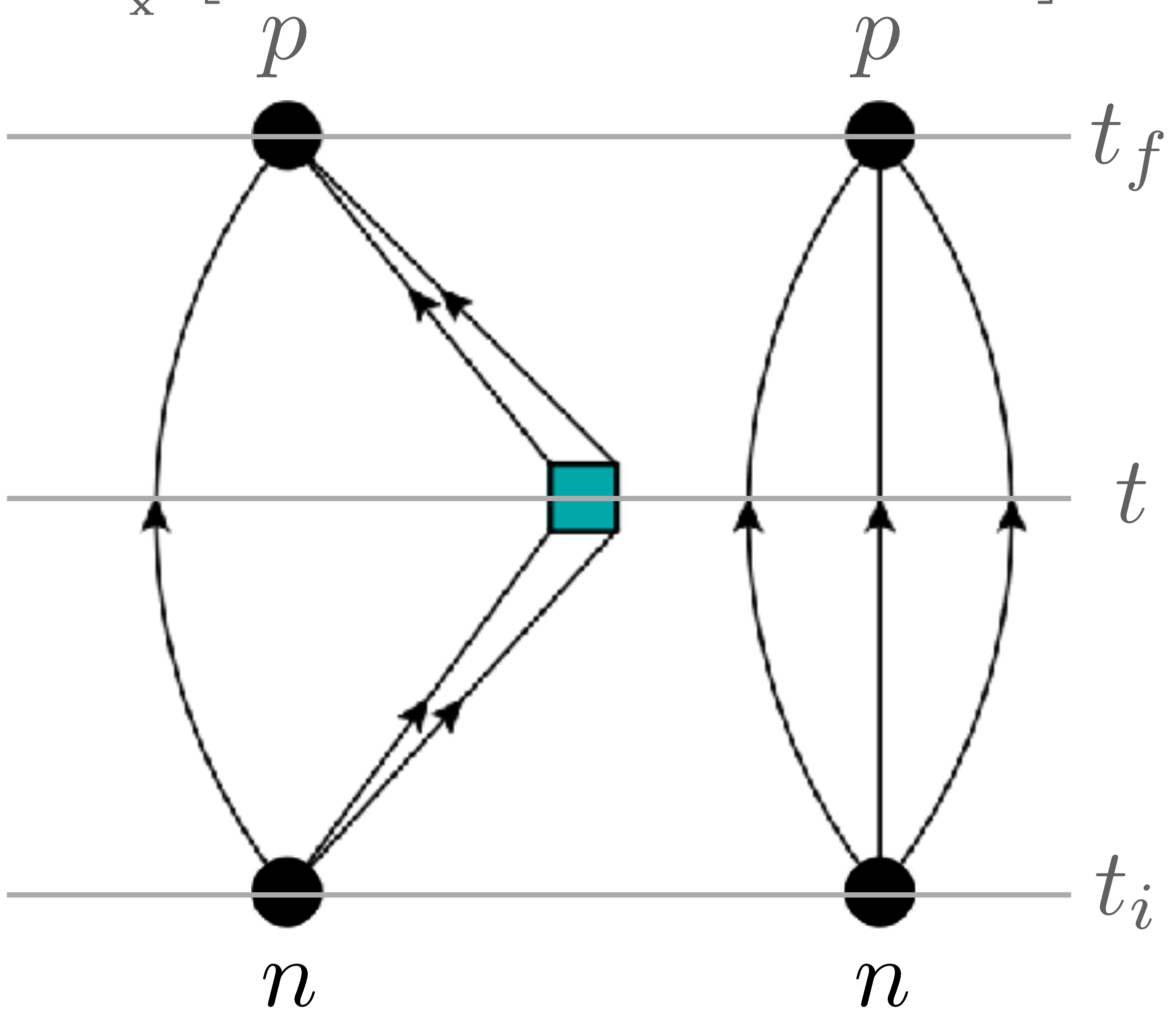

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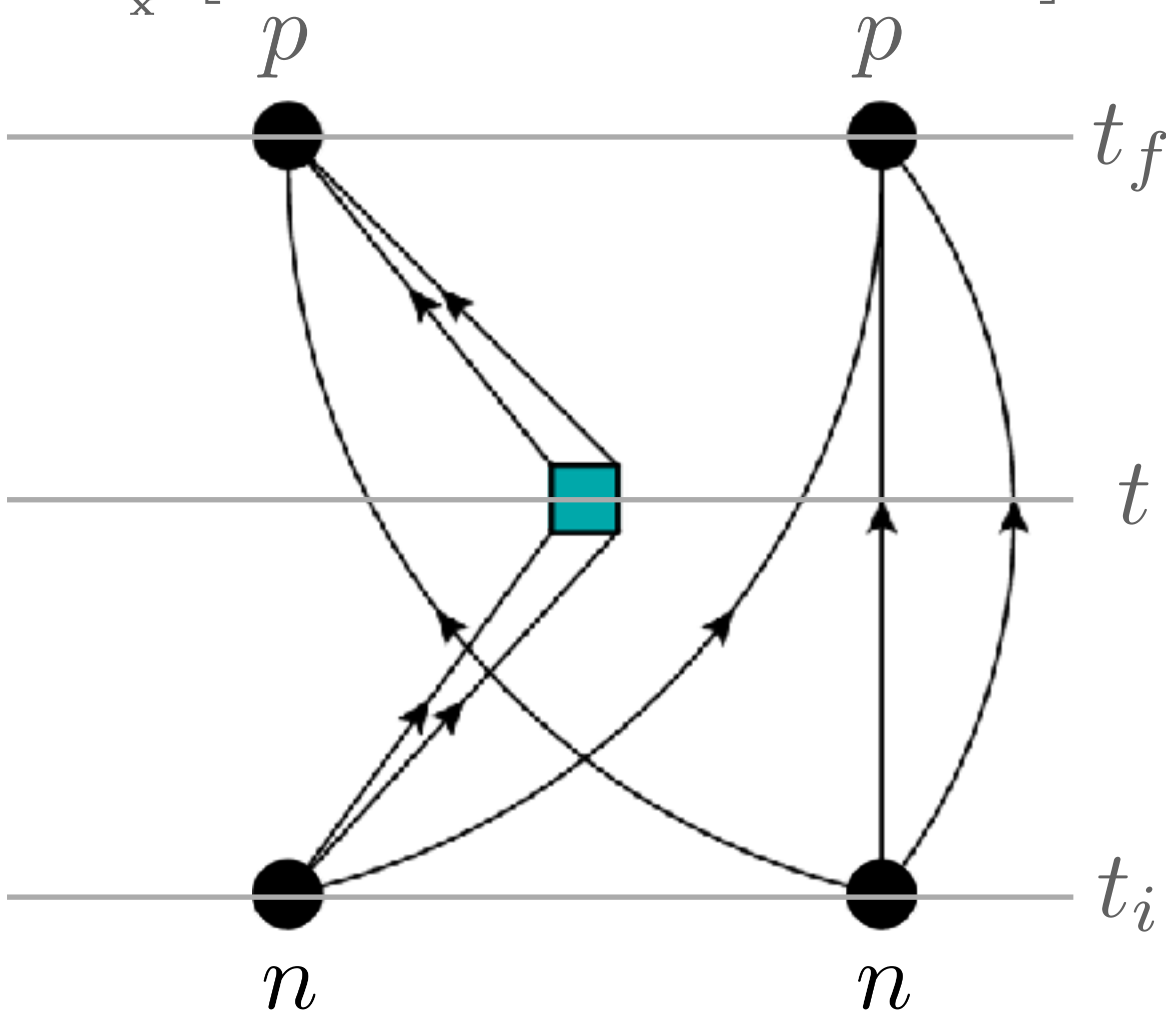

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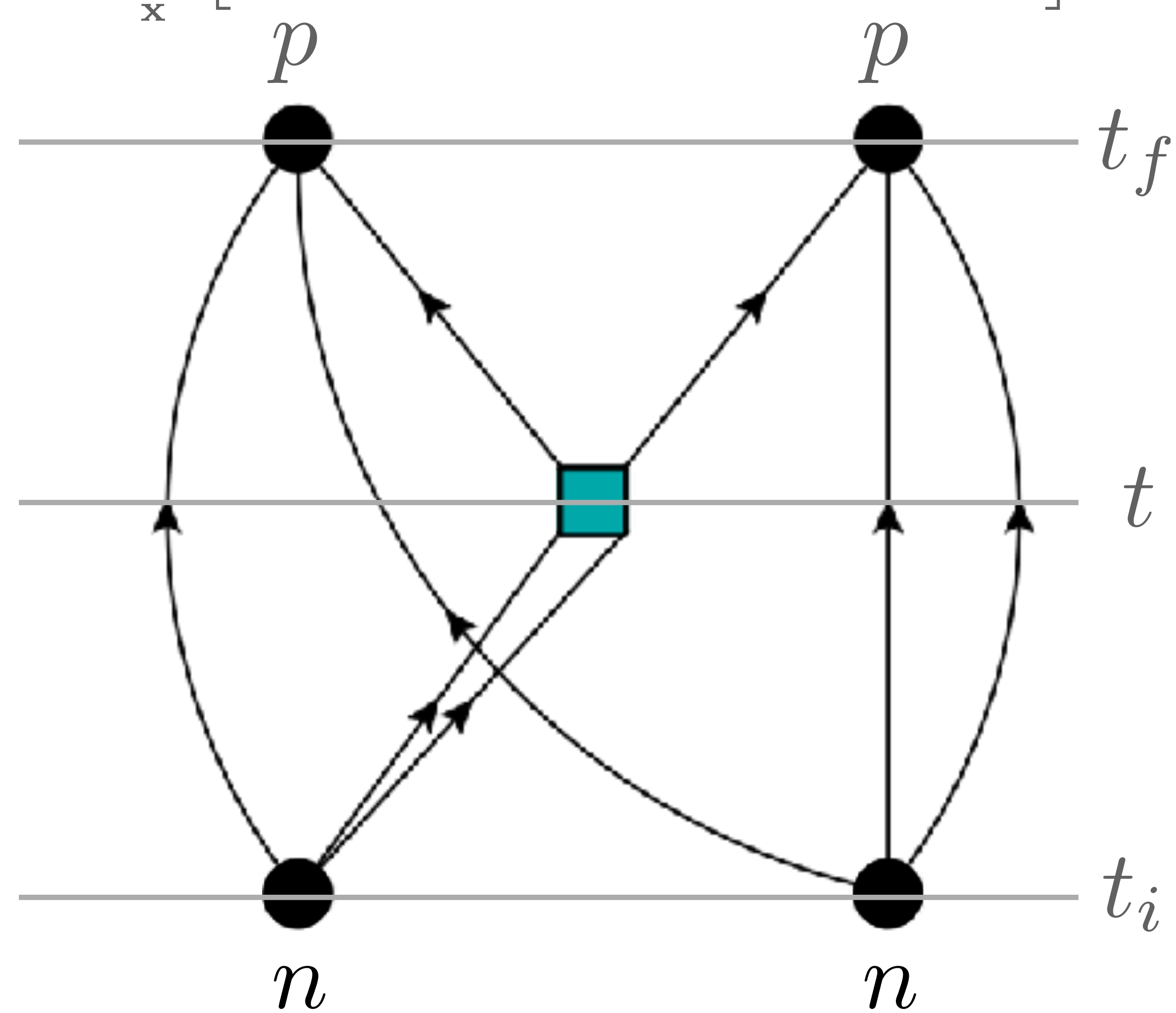

