Model-Independent Constraints on $R_{J/\Psi}$

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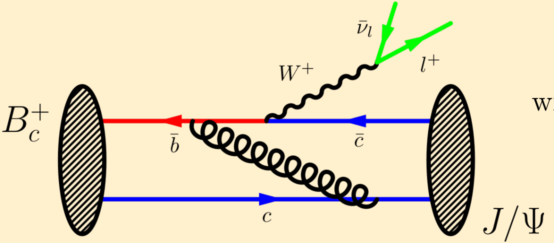


Hank Lamm

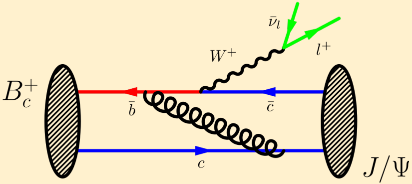
with Tom Cohen and Rich Lebed

29 May 2018

CIPANP2018

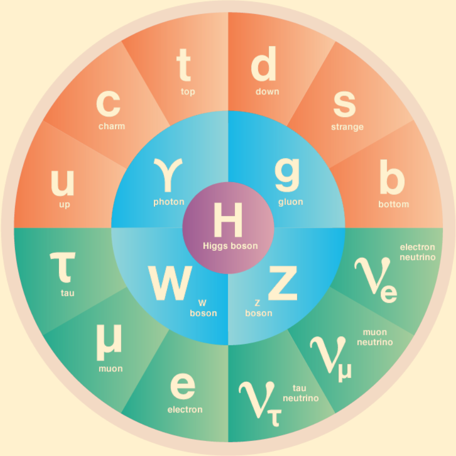
UNIVERSITY OF MARYLAND

Outline



Who ordered that?

Within the Standard Model, *lepton universality* is broken only by the Higgs interaction



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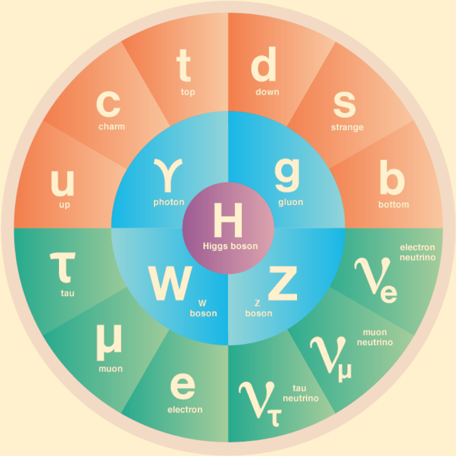
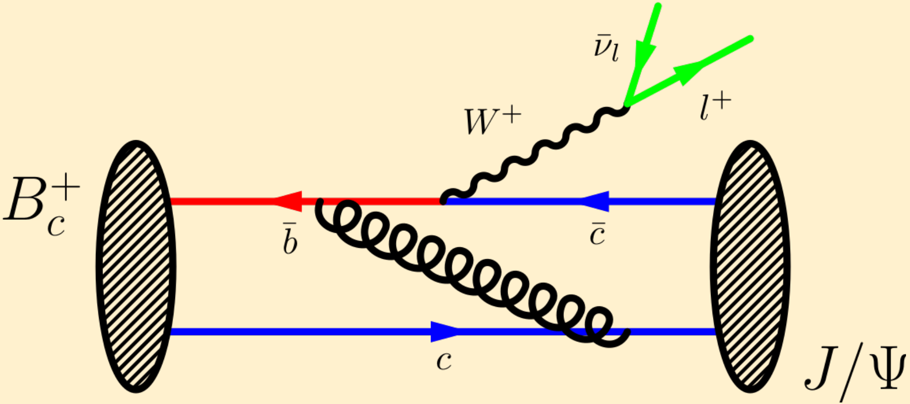


...but m_ν implies this isn't the end of the story

...so let's do some precision physics!



