

- **TMDs** -  
**transverse connections**  
between **particle** and **nuclear** physics

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Andrea Signori

13<sup>th</sup> conference on the  
intersections of  
**particle** and **nuclear** physics

CIPANP 2018  
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# Outline of the talk

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- 1) Transverse-momentum-distributions (TMDs)
- 2) “intersections of particle and nuclear physics”
- 3) predictive power of TMDs
- 4) impact on LHC physics



# Collaborations

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I will present some research directions, in collaboration with:

- J. Qiu (JLab)



- M. Grewal, Z. Kang (UCLA)



- A. Bacchetta, G. Bozzi, M. Radici (Pavia U., INFN)



- P. Mulders, M. Ritzmann (Nikhef)



# TMDs

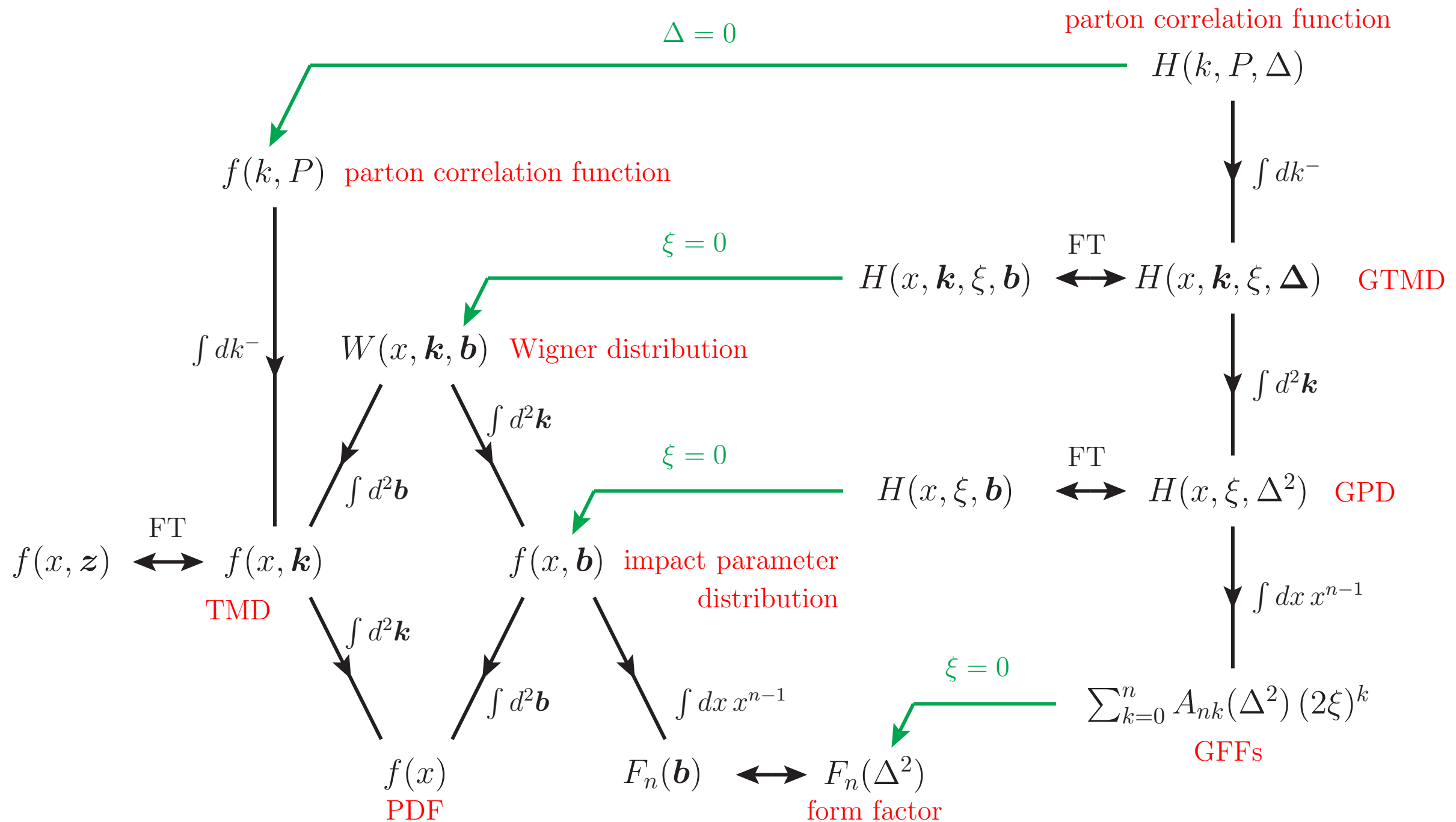
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References (intro and reviews) :

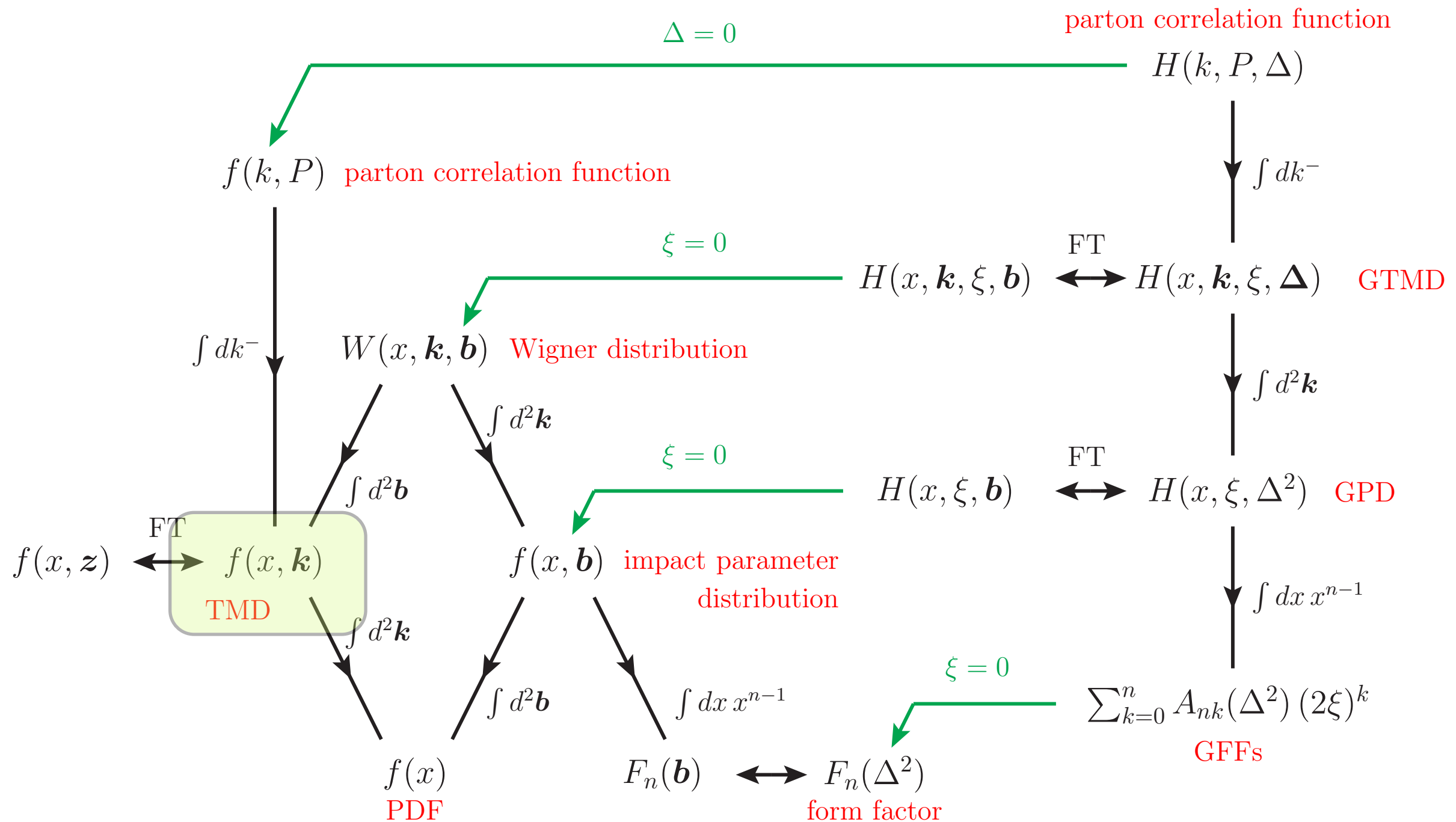
- “The 3D structure of the nucleon” **EPJ A (2016) 52**
- J.C. Collins “**Foundations of perturbative QCD**”
- material from the TMD collaboration **summer school**



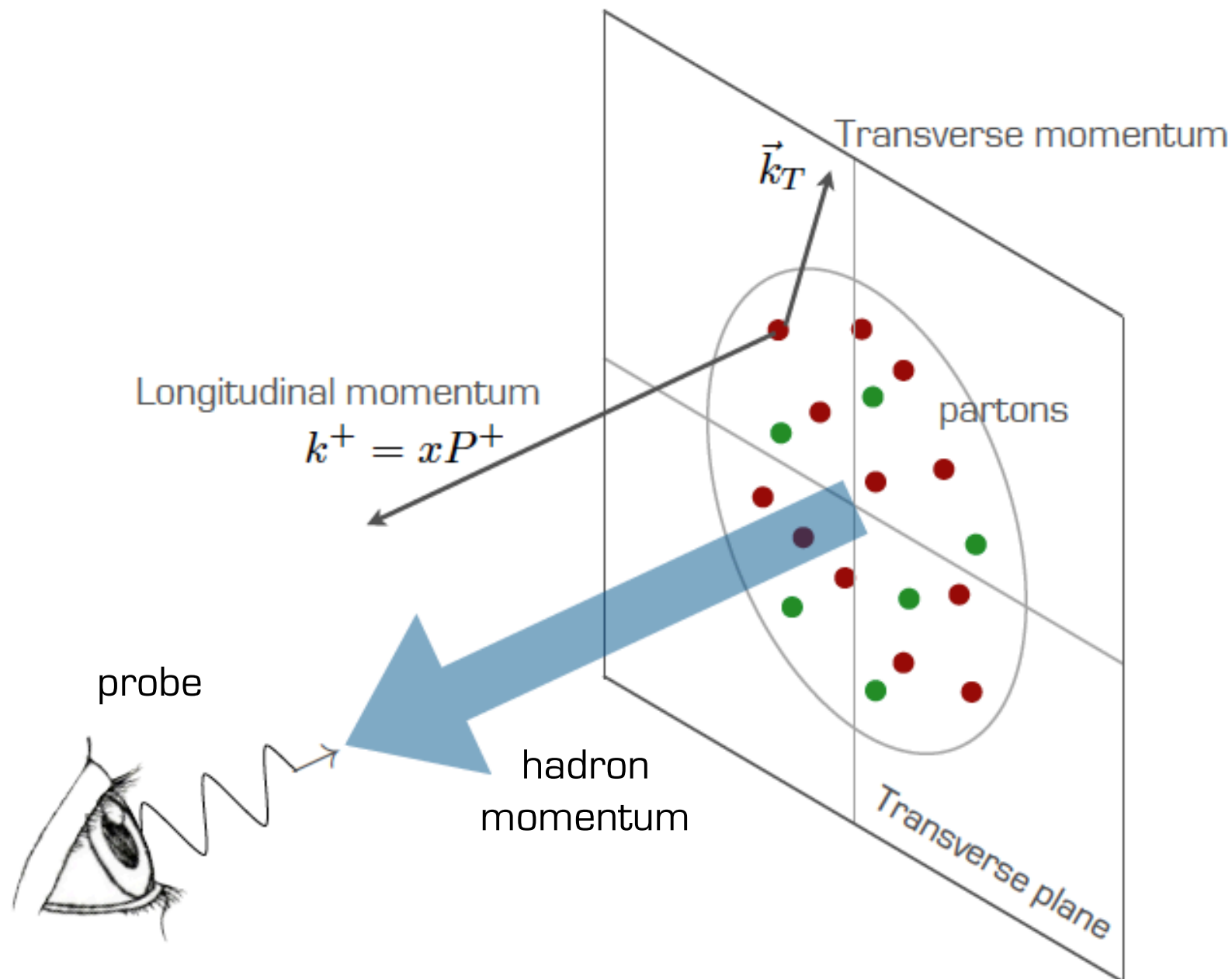
# The hadronic landscape



# The hadronic landscape



# TMDs



extraction of a **parton**  
whose momentum has  
**longitudinal** and  
**transverse components**  
with respect to the  
parent **hadron** momentum

richer than PDFs

How are TMDs **defined** ?