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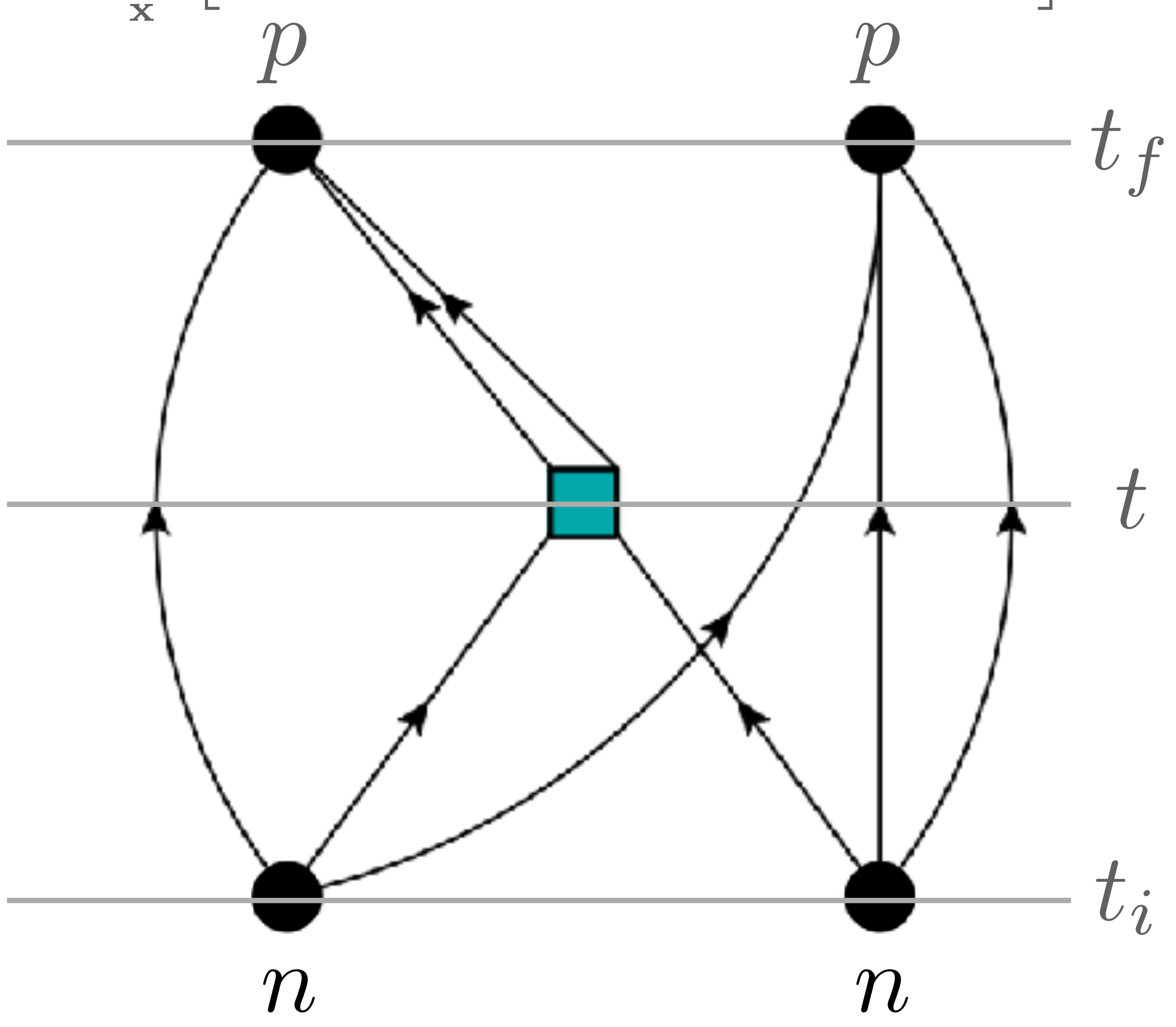

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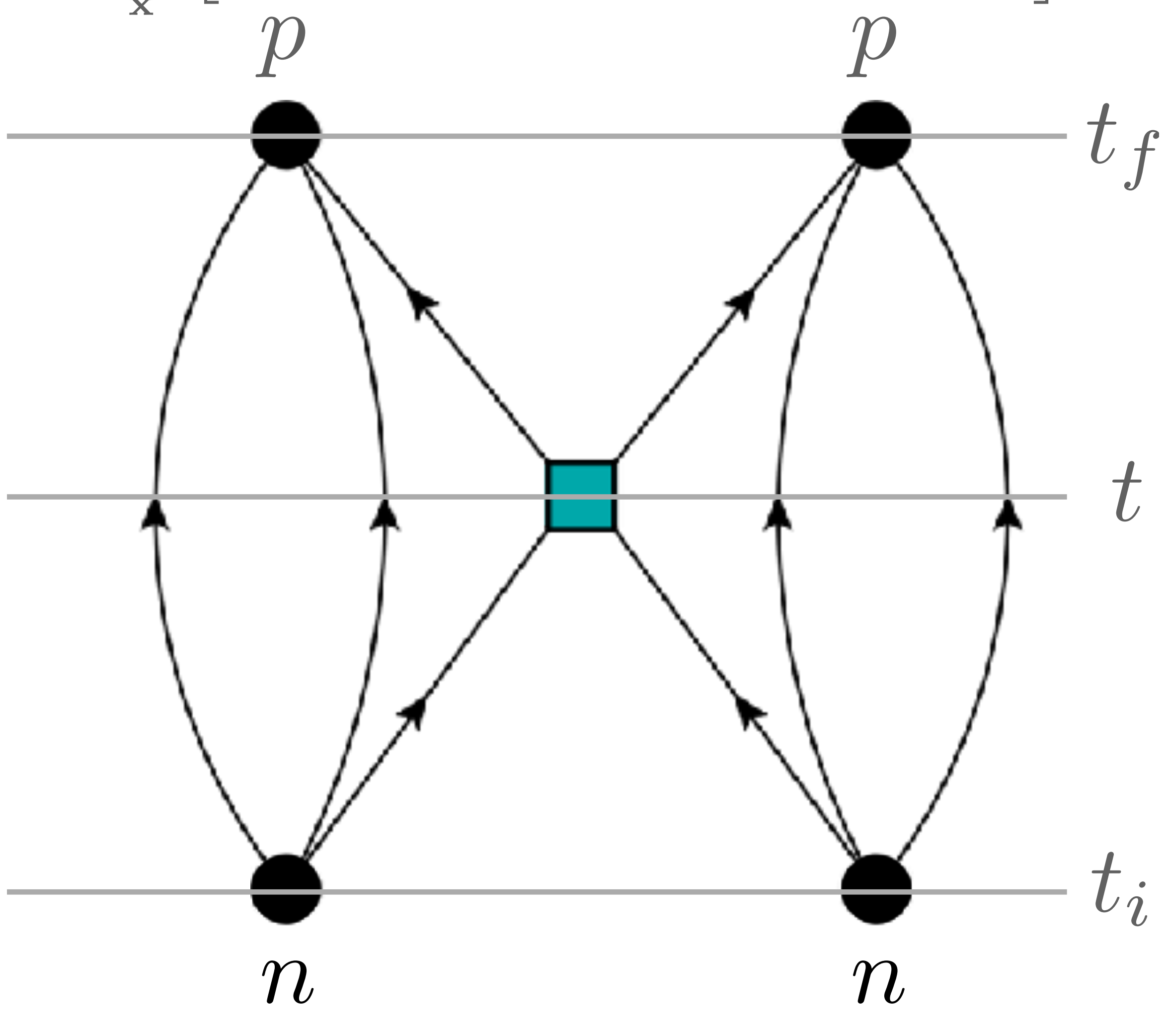

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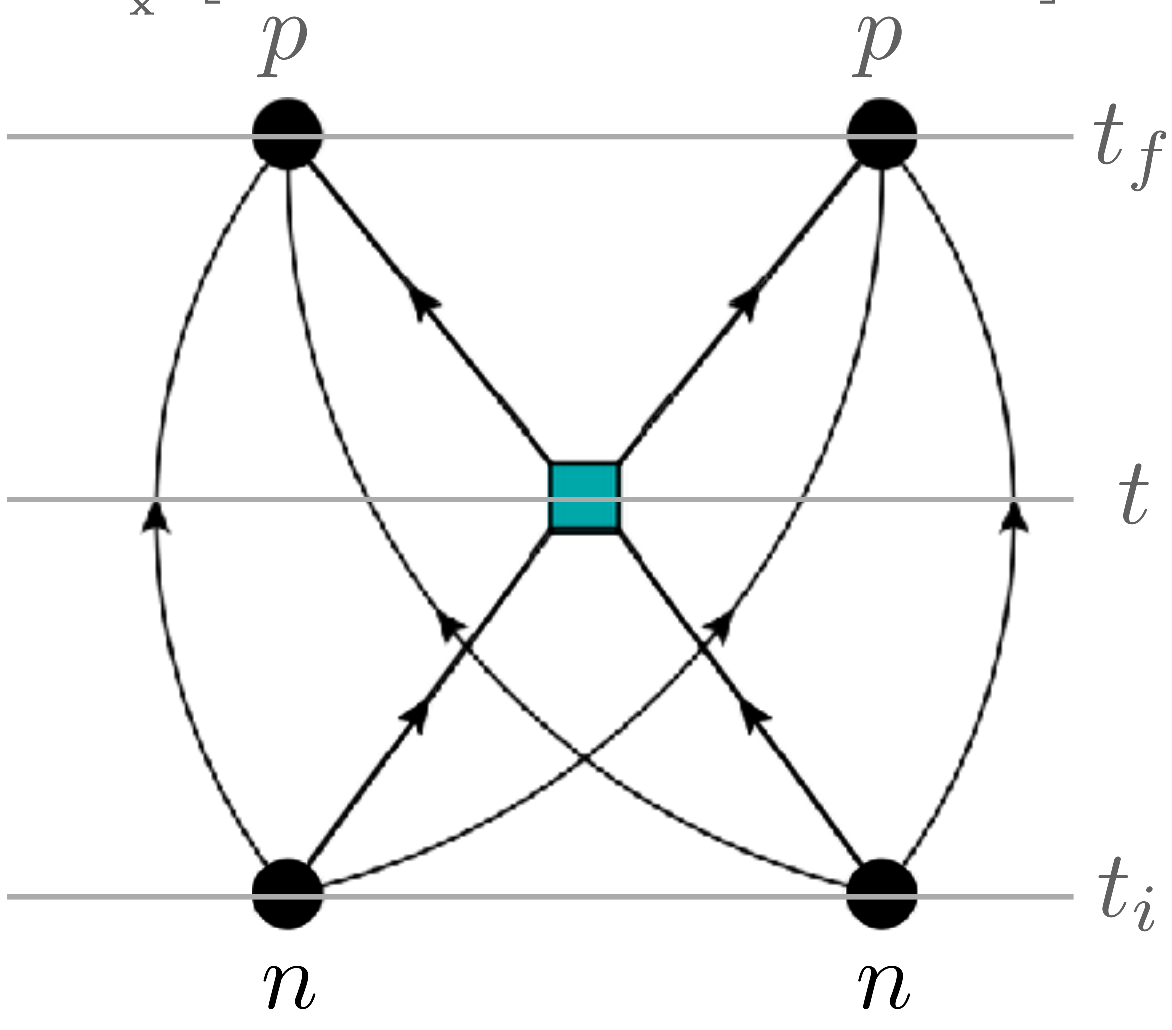

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- Extension of UCM, automatic code generation using Mathematica

  Doi & Endres, Originos et. al., Günther et. al.

