The nucleon axial coupling from Quantum Chromodynamics

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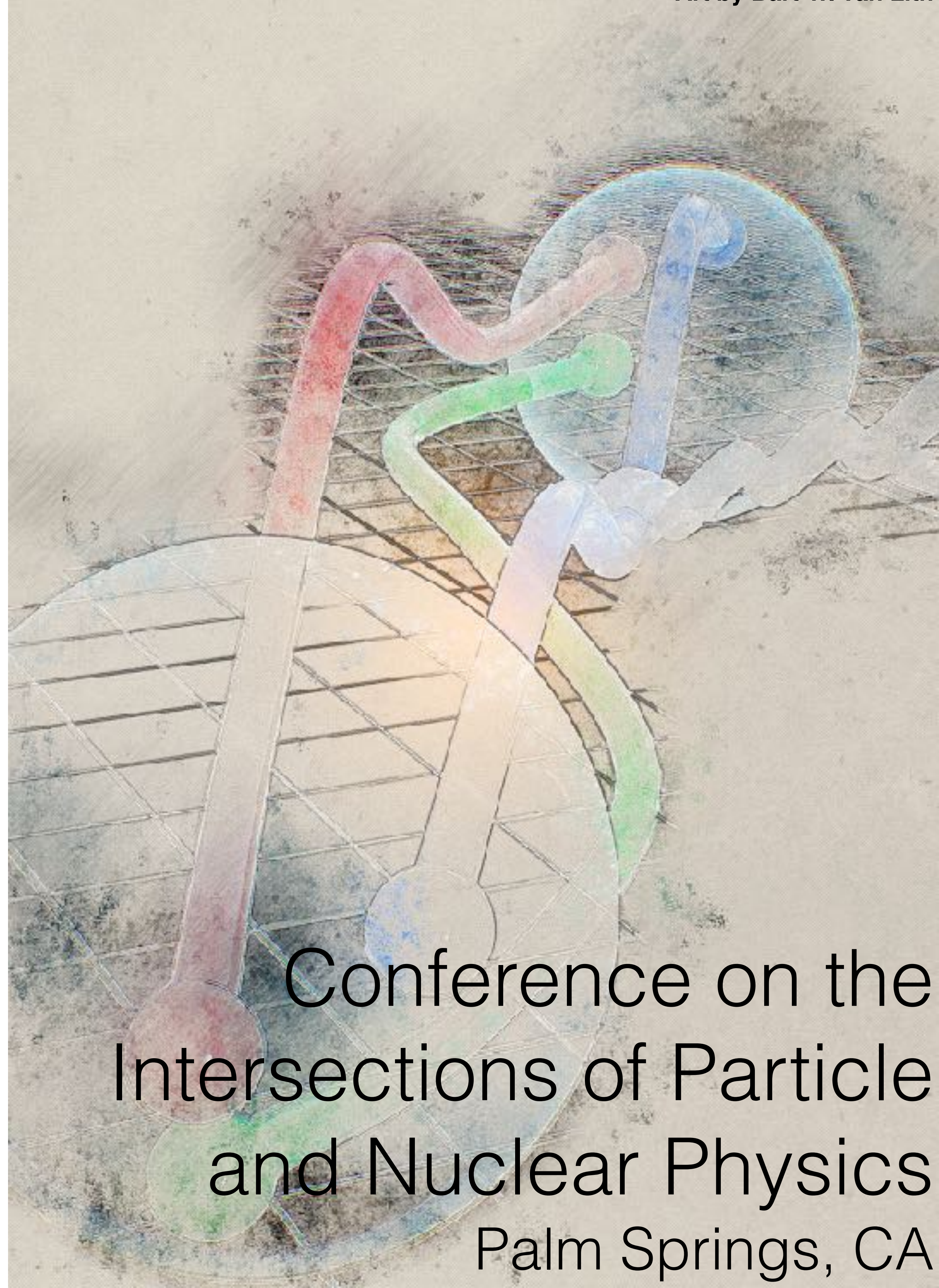
Art by Bart-W. van Lith

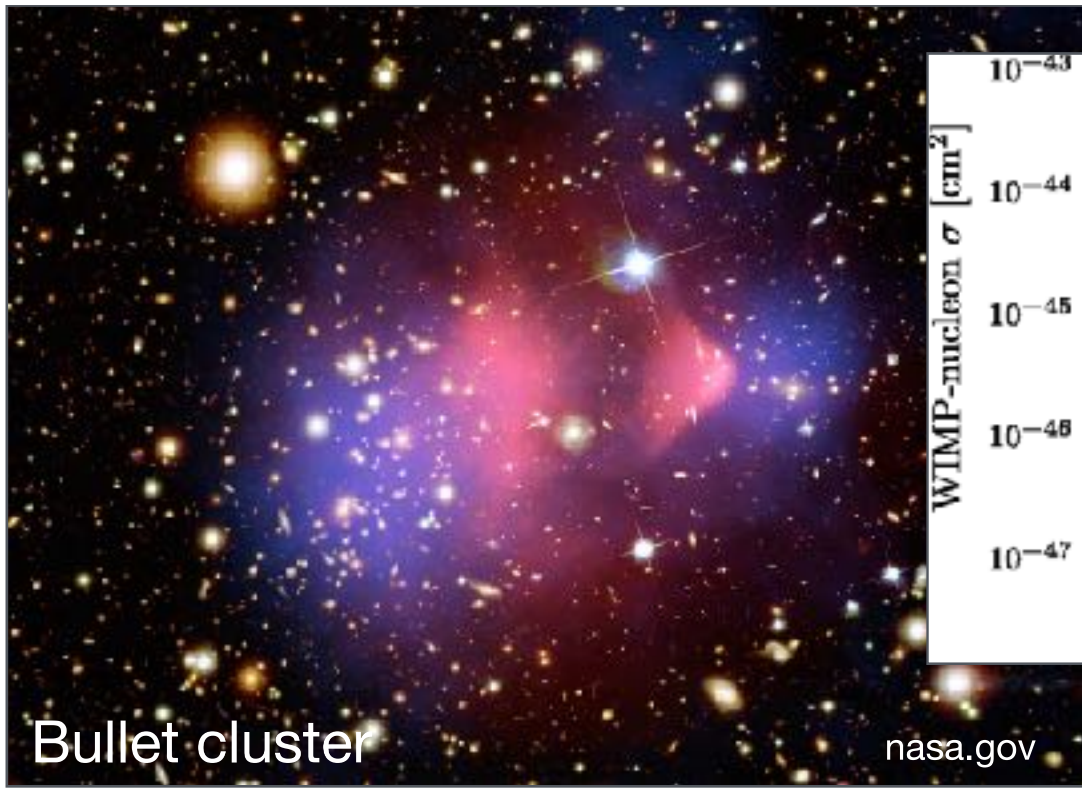


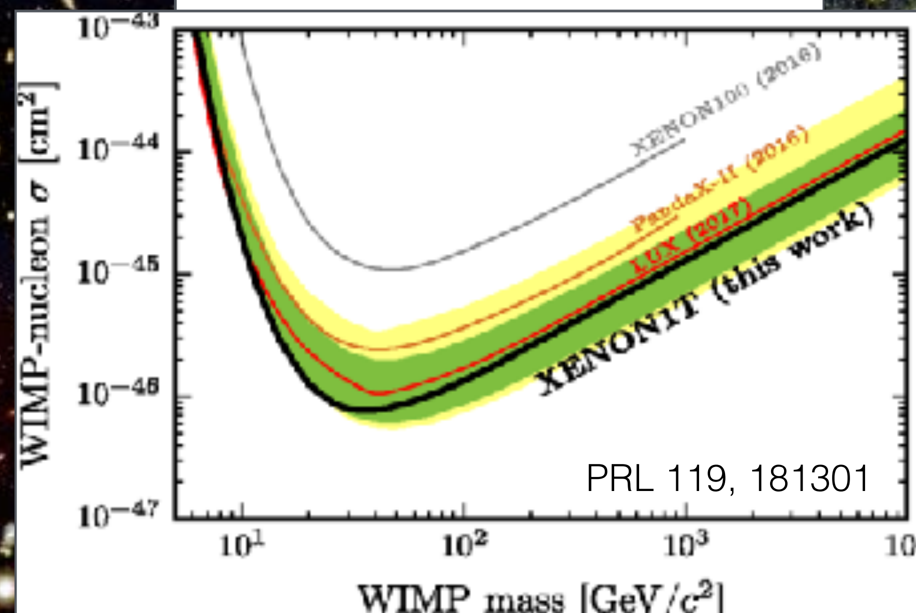
Conference on the Intersections of Particle and Nuclear Physics

Palm Springs, CA

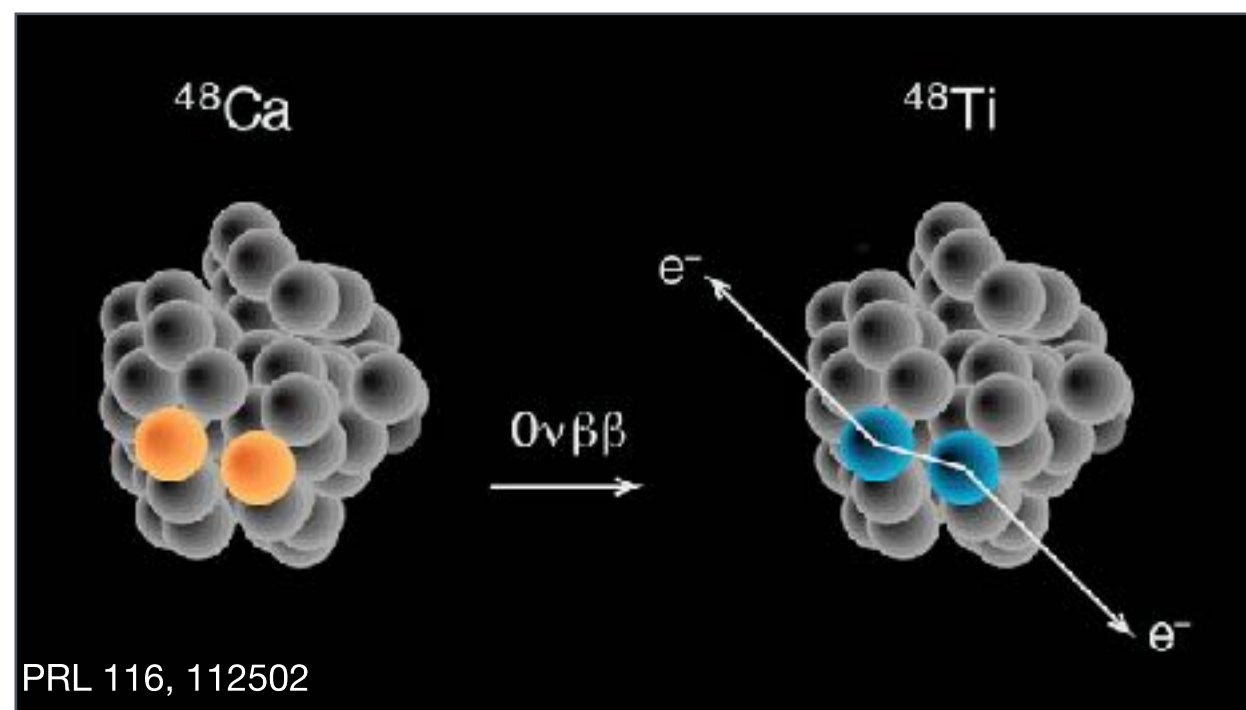
Some outstanding questions

The origin of matter
Mechanism of asymmetric matter vs. antimatter production
Recent status: Tokai 2 Kamioka (T2K)
Current effort: T2HK with Hyper-Kamiokade
DUNE @ Fermilab to Sanford





Searches for dark matter
Only 5% is regular matter
Many ongoing efforts
Recent result Xenon1T
@ Grand Sasso
Stronger bound on
WIMP dark matter



Properties of the neutrino
What is the mass of the neutrino?
What is the origin of the neutrino mass?
Searches for neutrino-less double beta decay
e.g. CANDLES @ Kamioka with Ca-48

Additionally, there are specific “puzzles”

The neutron lifetime puzzle

Experimental measurements

Bottle-type experiment

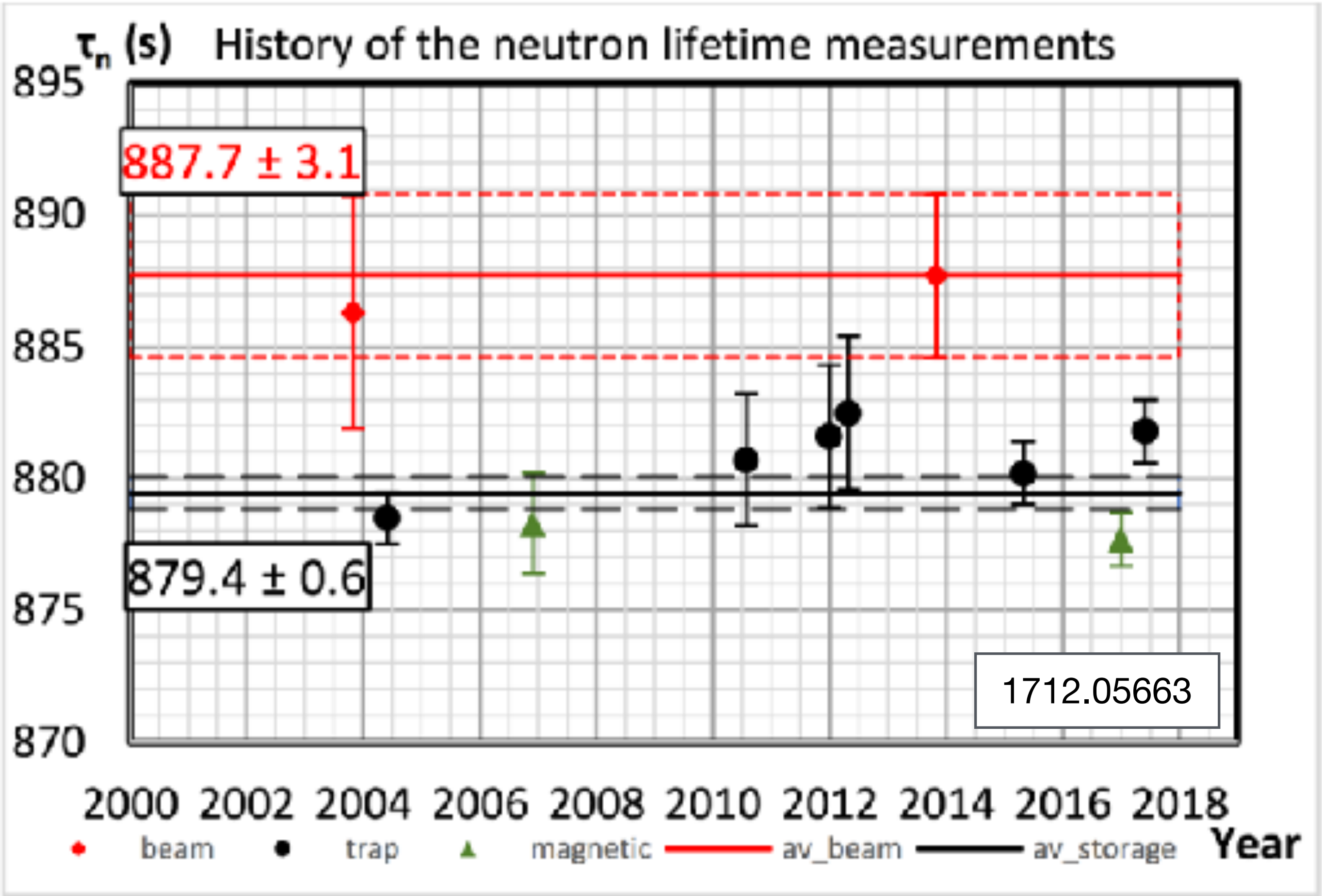
Traps neutrons in bottle and measures how many are left.

Beam-type experiment

Count protons emerging from a beam of neutrons.

If neutrons decay to dark matter, bottle lifetime will be **shorter** than beam lifetime!

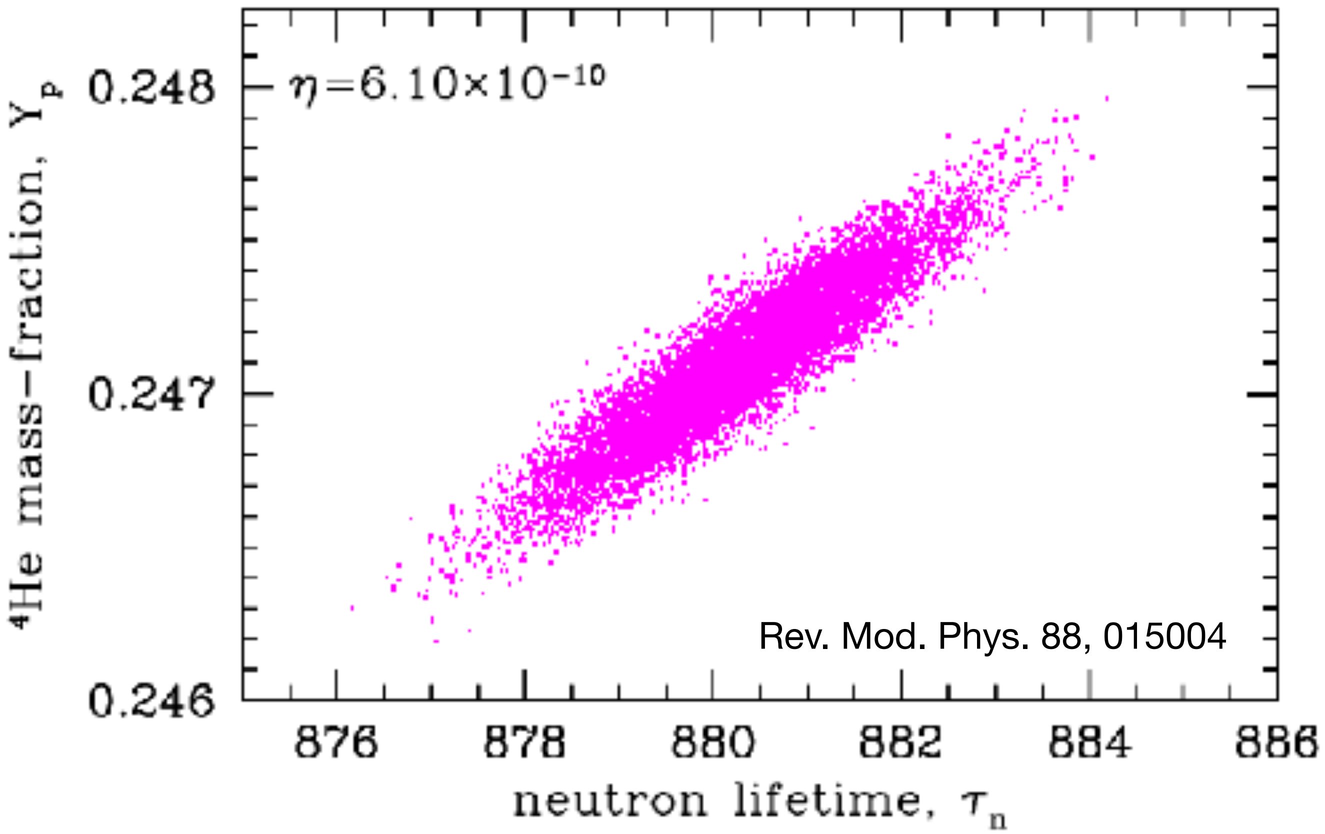
Precision LQCD can be use to discriminate between beam and bottle measurements.



Big Bang Nucleosynthesis

Prediction of the Helium-4 mass fraction is sensitive to the neutron lifetime.

Current observed values of Y_p have >1% uncertainty, but improvement in both quantities may shed light on BSM physics.



