How a Future Lepton Collider Indirectly Probe Neutralino DM?

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Outline

Based on arxiv:1712.07825, Huayong Han, RH, Minyuan Jiang, Jing Shu.

- Standard Model (SM) Effective Field Theory (EFT)
- One-Loop Diagram Integration out: Covariant Derivative Expansion (CDE) Technique, development till we can calculate the electroweakino sector.
- New MSSM electroweakino sector results
- Numerics

What Is SM EFT?

See Sally's and Chris' talk.

- Dimension-5 operators = "Seesaw" operator for neutrino. Irrelevant.
- Effectively the start: Dimension-6 operators = 59 baryon number conserved +5 baryon number violating (for one generation).
- Operator classification
  - Pure Gauge Boson: $\mathcal{O}_{3G}, \mathcal{O}_{3W}$ and their CP-odd counterparts, $\mathcal{O}_{2G}, \mathcal{O}_{2W}, \mathcal{O}_{2B}$
  - Pure Higgs: 6 Higgs $\mathcal{O}_6$, 4 Higgs 2 derivatives $\mathcal{O}_H, \mathcal{O}_T, \mathcal{O}_R$, 2 Higgs 4 derivative $\mathcal{O}_D$
  - Gauge Boson + Higgs: $\mathcal{O}_{GG}, \mathcal{O}_{WW}, \mathcal{O}_{BB}, \mathcal{O}_{WB}$ and CP-odd counterparts, $\mathcal{O}_{HW}, \mathcal{O}_{HB}, \mathcal{O}_W, \mathcal{O}_B$
  - Pure Fermionic
  - Higgs + Fermionic
  - Gauge Boson + Fermionic
  - gauge Boson + Higgs + Fermionic

  CDE can calculate the first three classes, pure bosonic ones.

Model $\leftrightarrow$ Operators.

The CDE Technique

Unable to calculate the tree-level integrating out, as well as their RGE running contribution. But models with such contribution are limited and can be enumerated.

Level:

- Coleman-Weinberg Potential
- Derivative Expansion of Spacetime Dependent Field
- Covariant Generalization for Derivatives
- Fermionic γ Matrices
- Nondegenerated Large Suppression Masses
- Chiral and Majorana Fermions Mixing

- Coleman-Weinberg potential is in momentum representation

$$iV_{1L} = \frac{n_{\mathrm{B/F}}}{2}\int \frac{d^4p}{(2\pi)^4}\ln(p^2 - V'').$$

But quantum mechanics is more frequently using position representation. For spacetime independent field such as the Higgs VEV there is no difference for the two representations, but not for the case if V'' has spacetime dependence.

- From Chan, PRL 54 1222, one cae take the ln inside the Coleman-Weinberg potential to be actually

$$\int \frac{d^dp}{(2\pi)^d} e^{-ipx} \ln\left(p^2 - V''(x)\right) e^{ipy} = \int \frac{d^dp}{(2\pi)^d} \ln\left(p^2 - V''(x - i\tfrac{\partial}{\partial p})\right) e^{-ip(x-y)}$$

Consistent with Fourier transformation $\int \frac{d^dp}{(2\pi)^d} e^{-ipx} = \delta(x)$ which gives Green function.

- Spacetime dependent $V''(x)$ does not commute with p, so after Taylor expansion

$$V''(x - i\tfrac{\partial}{\partial p}) = M^2 + \delta V''(x) + \sum_{n=1}^{\infty}\frac{(-i)^n}{n!}\partial_{\mu_1}\cdots\partial_{\mu_n}\delta V''(x)\frac{\partial}{\partial p}^{\mu_1}\cdots\frac{\partial}{\partial p}^{\mu_n}$$

which makes key difference.