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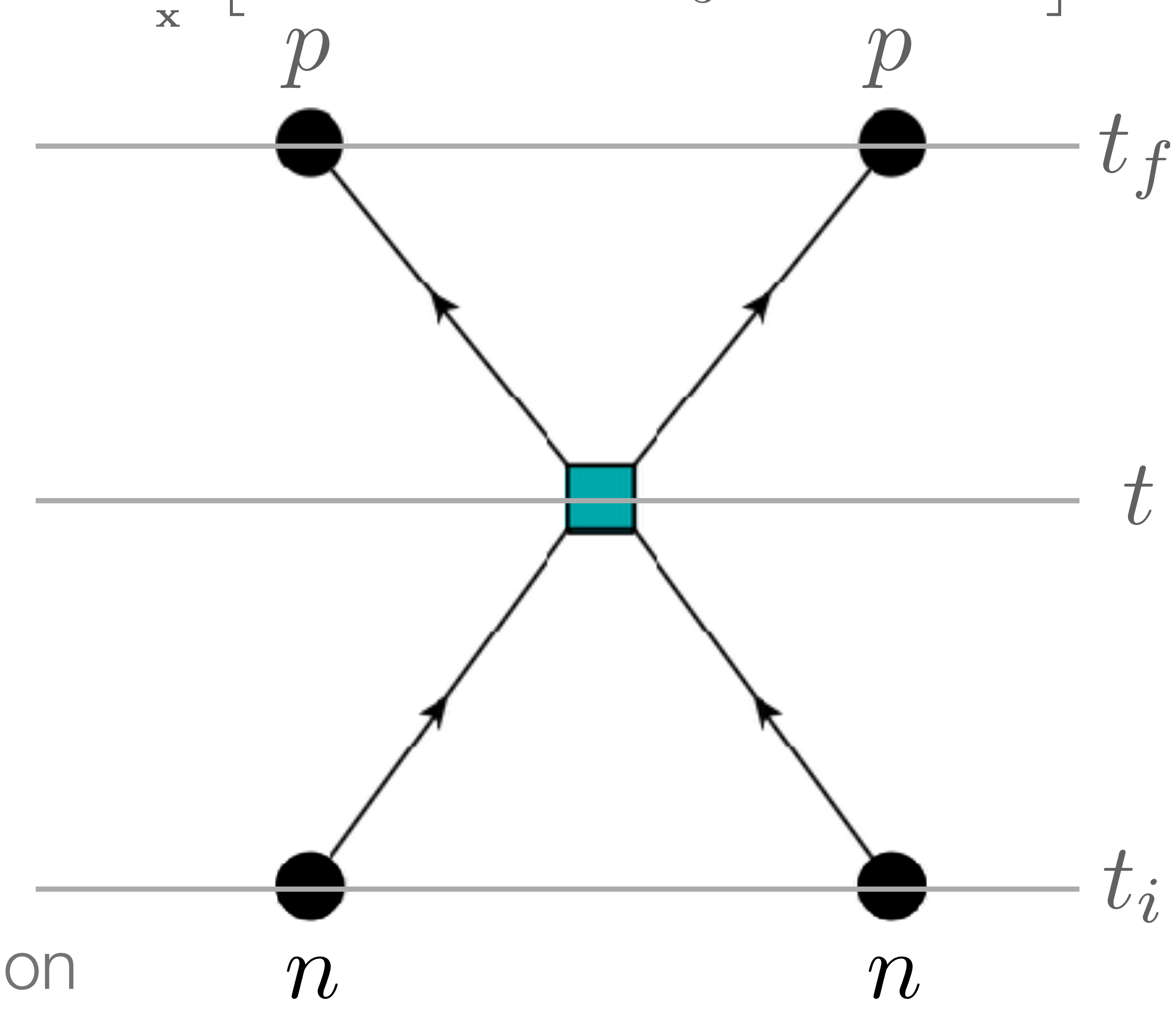

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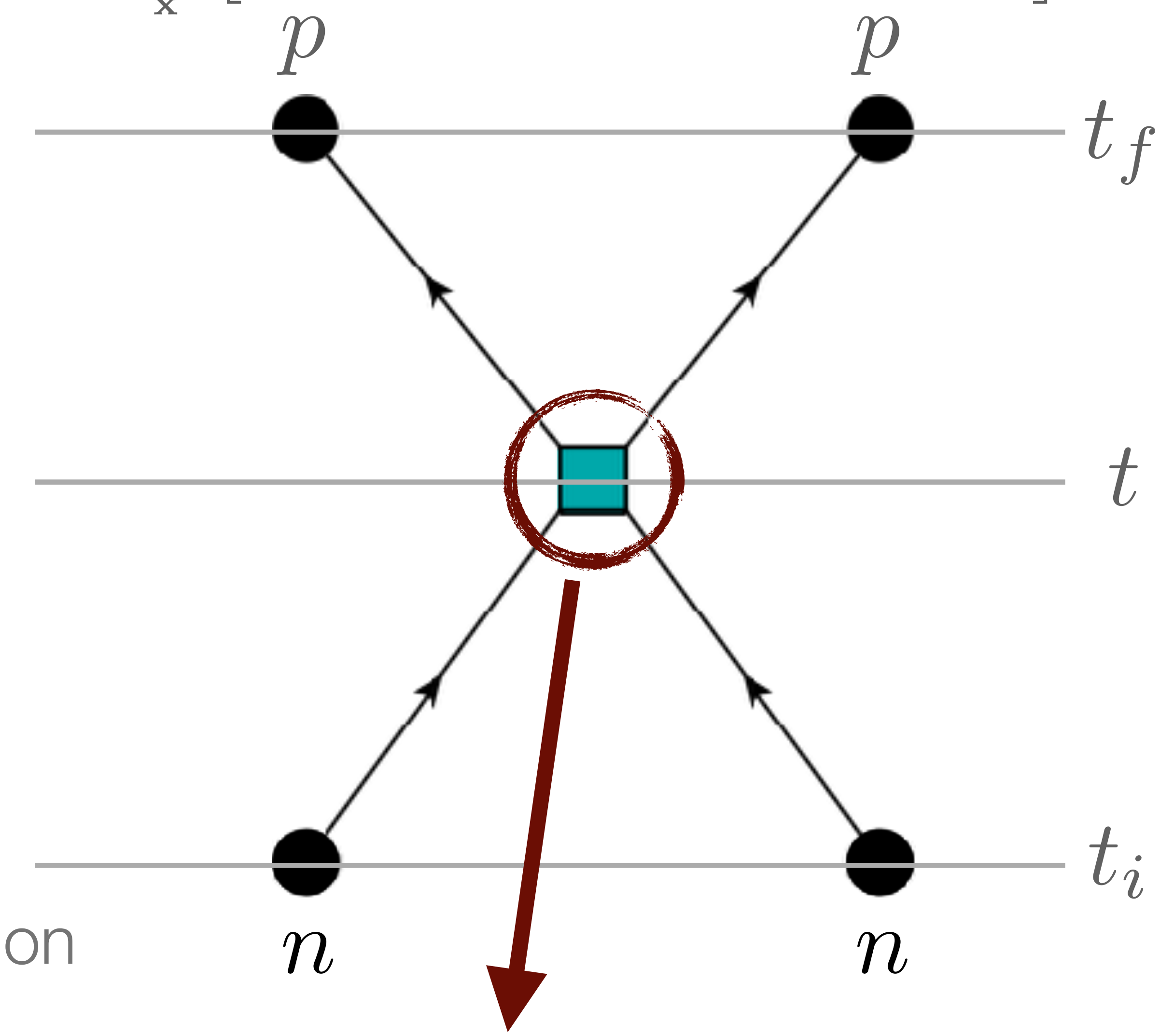
- Partial wave scattering needed as well

$$\mathcal{O}^{\Delta I=2} \equiv \sum_{\mathbf{x}} \left[(\bar{q}\tau^3 q)_A (\bar{q}\tau^3 q)_V - \frac{1}{3} (\bar{q}\vec{\tau} q)_A (\bar{q}\vec{\tau} q)_V \right] (\mathbf{x})$$

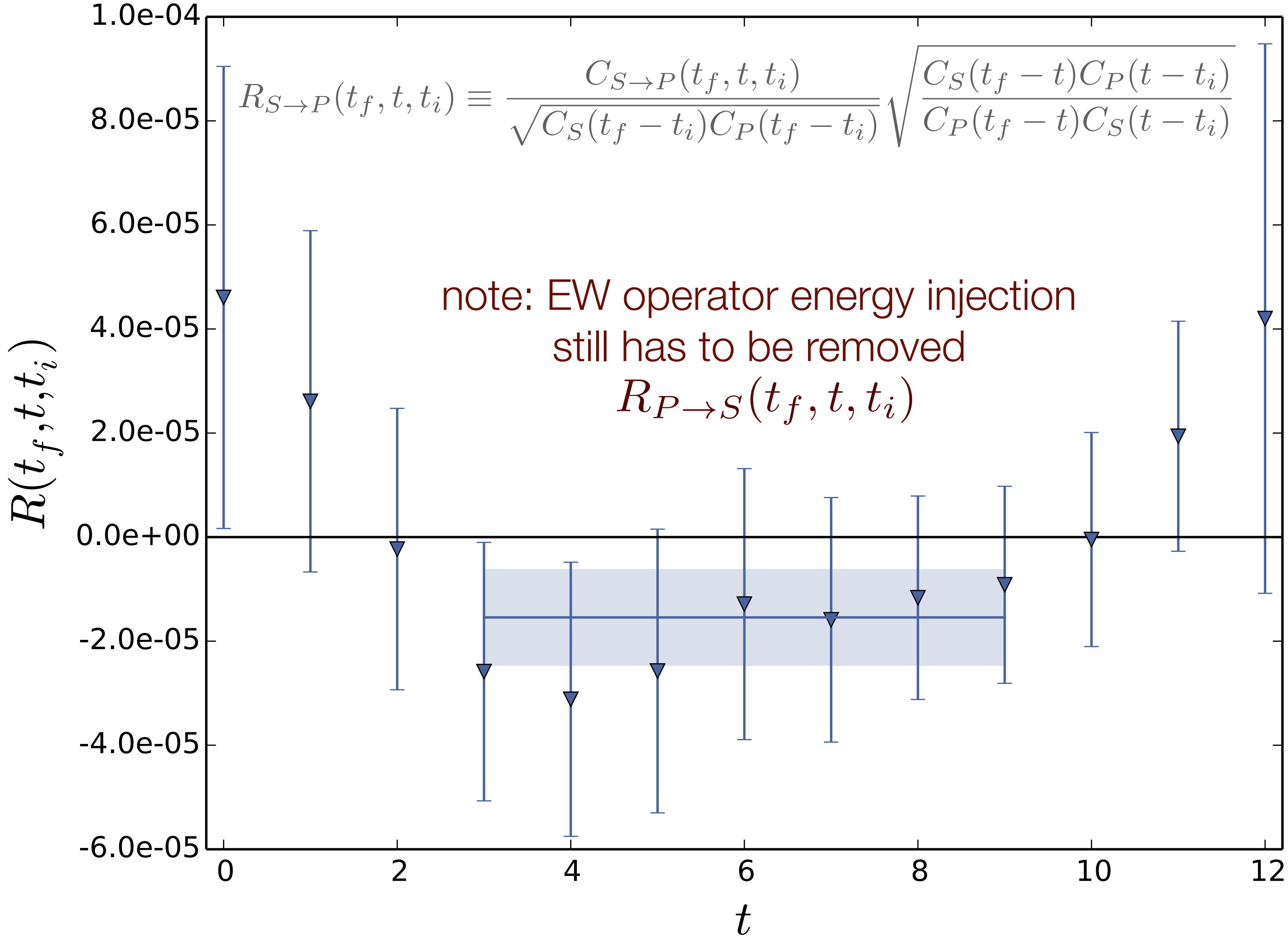


$$\langle {}^3P_0 | H_{\mathrm{EW}} | {}^1S_0 \rangle_\infty = f\left(\delta^{(S)}, \partial_E \delta^{(S)}, \delta^{(P)}, \partial_E \delta^{(P)}\right) \times \langle {}^3P_0 | H_{\mathrm{EW}} | {}^1S_0 \rangle_{\mathrm{FV}}$$