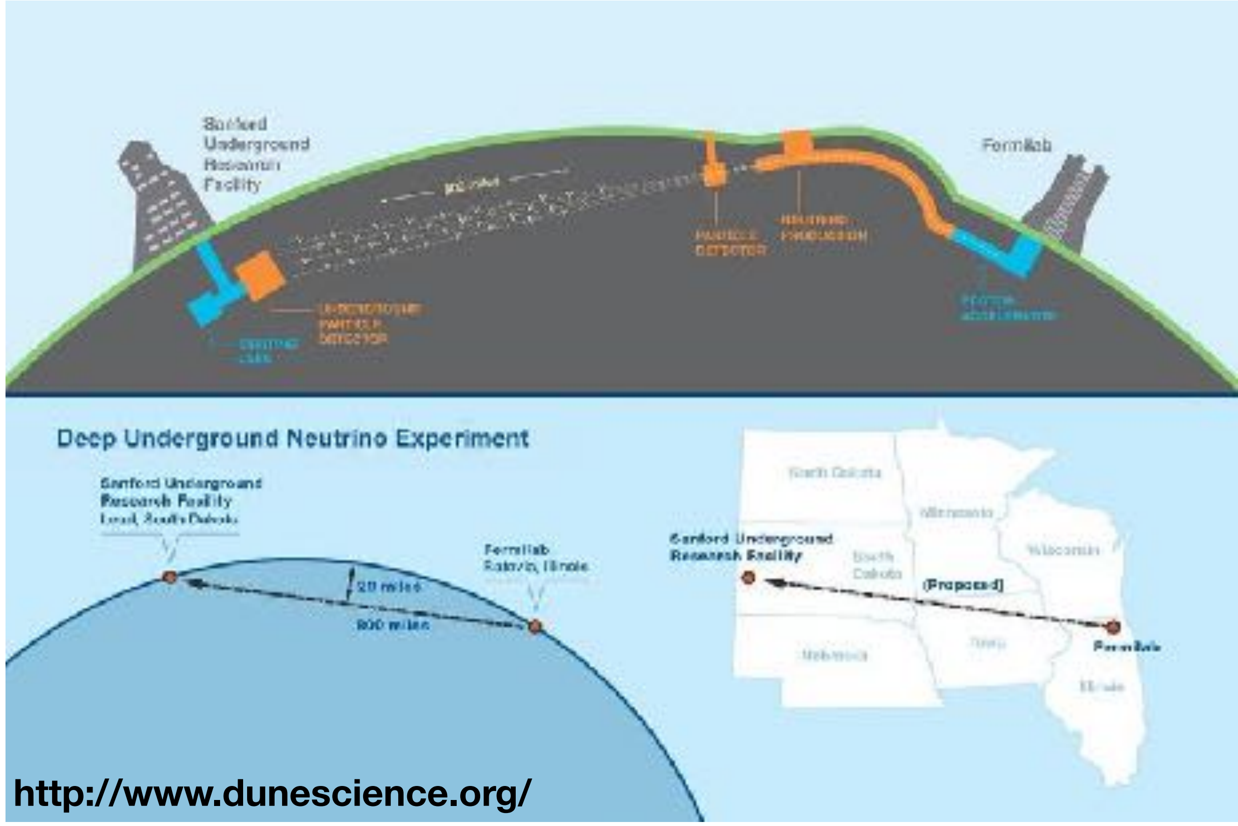
Nucleon axial form factor

What is it?

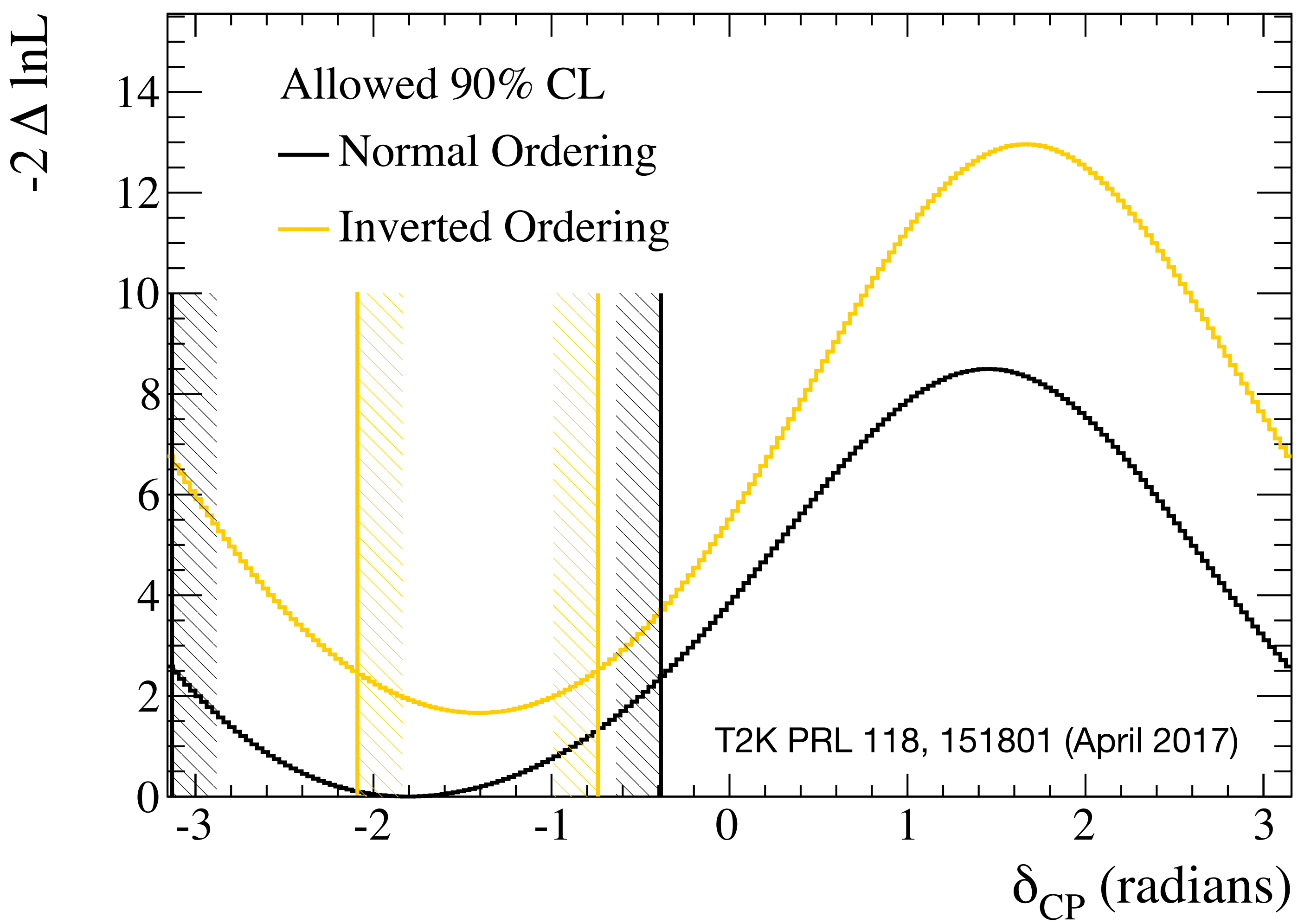
Axial coupling as function of momentum transfer
Fourier Transform of axial-charge density
Dictates quasi-elastic scattering of T2HK, DUNE

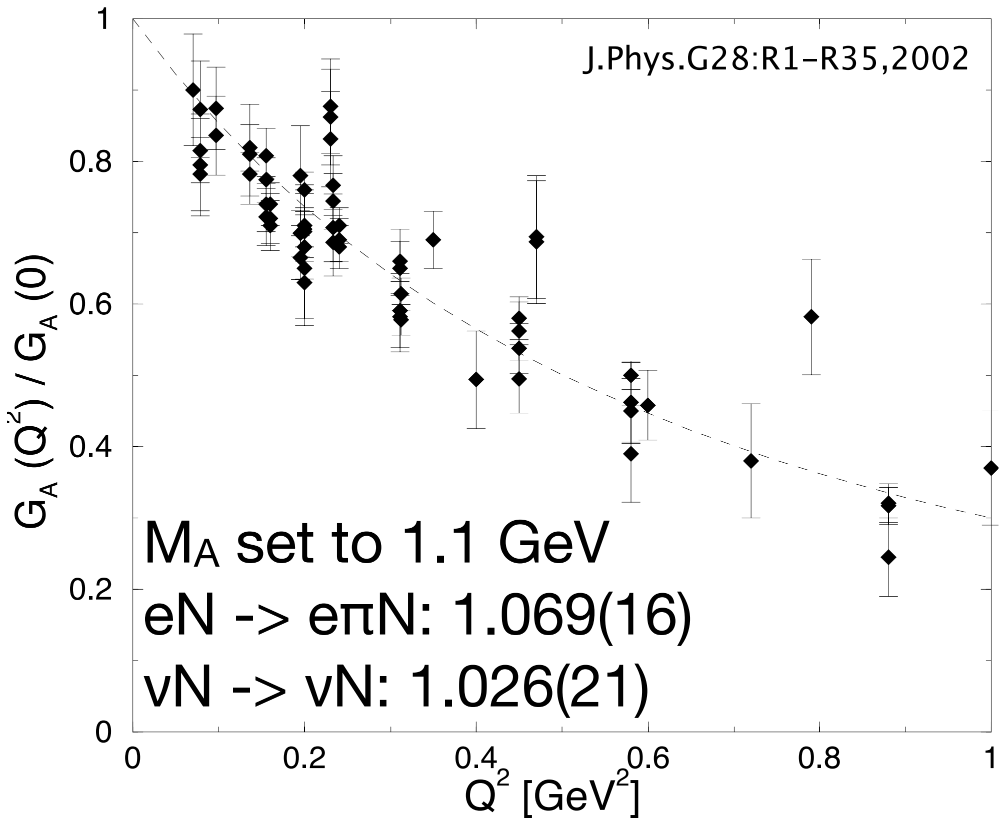
T2K: CP conservation excluded at 90% CI

Hint of mechanism for leptogenesis?
Need more precise determination at T2HK, DUNE



Precision axial form factor from LQCD





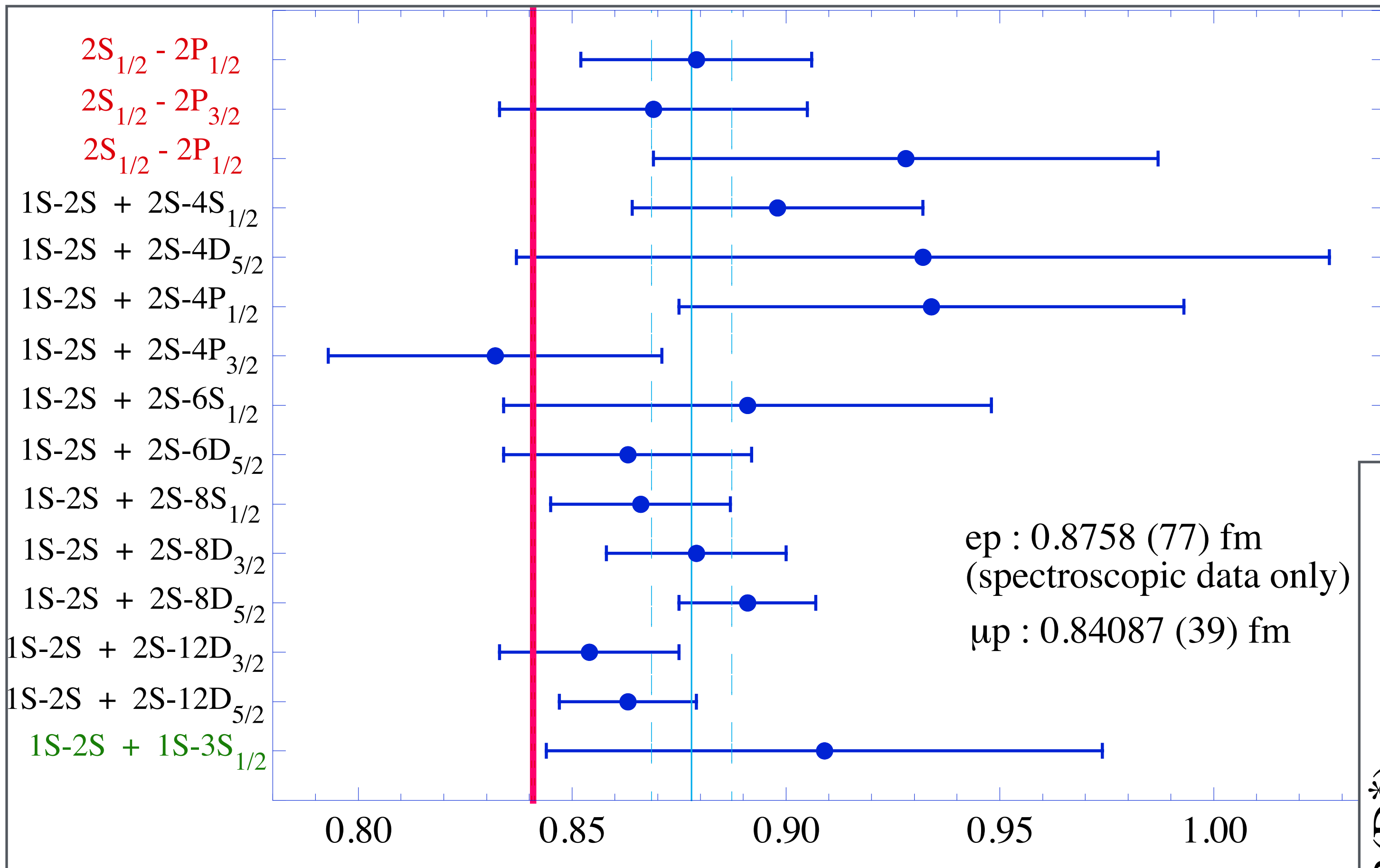
Dipole is over-constraining
Experimental result noisy at Q^2 != 0
Use LQCD to calculate $G_A(Q^2)$

Proton charge radius puzzle

In 2010 the size of the proton was measured in muonic Hydrogen
Radius shrank by 4% with 5σ tension with atomic Hydrogen
Result published in Nature 466, 213-216 (08 July 2010)
New York Times ran an article four days later gaining popularity

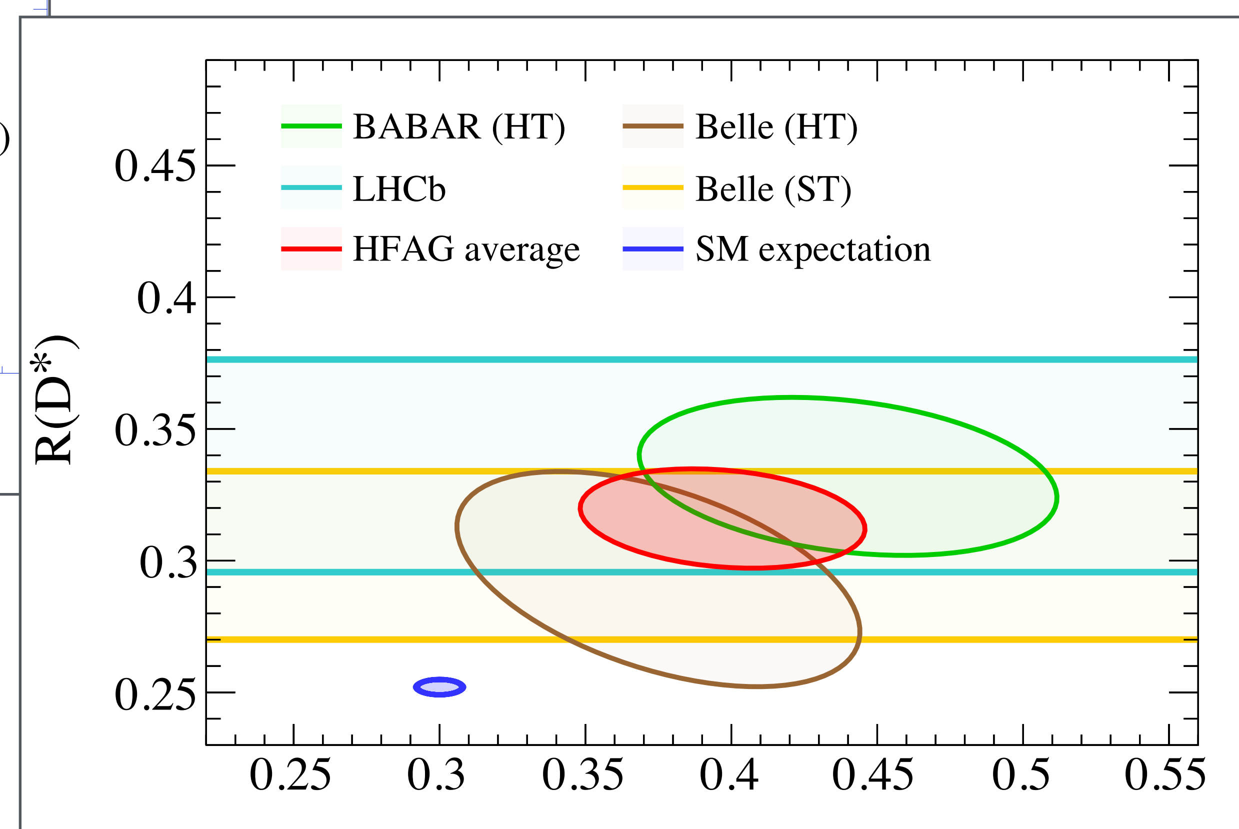
New York Times Jul 12, 2010

Result challenges *lepton universality*



PPNP 82, 2015, pp. 59-77

Lepton universality is also challenged in recent *B*-meson semileptonic decays experimental data @ ~4σ



Nature 546, 227-233 (2017)

Proton radius and multiple independent *B* decay discrepancy -> new physics?

Lattice QCD can directly calculate the radius.

Connecting QCD to nuclear physics

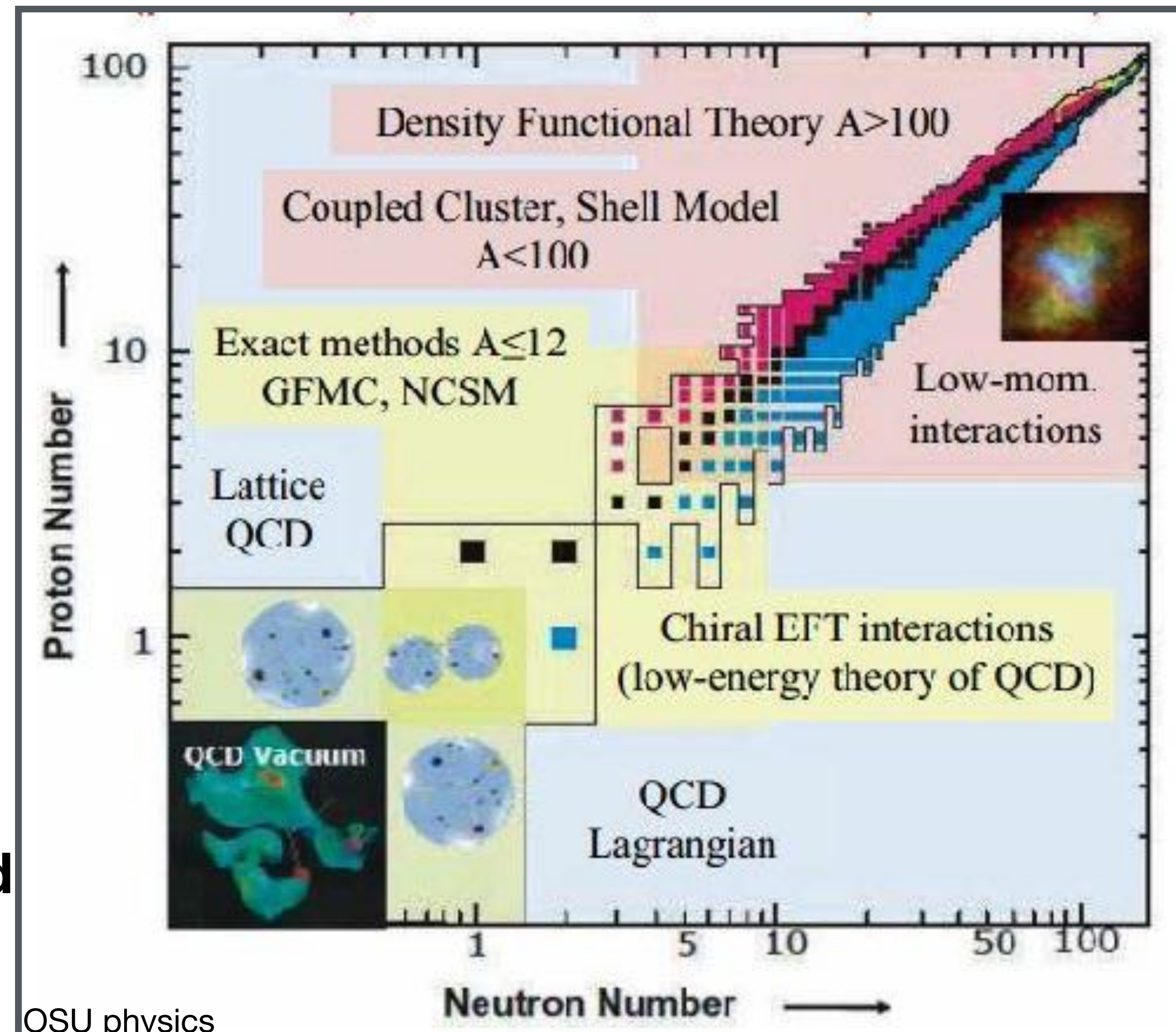
Experiments require fundamental understanding of nuclear physics
DUNE uses Argon target
Dark matter detectors use Xenon
0vbb use nuclear spectroscopy

Goal: Understand how nuclei interact from first principle theory

Quantum chromodynamics (QCD)
Modern fundamental description of the strong interaction
Much of nuclear physics is in principle described by approximately 1 parameter

Problem: elegant theory but hard to evaluate
Nuclear physics emerges from **non-perturbative** dynamics of QCD
Solution:
Discretize theory: **Lattice QCD** non-perturbatively regulates the theory
Discretization allows for **numerical evaluation**

LQCD can determine nuclear properties difficult or impossible to measure from experiment
Lattice QCD with many-body effective field theory is the only way to understand nuclear physics from first-principles