Regularization for $-i\frac{\partial}{\partial p}$ Orders

What is the order for the $-i\frac{\partial}{\partial p}$s in ln to find the ps to act on?

$$(A-B)^{-1} = A^{-1} + A^{-1}BA^{-1} + A^{-1}BA^{-1}BA^{-1} + \cdots$$

Let $A = p^2 - M^2$, $B = \delta V''(x) + \sum_{n=1}^{\infty} \frac{(-i)^n}{n!} \partial_{\mu_1}\cdots\partial_{\mu_n}\delta V''(x) \frac{\partial}{\partial p}^{\mu_1} \cdots \frac{\partial}{\partial p}^{\mu_n}$, then $-i\frac{\partial}{\partial p}$s have unambiguous orders to the ps.

There are two popular ways (regularizations) to reach $\left(p^2 - M^2 - \delta V''(x - i\frac{\partial}{\partial p})\right)^{-1}$ from ln

- With dimension-2 parameter u, at last drop $u \to \infty$ terms

$$2iV_{1L} = n_{B/F} \int_0^{\infty} -du \int \frac{d^d p}{(2\pi)^d} \frac{d}{du} \ln(p^2 - M^2 - \delta V''(x - i\frac{\partial}{\partial p}) - u)$$

$$= n_{B/F} \int \frac{d^d p}{(2\pi)^d} \int_0^{\infty} du \left(p^2 - M^2 - \delta V''(x - i\frac{\partial}{\partial p}) - u\right)^{-1} = n_{B/F} \int \frac{d^d p}{(2\pi)^d} \ln\left(p^2 - M^2 - \delta V''(x)\right)$$

$$+ n_{B/F} \int_0^{\infty} du \int \frac{d^d p}{(2\pi)^d} \sum_{n=1}^{\infty} \left(\frac{1}{p^2 - M^2 - u} \left[\delta V'' + \sum_{n=1}^{\infty} \frac{(-i)^n}{n!} \partial_{\mu_1} \cdots \partial_{\mu_n} \delta V'' \frac{\partial}{\partial p}^{\mu_1} \cdots \frac{\partial}{\partial p}^{\mu_n}\right]\right)^n \frac{1}{p^2 - M^2 - u}$$

- With dimensionless ξ parameter, at last drop $\xi \to \infty$ terms

$$2iV_{1L} = n_{B/F} \int_1^{\infty} d\xi \int \frac{d^d p}{(2\pi)^d} \frac{d}{d\xi} \ln(p^2 - \xi M^2 - \delta V''(x - i\frac{\partial}{\partial p}))$$

We have checked that the two regularizations give the same results, but they can differ in intermediate steps.

Covariant Generalization for Derivatives

- Mechanical momentum to canonical momentum $p^2 - V'' \to (p + gA)^2 - V''$ $(p + gA(x - i\frac{\partial}{\partial p}))^2 - V''(x - i\frac{\partial}{\partial p})$
- Multiplying $e^{-iD\frac{\partial}{\partial p}} e^{i\frac{\partial}{\partial p}\partial}$ on the left and its inverse on the right is invariant.

$$e^{-iD\frac{\partial}{\partial p}} e^{i\frac{\partial}{\partial p}\partial} \left((p + gA(x - i\frac{\partial}{\partial p}))^2 - V''(x - i\frac{\partial}{\partial p}) \right) e^{-i\frac{\partial}{\partial p}\partial} e^{iD\frac{\partial}{\partial p}} = e^{-iD\frac{\partial}{\partial p}} \left((p + i\partial + gA(x))^2 - V''(x) \right) e^{iD\frac{\partial}{\partial p}}$$

$$\delta\tilde{V}'' = e^{-iD\frac{\partial}{\partial p}} V''(x) e^{iD\frac{\partial}{\partial p}} = M^2 + \delta V''(x) + \sum_{n=1}^{\infty} \frac{(-i)^n}{n!} D_{\mu_1} \cdots D_{\mu_n} \delta V''(x) \ \frac{\partial}{\partial p}^{\mu_1} \cdots \frac{\partial}{\partial p}^{\mu_n}$$

$$\tilde{G} = e^{-iD\frac{\partial}{\partial p}} (p_\mu + iD_\mu)^2 e^{iD\frac{\partial}{\partial p}} = \left(\sum_{n=0}^{\infty} \frac{1}{n!} [-iD\frac{\partial}{\partial p}, [-iD\frac{\partial}{\partial p}, [\cdots [-iD\frac{\partial}{\partial p}, p + iD] \cdots]]] \right)^2$$