Slide adapted from T. Kurth

Hadronic Parity Violation

Normalized Ratio



Preliminary!
m_π=800 MeV
only 200 samples

The most challenging aspect of this calculation is the NN interaction

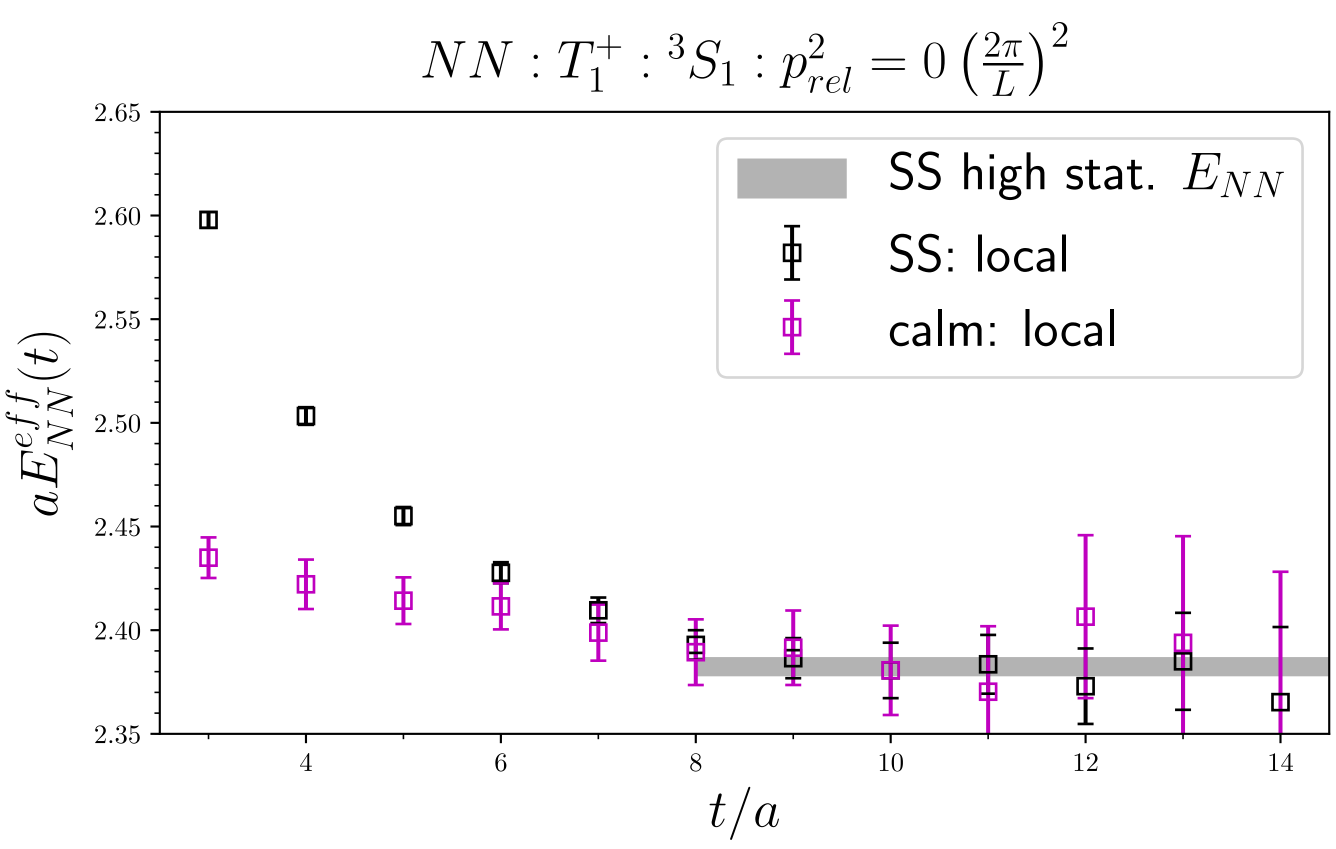
Inspiration for LQCD calculations of ΔI=2 PNC

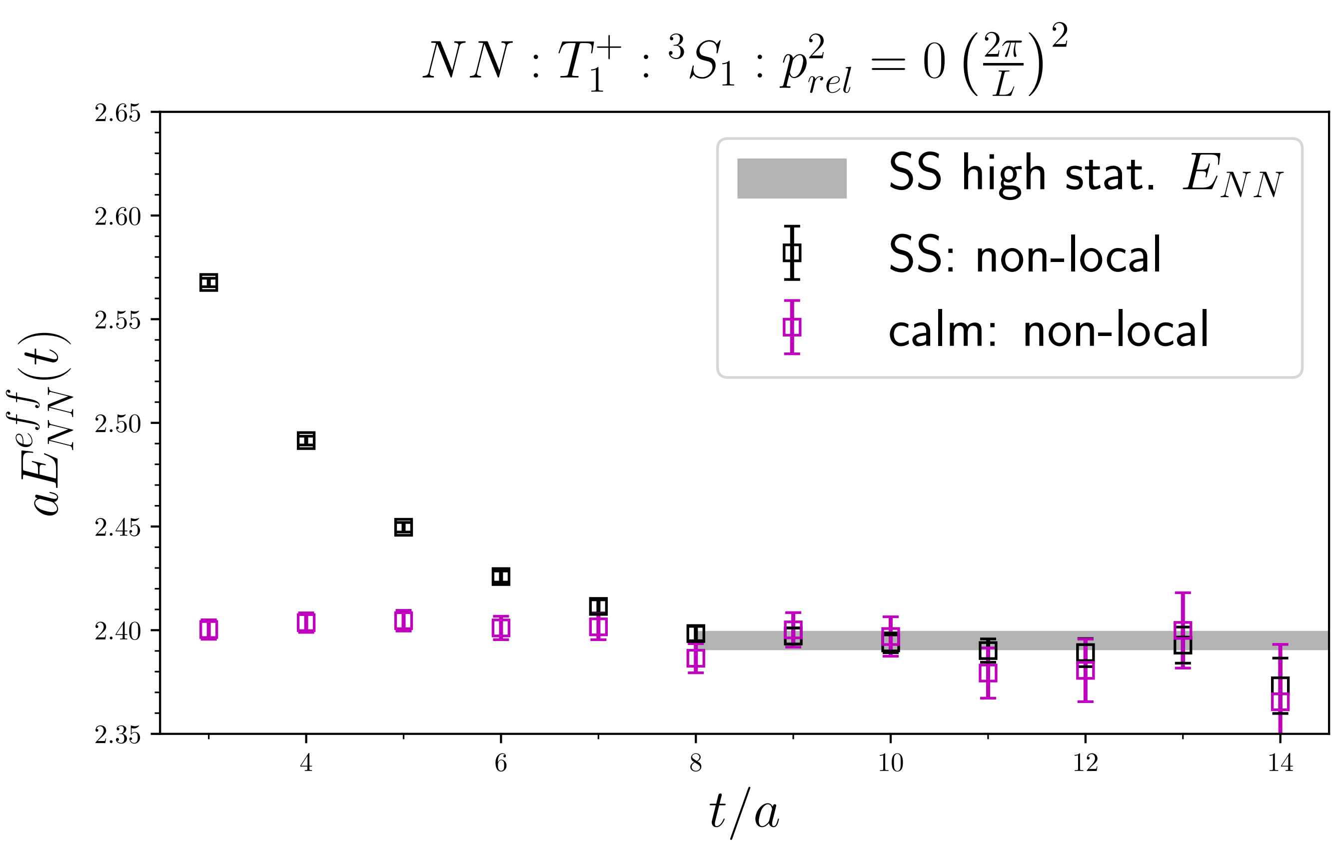
- We have cooked up a simple idea that offers the promise of exponentially improving the calculations

Calm Multi-Baryon Operators

E.Berkowitz, A. Nicholson, C.C. Chang, E. Rinaldi, M.A. Clark, B. Joo, T. Kurth, P. Vranas, AWL

arXiv:1710.05642





Matrix Prony

arXiv.org > hep-lat > arXiv:0903.2990

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High Energy Physics – Lattice

High Statistics Analysis using Anisotropic Clover Lattices: (I) Single Hadron Correlation Functions

Silas R. Beane, William Detmold, Thomas C. Luu, Kostas Orginos, Assumpta Parreno, Martin J. Savage, Aaron Torok, Andre Walker-Loud

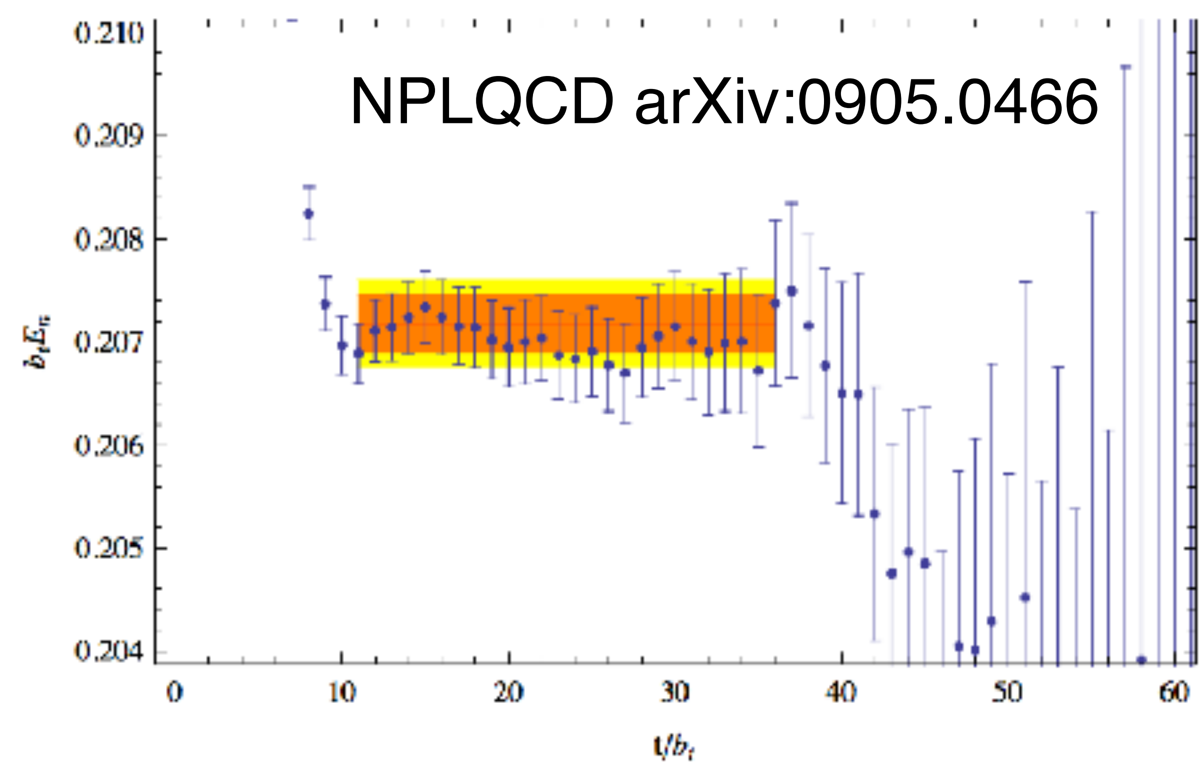
- In this work, we applied for the first time, the Matrix Prony method for analyzing two-point correlation functions.
- Idea: construct linear combination of correlation functions to remove excited state contamination

$$C_A(t) = A_0 e^{-E_0 t} + A_1 e^{-E_1 t} + \cdots$$

$$C_B(t) = B_0 e^{-E_0 t} + B_1 e^{-E_1 t} + \cdots$$

$$C_0 = \alpha_0 C_A + \beta_0 C_B$$

$$\alpha_0 A_1 + \beta_0 B_1 = 0$$



Find α_0 and β_0 with black-box method (Matrix Prony)