OSU physics

The nucleon axial coupling

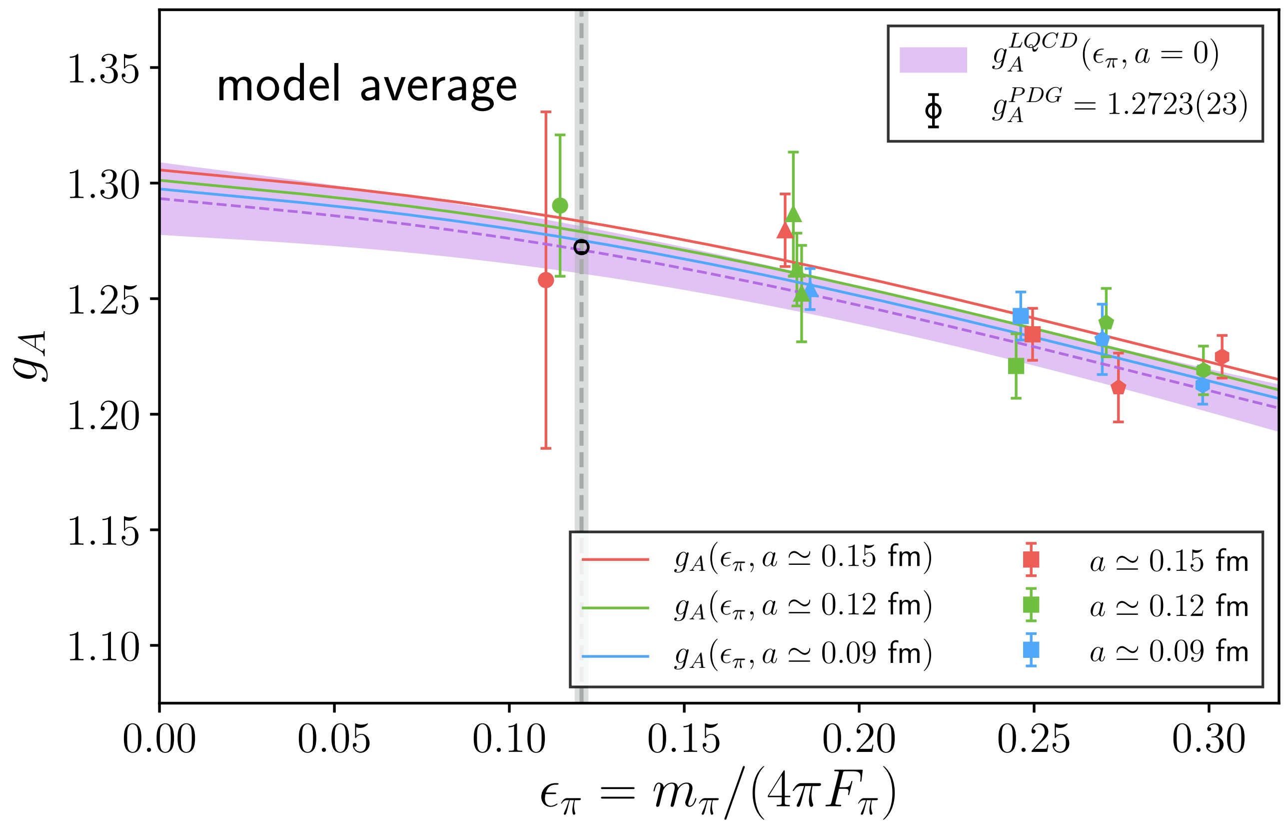
Fundamental parameter to much of nuclear physics
Benchmark calculation for Lattice QCD

Today I will present the first percent-level determination of g_A from QCD

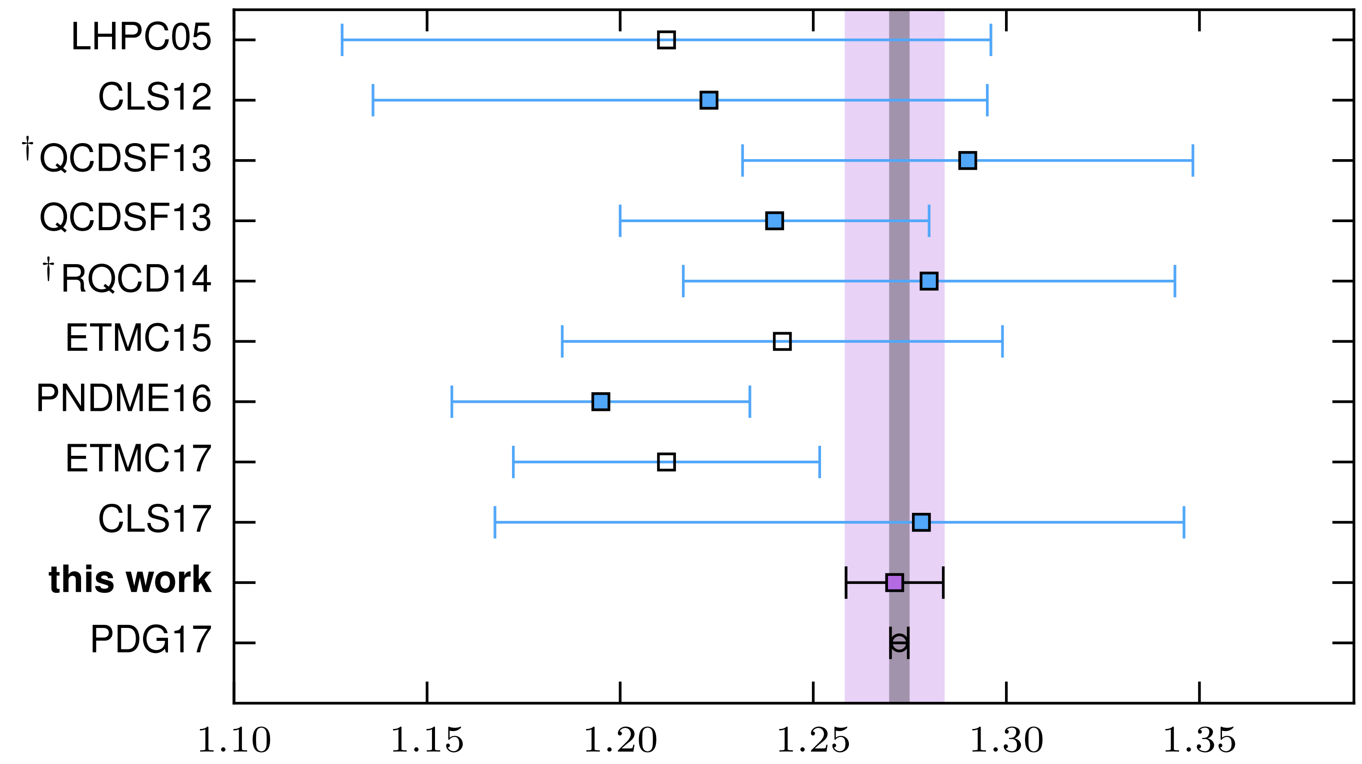
$$g_A^{\mathrm{QCD}} = 1.2711(126)$$
$$g_A^{\mathrm{UNCA}} = 1.2772(020)$$

Phys. Rev. C 97, 035505

(experiment is still 6 times more precise, but we are catching up!)



chiral, continuum, and infinite volume



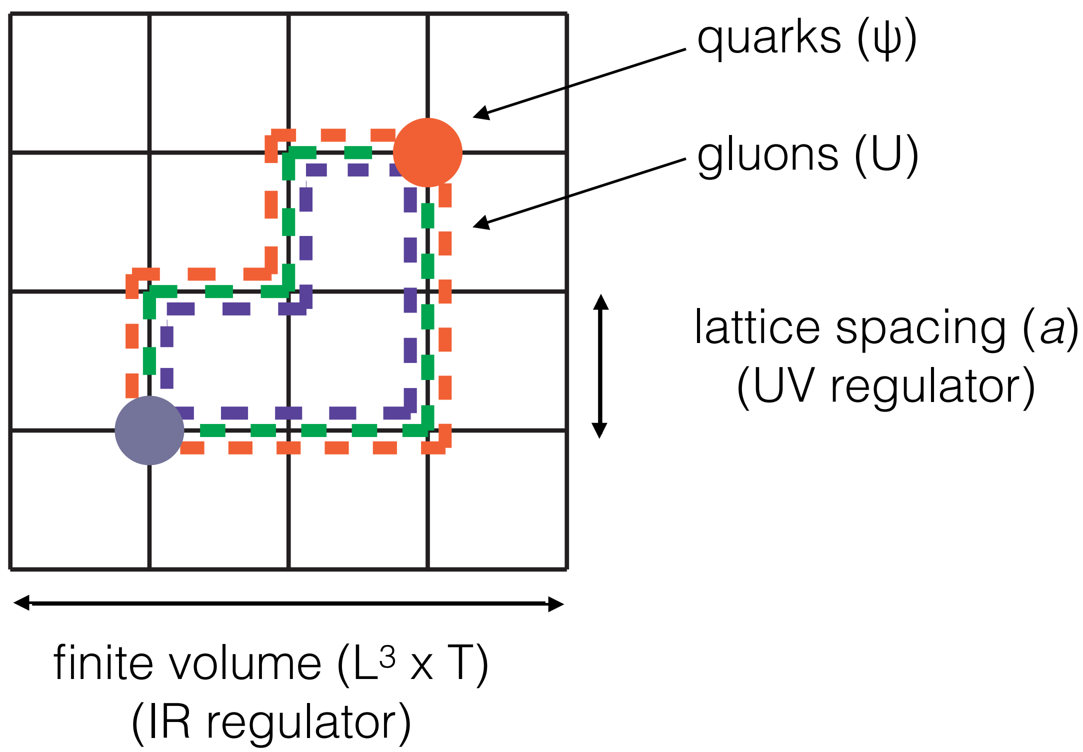
g_A from LQCD in chronological order


TITAN
OAK RIDGE
National Laboratory
LEADERSHIP
COMPUTING
FACILITY

IBM
Lawrence Livermore
National Laboratory

Introduction to Lattice QCD

Lattice QCD is QCD with non-perturbative (lattice) regularization
Allows for first-principles approach to calculating hadronic observables



Evaluate Feynman path integral on the lattice
Wick rotate so domain of integration is finite

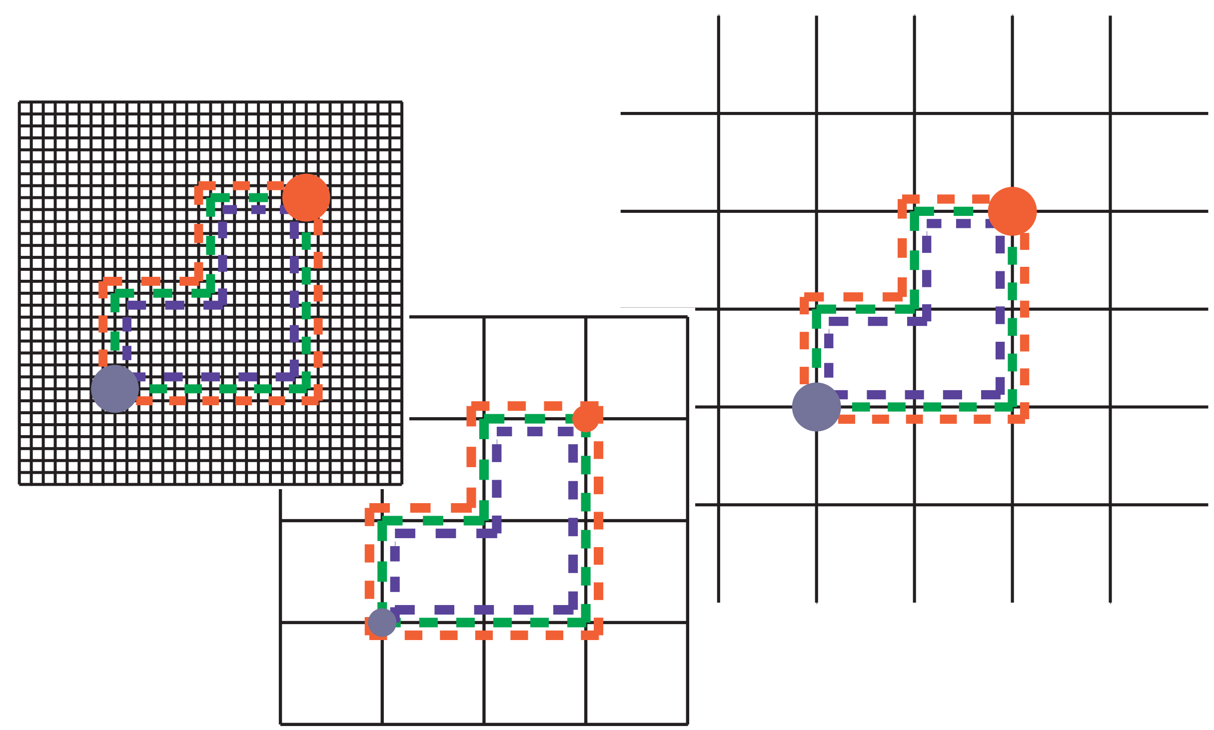
$$\begin{aligned}\langle \mathcal{A} \rangle &= \frac{1}{\mathcal{Z}} \int [d\psi][d\bar{\psi}][dU] \mathcal{A} e^{-S[\bar{\psi},\psi,U]} \\ &= \frac{1}{\mathcal{Z}} \int [dU] \det\left(\not{D} + m\right) e^{-S[U]} \mathcal{A}\end{aligned}$$

Importance sample gauge field ~ $e^{-S[U] - \ln \det \not{D}}$

Observables from simple average $\langle \mathcal{A} \rangle \approx \frac{1}{N} \sum_{i=1}^{N} \mathcal{A}[U_i]$

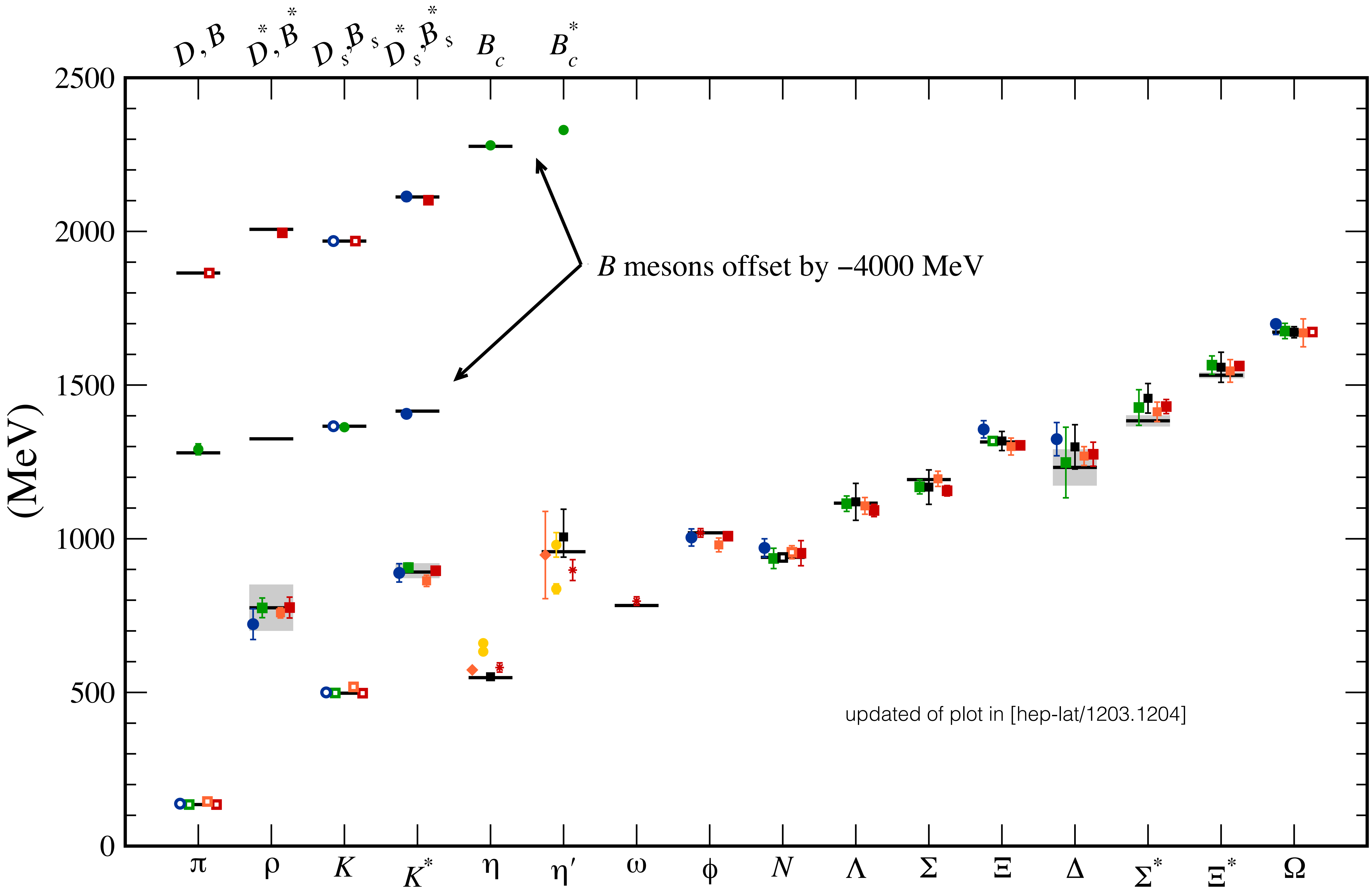
Major lattice uncertainties and related issues

- continuum limit $t_{\text{comp.}} \propto 1/a^6$
- infinite volume $t_{\text{comp.}} \propto V^{5/4}$
- light pion mass exponentially bad
  - condition number
  - signal-to-noise



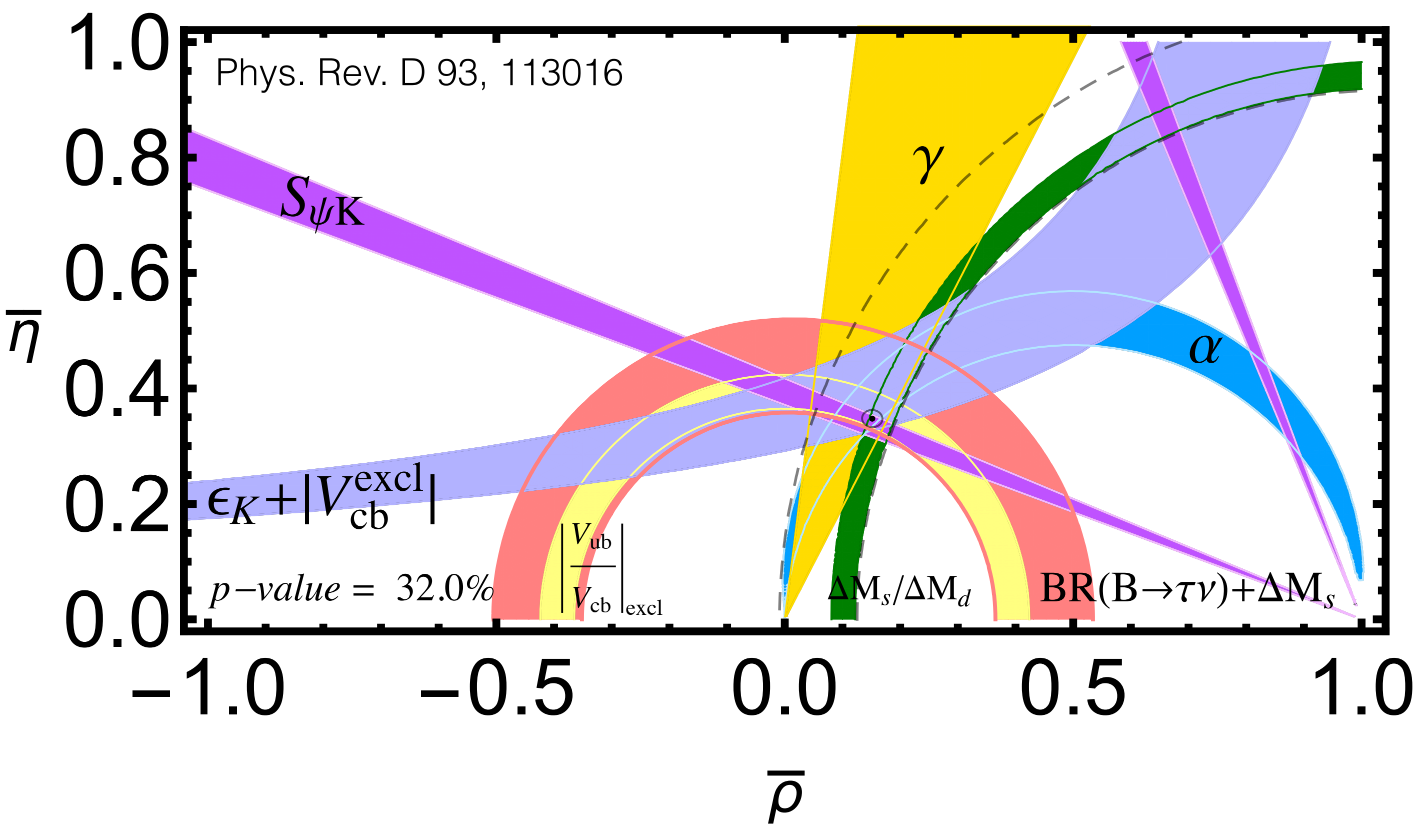
Hadron spectroscopy on the lattice

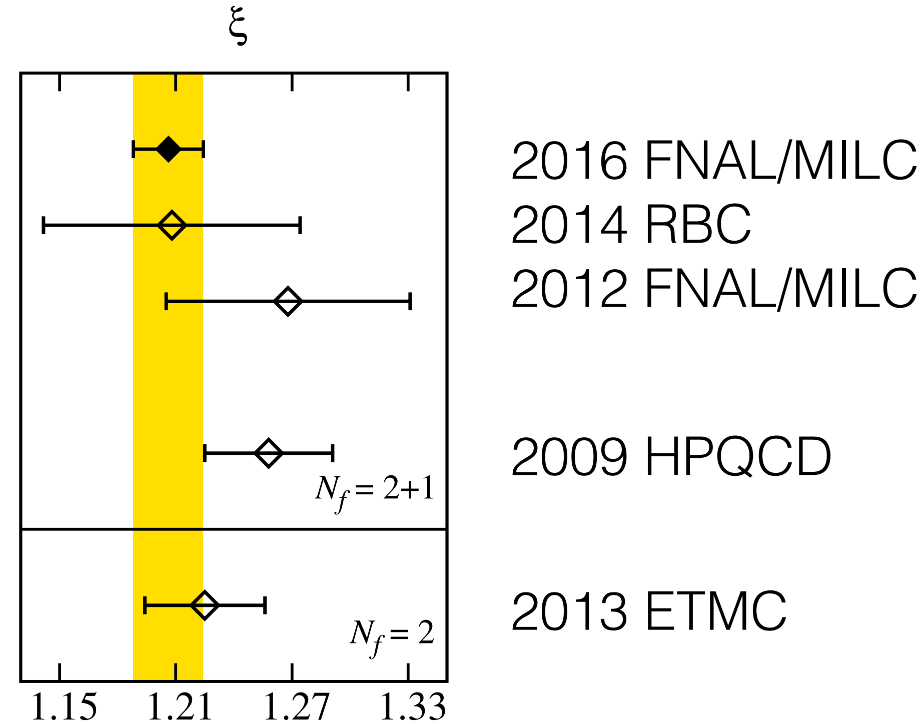
Very successful history in hadron spectroscopy