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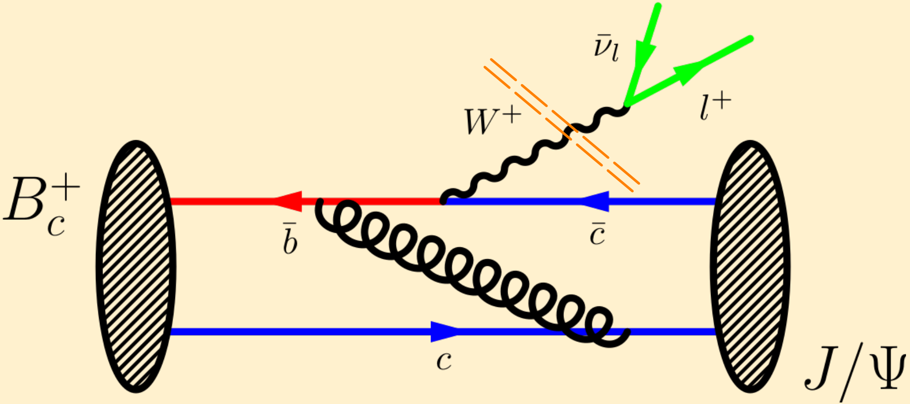

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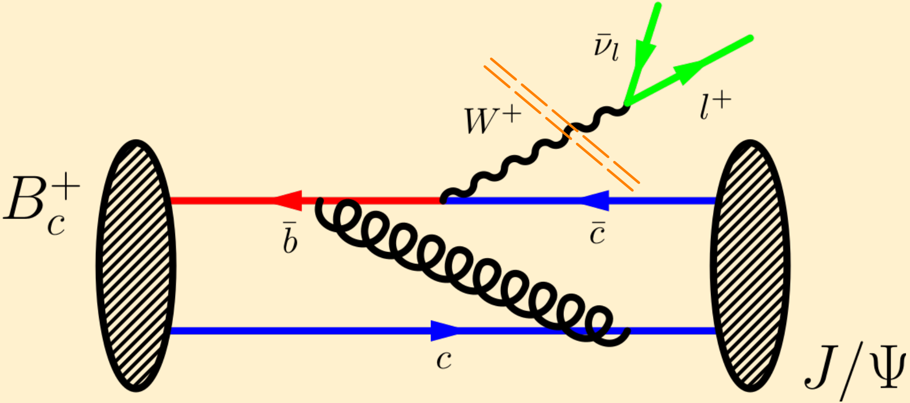

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$$R(h_b \to h_c) \equiv \frac{\mathcal{B}(h_b\to h_c\tau\bar{\nu}_\tau)}{\mathcal{B}(h_b\to h_c l\bar{\nu}_l)} = ???$$

Ratios of semileptonic $b-$quark decays, they persisted...

Ratio	Exp	R_{exp}	R_{theory}
$R(B \to \pi^-)$	BELLE	< 1.93 (95% CL)	0.641(17)
$R(B \to D)$	BaBaR	$0.469(84)_{stat}(53)_{syst}$	0.300(8)
$R(B \to D^*)$	PDG	0.328(22)	0.252(3)
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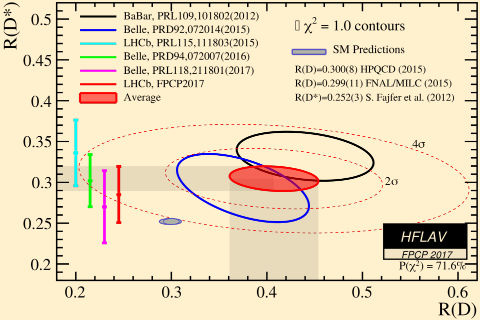

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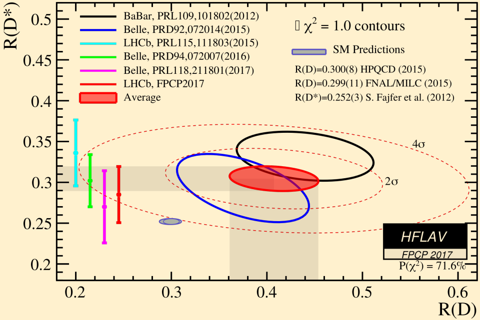

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Only model-dependent predictions exist