Matrix Prony

- Matrix Prony is a very clean idea

  Suppose you have a vector of correlation functions $y(t) = \begin{pmatrix} y_{SS}(t) \\ y_{PS}(t) \end{pmatrix}$

  There will be a **transfer** operator for this set of correlation functions

$$y(t+\tau) = T(\tau)y(t)$$

  Let us factorize the transfer operator as $T = M^{-1}V$

$$My(t+\tau) = Vy(t)$$

  We can take the outer product with $y(t)$

$$My(t+\tau)y^T(t) = Vy(t)y^T(t)$$

Matrix Prony

$$My(t+\tau)y^T(t) = Vy(t)y^T(t)$$

Let us make the ansatz that under a window of time, there are only 2 states which contribute meaningfully to both correlation functions. Then T (and M and V) will be independent of t

$$M\sum_{t=t_i}^{t_f} y(t+\tau)y^T(t) = V\sum_{t=t_i}^{t_f} y(t)y^T(t)$$

Up to overall normalizations, the solution to this system is

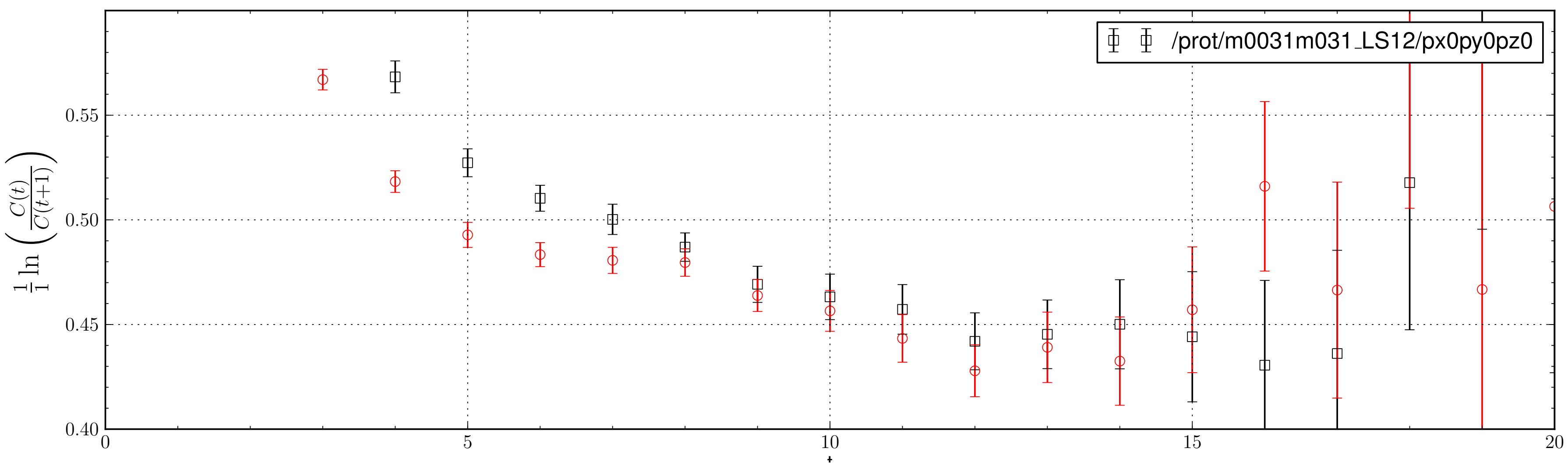
$$M = \left[\sum_{t=t_i}^{t_f} y(t+\tau)y^T(t)\right]^{-1} \qquad V = \left[\sum_{t=t_i}^{t_f} y(t)y^T(t)\right]^{-1}$$

We then solve the eigenvalue system

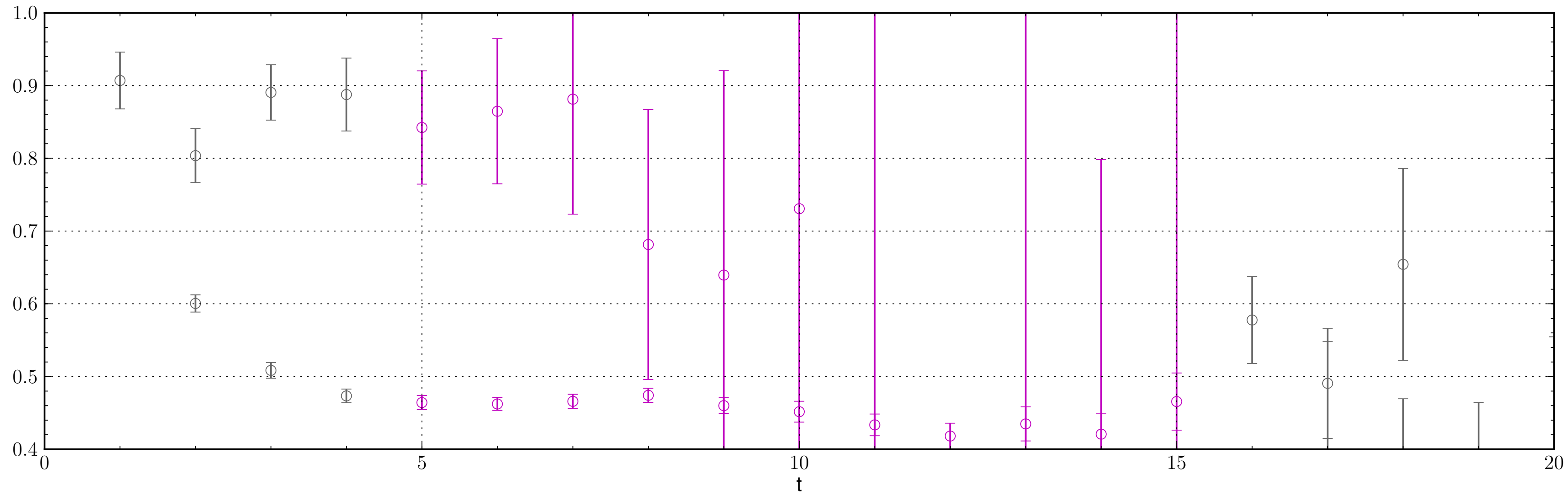
$$T(\tau)q_\lambda = \lambda^\tau q_\lambda \qquad \lambda = e^{-E_\lambda} \qquad q_\lambda = \alpha_\lambda y_{SS} + \beta_\lambda y_{SP}$$

If the ansatz is satisfied - the resulting q_λ will be free from excited state contamination over the range of times used

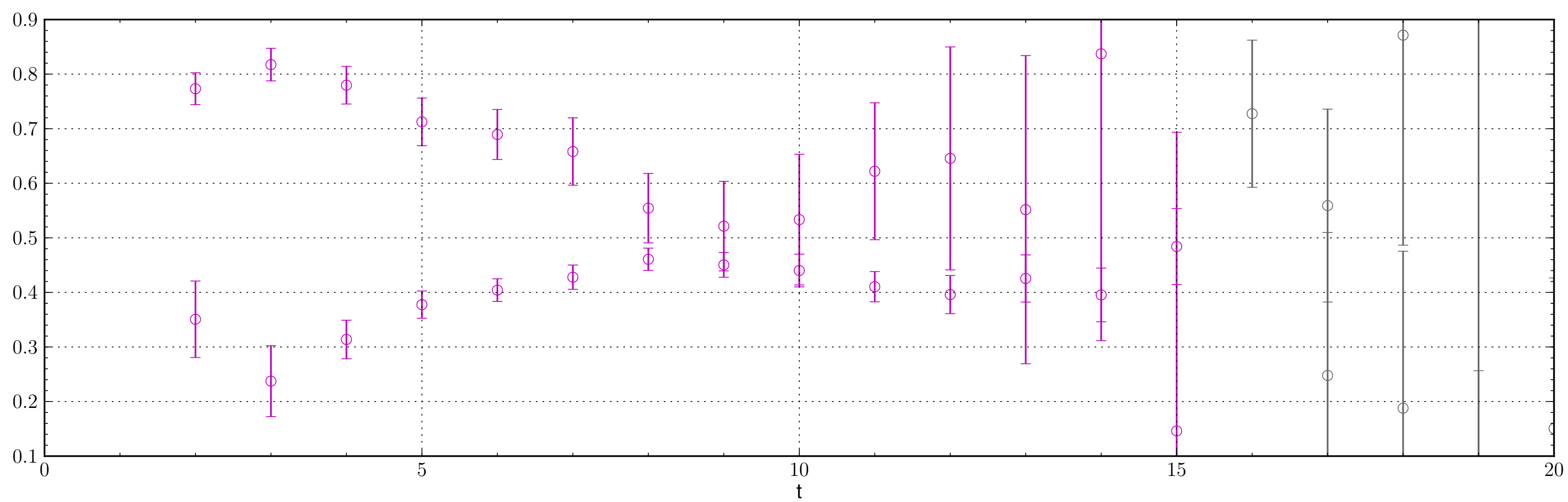
Matrix Prony



original correlation functions



Matrix Prony t=5-15
2 correlation functions with minimal excited state contamination

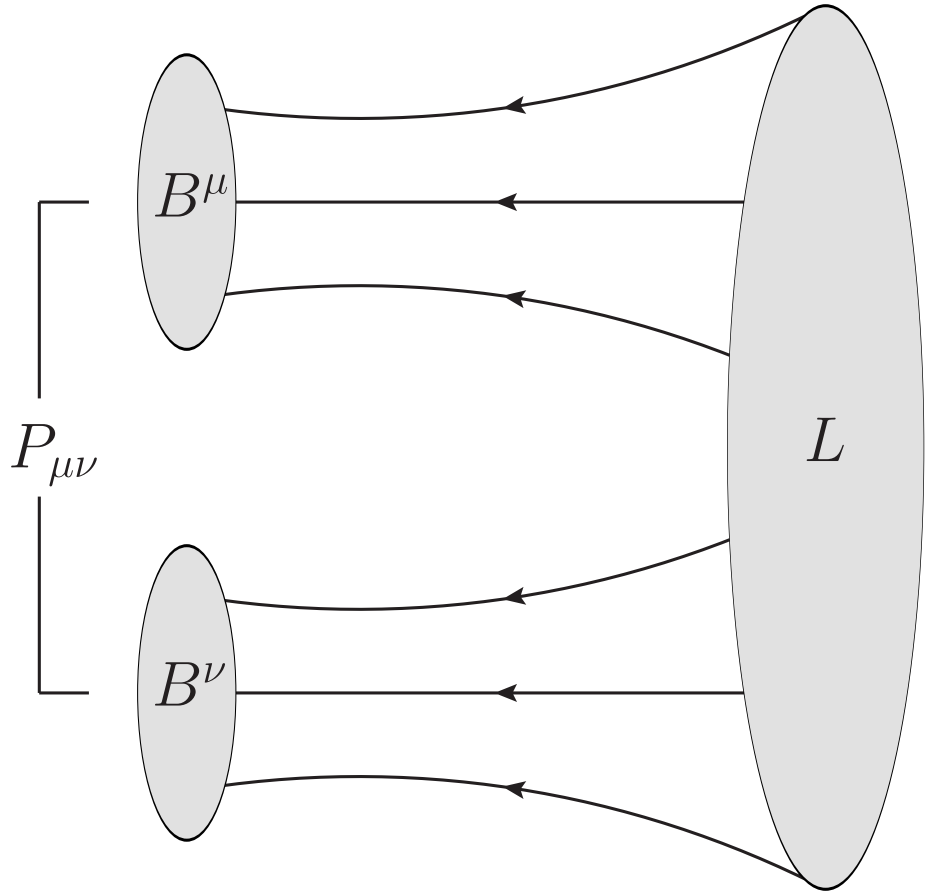


Matrix Prony t=2-15
too aggressive - clear excited state contamination

Matrix Prony: Improved NN

What is the new idea?