- Several useful terms

$$\tilde{G} = gp^\mu t^a \left(iF^a_{\nu\mu} \frac{\partial}{\partial p}^\nu + \frac{4}{3!} D_\rho F^a_{\nu\mu} \frac{\partial}{\partial p}^\rho \frac{\partial}{\partial p}^\nu + \cdots \right) + gt^a \left(\frac{2}{3!} D^\mu F^a_{\nu\mu} \frac{\partial}{\partial p}^\nu + \cdots \right) + g^2 t^a t^b \left(\frac{1}{4} F^a_{\nu\mu} F^{b\rho\mu} \frac{\partial}{\partial p}^\nu \frac{\partial}{\partial p}_\rho + \cdots \right)$$

- Fermionic CW potential

$$V_{\rm CW} \propto \frac{in_F}{2}\int \frac{d^4p}{(2\pi)^4}\ln(\not{p} - V_F'') \to \frac{in_F}{4}\int \frac{d^4p}{(2\pi)^4}\ln(\not{p} - M - \delta V_F'') + \ln(\not{p} + M + \delta V_F'')$$

- The bosonic correspondence

$$\delta V'' = \{M, \delta V_F''\} + (\delta V_F'')^2$$

- γ matrices

$$e^{-iD\frac{\partial}{\partial p}}\Big((\not{p} + i\not{D})(\not{p} + i\not{D}) + (\not{p} + i\not{D})(M + \delta V_F'') - (M + \delta V_F'')(\not{p} + i\not{D}) - (M + \delta V_F'')^2\Big)e^{iD\frac{\partial}{\partial p}}$$

Even number of γ matrices contribute.

- γ matrices induced pure Fermionic terms

$$\pm\tilde{\Gamma}_1 = i\not{D}\delta V_F'' + \frac{1}{2}(\not{D}D_\mu + D_\mu\not{D})\delta V_F''\frac{\partial}{\partial p}^\mu - \frac{i}{6}(\not{D}D_\mu D_\nu + D_\mu\not{D}D_\nu + D_\mu D_\nu\not{D})\delta V_F''\frac{\partial}{\partial p}^\mu\frac{\partial}{\partial p}^\nu + \cdots$$
$$+\frac{i}{2}g\gamma^\mu[\delta V_F'', F^a_{\nu\mu}t^a]\frac{\partial}{\partial p}^\nu + \frac{1}{2}g\gamma^\mu[D_\rho\delta V_F'', F^a_{\nu\mu}t^a]\frac{\partial}{\partial p}^\nu\frac{\partial}{\partial p}^\rho + \frac{2}{3!}g\gamma^\mu[\delta V_F'', D_\rho F^a_{\nu\mu}t^a]\frac{\partial}{\partial p}^\nu\frac{\partial}{\partial p}^\rho + \cdots$$
$$\tilde{\Gamma}_2 = -\frac{i}{4}g[\gamma^\mu, \gamma^\nu]F^a_{\mu\nu}t^a + \cdots$$

$$\mathcal{L}_{\rm CDE} = \frac{n_{\rm B/F}}{2}\int_0^\infty du\int\frac{d^dp_E}{(2\pi)^d}\sum_{m=1}^\infty(-1)^m{\rm tr}\left[\left(\frac{1}{p_E^2 + M^2 + u}\left[\delta\tilde{V}'' + \tilde{G} + \tilde{\Gamma}_1 + \tilde{\Gamma}_2\right]\right)^m\frac{1}{p_E^2 + M^2 + u}\right]$$