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RCQM	0.26	2000
QCDSR	0.25	2003
RCQM	0.24	2006
NRQM	$0.27^{+0.02}_{-0}$	2006
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pQCD	$0.29^{+0.09}_{-0.09}$	2013
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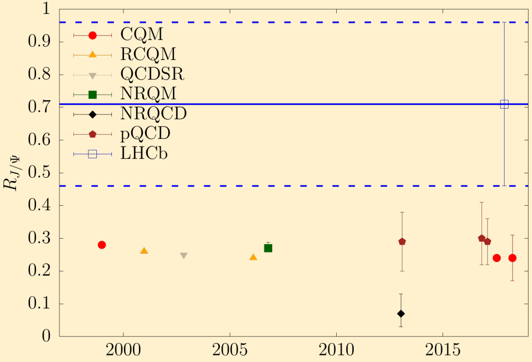

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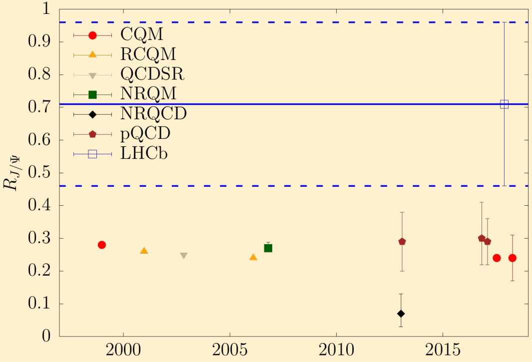

Taking the largest/smallest $\mathcal{B}(B_c^+ \to J/\psi \tau^+ \bar{\nu}_\tau)$ and $\mathcal{B}(B_c^+ \to J/\psi l^+ \bar{\nu}_l)$ and compute a **worst-case** scenario $R_{J/\psi} = [0, 3]$

What's the worst that can happen?

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$$\langle V(p',\epsilon)|V^\mu - A^\mu|P(p)\rangle = \frac{2i\epsilon^{\mu\nu\rho\sigma}}{M+m}\epsilon^*_\nu p'_\rho p_\sigma V(q^2) - (M+m)\epsilon^{*\mu}A_1(q^2)$$
$$+\frac{\epsilon^*\cdot q}{M+m}(p+p')^\mu A_2(q^2) + 2m\frac{\epsilon^*\cdot q}{q^2}q^\mu A_3(q^2) - 2m\frac{\epsilon^*\cdot q}{q^2}q^\mu A_0(q^2) \quad (1)$$

$$A_3(q^2) = \frac{M+m}{2m}A_1(q^2) - \frac{M-m}{2m}A_2(q^2) \quad (2)$$

where $A_3(0) = A_0(0)$ and the masses are given by $M = m_P, m = m_V$

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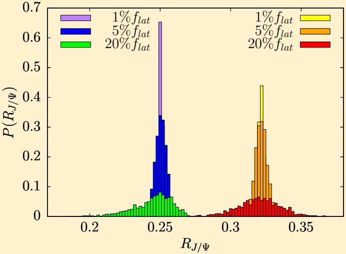
Strict prediction would require **additional assumptions** about priors, but min/max values are **independent** of this

So what can the Standard Model allow?



95% CL Upper and Lower Bounds on $R_{J/\psi}$

$\% f_{lat}$	$n = 1$	$n = 2$
1	[0.257,0.314]	[0.2495,0.3256]
5	[0.252,0.317]	[0.2442,0.3294]
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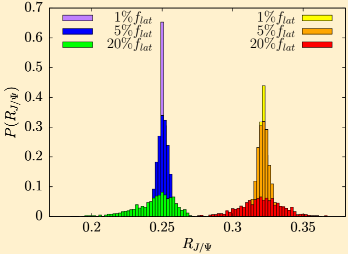