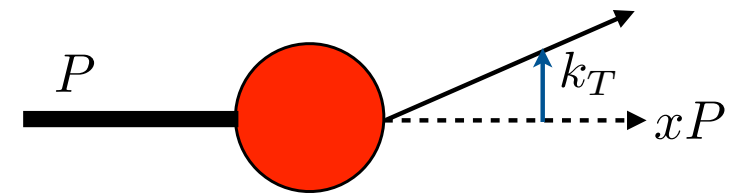
# Quark TMD PDFs

$$\Phi_{ij}(k, P; S, T) \sim \text{F.T.} \langle PS | \bar{\psi}_j(0) U_{[0,\xi]} \psi_i(\xi) | PS \rangle_{LF}$$

|        | U              | L                   | T                       |
|--------|----------------|---------------------|-------------------------|
| Quarks | $\gamma^+$     | $\gamma^+ \gamma^5$ | $i\sigma^{i+} \gamma^5$ |
| U      | $f_1$          |                     | $h_1^\perp$             |
| L      |                | $g_1$               | $h_{1L}^\perp$          |
| T      | $f_{1T}^\perp$ | $g_{1T}$            | $h_1, h_{1T}^\perp$     |

Sivers TMD PDF

unpolarized TMD PDF



extraction of a **quark**  
**not** collinear with the proton

encode all the possible  
**spin-spin** and **spin-momentum**  
**correlations**  
between the proton  
and its constituents

similar table for **gluons** and for **fragmentation**

**bold** : also collinear

**red** : time-reversal odd (universality properties)

# Quark TMD PDFs

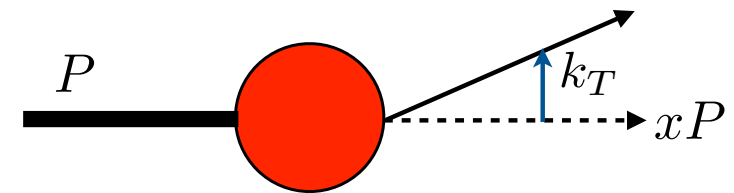
$$\Phi_{ij}(k, P; S, T) \sim \text{F.T.} \langle PST | \bar{\psi}_j(0) U_{[0,\xi]} \psi_i(\xi) | PST \rangle_{LF}$$

|        | U                           | L                       | T                                     |
|--------|-----------------------------|-------------------------|---------------------------------------|
| Quarks | $\gamma^+$                  | $\gamma^+ \gamma^5$     | $i\sigma^{i+} \gamma^5$               |
| U      | <b><math>f_1</math></b>     |                         | $h_1^\perp$                           |
| L      |                             | <b><math>g_1</math></b> | $h_{1L}^\perp$                        |
| T      | $f_{1T}^\perp$              | $g_{1T}$                | <b><math>h_1, h_{1T}^\perp</math></b> |
| LL     | <b><math>f_{1LL}</math></b> |                         | $h_{1LL}^\perp$                       |
| LT     | $f_{1LT}$                   | $g_{1LT}$               | $h_{1LT}, h_{1LT}^\perp$              |
| TT     | $f_{1TT}$                   | $g_{1TT}$               | $h_{1TT}, h_{1TT}^\perp$              |

similar table for **gluons** and for **fragmentation**

**bold** : also collinear

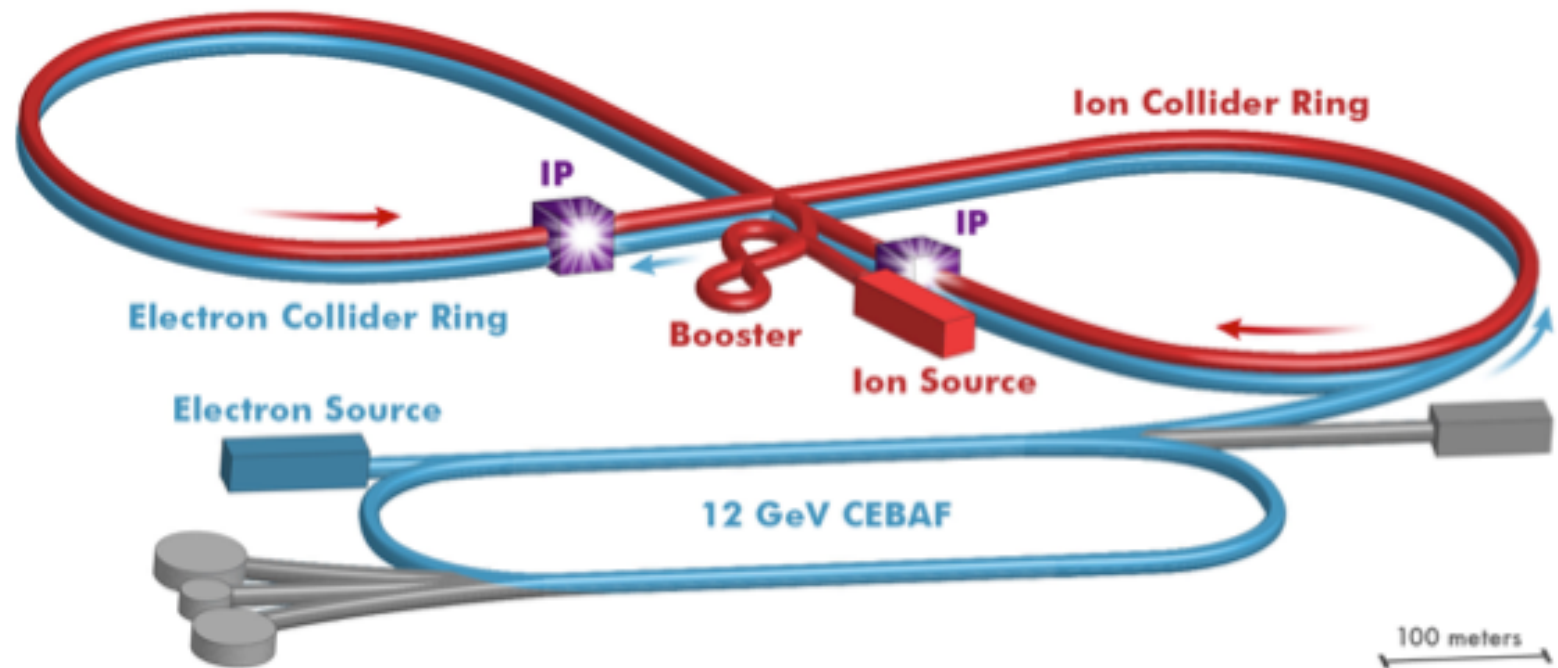
**red** : time-reversal odd (universality properties)



extraction of a **quark**  
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# Motivations

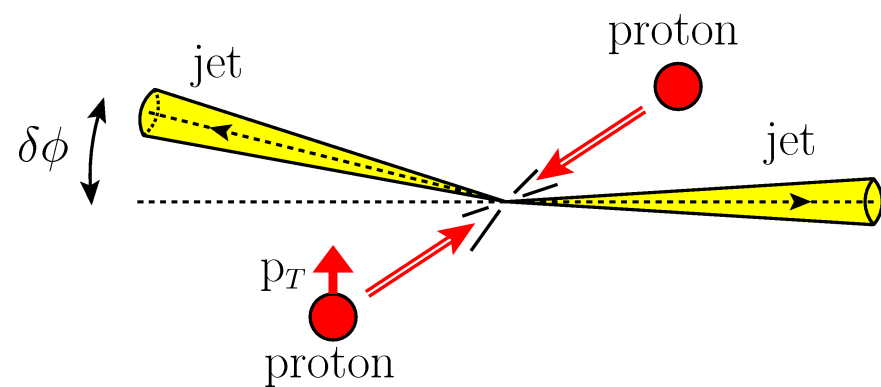
Some references :

- Dudek et al. [“Physics opportunities with the 12 GeV upgrade at Jefferson Lab”](#)
- Accardi et al. [“Electron-Ion Collider: the next QCD frontier”](#)
- AFTER@LHC study group [“Physics opportunities with a fixed-target experiment at the LHC”](#)
- ... other existing and future facilities ...

# The frontier

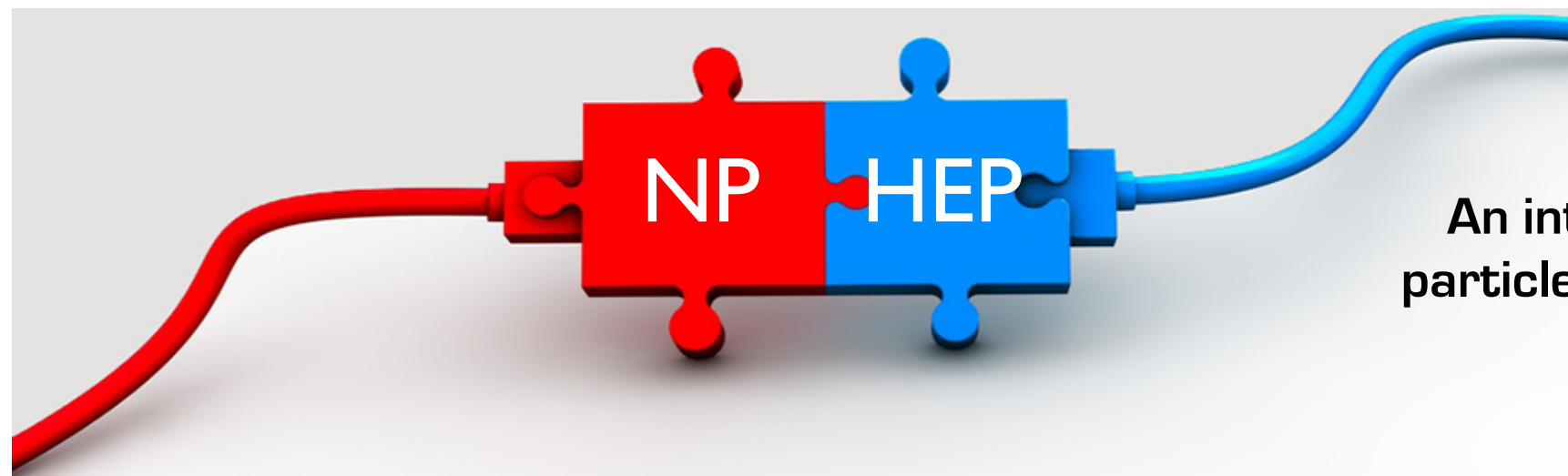
## Nucleon/nuclear tomography in momentum space:

aimed at understanding how hadrons are built in terms of the elementary degrees of freedom of QCD



## High-energy phenomenology:

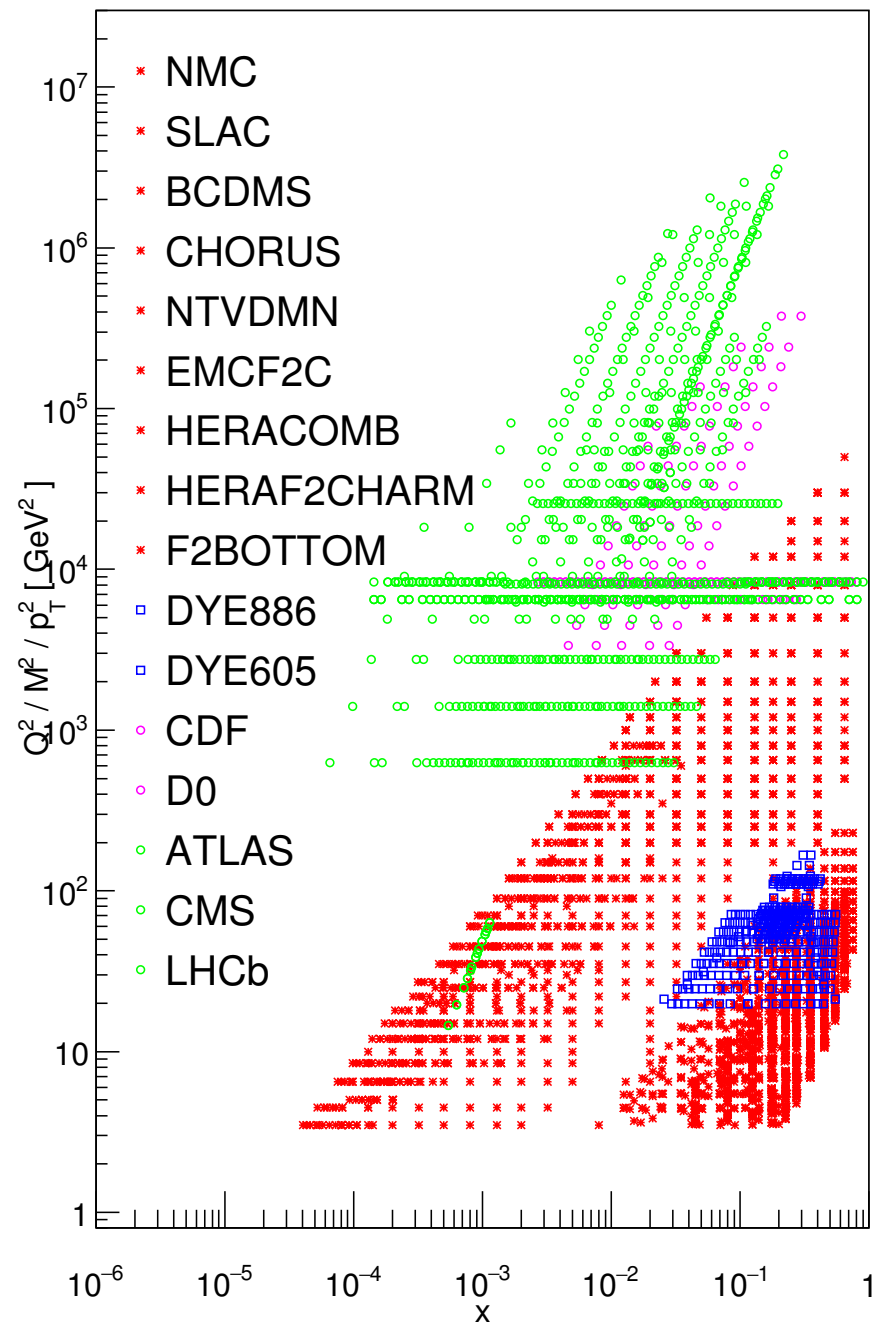
aimed at improving our understanding of high-energy scattering experiments and their potential to explore BSM physics assuming a certain degree of knowledge of hadron structure



An intersection between particle and nuclear physics!