- $\delta\tilde{V}'' + \tilde{G} + \tilde{\Gamma}_1 + \tilde{\Gamma}_2$ are matrices, corresponding to BSM particles' contribution in the basis of $SU(3)_c \times SU(2)_L \times U(1)_Y$.
- $\frac{1}{p_E^2+M^2}$ should be viewed as matrix $(p_E^2\mathbf{1} + (M^2)_{ii})^{-1}$, the diagonal spacetime independent $(M^2)_{ii}$s may correspond to different new particles and adopt different values.
- The terms contributing to dimension-6 operators are actually calculating

$$\frac{1}{\prod_{\sum n_i=4}(p_E^2+M_i^2+u)^{n_i}}, \quad \frac{p_E^2}{\prod_{\sum n_i=5}(p_E^2+M_i^2+u)^{n_i}},$$

$$\frac{X_iX_j}{\prod_{\sum n_i=5}(p_E^2+M_i^2+u)^{n_i}}, \quad \frac{p_E^2X_iX_j}{\prod_{\sum n_i=6}(p_E^2+M_i^2+u)^{n_i}}, \quad \frac{p_E^4X_iX_j}{\prod_{\sum n_i=7}(p_E^2+M_i^2+u)^{n_i}},$$

$$\frac{X_iX_jX_kX_l}{\prod_{\sum n_i=6}(p_E^2+M_i^2+u)^{n_i}}, \quad \frac{p_E^2X_iX_jX_kX_l}{\prod_{\sum n_i=7}(p_E^2+M_i^2+u)^{n_i}},$$

$$\frac{X_iX_jX_kX_lX_mX_n}{\prod_{\sum n_i=7}(p_E^2+M_i^2+u)^{n_i}}.$$

  There is similar form for ξ regularization.
- Repeated usage of Feynman parameterization.

The job is to find the correct input matrix.
We use Van der Waerden spinor notation $\chi = \binom{\chi_\alpha}{\chi^{\dagger\dot{\alpha}}}$

- Both for gaugino/Majorana spinors and for higgsino/chiral spinors, in pure kinetic terms one can flip chiralities since $\xi^\dagger i\bar{\sigma}^\mu \partial_\mu \chi = \chi i \sigma^\mu \partial_\mu \xi^\dagger$
- For Non-Abelian gauge group, such chiralities flip induces a transpose of gauge field associated to the fundamental representation. $g\psi_i^\dagger \bar{\sigma}^\mu \psi_j A_\mu^a t_{ij}^a = -g\psi_j \sigma^\mu \psi_i^\dagger A_\mu^a t_{ij}^a$
- But it will be invariant under the rising and lowing operator basis, or the basis which respect electric charge. Then the 4 component spinors should be packaged as

$$\chi^B = \begin{pmatrix} \widetilde{B} \\ \widetilde{B}^\dagger \end{pmatrix}, \ \chi^1 = \begin{pmatrix} \widetilde{W}^1 \\ \widetilde{W}^{1\dagger} \end{pmatrix}, \ \chi^2 = \begin{pmatrix} \widetilde{W}^2 \\ \widetilde{W}^{2\dagger} \end{pmatrix}, \ \chi^3 = \begin{pmatrix} \widetilde{W}^3 \\ \widetilde{W}^{3\dagger} \end{pmatrix},$$

$$\chi^+ = \begin{pmatrix} \widetilde{H}_u^+ \\ \widetilde{H}_d^+ \end{pmatrix}, \ \chi^0 = \begin{pmatrix} \widetilde{H}_u^0 \\ \widetilde{H}_d^0 \end{pmatrix}, \ \chi^{0*} = \begin{pmatrix} \widetilde{H}_d^{0*} \\ \widetilde{H}_u^{0*} \end{pmatrix}, \ \chi^- = \begin{pmatrix} \widetilde{H}_d^- \\ \widetilde{H}_u^- \end{pmatrix}.$$

- $\chi = (\chi^B, \ \chi^1, \ \chi^2, \ \chi^3, \ \chi^+, \chi^0, \ \chi^{0*}, -\chi^-)^T$