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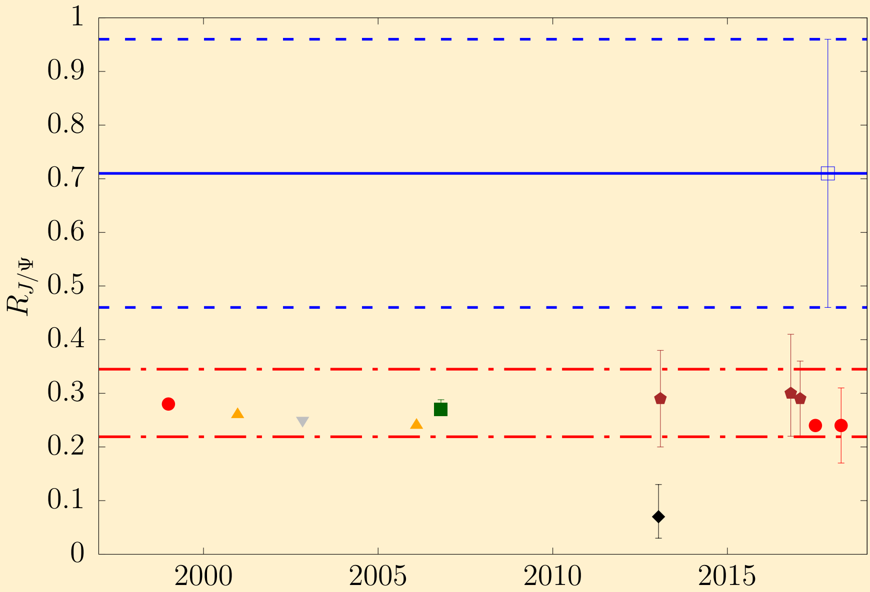
$n > 2$ **unlikely** to strongly affect bound, because $\frac{a_{n+1}}{a_n} \geq z_{\max} = 0.027$ and $\sum a_{ni}^2 \leq 1$ heavily penalize larger n

Updated $R_{J/\psi}$ Plot



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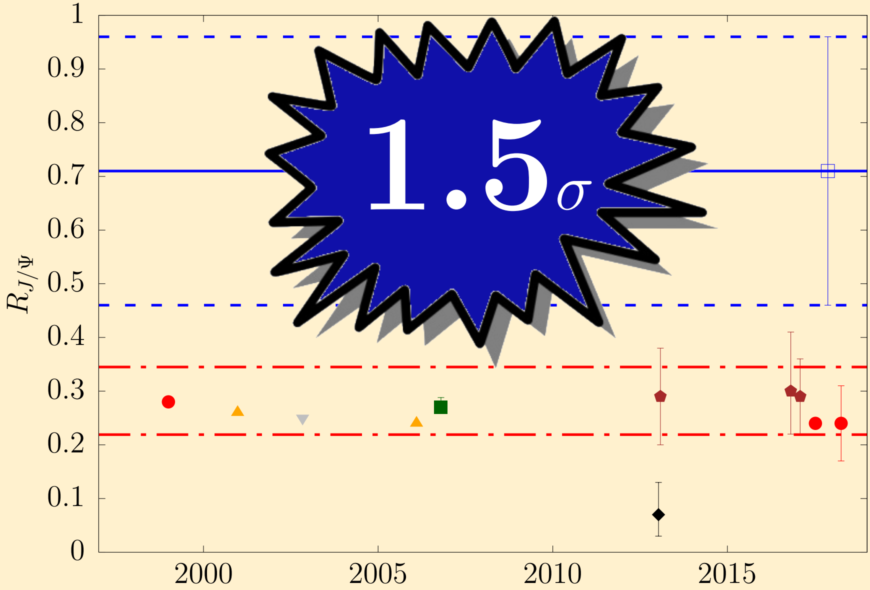
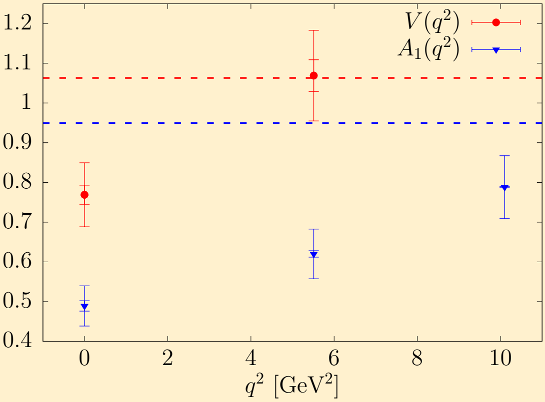


Lattice NRQCD results provide limited input[2]



2+1+1 HISQ, $a = 0.09$ fm, $m_s/m_l \approx 5$ from MILC with NRQCD for b

[2] **Colquhoun:2016osw**.