# Collinear vs TMD PDFs

see E. Nocera - POETIC2016

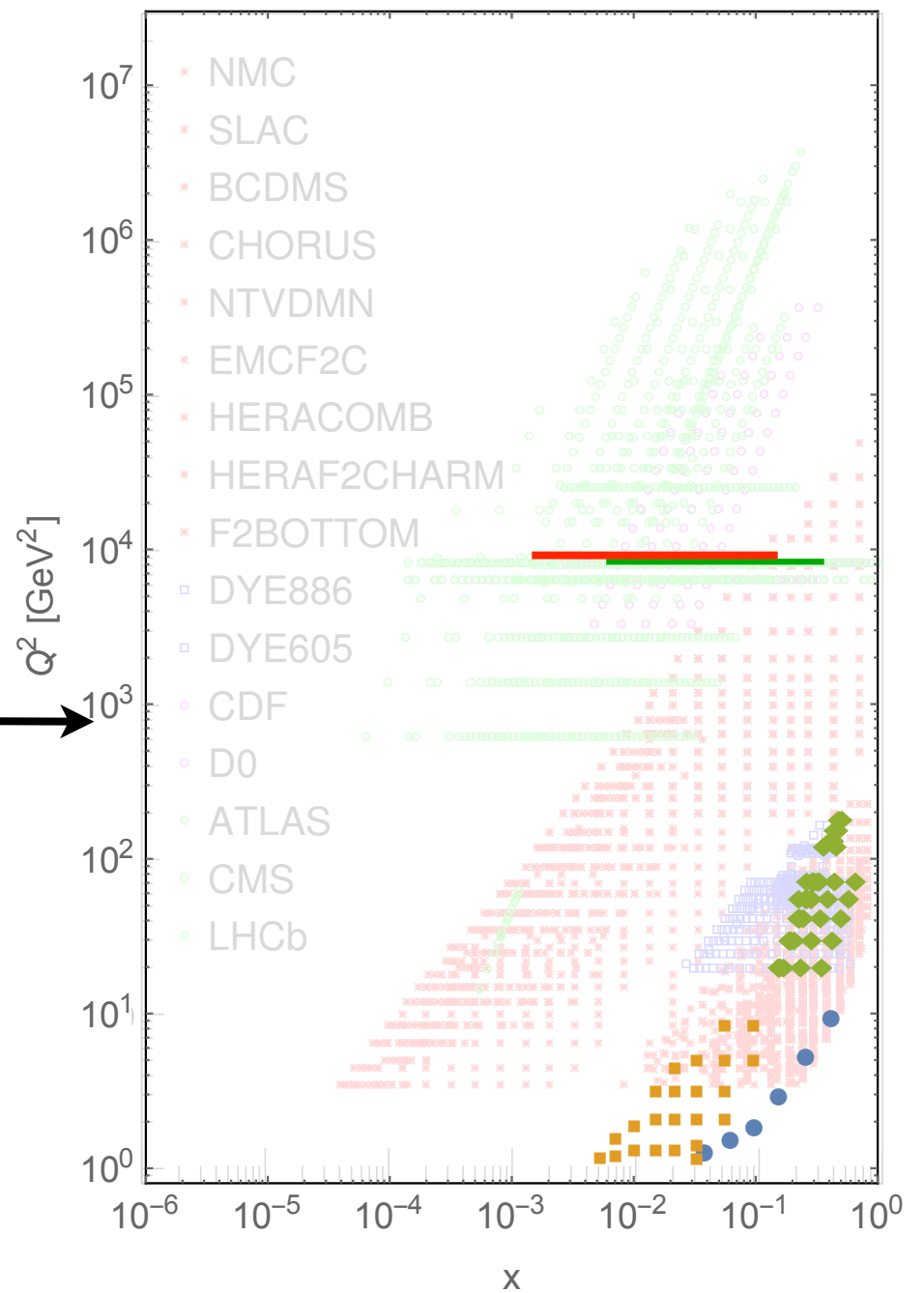


$Q$  : resolution of the probe

data driven science

data sets available:

← collinear PDFs  
vs  
TMD PDFs →



$x$  : momentum fraction carried by the parton



# Predictive power of TMDs

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## References :

- Parisi, Petronzio: Nucl. Phys. B154, 427 (1979)
- Collins, Soper, Sterman: Nucl. Phys. B250, 199 (1985)
- Qiu, Zhang: Phys. Rev. D63, 114011 (2001)
- Qiu, Berger: Phys. Rev. Lett. 91, 222003 (2003)
- Grewal, Kang, Qiu, **AS**: in preparation

# Evolution of TMDs

$$f_1^a(x, b_T^2, \mu_f, \zeta_f) = f_1^a(x, b_T^2, \mu_i, \zeta_i) \times \exp \left\{ \int_{\mu_i}^{\mu_f} \frac{d\mu}{\mu} \gamma_F \left[ \alpha_s(\mu), \frac{\zeta_f}{\mu^2} \right] \right\} \times \left( \frac{\zeta_f}{\zeta_i} \right)^{-K(b_T, \mu_i)}$$

two "evolution scales"

$b_T$ , Fourier conjugate of  $k_T$

evolution in  $\mu$   
 $\mu_i \rightarrow \mu_f$

evolution in  $\zeta$   
 $\zeta_i \rightarrow \zeta_f$

need correction  
at large  $b_T$

Input TMD distribution can be **expanded at low  $b_T$**  onto a basis of collinear distributions

$$f_1^a(x, b_T^2, \mu_i, \zeta_i) = \sum_b C_{a/b}(x, b_T^2, \mu_i, \zeta_i) \otimes f_b(x, \mu_i)$$

A sensible choice is to set the initial and final scale as:

$$\zeta_i = \mu_i^2 = 4e^{-2\gamma_E} / b_T^2 \equiv \mu_b^2$$

$$\zeta_f = \mu_f^2 = Q^2$$

# TMD PDF with large $b_T$ corrections

$$f_1^a(x, b_T^2; Q) = \begin{cases} f_1^a(x, b_T^2; Q) \\ f_1^a(x, b_{max}^2; Q) F^{NP}(x, b_T, Q; b_{max}) \end{cases}$$

$b_T \leq b_{max}$   
 $b_T > b_{max}$

Calculable in pQCD  
(modulo PDFs)

To be modeled and  
fit to data!

Which one is the most  
**relevant** part and in  
which **kinematic**  
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$$F^{NP}(x, b_T, Q; b_{max}) = \exp \left\{ \begin{aligned} & - \ln \left( \frac{Q^2 b_{max}^2}{c^2} \right) \{ g_1 [(b^2)^\alpha - (b_{max}^2)^\alpha] \} \\ & - \ln \left( \frac{Q^2 b_{max}^2}{c^2} \right) \{ g_2 (b^2 - b_{max}^2) \} \\ & - \bar{g}_2 (b^2 - b_{max}^2) \} \end{aligned} \right.$$

“extrapolation term”  
(see also Qiu-Zhang  
PRD63 114011)

$g_1, \alpha$   
fixed as a function of the other  
parameters, requiring continuity of the  
first and second derivatives

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Correction to evolution

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“extrapolation term”  
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PRD63 114011)

Correction to evolution

Correction to OPE at small  $b_T$   
(intrinsic transverse momentum)

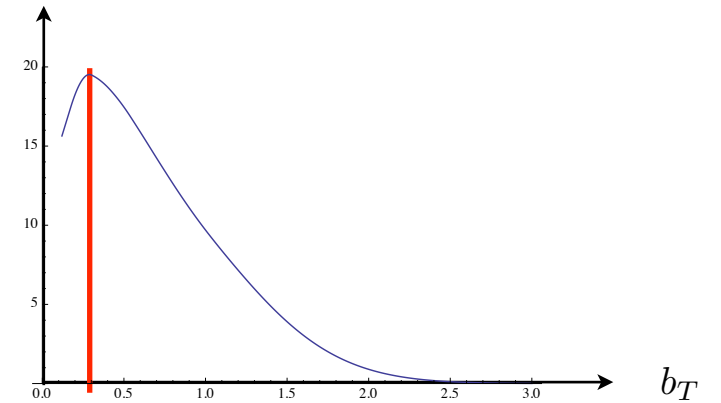
$g_1, \alpha$   
fixed as a function of the other  
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first and second derivatives

# Saddle point approximation

Given a generic function  $f \in C^2(a, b)$  and a positive constant  $A$

Given  $x_0$ , maximum in  $[a, b]$  for  $f$ :

$$I(x_0, A) = \int_a^b dx e^{Af(x)} = e^{Af(x_0)} \sqrt{\frac{2\pi}{A(-f''(x_0))}} \left(1 + \mathcal{O}\left(\frac{1}{A}\right)\right)$$



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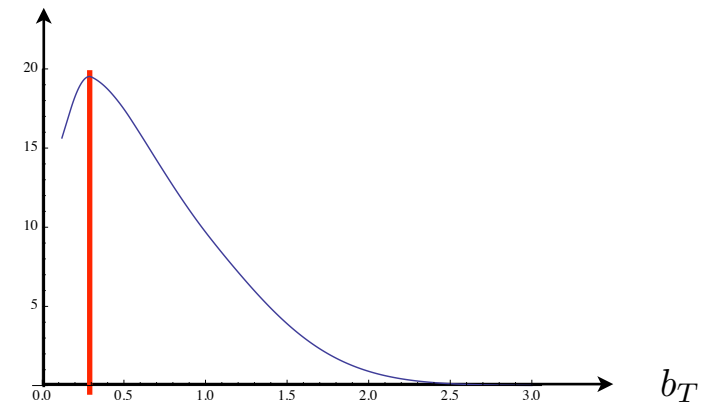
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Let's apply this to a TMD PDF evaluated at  $kT = 0$ :

$$f_1^a(x, k_T; \mu_f, \zeta_f) = \text{F.T.} [f_1^a(x, b_T; \mu_f, \zeta_f)]$$

$$f_1^a(x, k_T = 0; \mu_f, \zeta_f) = \frac{1}{4\pi} \int_{-\infty}^{+\infty} d(\ln b_T^2) \exp \left\{ \int_{\mu_i}^{\mu_f} \frac{d\mu}{\mu} \gamma_F \left[ \alpha_s(\mu), \frac{\zeta_f}{\mu^2} \right] - K(b_T, \mu_i) \ln \frac{\zeta_f}{\zeta_i} + \ln b_T^2 + \ln \left[ \sum_b C_{a/b} \otimes f_b \right] \right\}$$

To determine the **saddle point  $b_{Tsp}$  of the TMD PDF** we have to find the **stationary point of the exponent**





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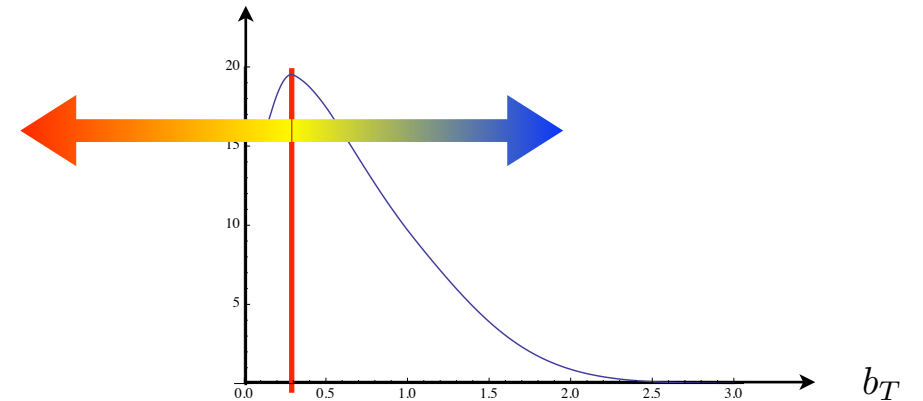
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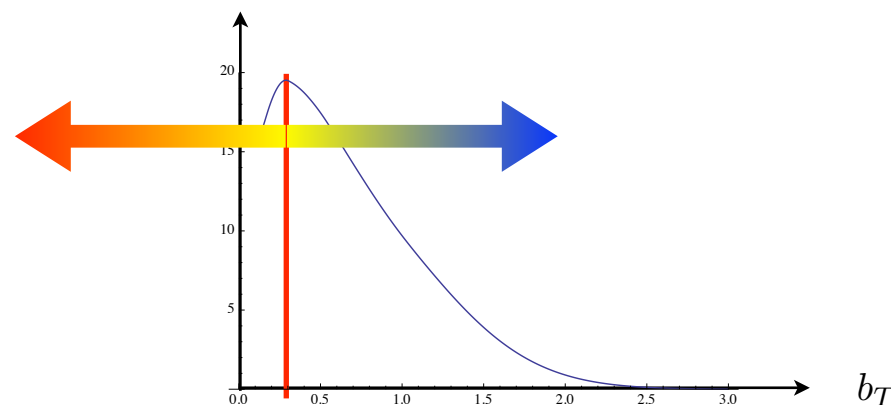
# Determination of the saddle point

$$\frac{d}{db_T} \left\{ \int_{\mu_i}^{\mu_f} \frac{d\mu}{\mu} \gamma_F \left[ \alpha_s(\mu), \frac{\zeta_f}{\mu^2} \right] - K(b_T, \mu_i) \ln \frac{\zeta_f}{\zeta_i} + \ln b_T^2 + \ln \left[ \sum_b C_{a/b}(x, b_T^2, \mu_i, \zeta_i) \otimes f_b(x, \mu_i) \right] \right\}_{b_T=b_T^{sp}} = 0$$

Generate the **Q-dependence** of the saddle point

Generate the **x-dependence** of the saddle point (new term)

The closer the **saddle point is to the small bT region**, the more the **TMD PDF is determined by the perturbative part** only and thus there is predictive power



At the same time, we can understand in which kinematic regions the **saddle point drifts towards the large bT region** ( $b_T > b_{max}$ ) and thus the **nonperturbative corrections become more important**

# Determination of the saddle point

$$\begin{aligned}
 & \frac{d}{db_T} \left\{ \int_{\mu_i}^{\mu_f} \frac{d\mu}{\mu} \gamma_F \left[ \alpha_s(\mu), \frac{\zeta_f}{\mu^2} \right] \right. \\
 & \quad - K(b_T, \mu_i) \ln \frac{\zeta_f}{\zeta_i} \\
 & \quad + \ln b_T^2 \\
 & \quad \left. + \ln \left[ \sum_b C_{a/b}(x, b_T^2, \mu_i, \zeta_i) \otimes f_b(x, \mu_i) \right] \right\}_{b_T=b_T^{sp}} = 0
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Generate the **Q-dependence** of the saddle point  
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**Parisi and Petronzio (1979) and Collins, Soper, Sterman (1982) :**  
 the same analysis at the level of the cross section, **neglecting the x-dependent part:**

# Determination of the saddle point

$$\frac{d}{db_T} \left\{ \int_{\mu_i}^{\mu_f} \frac{d\mu}{\mu} \gamma_F \left[ \alpha_s(\mu), \frac{\zeta_f}{\mu^2} \right] - K(b_T, \mu_i) \ln \frac{\zeta_f}{\zeta_i} + \ln b_T^2 + \ln \left[ \sum_b C_{a/b}(x, b_T^2, \mu_i, \zeta_i) \otimes f_b(x, \mu_i) \right] \right\}_{b_T=b_T^{sp}} = 0$$

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**Parisi and Petronzio (1979) and Collins, Soper, Sterman (1982) :**

the same analysis at the level of the cross section, **neglecting the x-dependent part:**