With Dirac type kinetic and mass matrix, then the input to CDE is

$$M = \begin{pmatrix} M_1 &&&&&&& \\ & M_2 &&&&&& \\ && M_2 &&&&& \\ &&& M_2 &&&& \\ &&&& \mu &&& \\ &&&&& \mu && \\ &&&&&& \mu & \\ &&&&&&& \mu \end{pmatrix},$$

$$U_L = \begin{pmatrix} &&&& -\frac{g'}{\sqrt{2}}H_u^- & -\frac{g'}{\sqrt{2}}H_u^{0*} & \frac{g'}{\sqrt{2}}H_d^0 & -\frac{g'}{\sqrt{2}}H_d^+ \\ &&&& -\frac{g}{\sqrt{2}}H_u^{0*} & -\frac{g}{\sqrt{2}}H_u^- & \frac{g}{\sqrt{2}}H_d^+ & -\frac{g}{\sqrt{2}}H_d^0 \\ &&&& -\frac{ig}{\sqrt{2}}H_u^{0*} & \frac{ig}{\sqrt{2}}H_u^- & \frac{ig}{\sqrt{2}}H_d^+ & \frac{ig}{\sqrt{2}}H_d^0 \\ &&&& -\frac{g}{\sqrt{2}}H_u^- & \frac{g}{\sqrt{2}}H_u^{0*} & -\frac{g}{\sqrt{2}}H_d^0 & -\frac{g}{\sqrt{2}}H_d^+ \\ -\frac{g'}{\sqrt{2}}H_d^+ & -\frac{g}{\sqrt{2}}H_d^0 & \frac{ig}{\sqrt{2}}H_d^0 & -\frac{g}{\sqrt{2}}H_d^+ &&&& \\ -\frac{g'}{\sqrt{2}}H_d^0 & -\frac{g}{\sqrt{2}}H_d^+ & -\frac{ig}{\sqrt{2}}H_d^+ & \frac{g}{\sqrt{2}}H_d^0 &&&& \\ \frac{g'}{\sqrt{2}}H_u^{0*} & \frac{g}{\sqrt{2}}H_u^- & -\frac{ig}{\sqrt{2}}H_u^- & -\frac{g}{\sqrt{2}}H_u^{0*} &&&& \\ -\frac{g'}{\sqrt{2}}H_u^- & -\frac{g}{\sqrt{2}}H_u^{0*} & -\frac{ig}{\sqrt{2}}H_u^{0*} & -\frac{g}{\sqrt{2}}H_u^- &&&& \end{pmatrix},$$

$$U_R = U_L^\dagger .$$

Full Analytical Results 1

$$
\begin{aligned}
r_1 &= \frac{M_1}{\mu}, \\
r_2 &= \frac{M_2}{\mu}, \\
s_{2\beta} &= \sin(2\beta), \\
c_{4\beta} &= \cos(4\beta).
\end{aligned}
$$