- Previously, Matrix Prony has been used to analyze linear combinations of correlation functions in B, BB, BBB, BBBB systems after they were generated
- We realized we could instead use Matrix Prony to form an optimal linear combination of single-nucleon correlation sinks, that could then be inserted into the two(multi)-nucleon contraction code

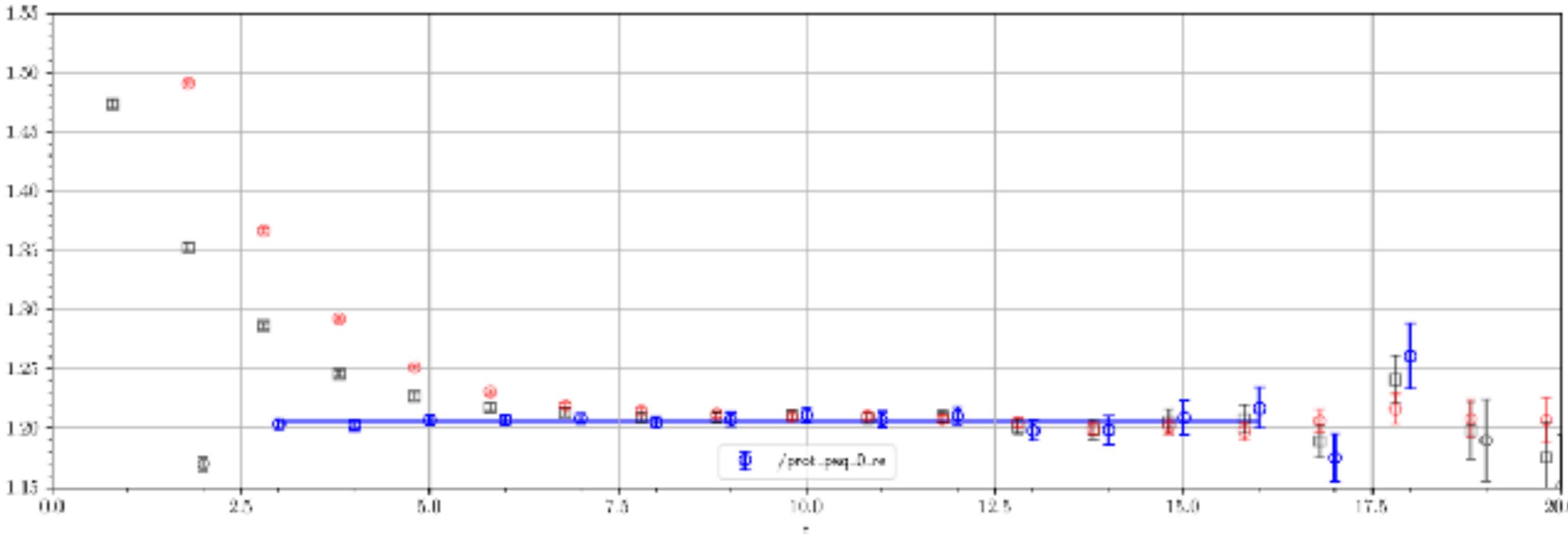


$$B^\mu = \alpha_0 B^\mu_{SS} + \beta_0 B^\mu_{PS}$$

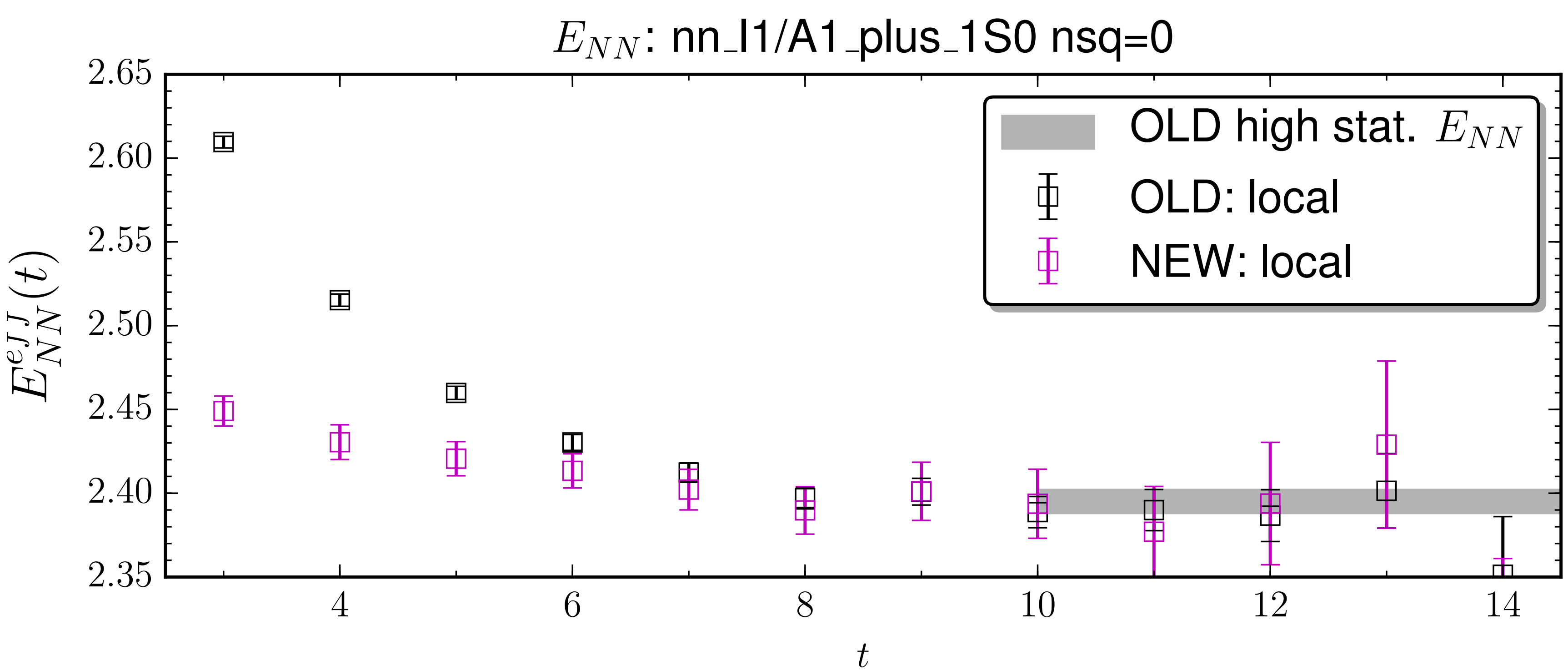
- This is a "poor man's" version of the more sophisticated variational methods used by, Bulava et. al., Hadspec, etc.
- The numerical cost is less than the standard method in which both SS and PS two-nucleon correlation functions are generated
  - only single set of contractions/FFT required

Matrix Prony: Improved NN

- How well does it work?
  - Test the idea out on mπ~800 MeV data - iso-clover WM/JLab cfgs



single nucleon pulled in 6 time slices



used with local NN interpolating fields

$$N^\dagger(0, x_i) N^\dagger(0, x_i)$$

we observe a reduction in the excited state contamination

Matrix Prony: Improved NN

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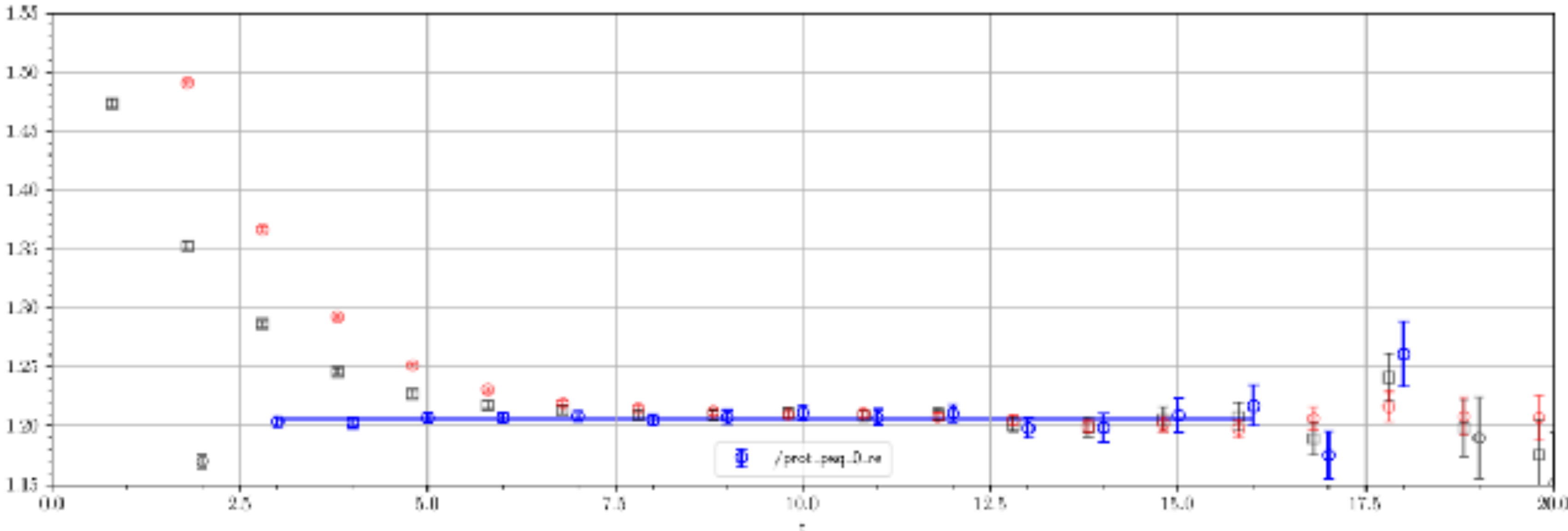
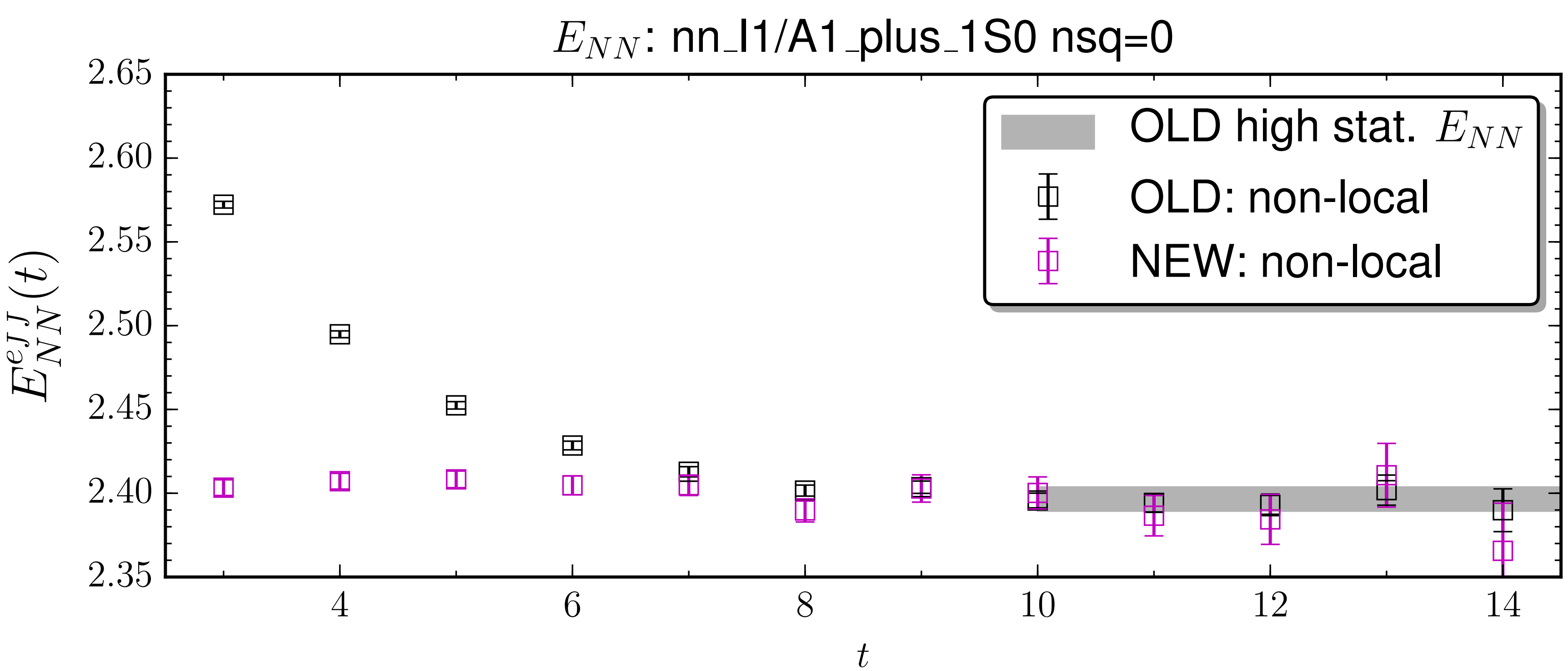