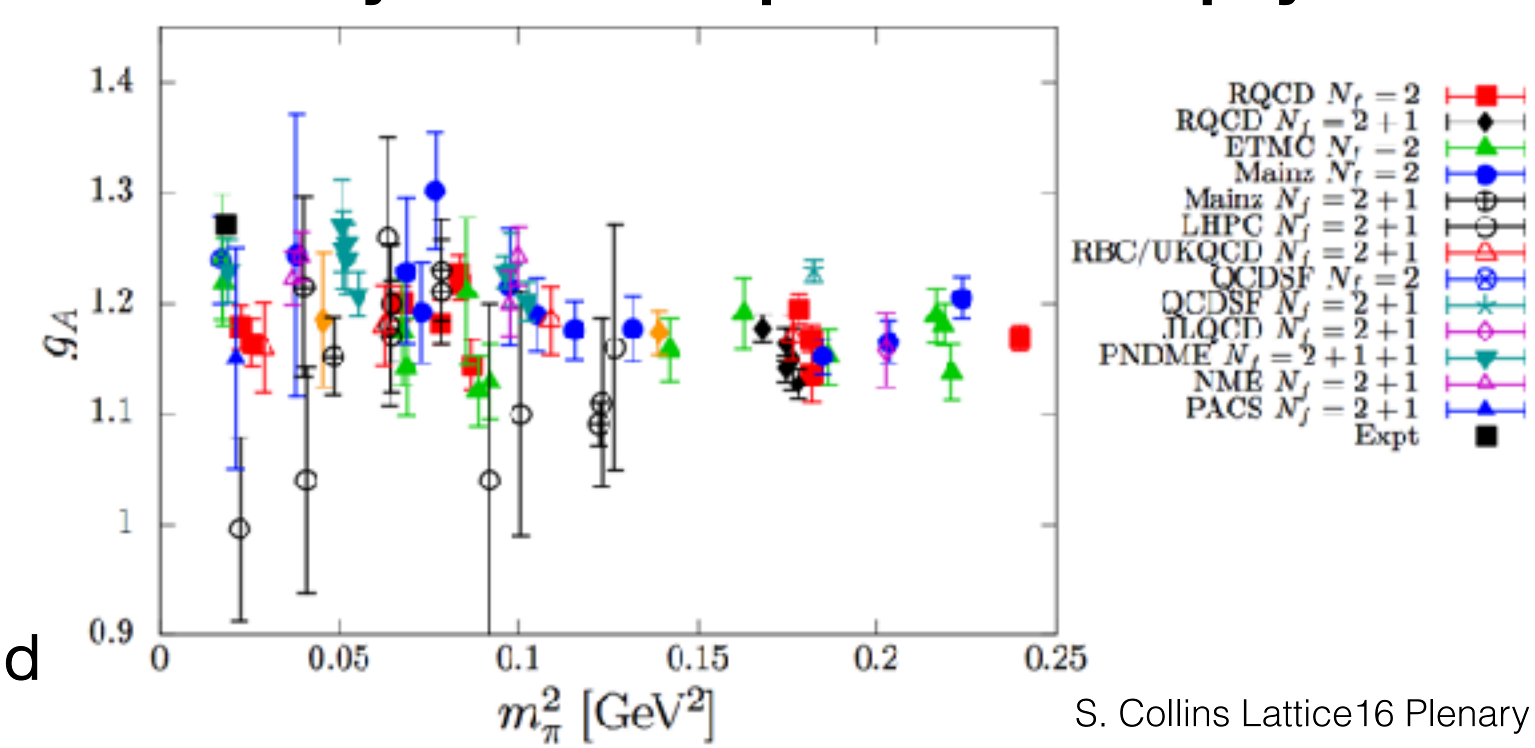
Flavor physics from Lattice QCD

Over-constrain CKM unitarity with great success





and many other examples in meson physics…



S. Collins Lattice16 Plenary

Why is g_A different?

- Large statistical uncertainty
- Large systematic uncertainty
- g_A vs. pion mass show no clear trend

Nucleon signal-to-noise problem

Exponentially larger signal-to-noise compared to mesonic systems

For an nucleon annihilation operator *N*, the time evolution of correlator (**signal**) is

$$\langle N\bar{N}\rangle = \sum_i \langle N|i\rangle\langle i|\bar{N}\rangle e^{-E_i t} \propto e^{-M_N t}$$

(Euclidean spacetime, long time limit)

The variance of the correlator (**noise**) is

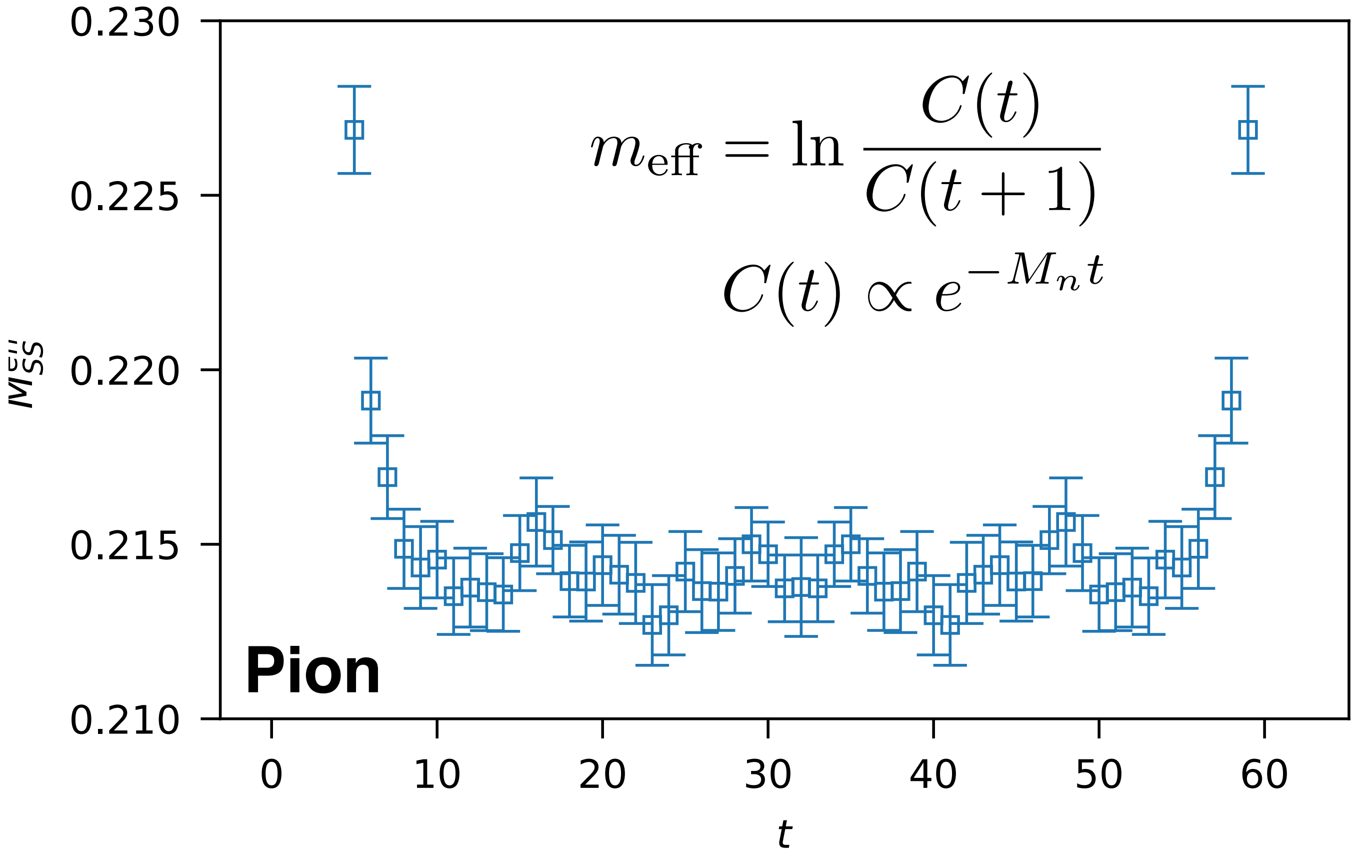
$$\begin{aligned}\mathrm{Var}\langle N\bar{N}\rangle =& \langle|N\bar{N}|^2\rangle - |\langle N\bar{N}\rangle|^2 \\ =& \langle|\pi\pi|^3\rangle - |\langle N\bar{N}\rangle|^2 \propto e^{-3M_\pi t}\end{aligned}$$

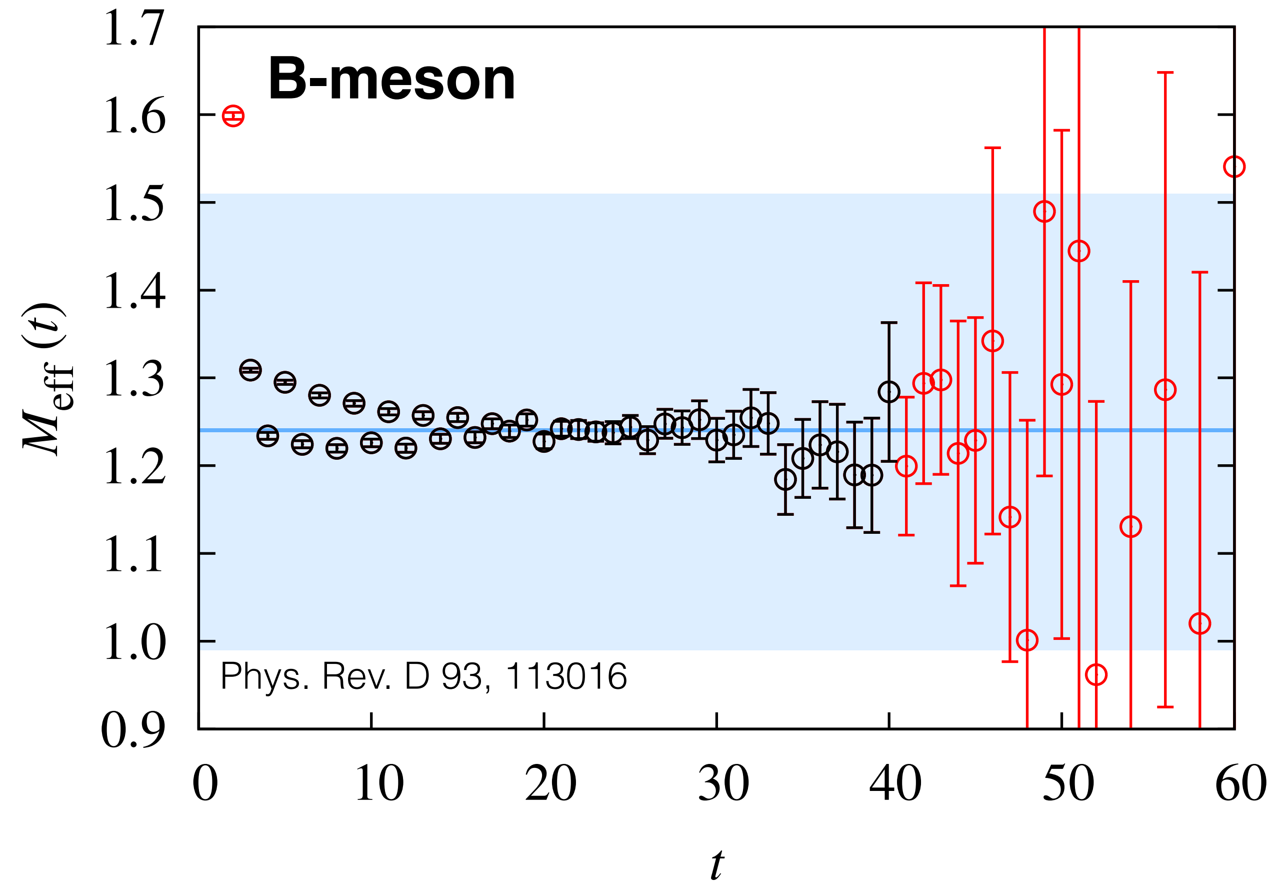
(long time limit only the lightest mode survives)

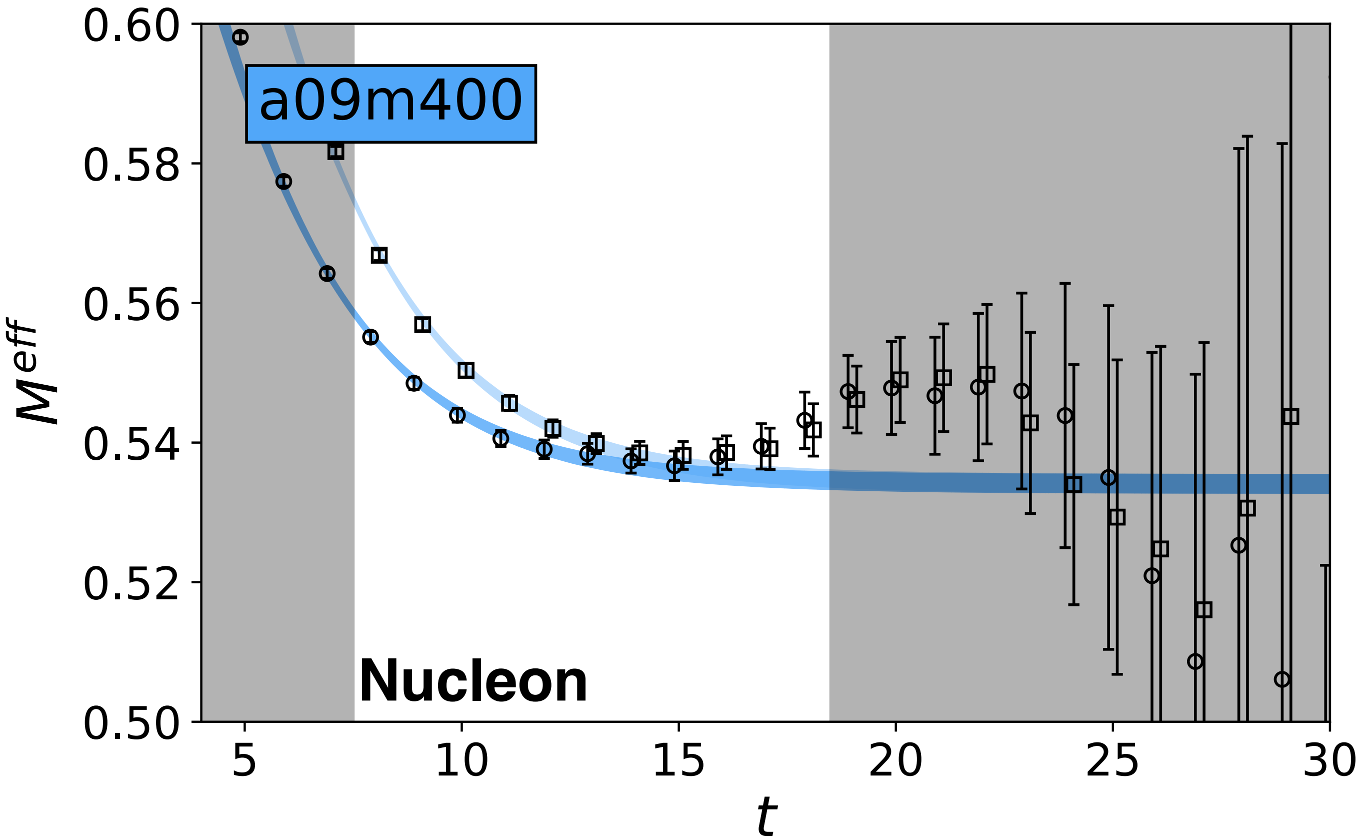
Signal-to-noise between mesonic and baryonic systems

light (pion/kaon)	$\mathrm{s/n} \propto e^{-[\mathrm{MeV}]t}/e^{-[\mathrm{MeV}]t}$
heavy-light (B/D)	$\mathrm{s/n} \propto e^{-[\mathrm{GeV}]t}/e^{-[\mathrm{GeV}]t}$
nucleon	$\mathrm{s/n} \propto e^{-[\mathrm{GeV}]t}/e^{-[\mathrm{MeV}]t}$

Signal-to-noise in data







Pion
Relative uncertainty **constant** with time.

B-meson
Rel. uncertainty grows but controlled.

Nucleon
Rel. uncertainty *may* have correlated fluctuations before overwhelmed by noise.

(Light) baryons are most susceptible to systematic errors

Overcoming noise

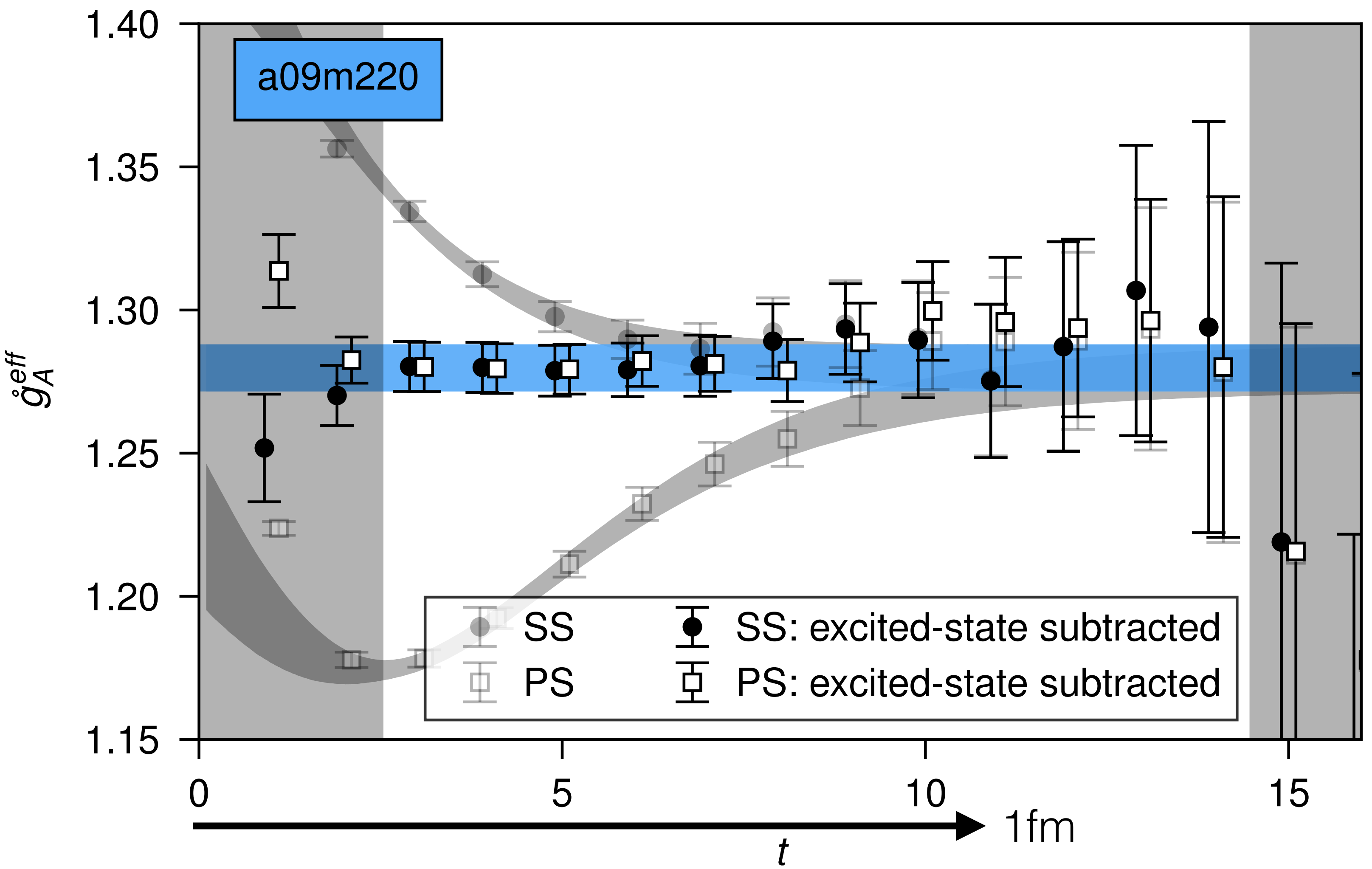
Get more statistics

Precision determination of g_A is believed to be an exascale problem.