$$
\begin{aligned}
c_{2W} &= \frac{g^2\left(r_2^2+2\right)}{120\pi^2\mu^2 r_2^2}, \\
c_{2B} &= \frac{g'^2}{120\pi^2\mu^2}, \\
c_{3W} &= -\frac{g^2\left(r_2^2+2\right)}{480\pi^2\mu^2 r_2^2}, \\
c_{WW} &= g'^2\left\{-\frac{\left(2\left(2r_1^4+5r_1^2-1\right)r_1 s_{2\beta}+13r_1^4-2r_1^2+1\right)}{1536\pi^2(r_1^2-1)^3\mu^2}+\frac{\left(2r_1 s_{2\beta}+r_1^2+1\right)r_1^4\mathrm{Log}[\frac{M_1^2}{\mu^2}]}{256\pi^2(r_1^2-1)^4\mu^2}\right\} \\
&+ g^2\left\{-\frac{2\left(6r_2^6-25r_2^4+5r_2^2-16\right)s_{2\beta}-17r_2^5+10r_2^3-53r_2}{1536\pi^2(r_2^2-1)^3\mu^2 r_2}-\frac{\left(r_2^4+4\right)\left(2r_2 s_{2\beta}+r_2^2+1\right)\mathrm{Log}[\frac{M_2^2}{\mu^2}]}{256\pi^2(r_2^2-1)^4\mu^2}\right\}, \\
c_{BB} &= g'^2\left\{-\frac{\left(2\left(2r_1^4+5r_1^2-1\right)r_1 s_{2\beta}+13r_1^4-2r_1^2+1\right)}{1536\pi^2(r_1^2-1)^3\mu^2}+\frac{r_1^4\left(2r_1 s_{2\beta}+r_1^2+1\right)\mathrm{Log}[\frac{M_1^2}{\mu^2}]}{256\pi^2(r_1^2-1)^4\mu^2}\right\} \\
&+ g^2\left\{-\frac{2\left(2r_2^4+5r_2^2-1\right)r_2 s_{2\beta}+13r_2^4-2r_2^2+1}{512\pi^2(r_2^2-1)^3\mu^2}+\frac{3r_2^4\left(2r_2 s_{2\beta}+r_2^2+1\right)\mathrm{Log}[\frac{M_2^2}{\mu^2}]}{256\pi^2(r_2^2-1)^4\mu^2}\right\}, \\
c_{WB} &= g'^2\left\{-\frac{\left(2\left(2r_1^4+5r_1^2-1\right)r_1 s_{2\beta}+13r_1^4-2r_1^2+1\right)}{768\pi^2(r_1^2-1)^3\mu^2}+\frac{r_1^4\left(2r_1 s_{2\beta}+r_1^2+1\right)\mathrm{Log}[\frac{M_1^2}{\mu^2}]}{128\pi^2(r_1^2-1)^4\mu^2}\right\} \\
&+ g^2\left\{\frac{\left(4r_2^5-38r_2^3+46r_2\right)s_{2\beta}-11r_2^4-2r_2^2+25}{768\pi^2(r_2^2-1)^3\mu^2}+\frac{\left(r_2^4-2\right)\mathrm{Log}[\frac{M_2^2}{\mu^2}]\left(2r_2 s_{2\beta}+r_2^2+1\right)}{128\pi^2(r_2^2-1)^4\mu^2}\right\}, \\
c_W &= g'^2\left\{\frac{4\left(7r_1^4-11r_1^2-2\right)r_1 s_{2\beta}+23r_1^6-17r_1^4-35r_1^2+5}{1152\pi^2(r_1^2-1)^4\mu^2}-\frac{r_1^3\left(2\left(r_1^4-3\right)s_{2\beta}+\left(r_1^4+4r_1^2-9\right)r_1\right)\mathrm{Log}[\frac{M_1^2}{\mu^2}]}{192\pi^2(r_1^2-1)^5\mu^2}\right\} \\
&+ g^2\left\{-\frac{4\left(5r_2^4+11r_2^2-10\right)r_2 s_{2\beta}+r_2^6+41r_2^4+11r_2^2-29}{384\pi^2(r_2^2-1)^4\mu^2}\right. \\
&\quad \left.+\frac{\left(2\left(r_2^6+12r_2^4-3r_2^2-4\right)r_2 s_{2\beta}+r_2^8+4r_2^6+27r_2^4-16r_2^2-4\right)\mathrm{Log}[\frac{M_2^2}{\mu^2}]}{192\pi^2(r_2^2-1)^5\mu^2}\right\}, \\