**Conclusion : the large bT corrections are more relevant at low Q**

Working at  $O(\alpha) + LL$  we can find the following solution :

$$b_T^{sp} \approx \frac{c}{\Lambda} \left( \frac{Q}{\Lambda} \right)^{-\Gamma_1^{cusp} / (\Gamma_1^{cusp} + 8\pi b_0)}$$

# Determination of the saddle point

$$\frac{d}{db_T} \left\{ \int_{\mu_i}^{\mu_f} \frac{d\mu}{\mu} \gamma_F \left[ \alpha_s(\mu), \frac{\zeta_f}{\mu^2} \right] - K(b_T, \mu_i) \ln \frac{\zeta_f}{\zeta_i} + \ln b_T^2 + \ln \left[ \sum_b C_{a/b}(x, b_T^2, \mu_i, \zeta_i) \otimes f_b(x, \mu_i) \right] \right\}_{b_T=b_T^{sp}} = 0$$

Generate the **Q-dependence** of the saddle point

Generate the **x-dependence** of the saddle point (new term)

**Qiu, Zhang** (2001) introduced the x-dependent term in the analysis at the level of the cross section.

**We** repeat the same at the level of the TMD PDF

# Determination of the saddle point

$$\frac{d}{db_T} \left\{ \int_{\mu_i}^{\mu_f} \frac{d\mu}{\mu} \gamma_F \left[ \alpha_s(\mu), \frac{\zeta_f}{\mu^2} \right] - K(b_T, \mu_i) \ln \frac{\zeta_f}{\zeta_i} + \ln b_T^2 + \ln \left[ \sum_b C_{a/b}(x, b_T^2, \mu_i, \zeta_i) \otimes f_b(x, \mu_i) \right] \right\}_{b_T=b_T^{sp}} = 0$$

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Generate the **x-dependence** of the saddle point (new term)

Working at  $O(\alpha) + LL$  we can find the following solution :

$$b_T^{sp} = \frac{c}{\Lambda} \left( \frac{Q}{\Lambda} \right)^{-\Gamma_1^{cusp} / \left[ \Gamma_1^{cusp} + 8\pi b_0 (1 - \mathcal{X}(x, \mu_b^*)) \right]}$$

$$\mathcal{X}(x, \mu) = \frac{d}{d \ln \mu^2} \ln f_a(x, \mu)$$

$$\mu_b^* = 2e^{-\gamma_E} / b_T^{sp}$$

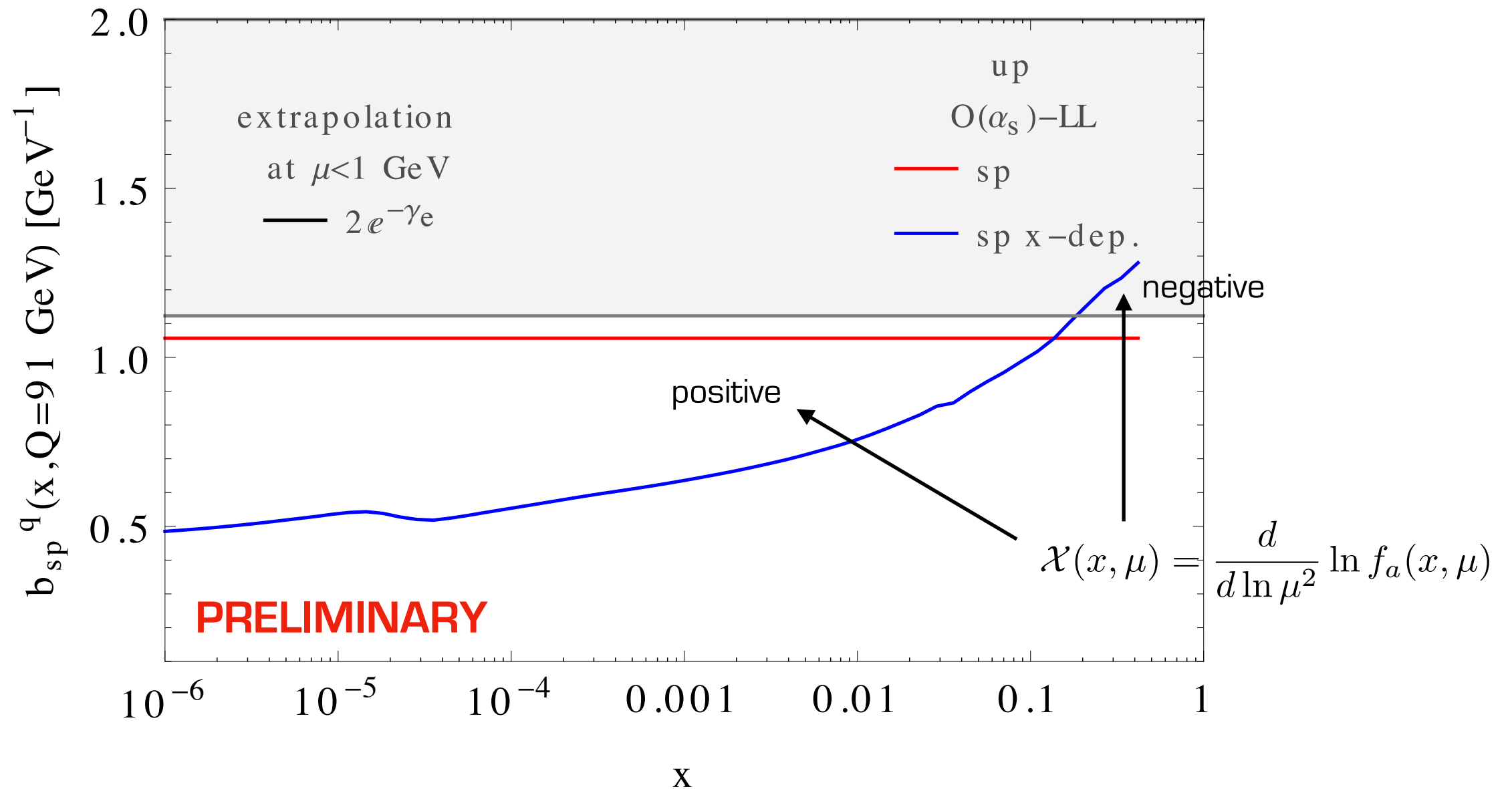
Requires iterative solution

**Qiu, Zhang** (2001) introduced the x-dependent term in the analysis at the level of the cross section.

**We** repeat the same at the level of the TMD PDF

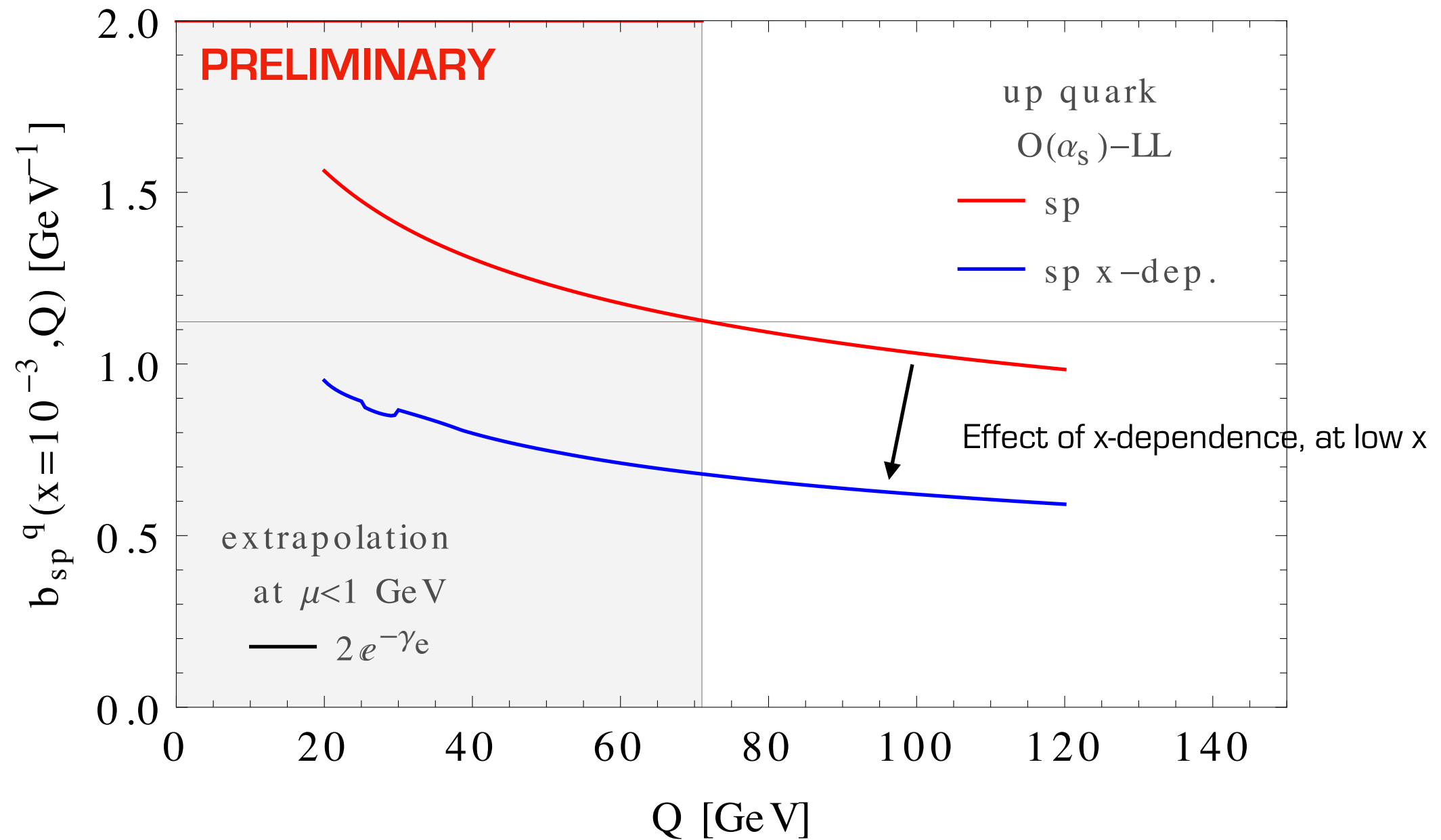
**Conclusion : the relevance of large  $b_T$  corrections is governed by both  $Q$  and  $x$ !**

# Determination of the saddle point



What happens if we include BFKL effects at very low  $x$ ?

# Determination of the saddle point

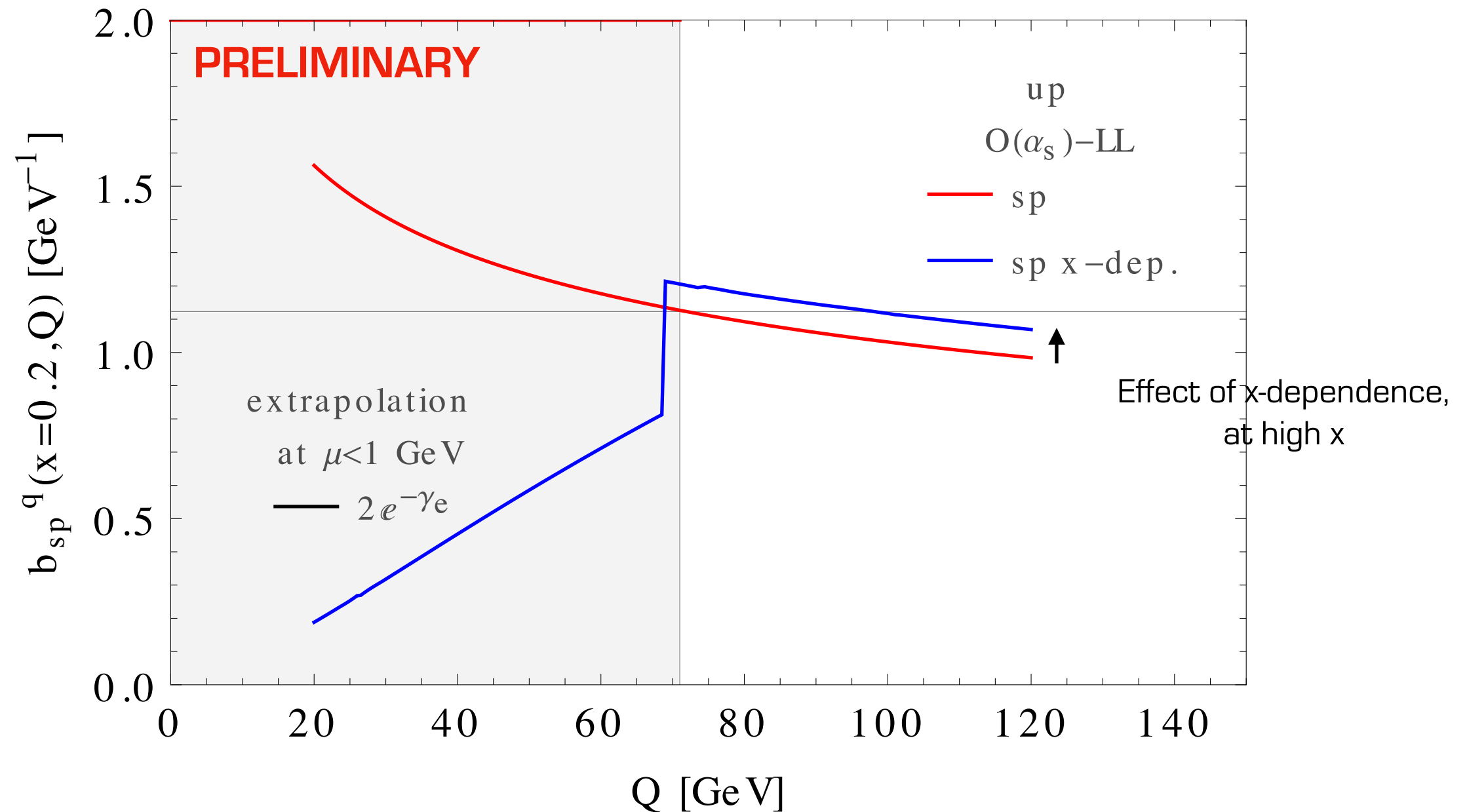


Fixed  $x = 0.001$ , change  $Q$

At **low x**, the x-dependent solution is **reduced** uniformly with respect to the CSS-like solution