2016 DOE OSTI Nuclear Physics Exascale Requirements Review, Fig. 3-40

Use a different computational strategy

Signal-to-noise is exponentially better at small time separations.
However, signal is polluted with systematics that needs to be fully controlled.



Standard computational strategy typically yields data > 1fm to suppress systematics

Feynman-Hellmann on the lattice

The Feynman-Hellmann theorem

$$\frac{\partial E_\lambda}{\partial \lambda} = \left\langle \psi_\lambda \left| \frac{\partial \hat{H}_\lambda}{\partial \lambda} \right| \psi_\lambda \right\rangle$$

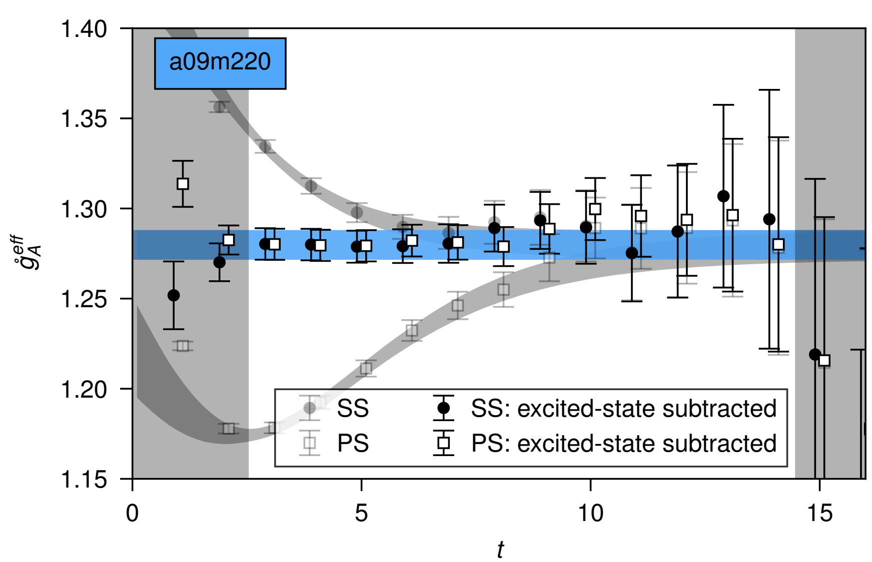
can be evaluated on the lattice

$$\frac{\partial m_{\rm eff}}{\partial \lambda} = \langle n | \mathcal{J} | n \rangle$$

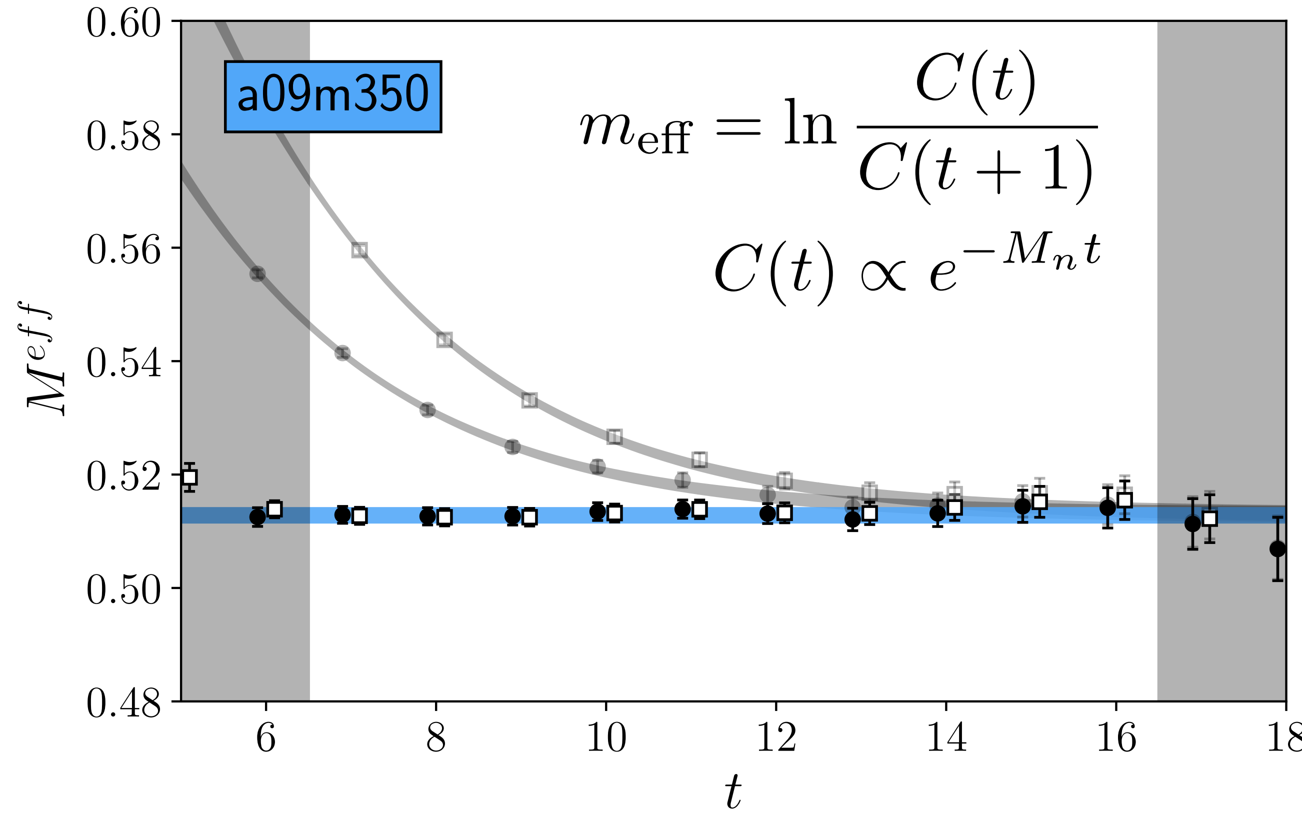
from the definition of $m_{\rm eff}$

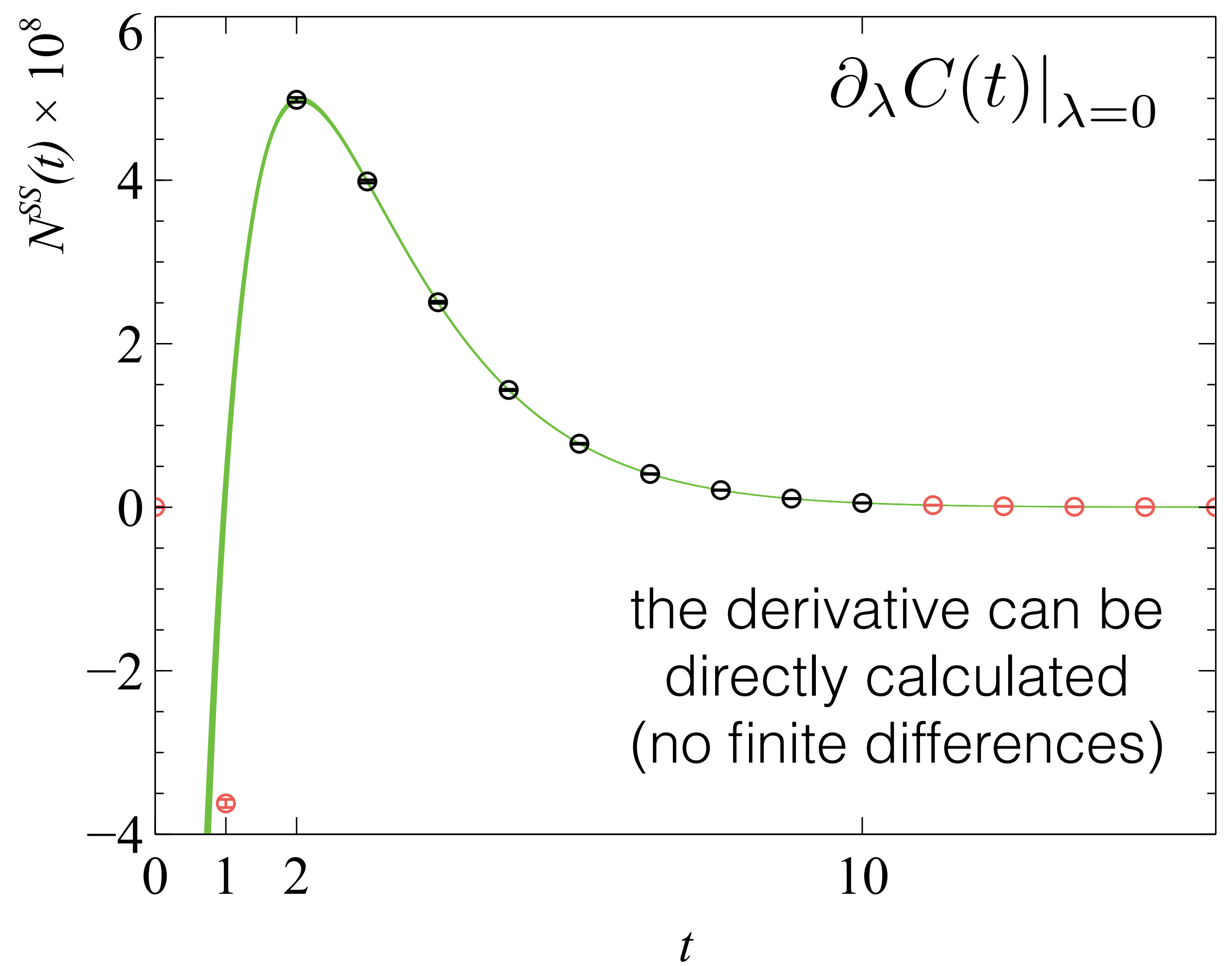
$$\left.\frac{\partial m_{\rm eff}}{\partial \lambda}\right|_{\lambda=0} = \left.\left[\frac{\partial_\lambda C(t)}{C(t)} - \frac{\partial_\lambda C(t+1)}{C(t+1)}\right]\right|_{\lambda=0}$$

Putting everything together



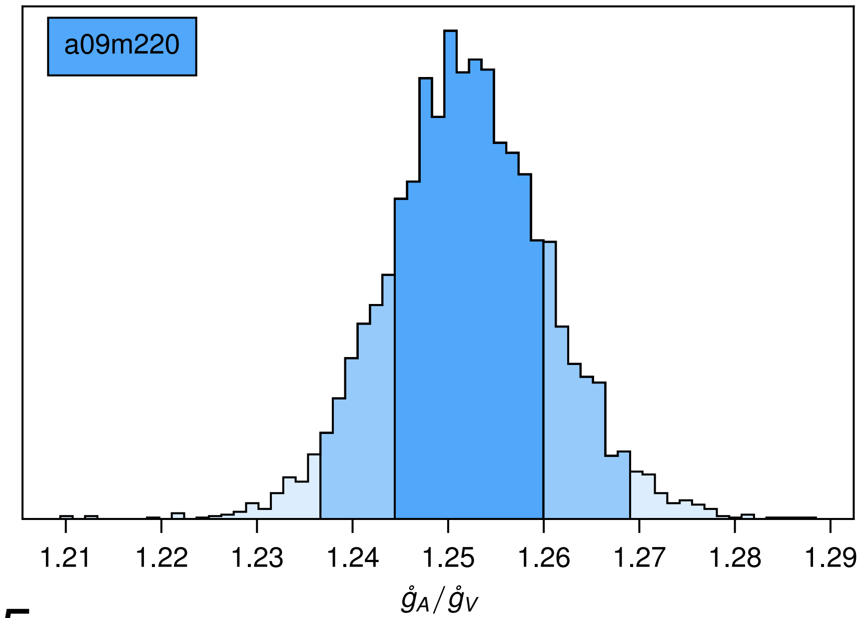
Good: Access small t where s/n is exp. improved.
Challenge: Control very large systematic effects.

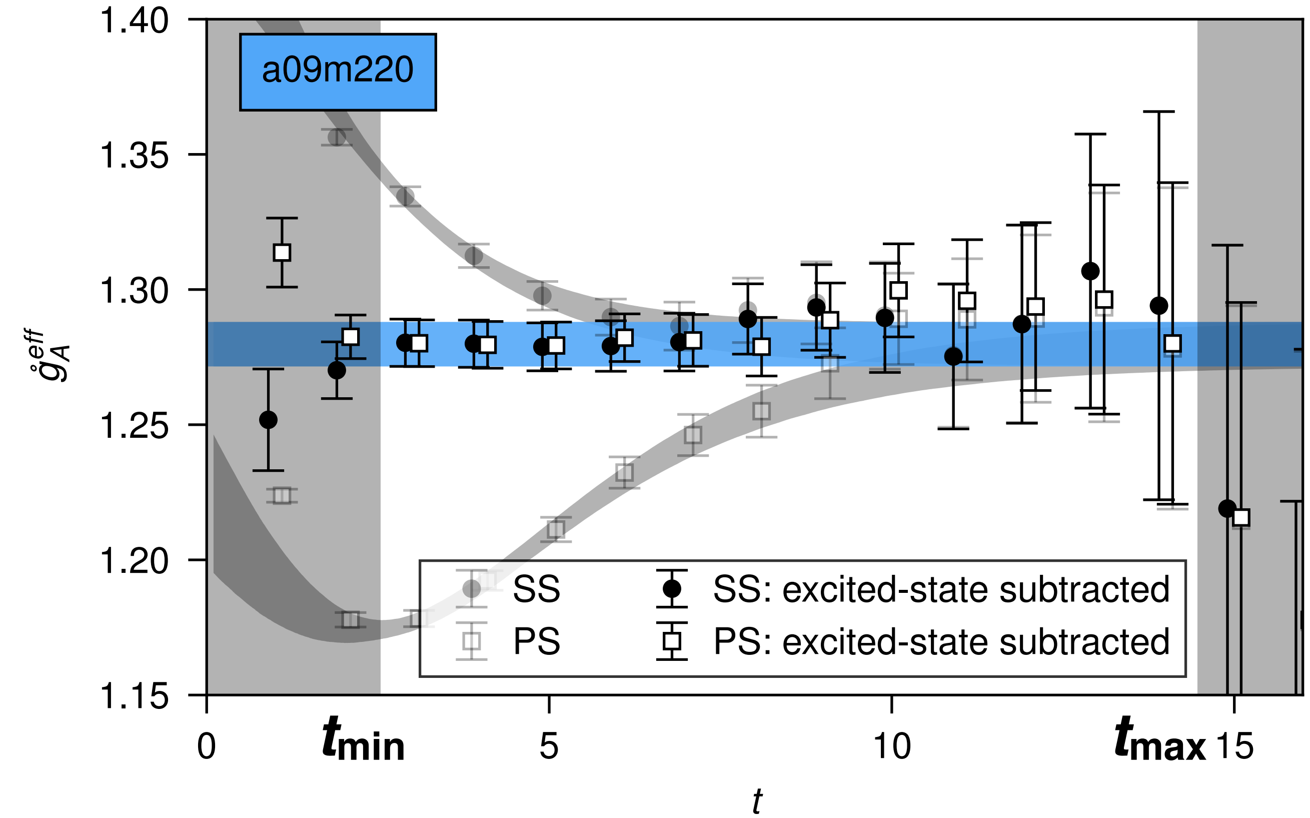




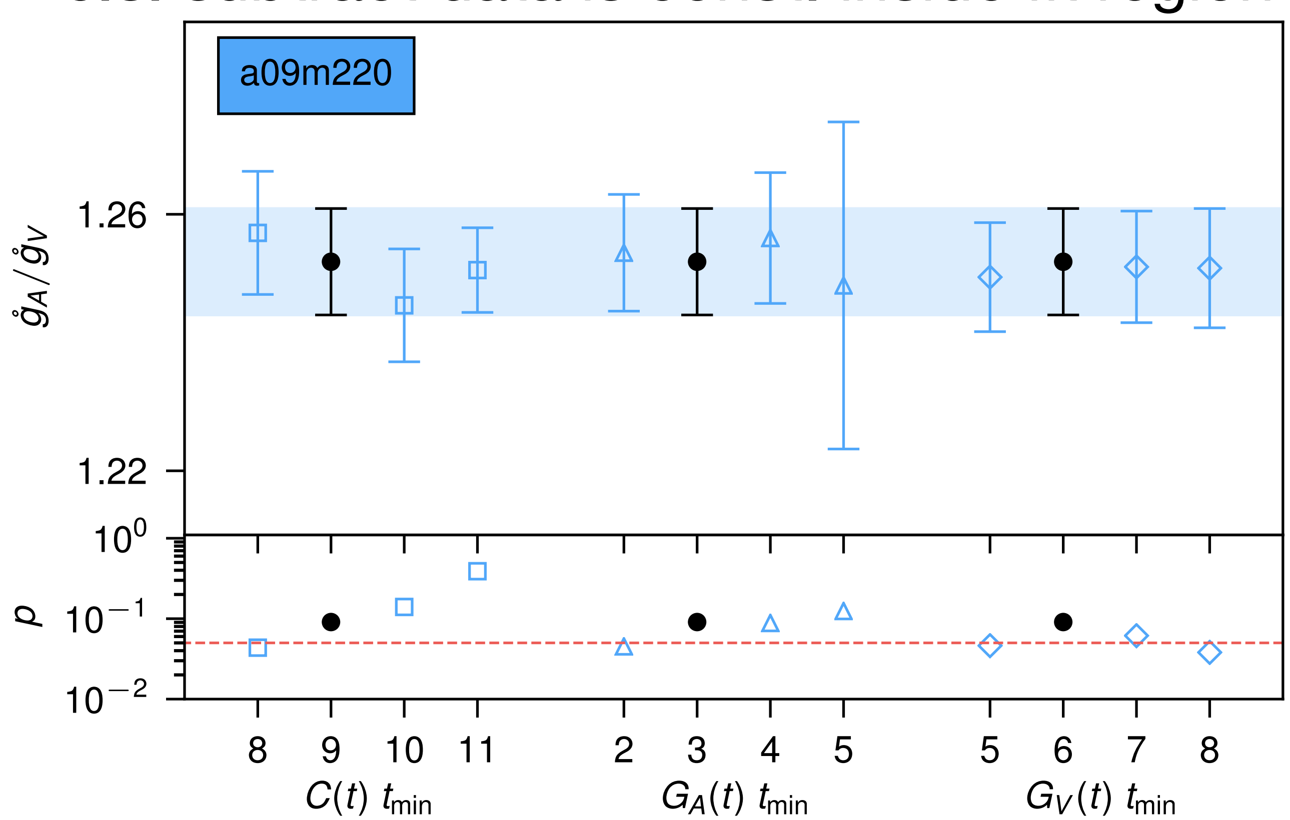
Sensitivity analysis

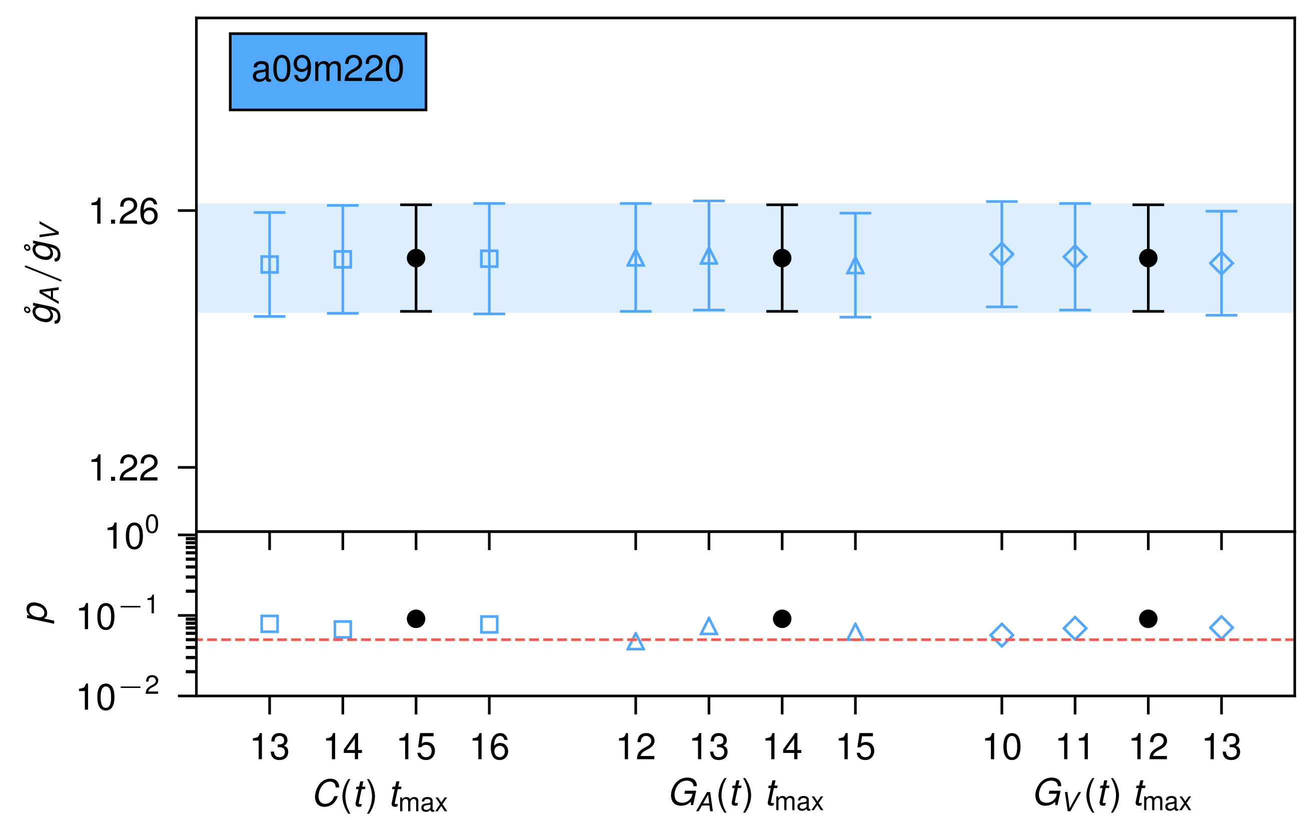
Excited-states present at small t
Correlated fluctuations at larger t