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$$\chi_J^L(q^2) \equiv \frac{\partial \Pi_J^L}{\partial q^2} = \frac{1}{\pi}\int_0^\infty dt\, \frac{\operatorname{Im}\Pi_J^L(t)}{(t-q^2)^2} \qquad (3)$$

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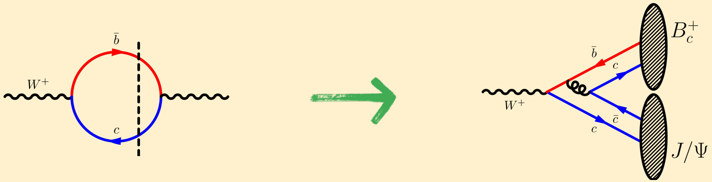

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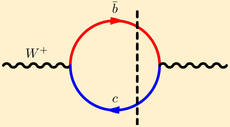
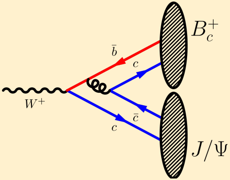
We **need** $\chi_J^{L,T}(q^2)$ computable in pQCD at $q^2=0$





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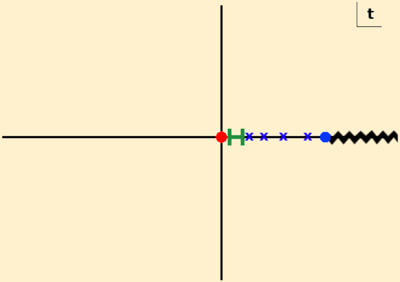

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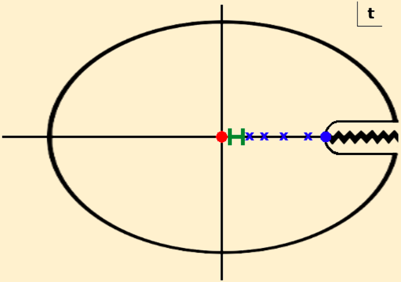

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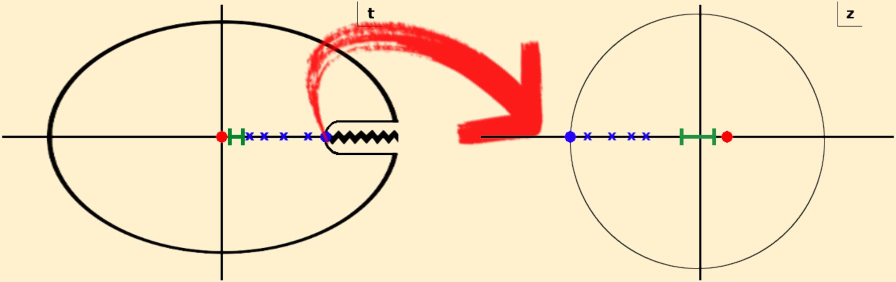

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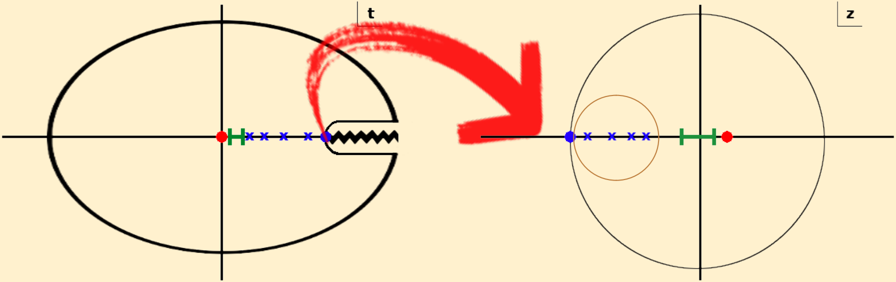

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Form factors **cannot** change arbitrarily fast!

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