# Determination of the saddle point



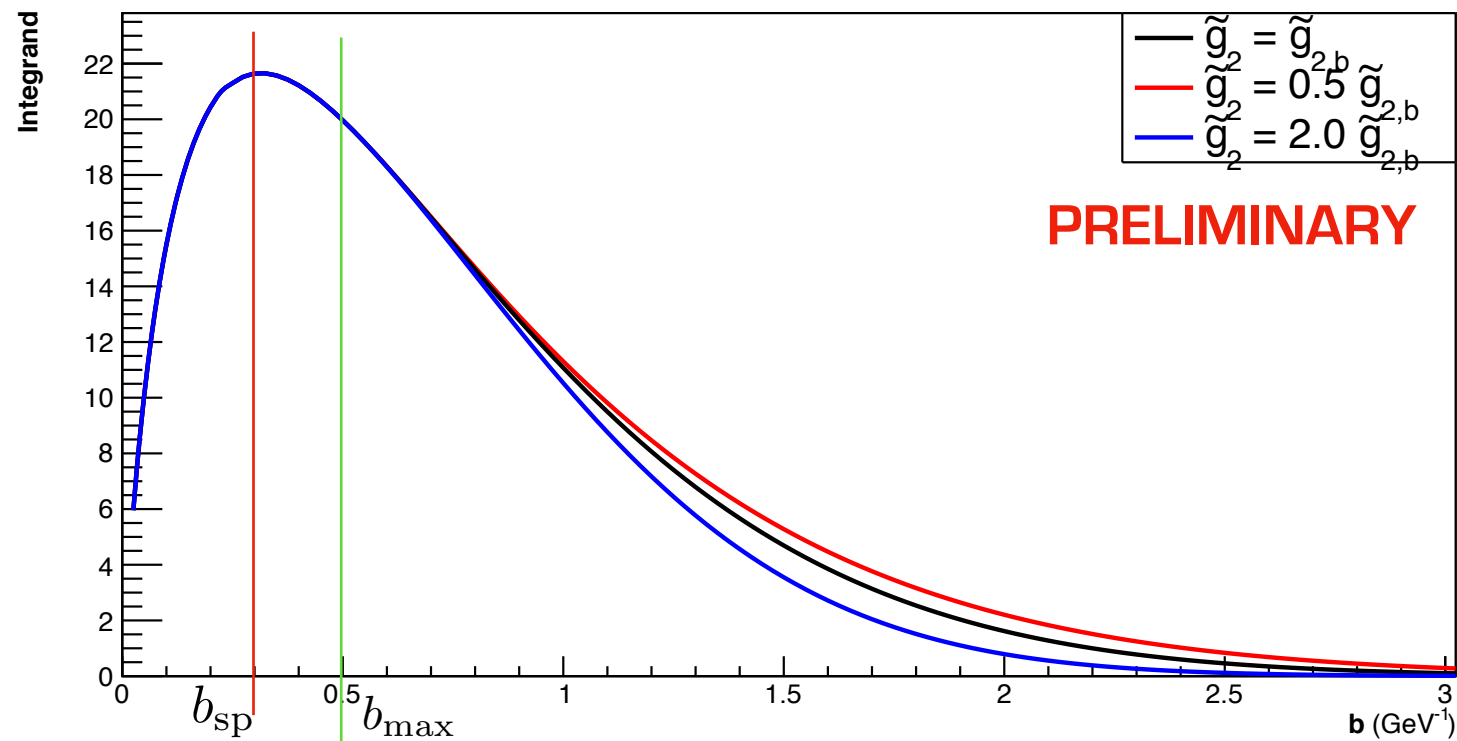
Fixed  $x = 0.2$ , change  $Q$

At **high x**, the x-dependent solution is **enhanced** uniformly with respect to the CSS-like solution

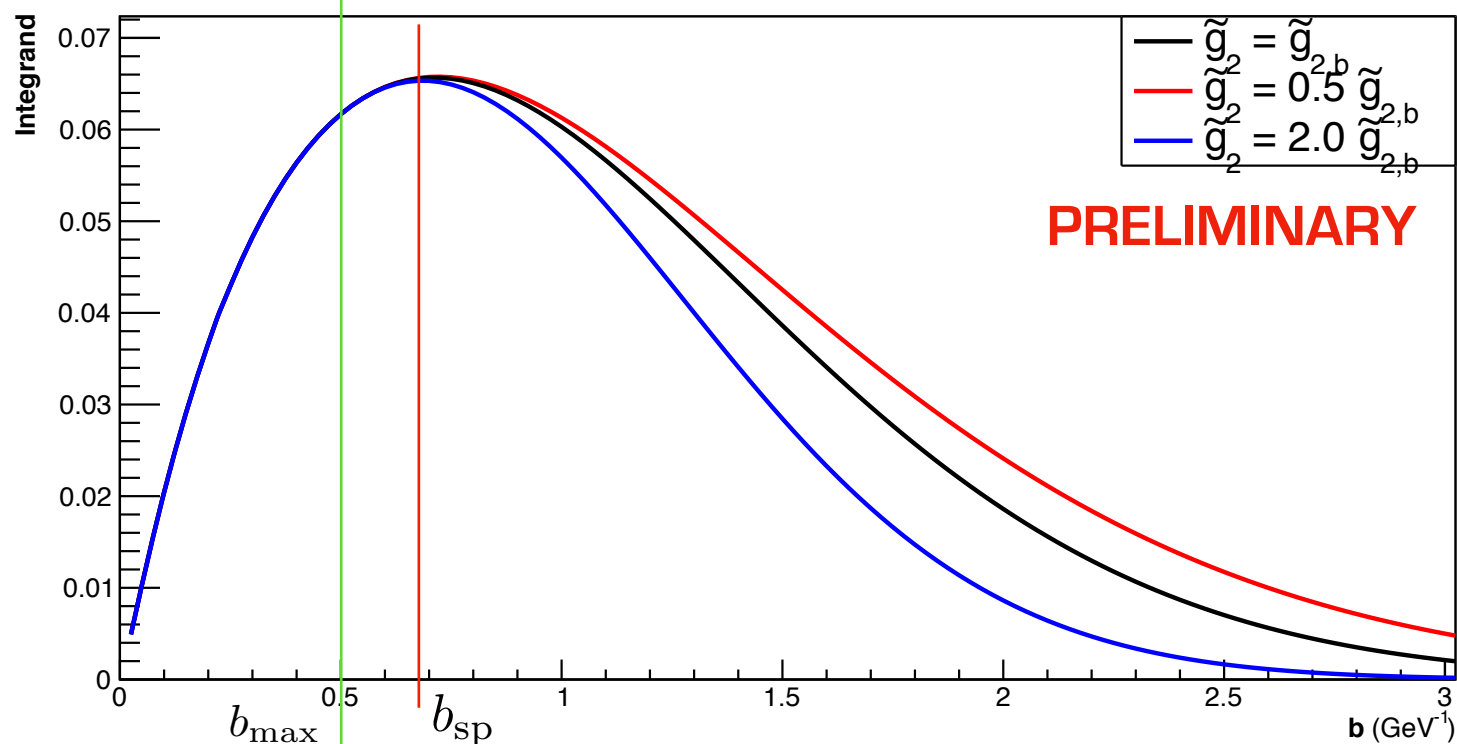
# Quark TMD PDFs

NNLO and NNLL

$Q = M_Z, x = 10^{-3}, k_T = 0.0 \text{ GeV}$



$Q = M_Z, x = 0.3, k_T = 0.0 \text{ GeV}$



Predictive power is maximum at large  $Q$  and small  $x$ .

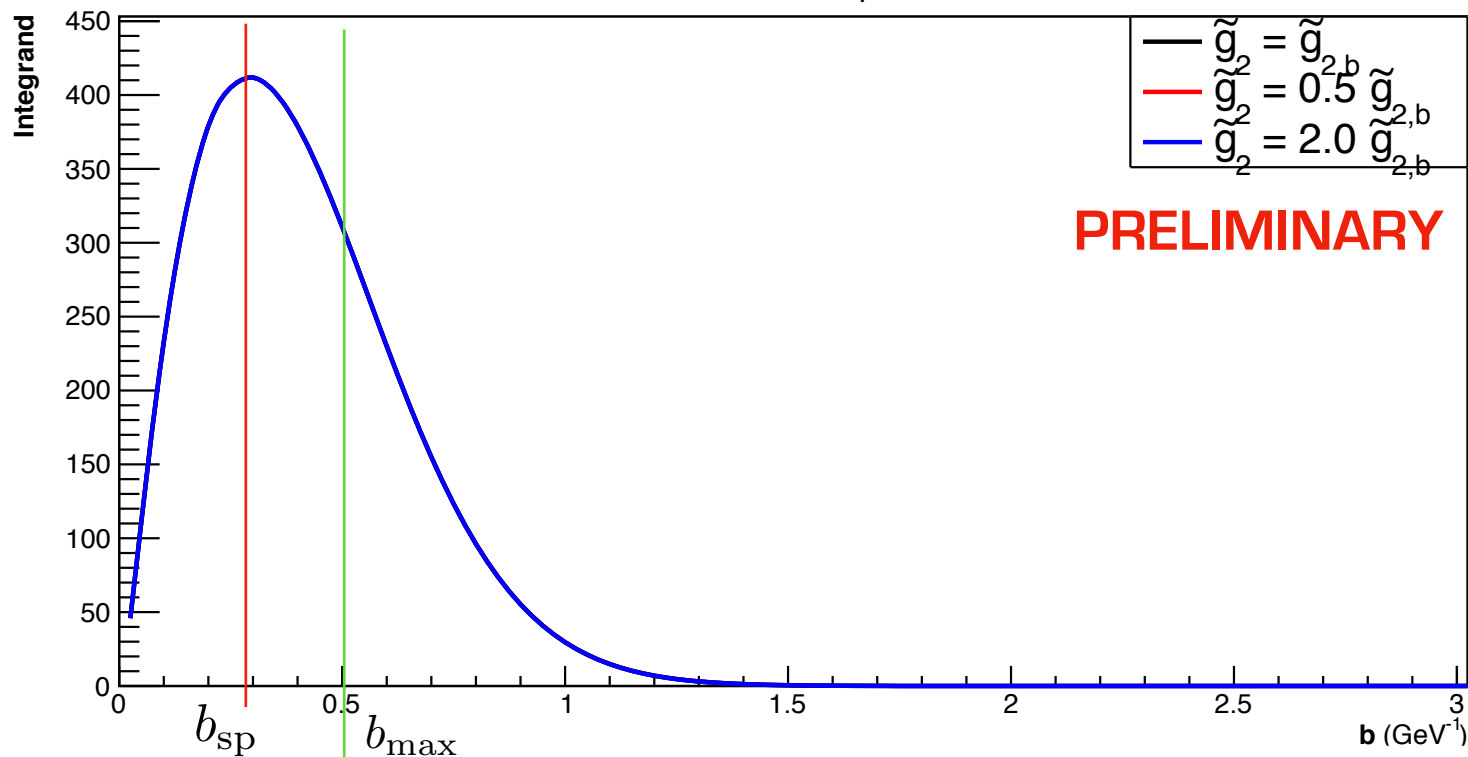
**But  $Q$  alone does not provide the full picture:**

at higher  $x$  the saddle point drifts towards the large  $b_T$  region and the nonperturbative corrections have a bigger impact on the TMD PDF

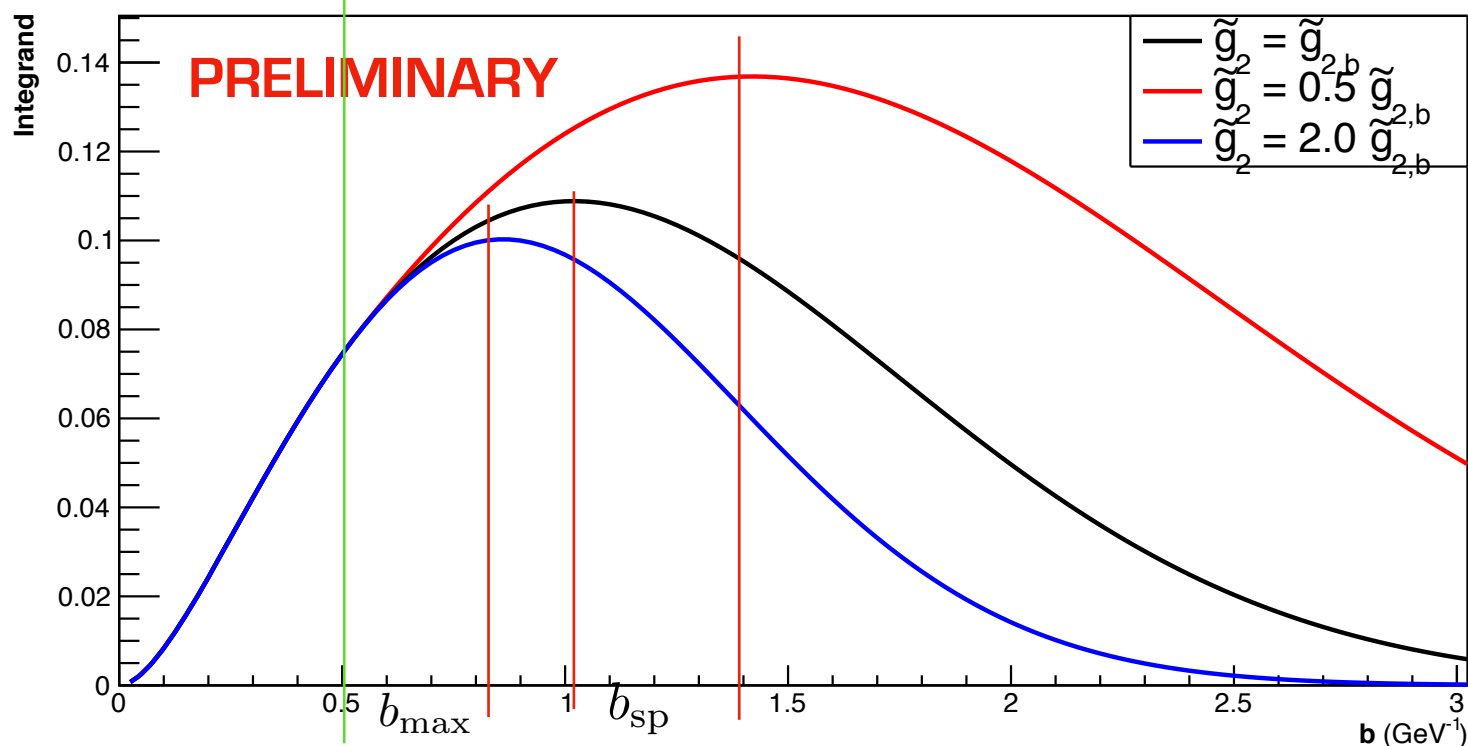
# Gluon TMD PDFs

NNLO and NNLL

$Q = M_Y, x = 10^{-3}, k_T = 0.0 \text{ GeV}$



$Q = M_Y, x = 0.3, k_T = 0.0 \text{ GeV}$



For **gluons the evolution is stronger** and the NP is less relevant already at lower  $Q$ .