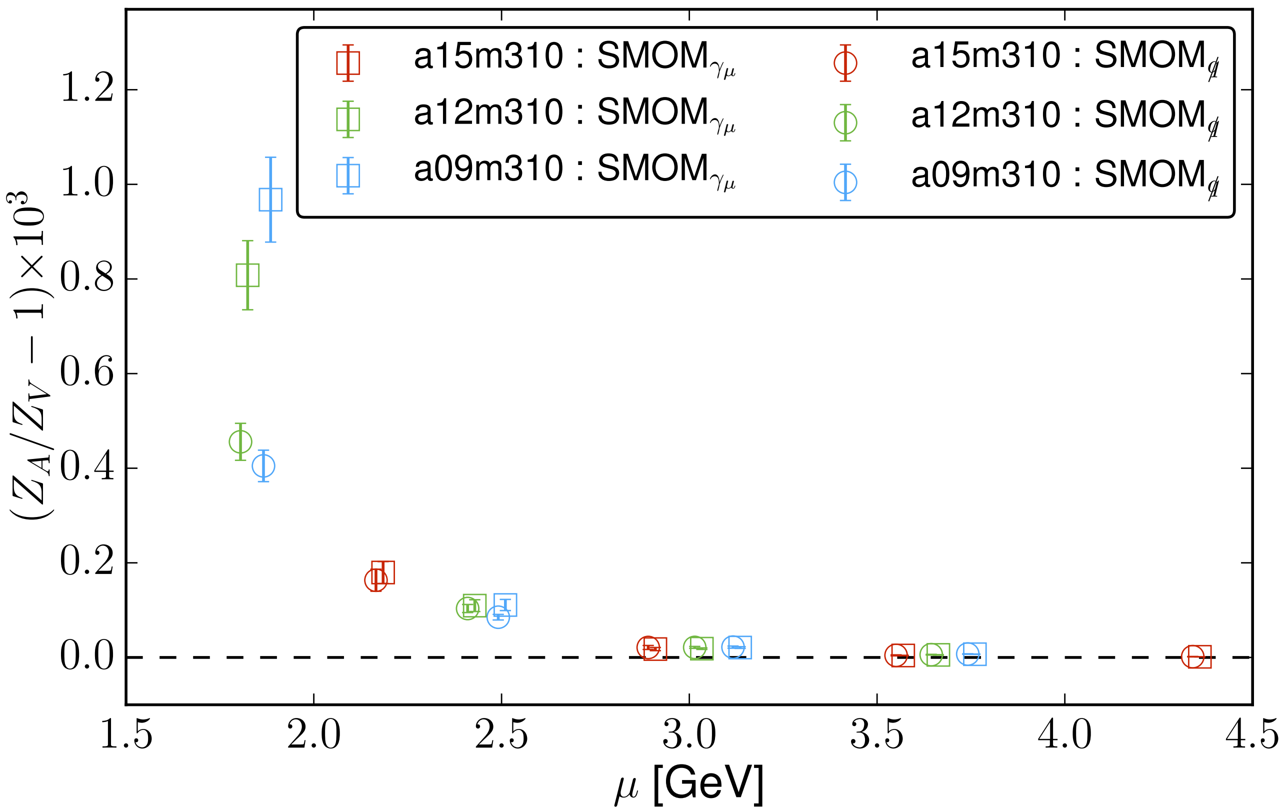


Additionally:
- p-value > 0.05
- Gaussian bootstrap dist.
- e.s. subtract data is const. inside fit region





Renormalization



Purpose

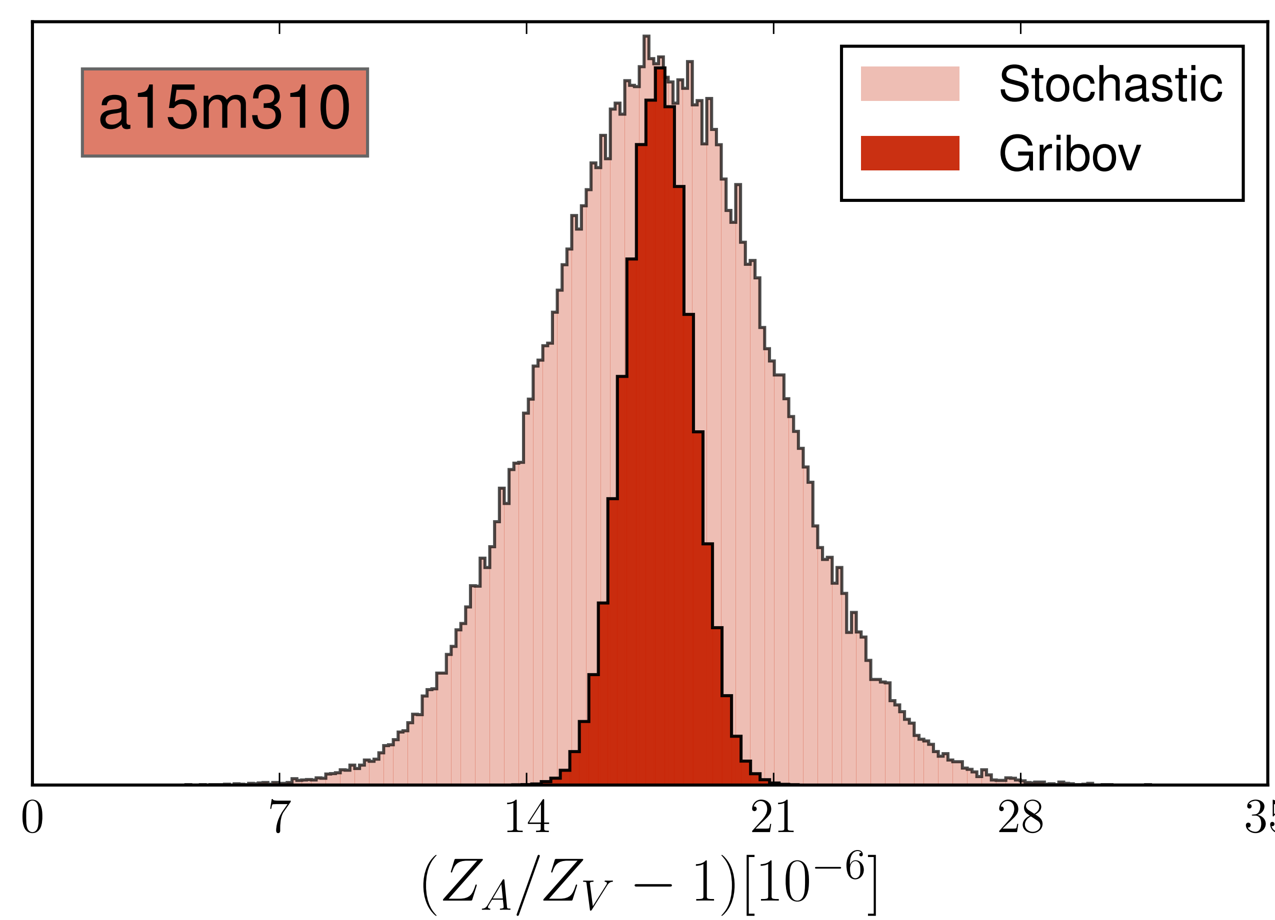
- the axial and vector currents have discretization errors
- correct for differences by matching lattice to continuum vertex functions

Details

- non-perturbative Rome-Southampton renormalization procedure
- non-exceptional kinematics (RI-SMOM)

calculate ratio $$\frac{Z_A}{Z_V}\frac{\mathring{g}_A}{\mathring{g}_V}$$

by definition $$Z_V \mathring{g}_V = 1$$



$Z_A / Z_V = 1$
@ one part in 10,000

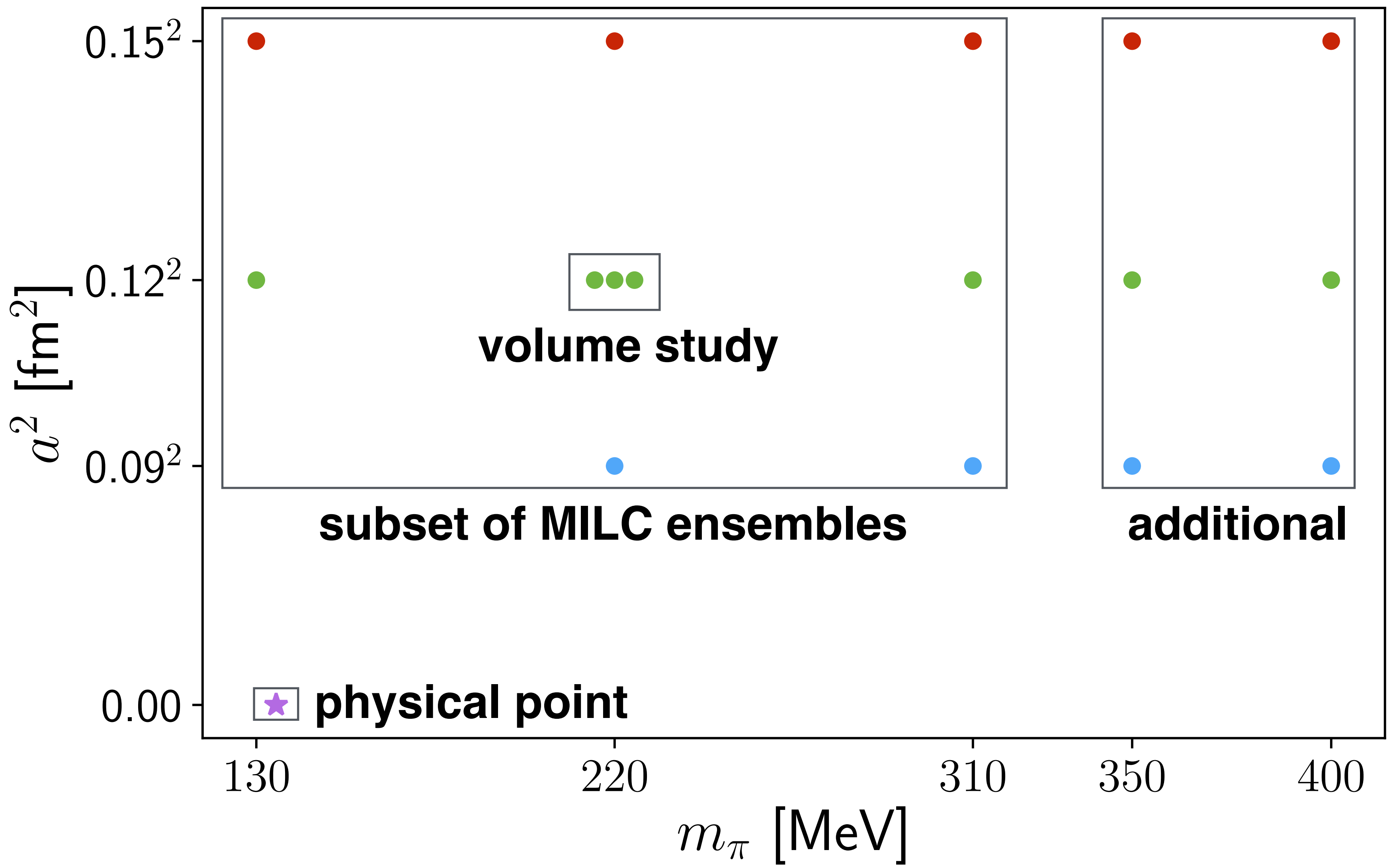
Conclusion

the ratio g_A/g_V is continuum-like

HISQ gauge configurations and mixed action

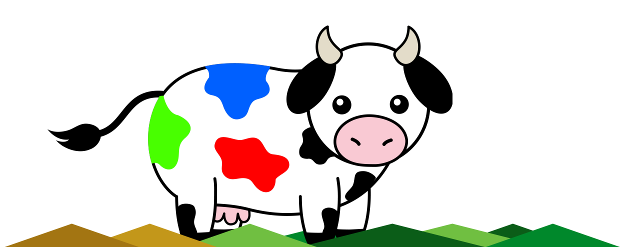
HISQ action
Errors starting at $\mathrm{O}(\alpha_s a^2, a^4)$
Lüscher-Weisz action
Errors starting at $\mathrm{O}(\alpha_s^2 a^2, a^4)$

Möbius domain-wall
tune $m_{\mathrm{res}} < 0.1 m_l$
Errors effectively start at $\mathrm{O}(a^2, \alpha_s a^2)$



MILC configurations are the only publicly available dataset capable of
- chiral extrapolation to physical pion mass
- continuum extrapolation
- infinite volume extrapolation

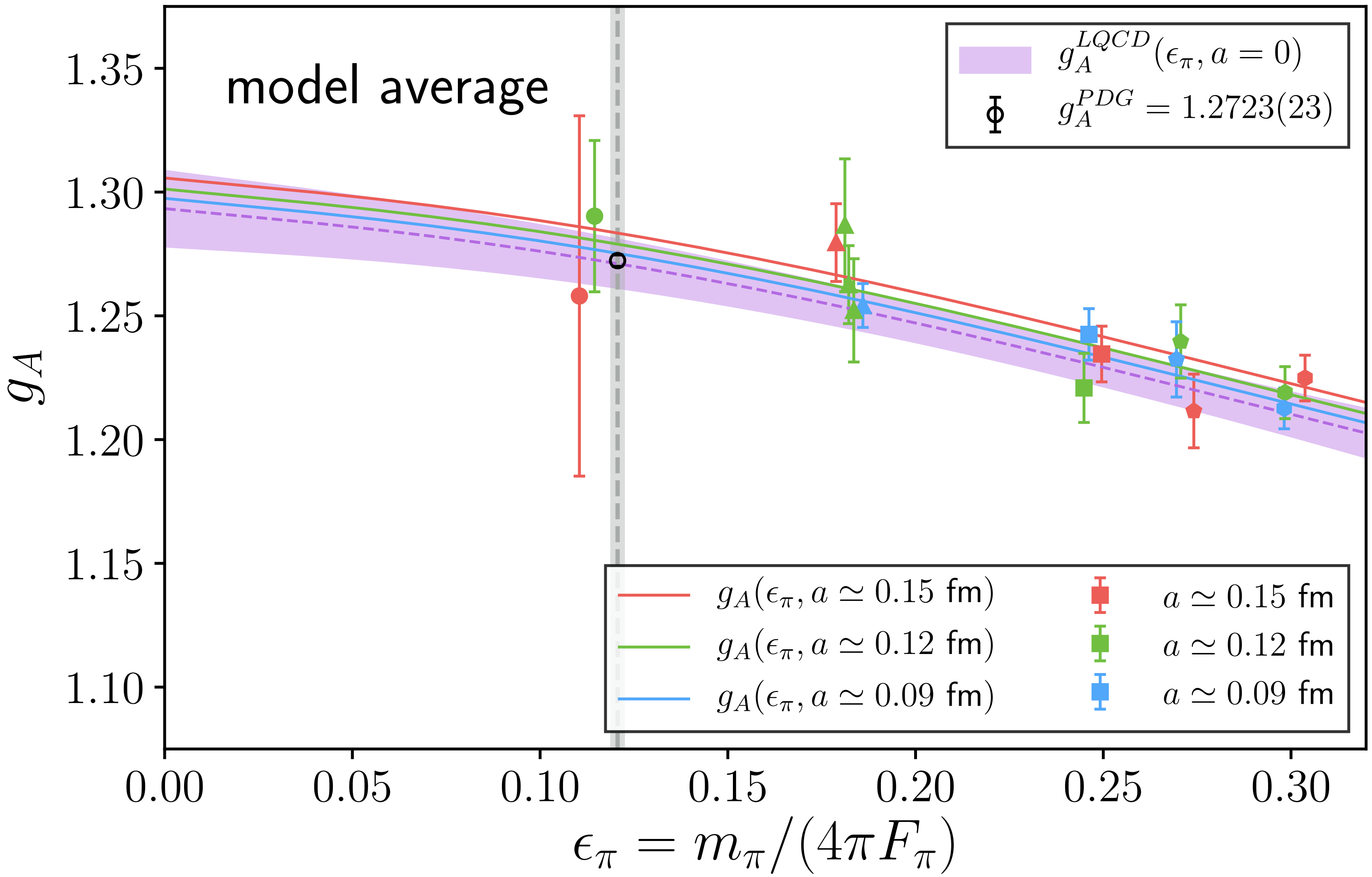
- all ensembles have add. 4D gradient-flow smearing



unofficial MILC cow
MILC = MIMD Lattice Computation
(the acronym has an acronym in the acronym)

MILC HISQ : Phys. Rev. D 87, 054505 (2013)

Extrapolation to the physical point



What goes in here?