single nucleon pulled in 6 time slices



used with non-local NN interpolating fields
$$N^\dagger(0,x_i)N^\dagger(0,x_i+\Delta)$$
CalLat: PLB765 [1508.00886]

we observe a more significant reduction in the excited state contamination

Matrix Prony: Improved NN

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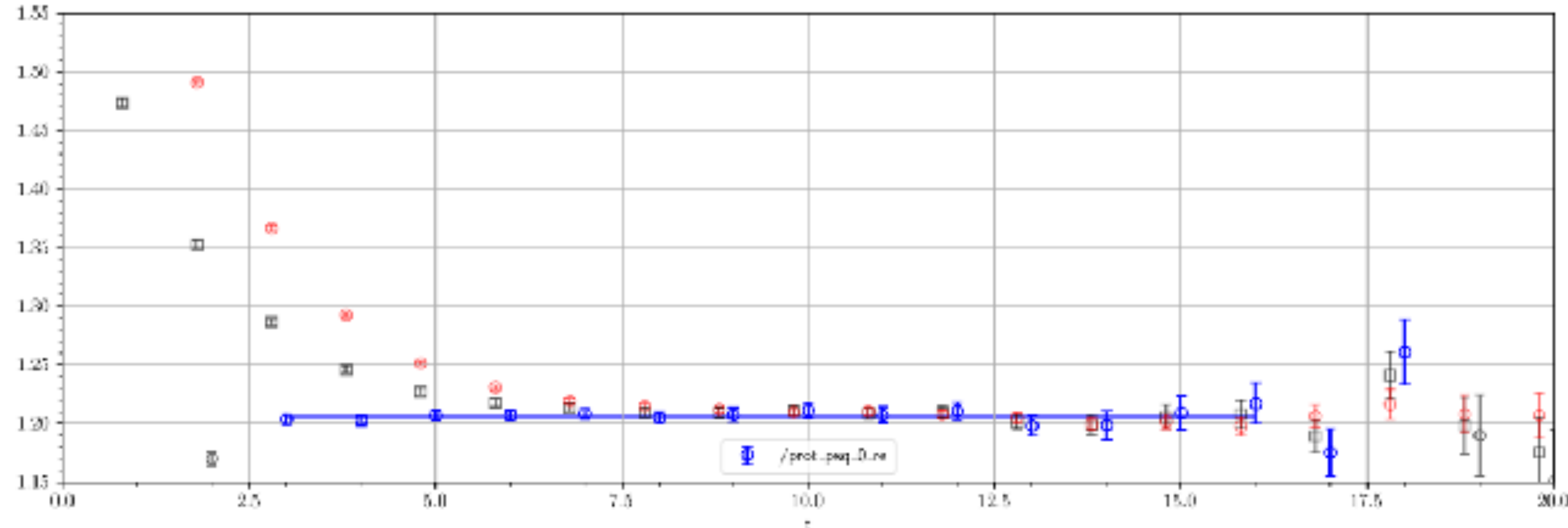

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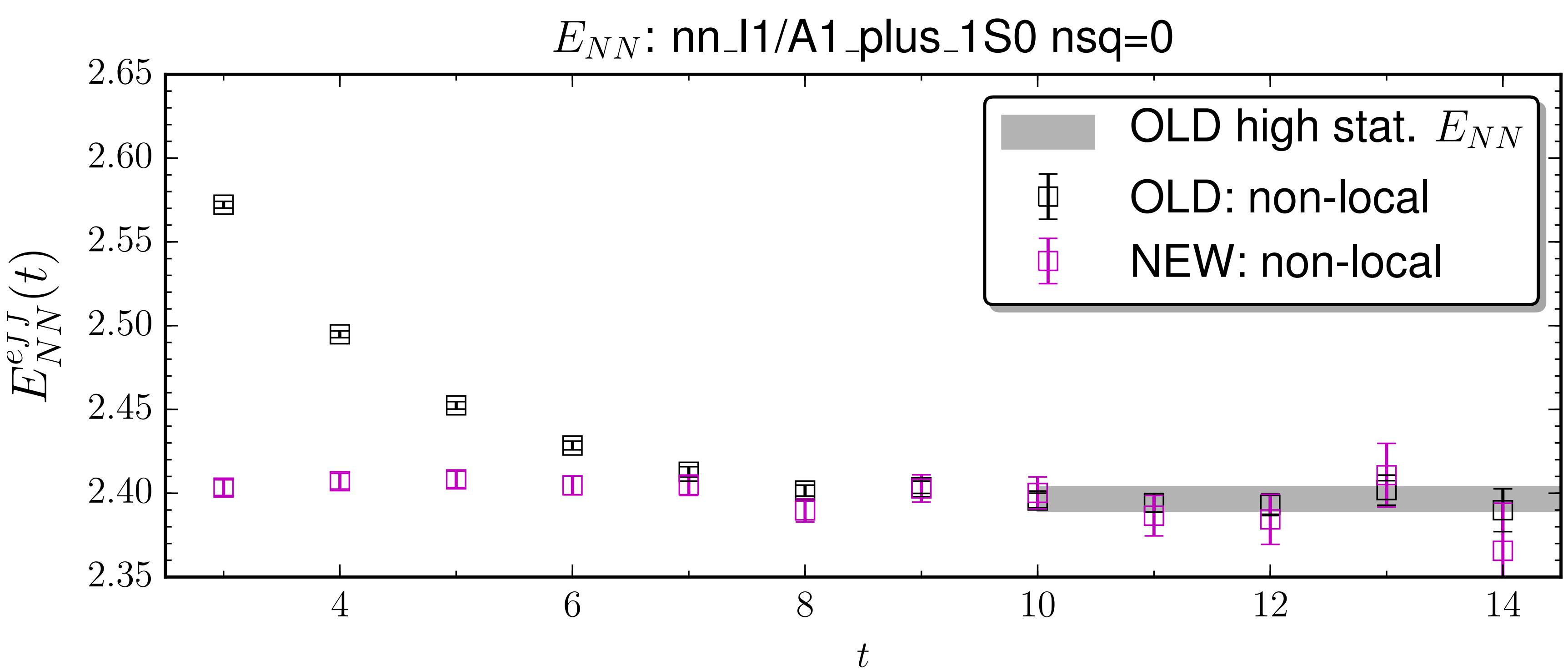

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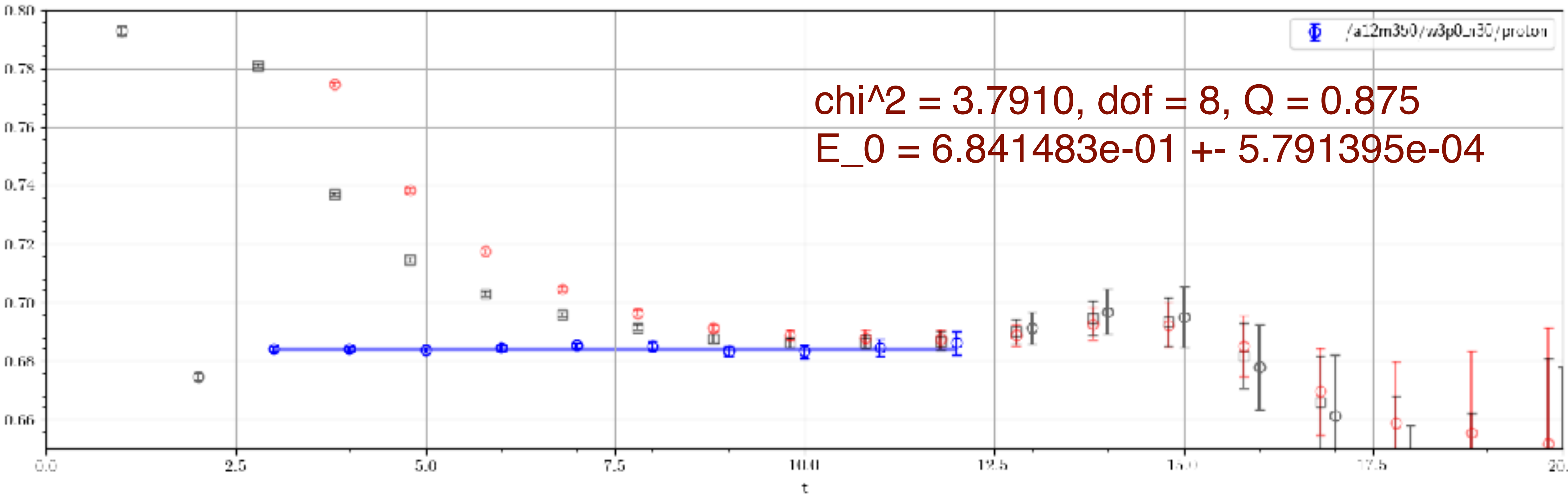
Our Baryons are ***calm*** because they are not excited

Matrix Prony: Improved NN

- How well does it work with an interesting pion mass?
  - Application with MDWF on gradient-flowed HISQ
    $m\pi$~350 MeV, Nsrc = 20,000 (2 srcs/cfg, 10K cfgs)



$$am_N = 0.68417(58)$$ ~ 0.085%, g.s. pulled in 8 time slices ~ 0.96 fm

NOTE: this is one of the ensembles used for our gA calculation - this demonstrates from the numerical data (no fits) that there are only 2 states meaningfully contributing to the correlation function all the way down to t = 3 (0.36 fm)