Predictive power is maximum at large  $Q$  and small  $x$ .

**But  $Q$  alone does not provide the full picture:**

at higher  $x$  the saddle point drifts towards the large  $b_T$  region and the nonperturbative corrections have a bigger impact on the TMD PDF

# Partial summary

---

The **predictive power of TMD PDFs** is maximum in the **high  $Q$  and small- $x$**  corner of the phase space (e.g. VB production at the LHC)

On the contrary, the relevance of the large  $bT$  corrections is maximum at low  $Q$  and high  $x$

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An example:  $W$  boson production at central rapidity at RHIC

We should also understand what happens in the region of high  $Q$  and small  $x$  (for example  $W$  boson production at the LHC), where the relevance should be “minimal”

**How small is “minimal” ?**

# W production at LHC

---

## References:

- Bozzi, Rojo, Vicini: Phys.Rev. D83 (2011) 113008
- Bozzi, Citelli, Vicini: Phys.Rev. D91 (2015) no.11, 113005
- AS, [PhD thesis](#)
- Bacchetta, Bozzi, Radici, Mulders, Ritzmann, AS - in preparation

# EW precision measurements

Eur.Phys.J. C74 (2014) 3046

Precise measurements of electroweak quantities allow:

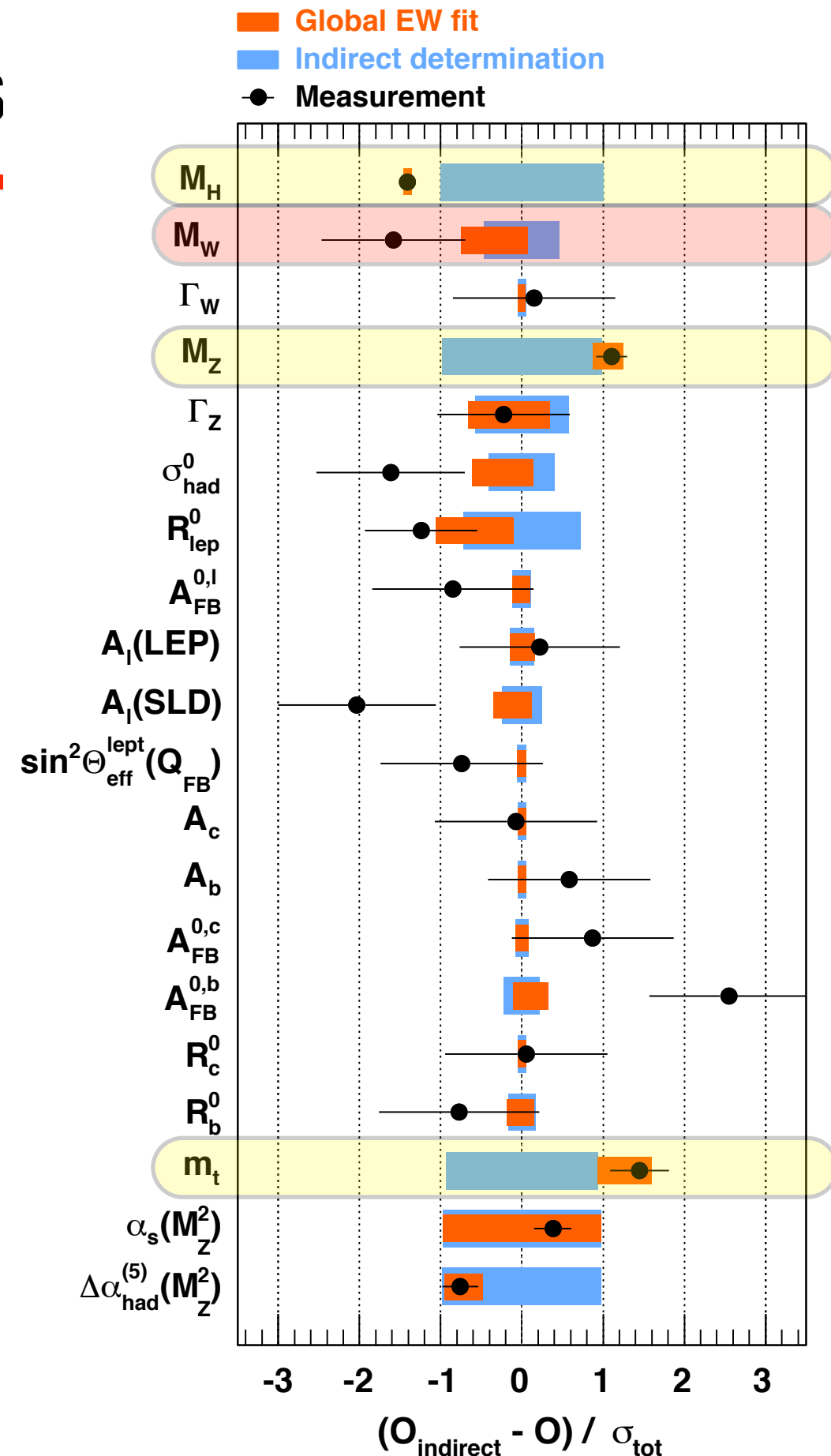
- 1) Stringent **tests** of the self consistency of the SM
- 2) Looking for hints of physics **beyond** the SM

In particular the values of the **masses** of the gauge bosons, the Higgs and the top quark can help in **discriminating among different BSM scenarios**.

H, Z, t : direct determinations more precise than indirect;  
**not for W !**

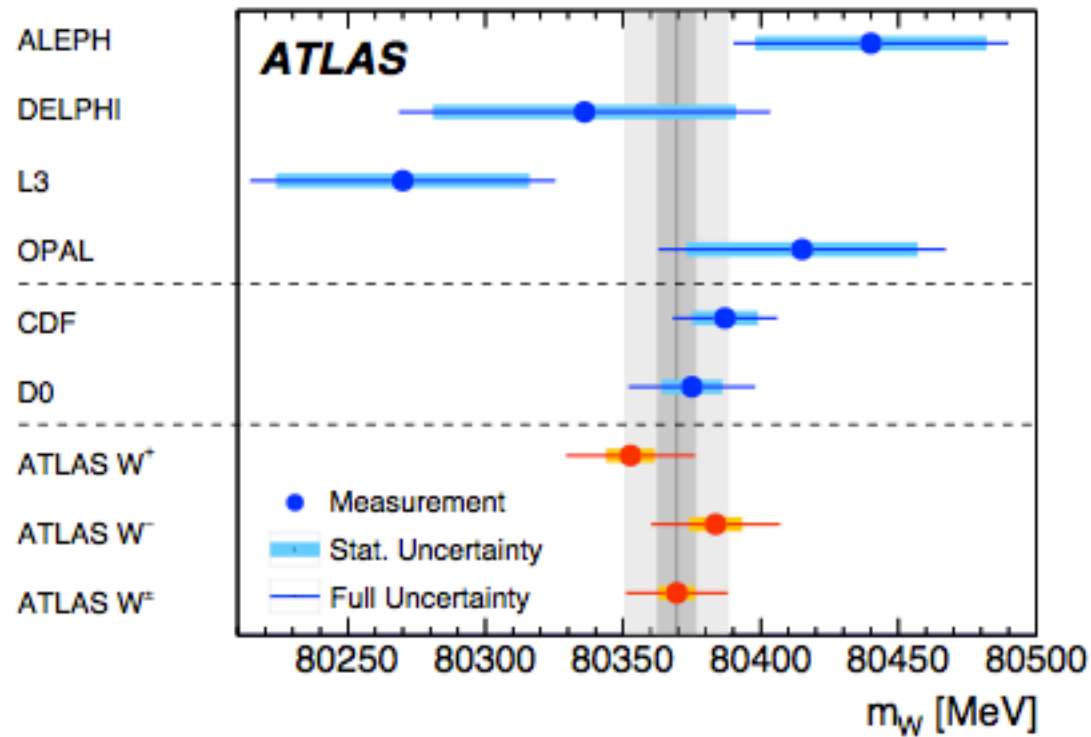
see:

\* S. Camarda - Measurement of the W mass with ATLAS  
EPS 2017



# W mass

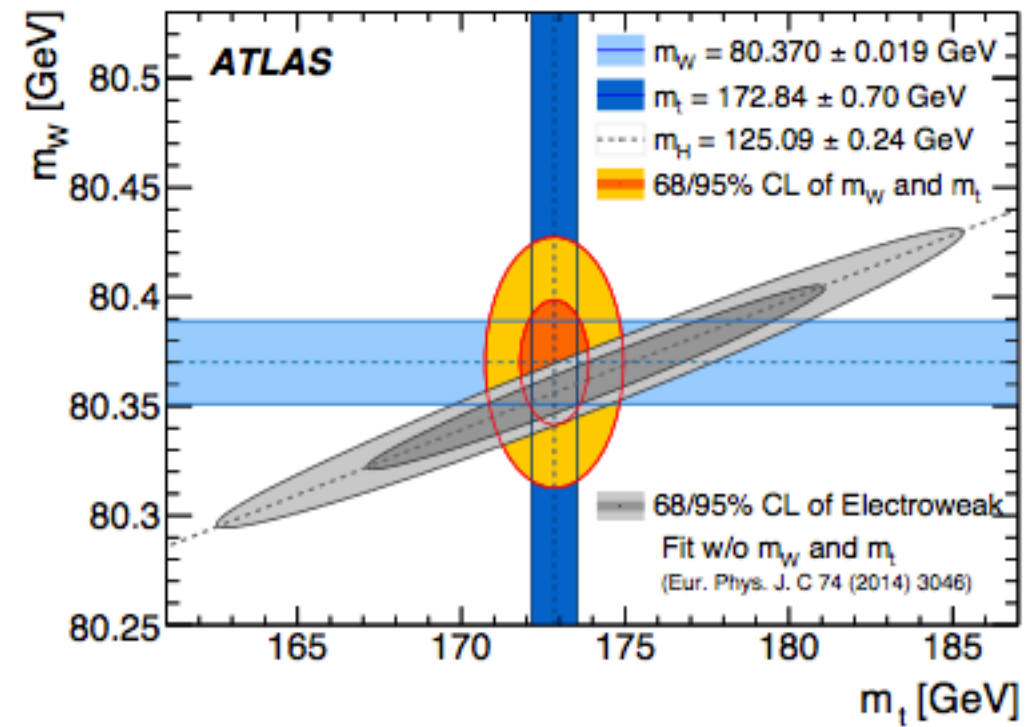
ATLAS, arxiv:1701.07240



## Experimental measurements

$$m_W = 80370 \pm 19 \text{ MeV}$$

(7 stat, 11 exp, 14 th)



## Global EW fit

$$m_W = 80356 \pm 8 \text{ MeV}$$

Need to **better control the uncertainties** associated to **direct** determinations of  $m_W$

Is it possible to reduce the uncertainty to less than 10 MeV ?

Are we estimating all the **uncertainties of hadronic nature** in the best way possible?

# Uncertainties on $W$ mass

Uncertainties on  $m_W$  [MeV] from  $p_T^\ell$  fit

| Source                   | $W \rightarrow \mu\nu$ | $W \rightarrow e\nu$ | Common |
|--------------------------|------------------------|----------------------|--------|
| Lepton energy scale      | 7                      | 10                   | 5      |
| Lepton energy resolution | 1                      | 4                    | 0      |
| Lepton efficiency        | 1                      | 2                    | 0      |
| Lepton tower removal     | 0                      | 0                    | 0      |
| Recoil scale             | 6                      | 6                    | 6      |
| Recoil resolution        | 5                      | 5                    | 5      |
| Backgrounds              | 5                      | 3                    | 0      |
| PDFs                     | 9                      | 9                    | 9      |
| $W$ boson $q_T$          | 9                      | 9                    | 9      |
| Photon radiation         | 4                      | 4                    | 4      |
| Statistical              | 18                     | 21                   | 0      |
| Total                    | 25                     | 28                   | 16     |

Tevatron case

sizable uncertainties from hadron structure

associated to  $\alpha_s$  and NP evolution; no intrinsic transverse momentum

ATLAS (arxiv:1701.07240)

| Combined categories            | Value [MeV] | Stat. Unc. | Muon Unc. | Elec. Unc. | Recoil Unc. | Bckg. Unc. | QCD Unc. | EW Unc. | PDF Unc. | Total Unc. |
|--------------------------------|-------------|------------|-----------|------------|-------------|------------|----------|---------|----------|------------|
| $m_{T-p_T^\ell}, W^\pm, e-\mu$ | 80369.5     | 6.8        | 6.6       | 6.4        | 2.9         | 4.5        | 8.3      | 5.5     | 9.2      | 18.5       |



# The extraction of physical quantities

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## Observables

- accessible via **counting experiments**: cross sections and asymmetries

## Pseudo-Observables

- functions of cross sections and symmetries
- **require a model** to be properly defined
  - $M_Z$  at LEP as pole of the Breit-Wigner resonance factor
  - $M_W$  at hadron colliders as fitting parameter of a template fit procedure (of  $m_T$ ,  $p_{Tlep}$ ,  $p_{Tmiss}$ )