- 5 pion masses
- 3 lattice spacing
- 3 to 7 fm box ($3.2 < m_\pi L < 5.8$)
- weighted average of different models

Strategy

- the final result is insensitive to a wide array of variations
- stability of the result is enhanced though a weighted average of different models

The final result for the nucleon axial coupling is g_A = 1.271(13)

Bayesian model averaging

Model averaging accounts for uncertainty from different physical-point extrapolations

- in general provides better out-of-sample forecast
- naturally expressed under the Bayesian framework

Marginalize over set of models

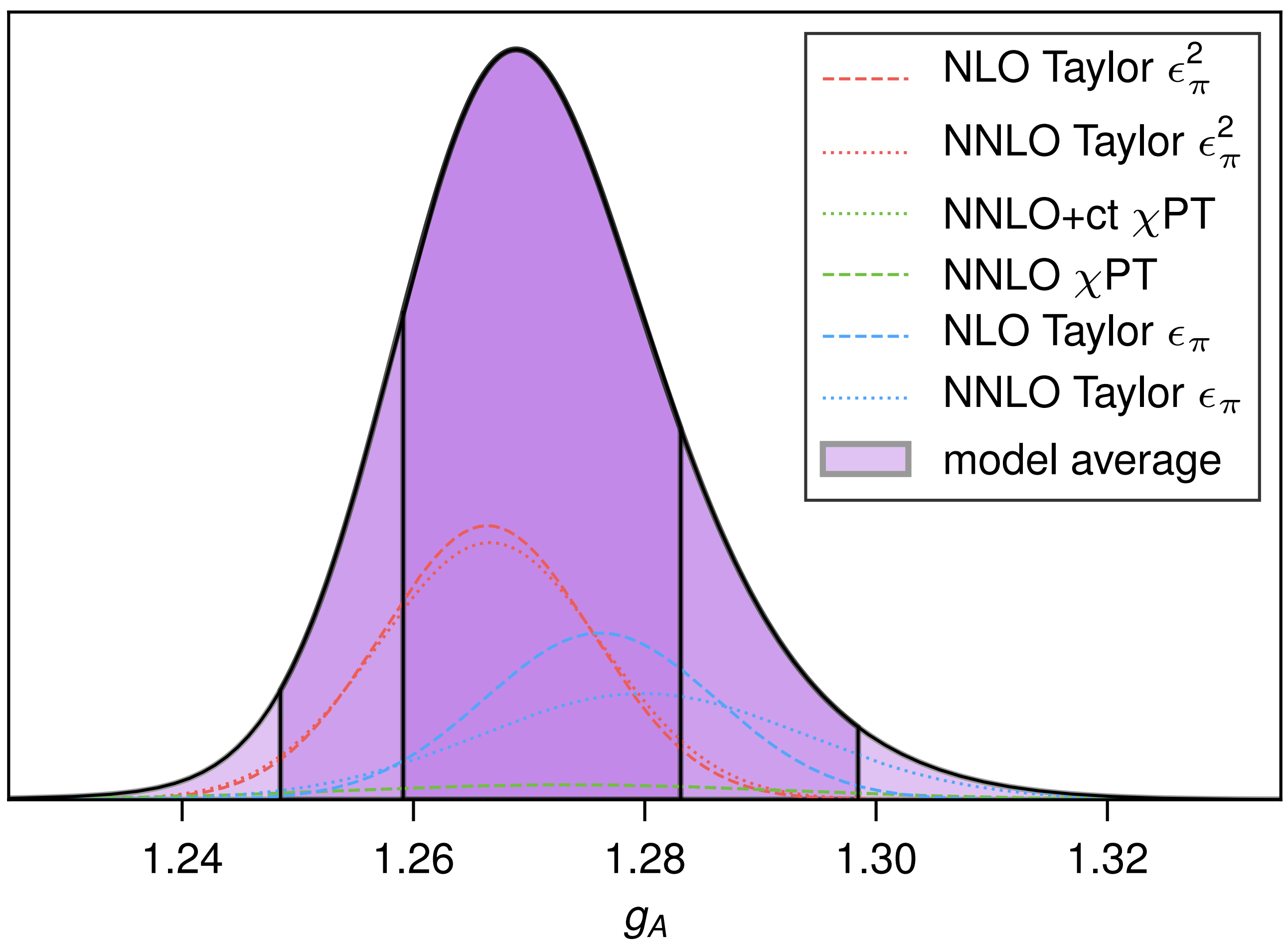
$$P(g_A|D) = \sum_k P(g_A|M_k, D)P(M_k|D)$$

Bayes' Theorem gives

$$P(M_k|D) = \frac{P(D|M_k)P(M_k)}{\sum_l P(D|M_l)P(M_l)}$$

where $P(D|M_k)$ is marginalized over params

$$P(D|M_k) = \int P(D|\theta_k, M_k)P(\theta_k|M_k)d\theta_k$$



Model averaged posterior distribution is

$$\mathrm{E}[g_A] = \sum_k \mathrm{E}[g_A|M_k]P(M_k|D)$$

$$\mathrm{Var}[g_A] = \sum_k \mathrm{Var}[g_A|M_k]P(M_k|D) + \left\{\sum_k \mathrm{E}^2[g_A|M_k]P(M_k|D)\right\} - \mathrm{E}^2[g_A|D]$$

Kass and Raftery *Bayes Factors* (1995)

Model set

(some) Baryon χPT : Phys. Lett. B259, 353 (1991)
Nucl. Phys. B388, 315 (1992)
JHEP 04, 031 (1999)

Taylor expansions around $m_\pi = 0$

$$\begin{aligned}
\text{NLO Taylor } \epsilon_\pi : &\quad c_0 + c_1\epsilon_\pi + \delta_a + \delta_L \\
\text{NNLO Taylor } \epsilon_\pi : &\quad c_0 + c_1\epsilon_\pi + c_2\epsilon_\pi^2 + \delta_a + \delta_L
\end{aligned}$$

$$\begin{aligned}
\text{NLO Taylor } \epsilon_\pi^2 : &\quad c_0 + c_2\epsilon_\pi^2 + \delta_a + \delta_L \\
\text{NNLO Taylor } \epsilon_\pi^2 : &\quad c_0 + c_2\epsilon_\pi^2 + c_4\epsilon_\pi^4 + \delta_a + \delta_L
\end{aligned}$$

Baryon chiral perturbation theory

$$\begin{aligned}
\text{NNLO } \chi\text{PT}: &\quad g_A^{\chi\text{PT}} + \delta_a + \delta_L \\
\text{NNLO+ct } \chi\text{PT}: &\quad g_A^{\chi\text{PT}} + c_4\epsilon_\pi^4 + \delta_a + \delta_L
\end{aligned}$$

$$\begin{aligned}
g_A^{\chi\text{PT}} =& g_0 + c_2\epsilon_\pi^2 - \epsilon_\pi^2(g_0 + 2g_0^3)\ln(\epsilon_\pi^2) \\
&+ g_0 c_3 \epsilon_\pi^3
\end{aligned}$$

Infinite volume extrapolation

leading order

$$\delta_L = 8/3\left(\epsilon_\pi^2\left[g_0^3 F_1(m_\pi L) + g_0 F_3(m_\pi L)\right]\right)$$

approximate NLO

$$\delta_{L_3} \equiv f_3 \epsilon_\pi^3 F_1(m_\pi L)$$

Continuum extrapolation

$$\delta_a = a_2\epsilon_a^2 + b_4\epsilon_a^2\epsilon_\pi^2 + a_4\epsilon_a^4$$

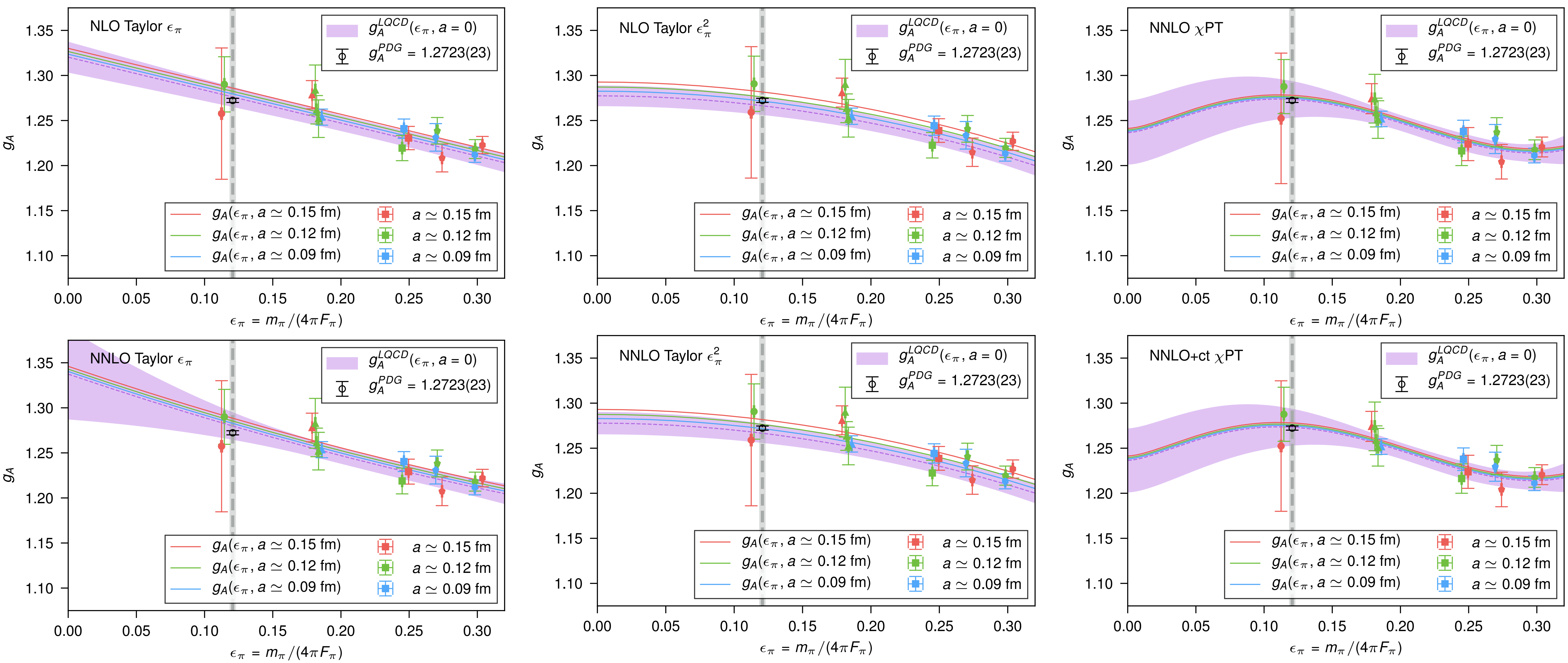
Dimensionless parameters

$$\epsilon_\pi = m_\pi/4\pi F_\pi \qquad \epsilon_a = a/\sqrt{4\pi w_0^2}$$

Summary of results

Fit	χ^2/dof	$\mathcal{L}(D\|M_k)$	$P(M_k\|D)$	$P(g_A\|M_k)$
NNLO χPT	0.727	22.734	0.033	1.273(19)
NNLO+ct χPT	0.726	22.729	0.033	1.273(19)
NLO Taylor ϵ_π^2	0.792	24.887	0.287	1.266(09)
NNLO Taylor ϵ_π^2	0.787	24.897	0.284	1.267(10)
NLO Taylor ϵ_π	0.700	24.855	0.191	1.276(10)
NNLO Taylor ϵ_π	0.674	24.848	0.172	1.280(14)
average				1.271(11)(06)

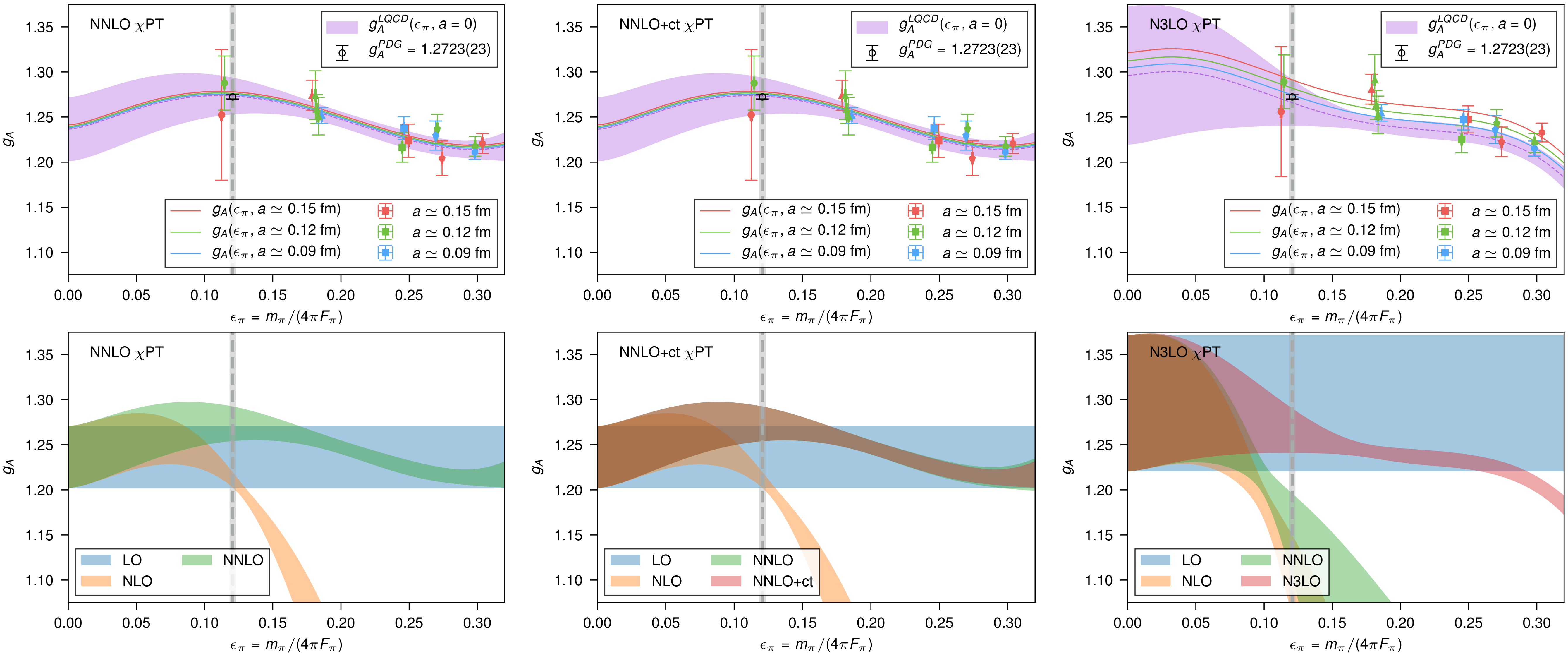
Chiral extrapolation models



Taylor in m_π | Taylor in $(m_\pi)^2$ | χPT

Convergence of chiral expansion

Double log : Phys. Lett. B639, 278 (2006)

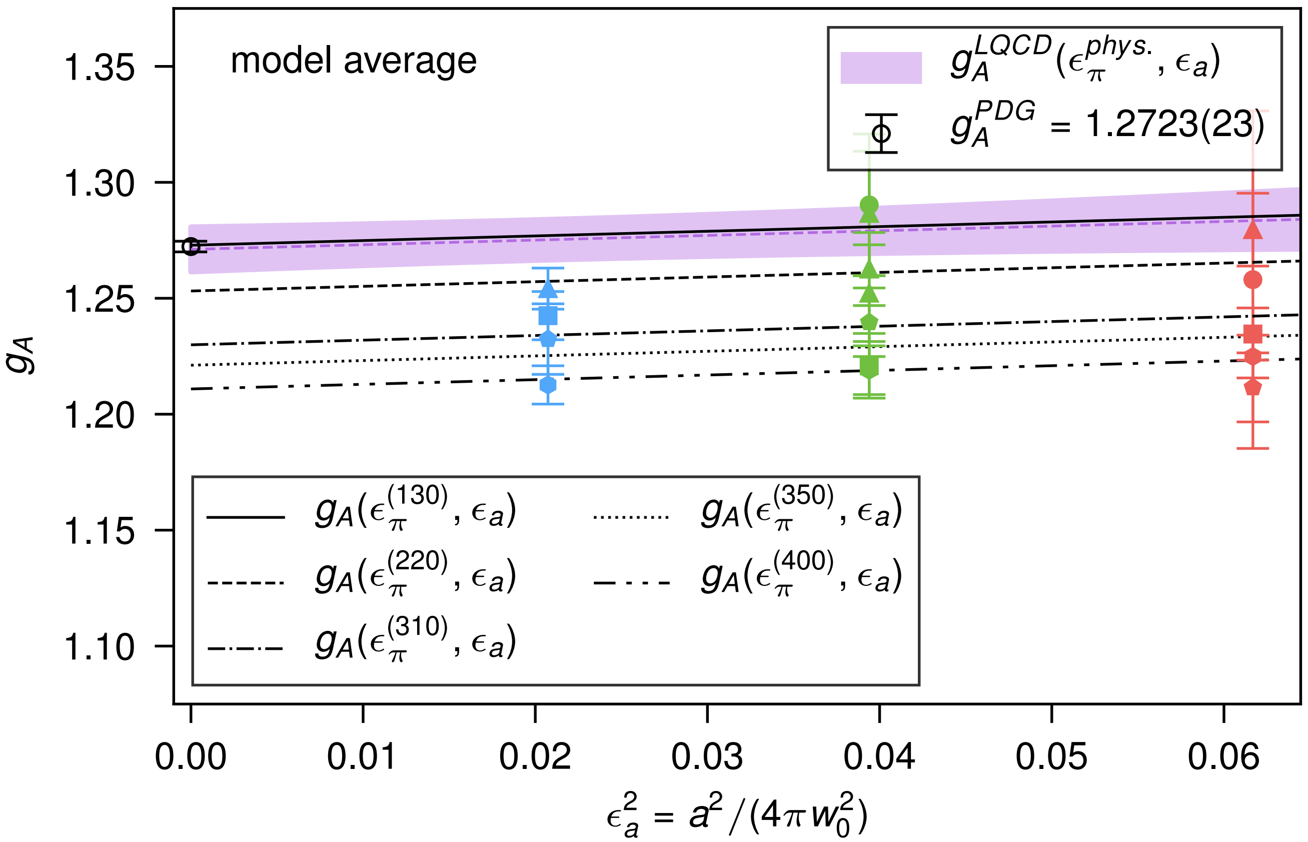


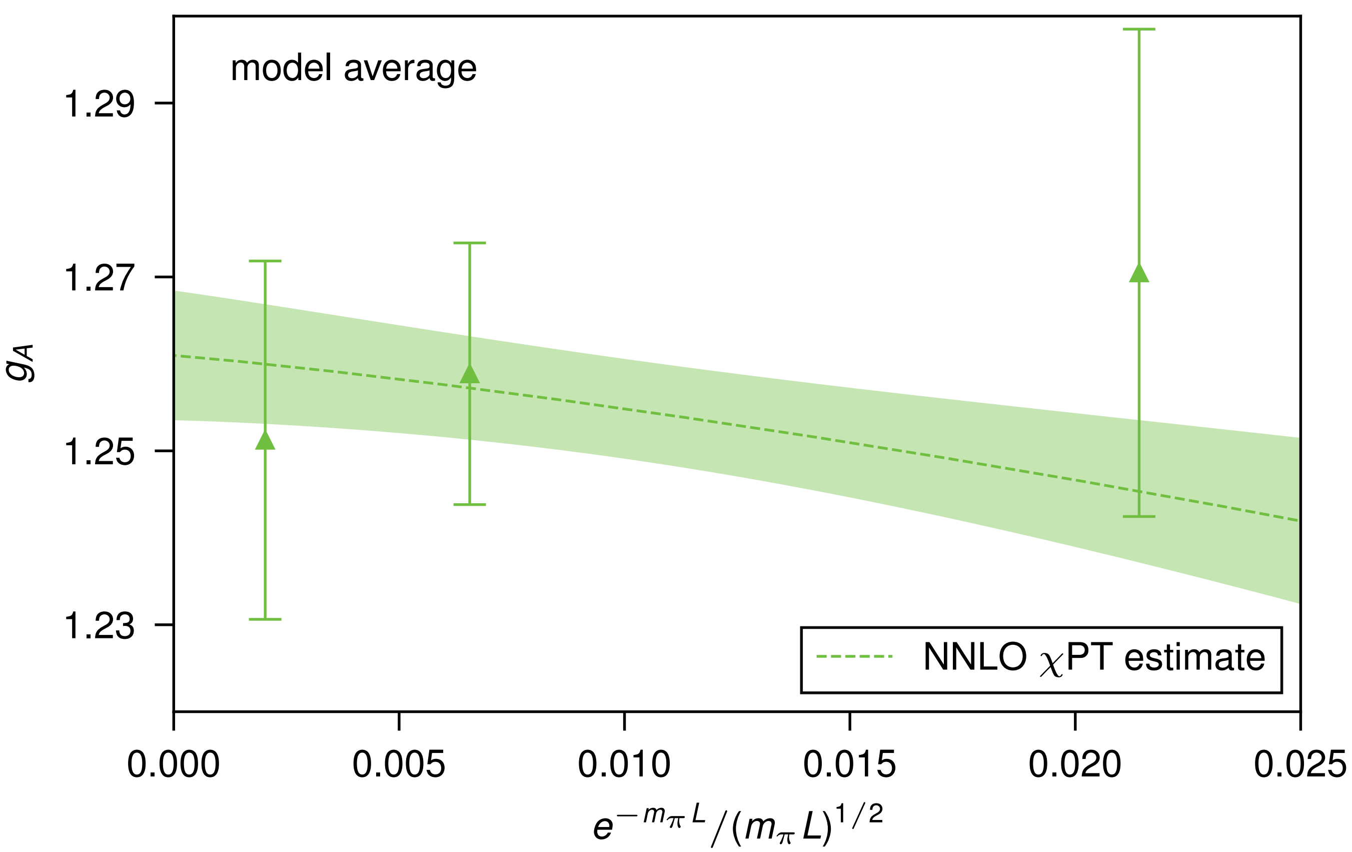
$$
\begin{aligned}
g_A = & g_0 \\
& + \epsilon_\pi^2 \left[(g_0 + 2g_0^3) \ln(\epsilon_\pi^2) + c_2 \right] \\
& + g_0 c_3 \epsilon_\pi^3
\end{aligned}
$$

$$
+ c_4 \epsilon_\pi^4
$$

$$
\begin{aligned}
& + \epsilon_\pi^4 \tilde{\gamma}_4 \ln(\epsilon_\pi^2) \\
& + \left(\frac{2}{3} g_0 + \frac{37}{12} g_0^3 + 4 g_0^5 \right) \ln^2(\epsilon_\pi^2)
\end{aligned}
$$