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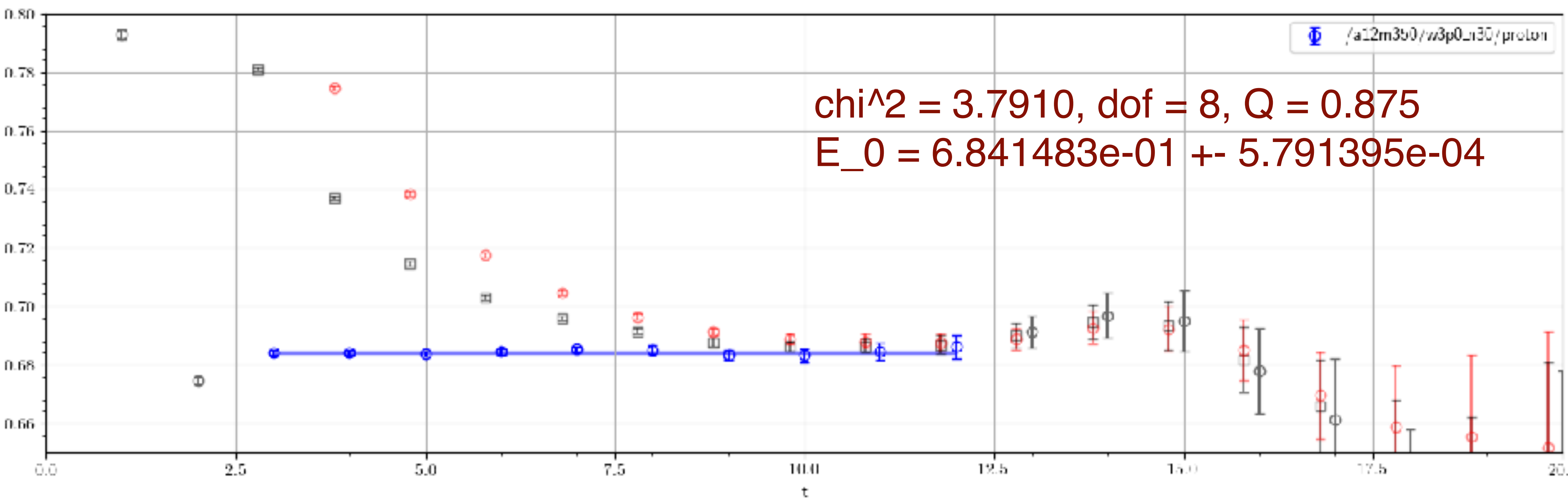

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NPLQCD [arXiv:0903.2990] mπ~400 MeV

$a_t m_N = 0.20693(33)$

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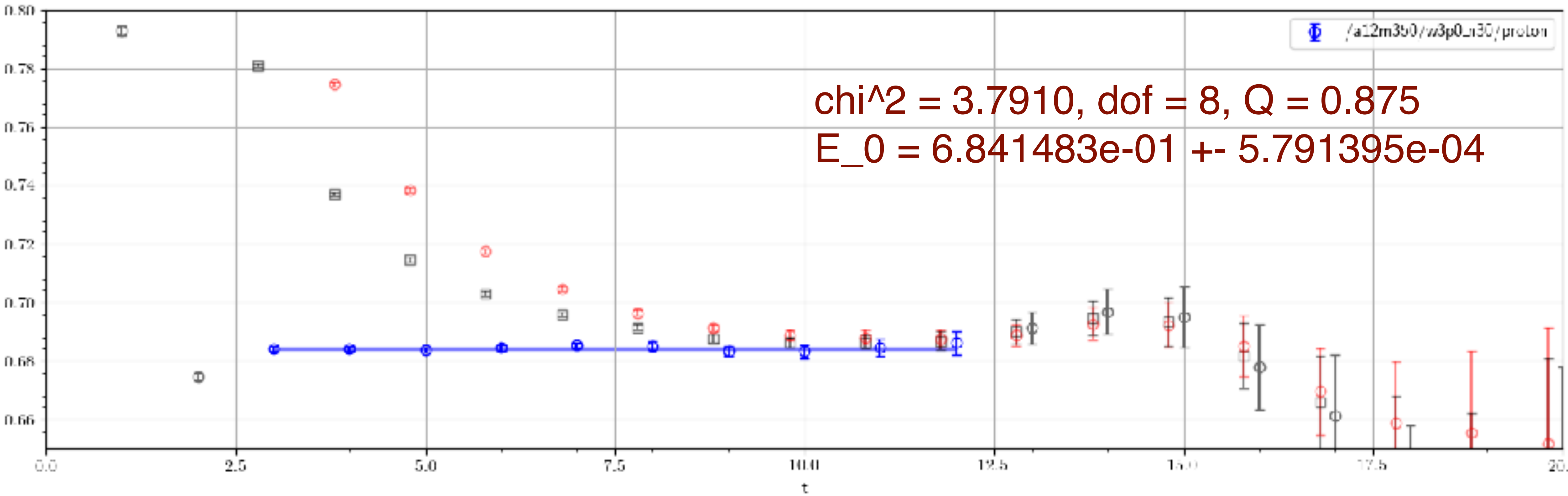

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We are very optimistic this idea will allow for a good LQCD calculations of NN at this interesting pion mass!

Matrix Prony: Improved NN

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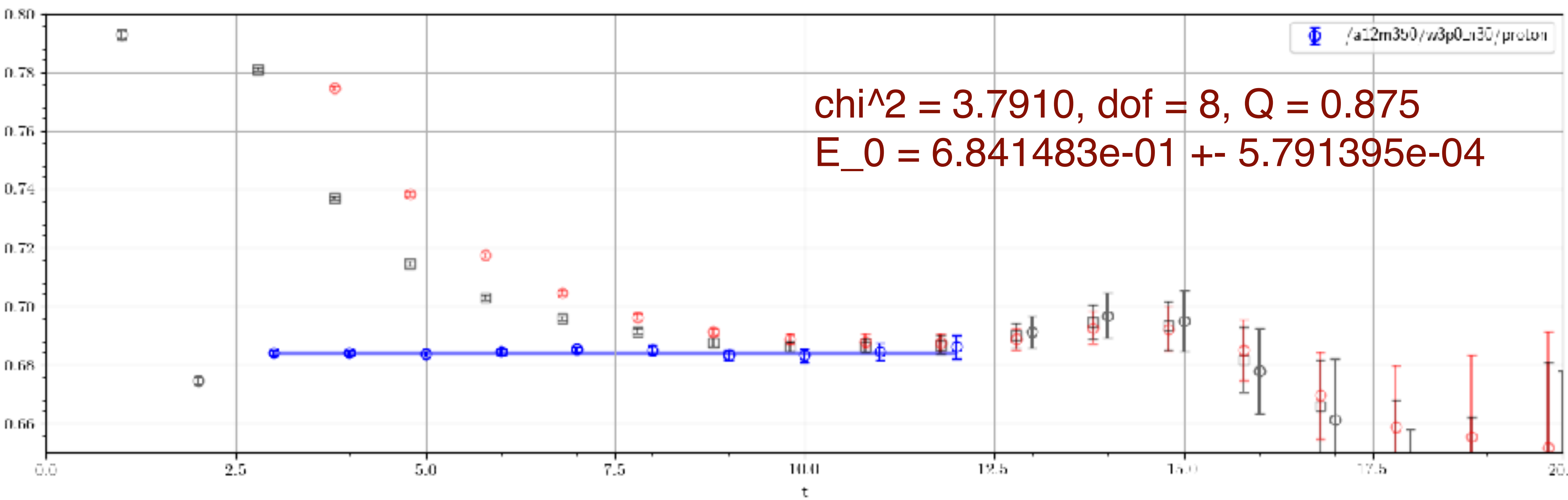

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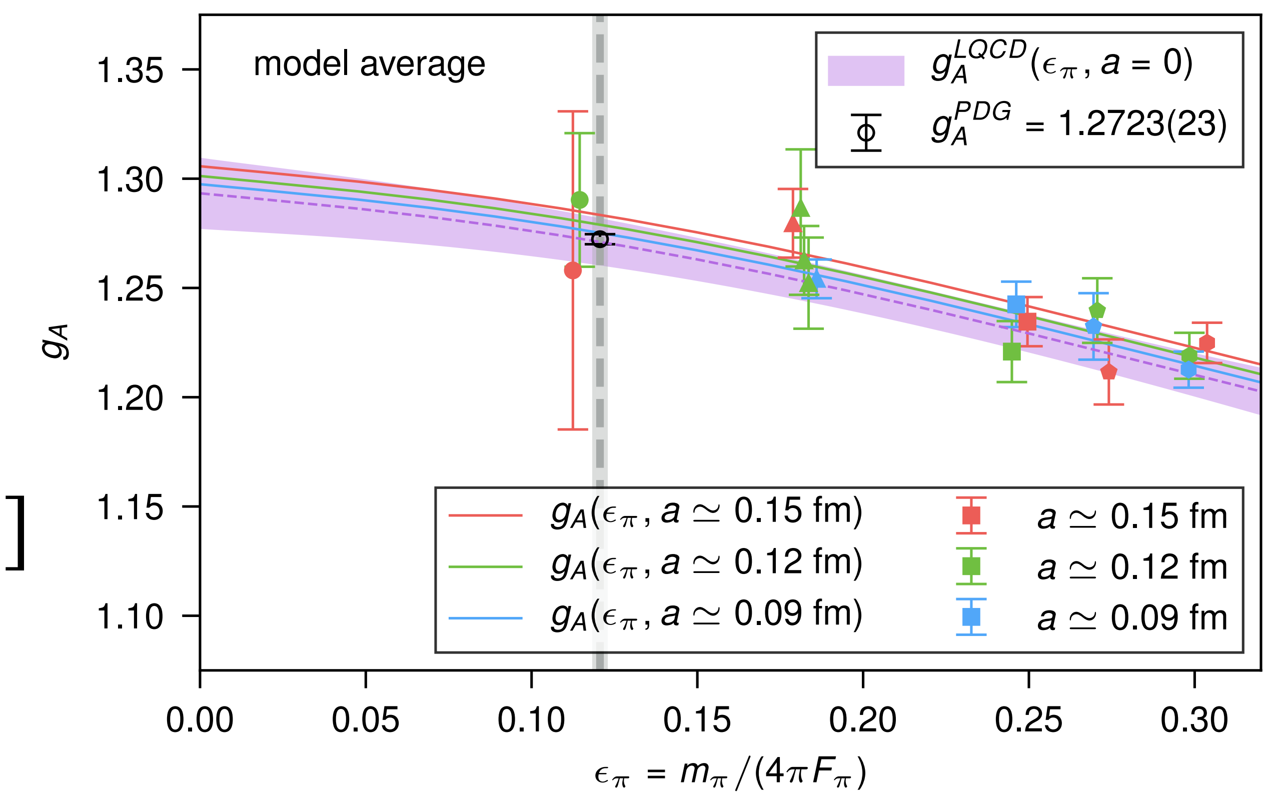
These calculations are so expensive - it is imperative to extract as much information from them as possible, which is optimally achieved early in Euclidean time where we have a clear theoretical understanding of the correlation functions and they are clean - before the noise sets in

Summary

- Applications of Lattice QCD to Nuclear Physics is Difficult
- With current generation of computers - we have finally made our first "nuclear physics" post diction of a benchmark quantity g_A - with a clear path to sub-percent precision

gA = 1.271(13) - Chang et. al., Nature 2018 [arXiv:1805.12130]



- The next generation of computers (appearing now at ORNL and LLNL) - plus new ideas - will enable us to begin computing *real* nuclear physics quantities (A=2 (3? 4?)), including ΔI=2 Hadronic Parity Nonconservation

Thank You