## Template fit

1. generate several histograms with the highest available theoretical accuracy and degree of realism in the detector simulation, and let the fit parameter (e.g.  $M_W$ ) vary in a range
2. the histogram that best describes data selects the preferred (i.e. measured)  $M_W$

- the result of the fit depends on the **hypotheses used to compute the templates** (PDFs, scales, non-perturbative, different prescriptions, ...)
- these hypotheses **should be treated as theoretical systematic errors**

# $p_{TW}$ and the modelling of intrinsic- $k_T$

- $p_{Tl} \Leftrightarrow p_{TW} \Leftrightarrow$  QCD initial state radiation + intrinsic  $k_T$  (usually, a Gaussian in  $k_T$ )
- PDF uncertainties and  $k_T$ -modelling entangled  
 $\Rightarrow$  no universal (flavour-independent) model
- Intrinsic  $k_T$  effects measured on  $Z$  data and used to predict  $W$  distributions, *assuming universality* Konychev, Nadolsky, PLB 633, 710 (2006)

but

*different flavour structure*

*different phase space available*

$\rightarrow$  *different Gaussian factors for different flavors*

$$f_1^a(x, k_T) = f_1^a(x) \frac{1}{\pi \langle k_T^2 \rangle_a(x)} e^{-\frac{k_T^2}{\langle k_T^2 \rangle_a(x)}}$$

$$\langle k_{\perp, u_v}^2 \rangle \neq \langle k_{\perp, d_v}^2 \rangle \neq \langle k_{\perp, sea}^2 \rangle$$

Flavor and kinematic  
dependent widths

ab

# Impact on $m_W$ : preliminary results

- Select 15 *flavour-dependent* NP sets for which  $\Delta(Z \text{ peak}) < 100 \text{ MeV}$  and compute *low-statistics*  $m_T$  and  $p_{Tl}$  distributions

➔ these are our **pseudodata**

- Select a universal (*flavour-independent*) NP parameter and compute *high-statistics*  $m_T$  and  $p_{Tl}$  distributions for 30 different values of  $M_W$

➔ these are our **templates**

- **perform the template fit procedure and compute the shifts induced by flavour effects**

- transverse mass: few MeV shifts, generally favouring lower values (**preferred by EW fit**)

- lepton pt & missing pt: quite important shifts (envelope: **21 MeV**)

| transverse mass |              | lepton pt |              | missing pt |              |
|-----------------|--------------|-----------|--------------|------------|--------------|
| Set             | $\Delta M_W$ | Set       | $\Delta M_W$ | Set        | $\Delta M_W$ |
| 1               | -3           | 1         | 2            | 1          | -6           |
| 2               | -3           | 2         | 2            | 2          | -6           |
| 3               | -1           | 3         | 2            | 3          | -3           |
| 4               | -1           | 4         | -4           | 4          | -13          |
| 5               | -3           | 5         | -11          | 5          | -15          |
| 6               | -1           | 6         | -4           | 6          | -13          |
| 7               | -3           | 7         | -14          | 7          | -15          |
| 8               | -2           | 8         | 1            | 8          | -4           |
| 9               | -2           | 9         | -15          | 9          | -15          |
| 10              | -1           | 10        | 5            | 10         | 1            |
| 11              | -3           | 11        | 1            | 11         | -4           |
| 12              | -2           | 12        | -1           | 12         | -4           |
| 13              | -2           | 13        | 6            | 13         | -5           |
| 14              | -3           | 14        | -3           | 14         | -10          |
| 15              | -3           | 15        | 0            | 15         | -6           |

NLL+LO QCD analysis obtained through a modified version of the **DYRes** code [Catani, deFlorian, Ferrera, Grazzini, JHEP 1512, 047 (2015)]

# Conclusions

---

The **predictive power of TMD PDFs** is maximum in the **high  $Q$  and small- $x$**  corner of the phase space (e.g. VB production at the LHC)

On the contrary, the relevance of the large  $b_T$  corrections is maximum at low  $Q$  and high  $x$

- the “**sweet spot**” to study the **nonperturbative contributions** to TMD PDFs (what usually we fit to data) could be the region at **high  $Q$**  (to well control the corrections to factorization) and **high  $x$**  (to enhance the sensitivity to the large  $b_T$  region) (but beware of thresholds effects, etc.);

An example:  $W$  boson production at central rapidity at RHIC

We should also understand what happens in the region of high  $Q$  and small  $x$  (for example  $W$  boson production at the LHC), where the relevance should be “minimal”

**How small is “minimal” ? “Minimal” is non-negligible!!**

# Backup

---

# Status of TMD phenomenology

Theory, data, fits : we are in a position to start validating the formalism

quark pol.

|              |   |                |                        |
|--------------|---|----------------|------------------------|
|              | U | L              | T                      |
| nucleon pol. | U | $f_1$          | $h_1^\perp$            |
|              | L |                | $h_{1L}^\perp$         |
|              | T | $f_{1T}^\perp$ | $g_{1T}, h_{1T}^\perp$ |

Twist-2 TMDs

Only first attempts

Limited data, theory, fits

see, e.g, Bacchetta, Radici, arXiv:1107.5755  
 Anselmino, Boglione, Melis, PRD86 (12)  
 Echevarria, Idilbi, Kang, Vitev, PRD 89 (14)  
 Anselmino, Boglione, D'Alesio, Murgia, Prokudin, arXiv:  
 1612.06413  
 Anselmino et al., PRD87 (13)  
 Kang et al. arXiv:1505.05589

Lu, Ma, Schmidt, arXiv:0912.2031  
 Lefky, Prokudin arXiv:1411.0580  
 Barone, Boglione, Gonzalez, Melis,  
 arXiv:1502.04214

# Structure functions

$$\ell P \rightarrow \ell' h X$$

$$\begin{aligned}
 \frac{d\sigma}{dx dy d\psi dz d\phi_h dP_{h\perp}^2} = & \frac{\alpha^2}{xyQ^2} \frac{y^2}{2(1-\varepsilon)} \left(1 + \frac{\gamma^2}{2x}\right) \left\{ F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\varepsilon(1+\varepsilon)} \cos\phi_h F_{UU}^{\cos\phi_h} \right. \\
 & + \varepsilon \cos(2\phi_h) F_{UU}^{\cos 2\phi_h} + \lambda_e \sqrt{2\varepsilon(1-\varepsilon)} \sin\phi_h F_{LU}^{\sin\phi_h} \\
 & + S_{\parallel} \left[ \sqrt{2\varepsilon(1+\varepsilon)} \sin\phi_h F_{UL}^{\sin\phi_h} + \varepsilon \sin(2\phi_h) F_{UL}^{\sin 2\phi_h} \right] \\
 & + S_{\parallel} \lambda_e \left[ \sqrt{1-\varepsilon^2} F_{LL} + \sqrt{2\varepsilon(1-\varepsilon)} \cos\phi_h F_{LL}^{\cos\phi_h} \right] \\
 & + |S_{\perp}| \left[ \sin(\phi_h - \phi_S) \left( F_{UT,T}^{\sin(\phi_h - \phi_S)} + \varepsilon F_{UT,L}^{\sin(\phi_h - \phi_S)} \right) \right. \\
 & + \varepsilon \sin(\phi_h + \phi_S) F_{UT}^{\sin(\phi_h + \phi_S)} + \varepsilon \sin(3\phi_h - \phi_S) F_{UT}^{\sin(3\phi_h - \phi_S)} \\
 & \left. + \sqrt{2\varepsilon(1+\varepsilon)} \sin\phi_S F_{UT}^{\sin\phi_S} + \sqrt{2\varepsilon(1+\varepsilon)} \sin(2\phi_h - \phi_S) F_{UT}^{\sin(2\phi_h - \phi_S)} \right] \\
 & + |S_{\perp}| \lambda_e \left[ \sqrt{1-\varepsilon^2} \cos(\phi_h - \phi_S) F_{LT}^{\cos(\phi_h - \phi_S)} + \sqrt{2\varepsilon(1-\varepsilon)} \cos\phi_S F_{LT}^{\cos\phi_S} \right. \\
 & \left. + \sqrt{2\varepsilon(1-\varepsilon)} \cos(2\phi_h - \phi_S) F_{LT}^{\cos(2\phi_h - \phi_S)} \right] \left. \right\}, \tag{2.7}
 \end{aligned}$$

Cross section  
expanded in terms of  
**structure functions**

criterion:  
Lorentz symmetry

indexes = **polarization** state  
first: lepton  
second: hadron  
(third: photon)

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 & + S_{\parallel} \lambda_e \left[ \sqrt{1-\varepsilon^2} F_{LL} + \sqrt{2\varepsilon(1-\varepsilon)} \cos\phi_h F_{LL}^{\cos\phi_h} \right] \\
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 & + \varepsilon \sin(\phi_h + \phi_S) F_{UT}^{\sin(\phi_h + \phi_S)} + \varepsilon \sin(3\phi_h - \phi_S) F_{UT}^{\sin(3\phi_h - \phi_S)} \\
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 & + |S_{\perp}| \lambda_e \left[ \sqrt{1-\varepsilon^2} \cos(\phi_h - \phi_S) F_{LT}^{\cos(\phi_h - \phi_S)} + \sqrt{2\varepsilon(1-\varepsilon)} \cos\phi_S F_{LT}^{\cos\phi_S} \right. \\
 & \left. + \sqrt{2\varepsilon(1-\varepsilon)} \cos(2\phi_h - \phi_S) F_{LT}^{\cos(2\phi_h - \phi_S)} \right] \left. \right\}, \tag{2.7}
 \end{aligned}$$

Cross section expanded in terms of **structure functions**

For each:  
different factorization theorems  
at low and high transverse  
momentum

How to match these?



# Structure functions

$$\ell P \rightarrow \ell' h X$$

$$\begin{aligned}
 \frac{d\sigma}{dx dy d\psi dz d\phi_h dP_{h\perp}^2} = & \frac{\alpha^2}{xyQ^2} \frac{y^2}{2(1-\varepsilon)} \left(1 + \frac{\gamma^2}{2x}\right) \left\{ F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\varepsilon(1+\varepsilon)} \cos\phi_h F_{UU}^{\cos\phi_h} \right. \\
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 \end{aligned}$$

structure functions :  
**convolutions of  
 TMD PDFs and FFs**

|              |            |                  |                       |
|--------------|------------|------------------|-----------------------|
|              | quark pol. |                  |                       |
|              | U          | L                | T                     |
| nucleon pol. | U          | $f_1$            | $h_1^{\perp}$         |
|              | L          |                  | $h_{1L}^{\perp}$      |
|              | T          | $f_{1T}^{\perp}$ | $h_1, h_{1T}^{\perp}$ |

Twist-2 TMDs

**+ higher-twist  
 contributions**

# Structure functions

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|              | T | $f_{1T}^{\perp}$ | $g_{1T}$ | $h_1, h_{1T}^{\perp}$ |
|              |   |                  |          |                       |

Twist-2 TMDs

**+ higher-twist  
contributions**

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e(x) - twist 3

$F_{LU}^{\sin\phi_h}$

structure functions :  
**convolutions of  
TMD PDFs and FFs**

quark pol.

|   |                  |          |                       |
|---|------------------|----------|-----------------------|
|   | U                | L        | T                     |
| U | $f_1$            |          | $h_1^{\perp}$         |
| L |                  | $g_{1L}$ | $h_{1L}^{\perp}$      |
| T | $f_{1T}^{\perp}$ | $g_{1T}$ | $h_1, h_{1T}^{\perp}$ |

nucleon pol.

Twist-2 TMDs

**+ higher-twist  
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structure functions :  
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|              |   | quark pol.       |          |                       |
|--------------|---|------------------|----------|-----------------------|
|              |   | U                | L        | T                     |
| nucleon pol. | U | $f_1$            |          | $h_1^{\perp}$         |
|              | L |                  | $g_{1L}$ | $h_{1L}^{\perp}$      |
|              | T | $f_{1T}^{\perp}$ | $g_{1T}$ | $h_1, h_{1T}^{\perp}$ |

Twist-2 TMDs

**+ higher-twist  
contributions**

# Structure functions

$$\ell P \rightarrow \ell' h X$$

$$\begin{aligned}
 \frac{d\sigma}{dx dy d\psi dz d\phi_h dP_{h\perp}^2} = & \frac{\alpha^2}{xyQ^2} \frac{y^2}{2(1-\varepsilon)} \left(1 + \frac{\gamma^2}{2x}\right) \left\{ F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\varepsilon(1+\varepsilon)} \cos\phi_h F_{UU}^{\cos\phi_h} \right. \\
 & + \varepsilon \cos(2\phi_h) F_{UU}^{\cos 2\phi_h} + \lambda_e \sqrt{2\varepsilon(1-\varepsilon)} \sin\phi_h F_{LU}^{\sin\phi_h} \\
 & + S_{\parallel} \left[ \sqrt{2\varepsilon(1+\varepsilon)} \sin\phi_h F_{UL}^{\sin\phi_h} + \varepsilon \sin(2\phi_h) F_{UL}^{\sin 2\phi_h} \right] \\
 & + S_{\parallel} \lambda_e \left[ \sqrt{1-\varepsilon^2} F_{LL} + \sqrt{2\varepsilon(1-\varepsilon)} \cos\phi_h F_{LL}^{\cos\phi_h} \right] \\
 & + |S_{\perp}| \left[ \sin(\phi_h - \phi_S) \left( F_{UT,T}^{\sin(\phi_h - \phi_S)} + \varepsilon F_{UT,L}^{\sin(\phi_h - \phi_S)} \right) \right. \\
 & + \varepsilon \sin(\phi_h + \phi_S) F_{UT}^{\sin(\phi_h + \phi_S)} + \varepsilon \sin(3\phi_h - \phi_S) F_{UT}^{\sin(3\phi_h - \phi_S)} \\
 & \left. + \sqrt{2\varepsilon(1+\varepsilon)} \sin\phi_S F_{UT}^{\sin\phi_S} + \sqrt{2\varepsilon(1+\varepsilon)} \sin(2\phi_h - \phi_S) F_{UT}^{\sin(2\phi_h - \phi_S)} \right] \\
 & + |S_{\perp}| \lambda_e \left[ \sqrt{1-\varepsilon^2} \cos(\phi_h - \phi_S) F_{LT}^{\cos(\phi_h - \phi_S)} + \sqrt{2\varepsilon(1-\varepsilon)} \cos\phi_S F_{LT}^{\cos\phi_S} \right. \\
 & \left. + \sqrt{2\varepsilon(1-\varepsilon)} \cos(2\phi_h - \phi_S) F_{LT}^{\cos(2\phi_h - \phi_S)} \right] \left. \right\}, \tag{2.7}
 \end{aligned}$$

structure functions :  
**convolutions of  
TMD PDFs and FFs**

|              |            |                  |                       |
|--------------|------------|------------------|-----------------------|
|              | quark pol. |                  |                       |
|              | U          | L                | T                     |
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|              | L          |                  | $h_{1L}^{\perp}$      |
|              | T          | $f_{1T}^{\perp}$ | $h_1, h_{1T}^{\perp}$ |
|              |            | $g_{1L}$         |                       |

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|              |   |                  |                       |
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pretzelosity

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Twist-2 TMDs

**+ higher-twist  
contributions**

# More motivations

---

$f_1$

**unpolarized TMD PDF:**

- test of factorization formalism
- improve our description of qT spectra (e.g. at **W at LHC**)
- baseline to extract polarized TMDs from asymmetries

$e$

**collinear twist 3 PDF  $e(x)$ :**

- insights in quark-gluon-quark correlations
- scalar charge of the nucleon

$h_1^\perp$  ,  $f_{1T}^\perp$

**T-odd Boer-Mulders and Sivers TMD PDFs:**

- rigorous tests of the symmetry properties of QCD  
(sign change between SIDIS and Drell-Yan)

$h_1$

**transversity (TMD) PDF:**

- access to the tensor charge of the nucleon
  - window on BSM physics
  - also accessible via jets ?