Continuum and infinite volume extrapolation

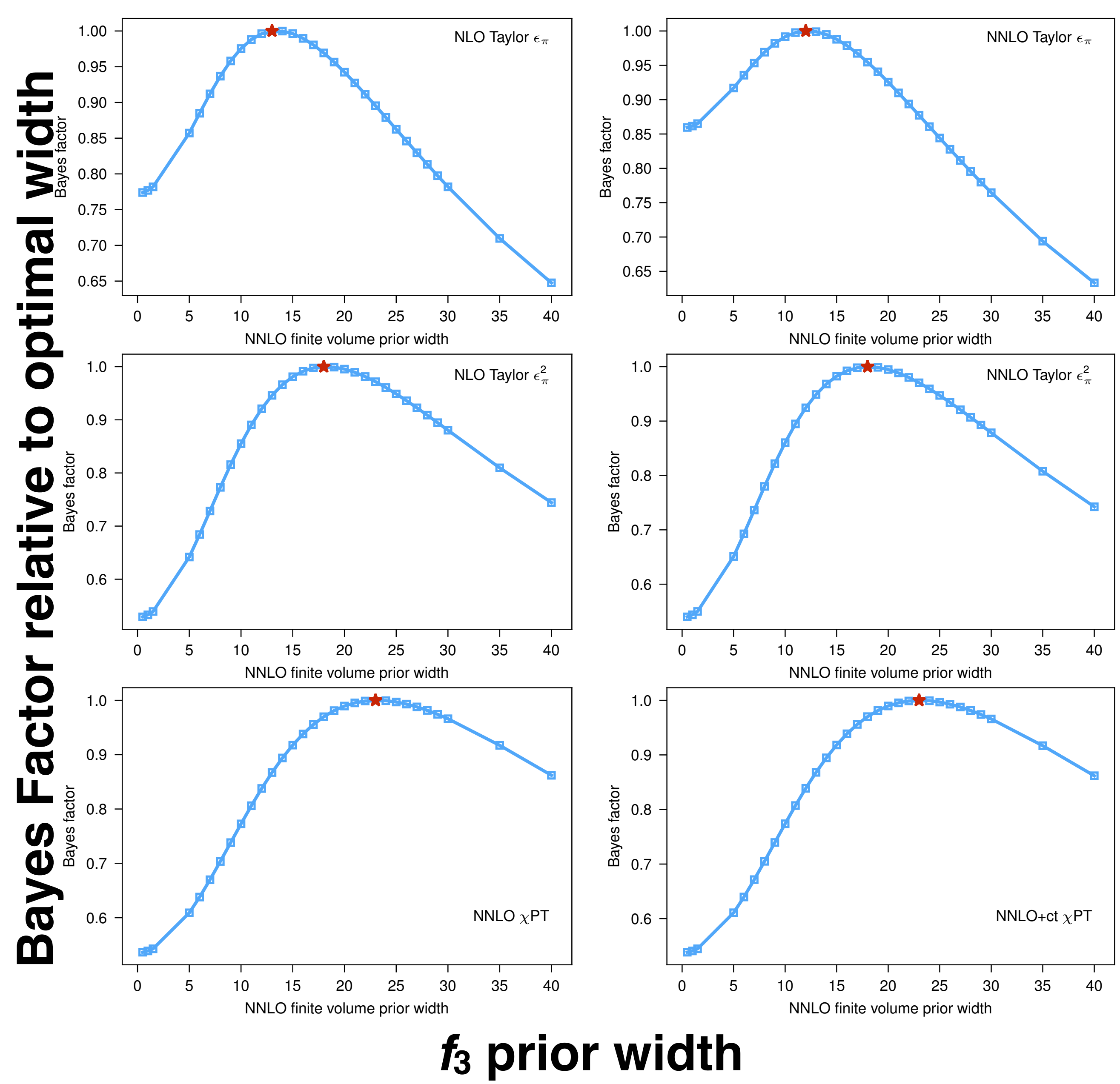




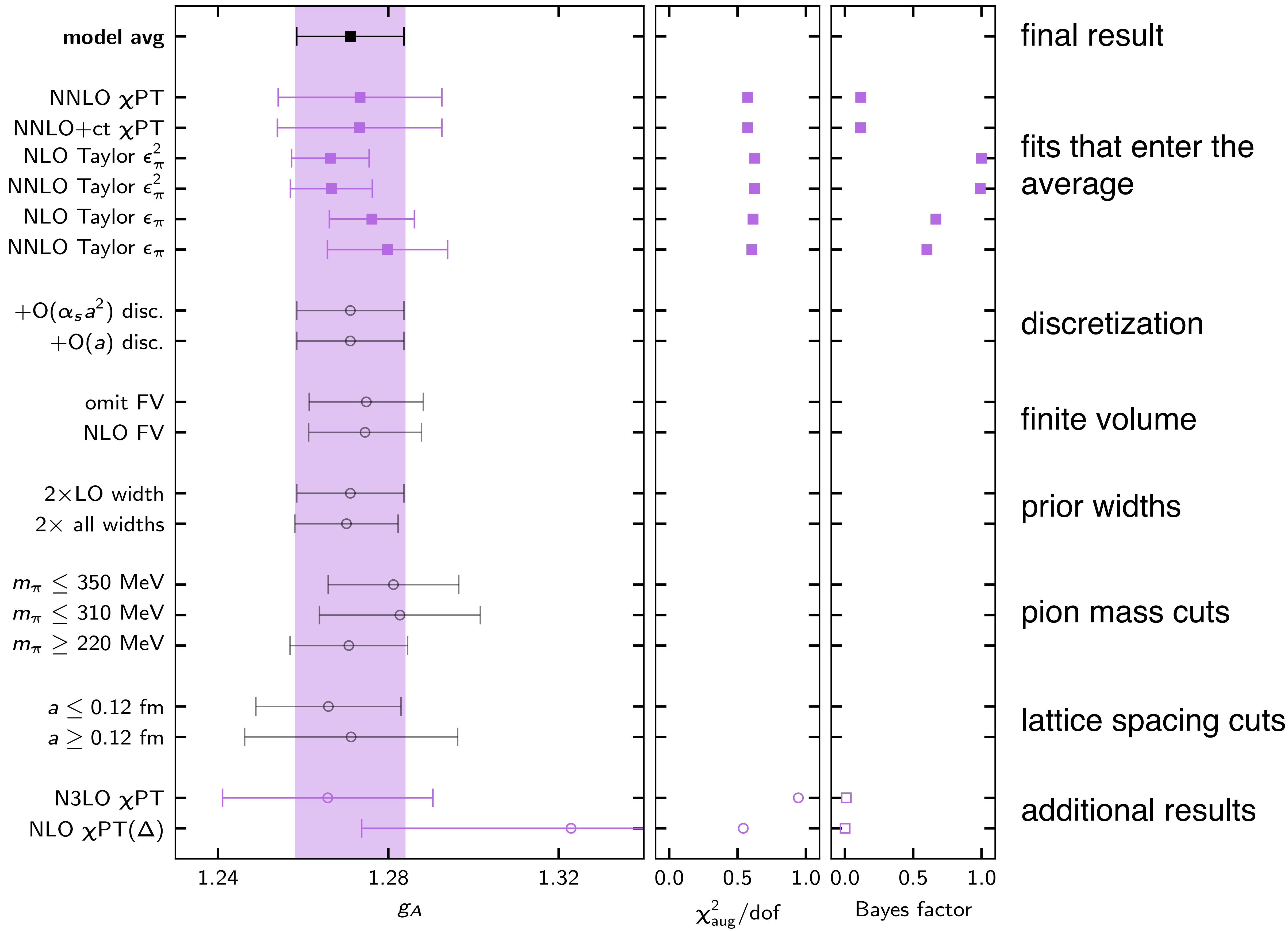
Continuum limit constant within >1σ

Volume extrap. constant within >1σ

Unknown coefficient for NLO FV is determined by Empirical Bayes method



Chiral-continuum sensitivity analysis



Systematic error budget

Sources of uncertainty

statistical
g_A, g_V, m_π, PDG m_π & F_π

chiral extrapolation
weighted ε_π coefficient uncertainties

continuum extrapolation
weighted ε_a coefficient uncertainties

finite volume
weighted f_3 coefficient uncertainties

isospin breaking
Largest uncertainty comes from

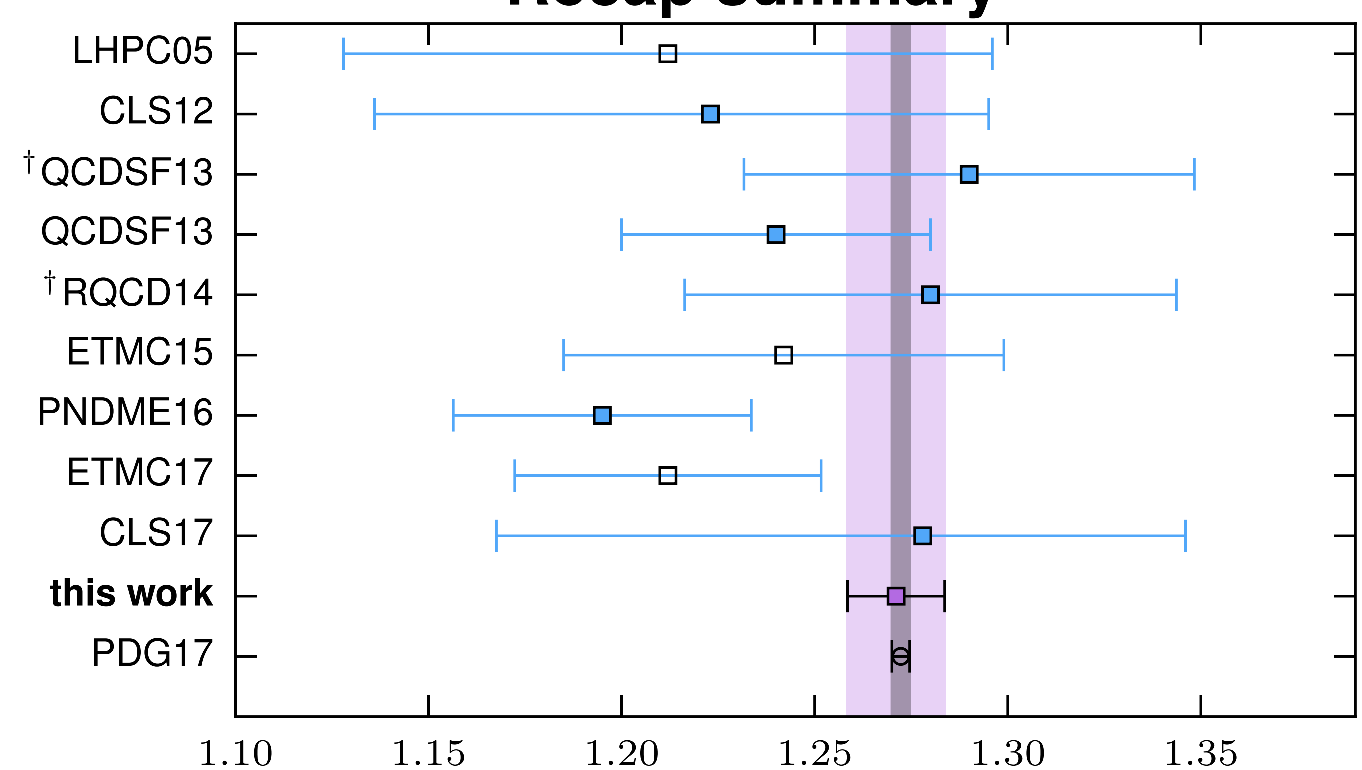
$$\left| \frac{g_A(\epsilon_{\pi^0}) - g_A(\epsilon_{\pi^\pm})}{2} \right|$$

Summary of uncertainties

statistical	0.81%
chiral extrapolation	0.15%
continuum extrapolation	0.12%
infinite volume extrap.	0.15%
isospin breaking	0.03%
model selection	0.43%
total (in quadrature)	0.99%

Final result
$g_A = 1.271(13)$

Recap summary



The lifetime of a free neutron from LQCD

$$\tau_n = \frac{4908.7(1.9)\text{sec}}{|V_{ud}|^2(1+3g_A^2)}$$

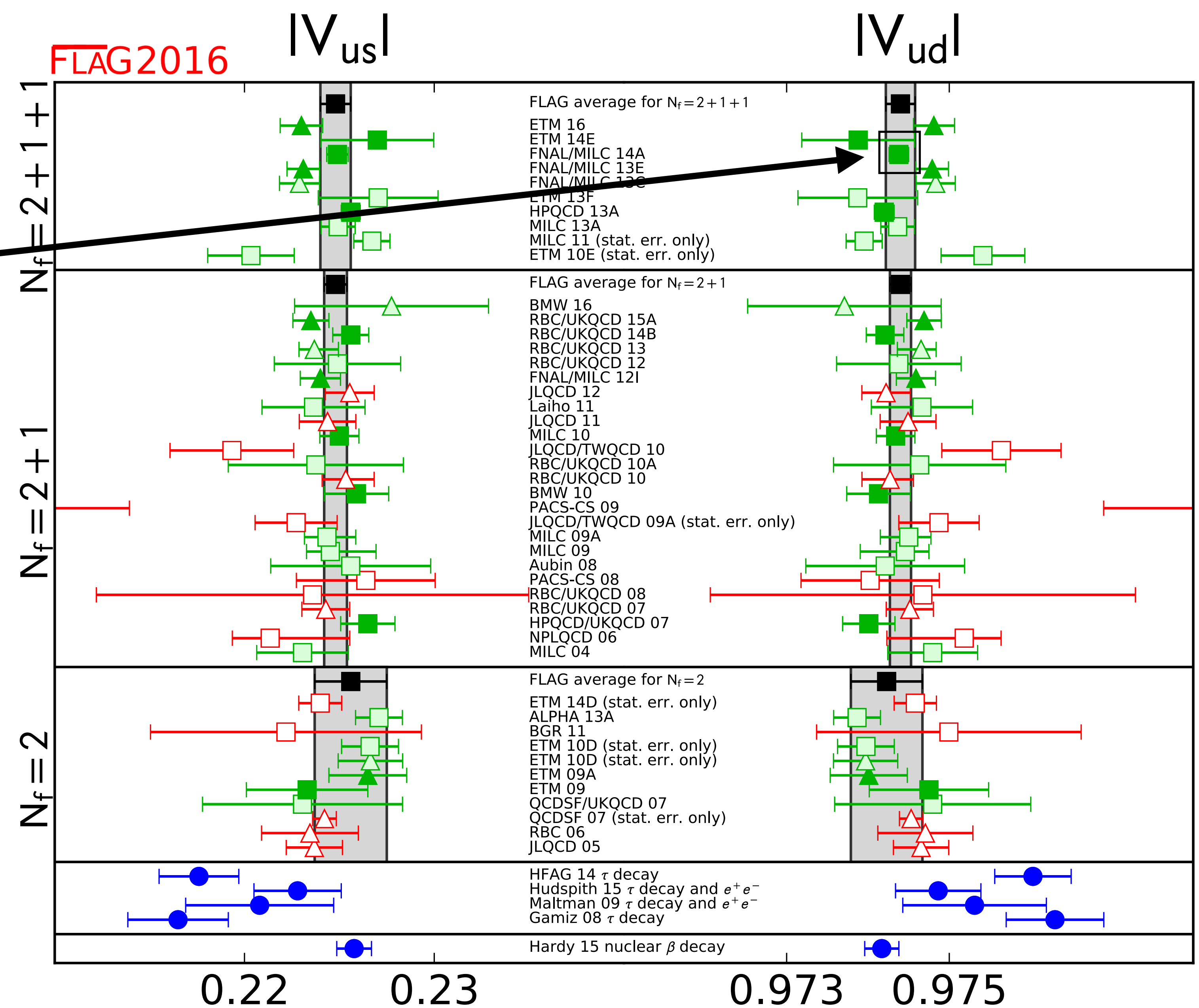
Numerator from one-loop electroweak contributions

V_{ud} from FNAL/MILC 14
$V_{ud} = 0.97438(12)$

free neutron lifetime
880(14) seconds
~ 14 minutes 40 seconds

PDG lifetime
880.2(1.0) seconds

consistent at 1.6%



Neutron lifetime : Phys. Rev. Lett 96 032002 (2006)
FNAL/MILC V_{ud} : Phys. Rev. D90 074509 (2014)

Outlook

https://www.nature.com/articles/s41586-018-0161-8
www.github.com/callat-qcd/project_gA

This work
First 1% determination of g_A from Lattice QCD.

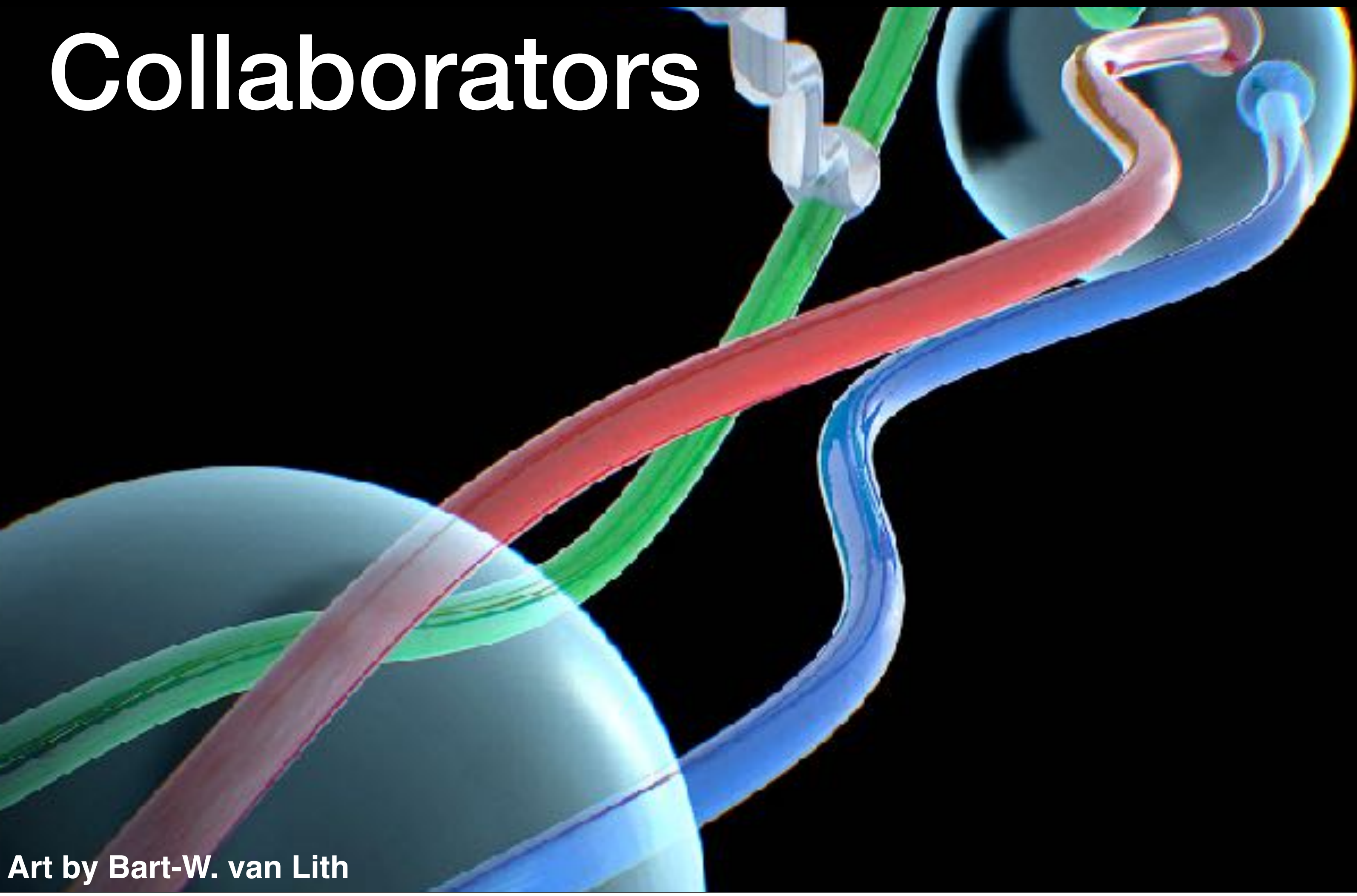
Neutron lifetime puzzle
The current determination of the axial coupling is statistics limited.
Neutron lifetime differ by 1% between beam vs. bottle.
A 0.3% determination of g_A can discriminate two results at 1σ (τ ~ 0.5%).
New 100+ PFLOP supercomputers will help achieve this goal.

Applications to other single nucleon observables (in no particular order)
Proton radius puzzle
Atomic and muonic Hydrogen radii differ by 4%.
Goal to directly determine the radius at 1%.
Development of new methods may let us achieve this goal.

Nucleon axial form factor
Long baseline neutrino experiments may uncover large sources of leptonic CP-violation.
Precise experiments with precise prediction of the entire axial form factor is needed.
The Feynman-Hellmann method may be applied to non-zero momentum (and other ideas).

Charm content of the nucleon
WIMP-N cross section is particularly sensitive to the charm content.
Need ~10% precision to motivate detector size. [PRL 112, 211602]

Collaborators



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