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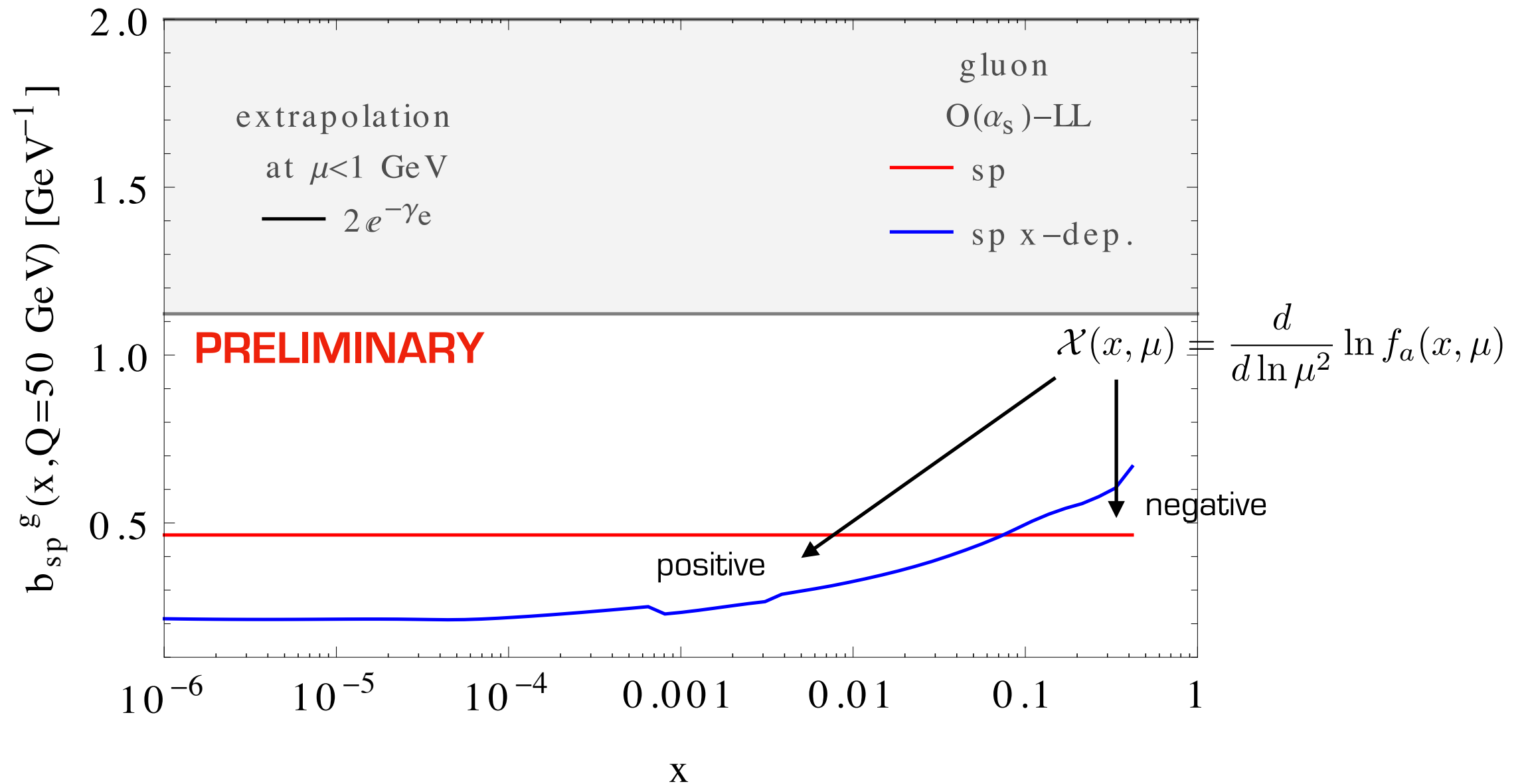
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$h_{1LT}$

**collinear spin-1 function:**

- another rigorous test of QCD symmetries
- T-odd effects in **spin-1** hadrons

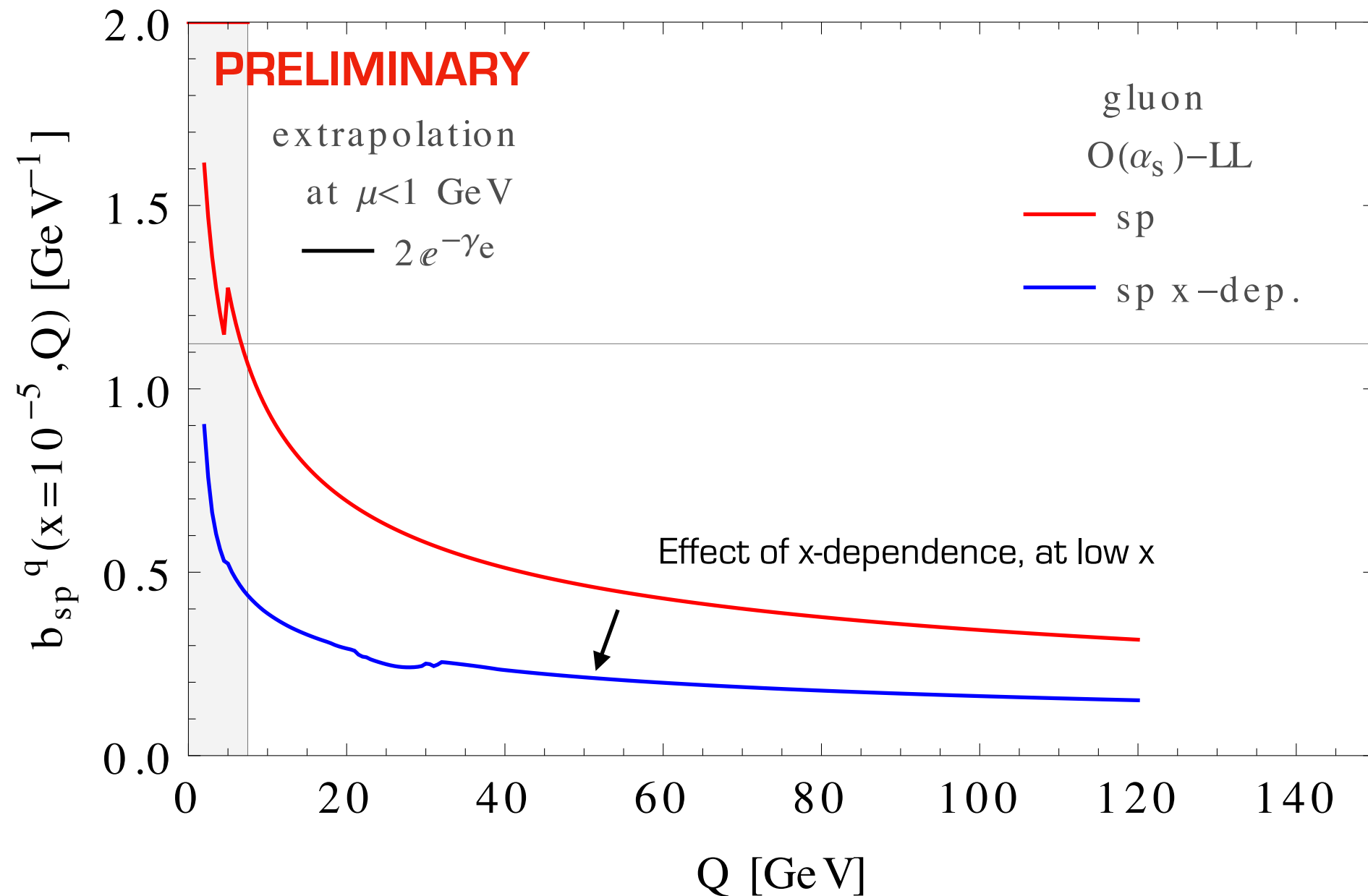
# Determination of the saddle point



Fixed  $Q = 50 \text{ GeV}$ , change  $x$   
 the  $x$  dependence determines a change with respect to CSS-like solution

What happens if we include BFKL effects at very low  $x$ ?

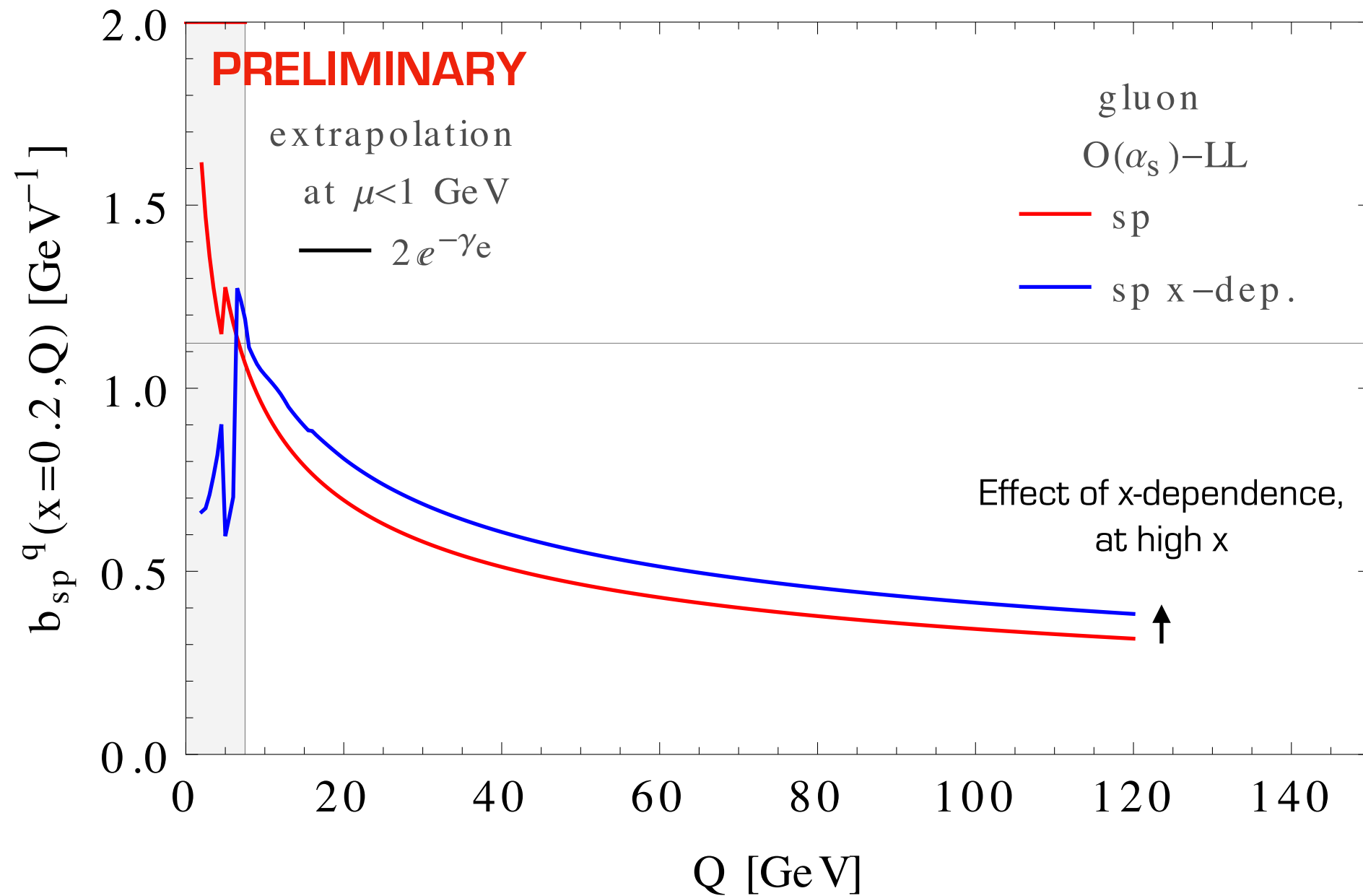
# Determination of the saddle point



Fixed  $x = 0.00001$ , change  $Q$

At low  $x$ , the  $x$ -dependent solution is reduced uniformly with respect to the CSS-like solution

# Determination of the saddle point



Fixed  $x = 0.2$ , change  $Q$

At high  $x$ , the  $x$ -dependent solution is enhanced uniformly with respect to the CSS-like solution

# Impact on Higgs physics

G. Ferrera, talk at REF 2014, Antwerp, <https://indico.cern.ch/event/330428/>

PDF uncertainties