c_B &= g'^2\left\{\frac{4\left(7r_1^4-11r_1^2-2\right)r_1 s_{2\beta}+23r_1^6-17r_1^4-35r_1^2+5}{1152\pi^2(r_1^2-1)^4\mu^2}-\frac{r_1^3\left(2\left(r_1^4-3\right)s_{2\beta}+\left(r_1^4+4r_1^2-9\right)r_1\right)\mathrm{Log}[\frac{M_1^2}{\mu^2}]}{192\pi^2(r_1^2-1)^5\mu^2}\right\} \\
&+ g^2\left\{\frac{4\left(7r_2^4-11r_2^2-2\right)r_2 s_{2\beta}+23r_2^6-17r_2^4-35r_2^2+5}{384\pi^2(r_2^2-1)^4\mu^2}-\frac{r_2^3\left(2\left(r_2^4-3\right)s_{2\beta}+\left(r_2^4+4r_2^2-9\right)r_2\right)\mathrm{Log}[\frac{M_2^2}{\mu^2}]}{64\pi^2(r_2^2-1)^5\mu^2}\right\}, \\
c_{HW} &= g'^2\left\{-\frac{\left(r_1^2-1\right)-2\left(r_1^5+10r_1^3+r_1\right)s_{2\beta}+11r_1^6-35r_1^4}{384\pi^2(r_1^2-1)^4\mu^2}+\frac{r_1^3\left(r_1^2+1\right)\left(r_1^3-3r_1-2s_{2\beta}\right)\mathrm{Log}[\frac{M_1^2}{\mu^2}]}{64\pi^2(r_1^2-1)^5\mu^2}\right\} \\
&+ g^2\left\{\frac{2\left(r_2^5+10r_2^3+r_2\right)s_{2\beta}+5r_2^6-13r_2^4+47r_2^2-15}{128\pi^2(r_2^2-1)^4\mu^2}-\frac{\left(r_2^2+1\right)\left(6r_2^3 s_{2\beta}+r_2^6-3r_2^4+12r_2^2-4\right)\mathrm{Log}[\frac{M_2^2}{\mu^2}]}{64\pi^2(r_2^2-1)^5\mu^2}\right\}, \\
c_{HB} &= g'^2\left\{-\frac{\left(r_1^2-1\right)-2\left(r_1^5+10r_1^3+r_1\right)s_{2\beta}+11r_1^6-35r_1^4}{384\pi^2(r_1^2-1)^4\mu^2}+\frac{r_1^3\left(r_1^2+1\right)\left(r_1^3-3r_1-2s_{2\beta}\right)\mathrm{Log}[\frac{M_1^2}{\mu^2}]}{64\pi^2(r_1^2-1)^5\mu^2}\right\} \\
&+ g^2\left\{\frac{2\left(r_2^5+10r_2^3+r_2\right)s_{2\beta}+5r_2^6-13r_2^4+47r_2^2-15}{128\pi^2(r_2^2-1)^4\mu^2}-\frac{\left(r_2^2+1\right)\left(6r_2^3 s_{2\beta}+r_2^6-3r_2^4+12r_2^2-4\right)\mathrm{Log}[\frac{M_2^2}{\mu^2}]}{64\pi^2(r_2^2-1)^5\mu^2}\right\}, \\
c_D &= g'^2\left\{\frac{-\left(r_1^5+10r_1^3+r_1\right)s_{2\beta}+r_1^6-7r_1^4-7r_1^2+1}{96\pi^2(r_1^2-1)^4\mu^2}+\frac{r_1^3\left(\left(r_1^2+1\right)s_{2\beta}+2r_1\right)\mathrm{Log}[\frac{M_1^2}{\mu^2}]}{16\pi^2(r_1^2-1)^5\mu^2}\right\} \\
&+ g^2\left\{\frac{-\left(r_2^5+10r_2^3+r_2\right)s_{2\beta}+r_2^6-7r_2^4-7r_2^2+1}{32\pi^2(r_2^2-1)^4\mu^2}+\frac{3r_2^3\left(\left(r_2^2+1\right)s_{2\beta}+2r_2\right)\mathrm{Log}[\frac{M_2^2}{\mu^2}]}{16\pi^2(r_2^2-1)^5\mu^2}\right\},
\end{aligned}
$$

$$c_H = g'^4\left\{\frac{1}{1536\pi^2(r_1^2-1)^4\mu^2}[(2r_1^6-45r_1^4-126r_1^2+25)c_{4\beta}+12(r_1^4+52r_1^2-5)r_1s_{2\beta}-28r_1^6+309r_1^4+192r_1^2-41]\right.$$
$$+\frac{\mathrm{Log}[\frac{M_1^2}{\mu^2}]}{256\pi^2(r_1^2-1)^5\mu^2}[(r_1^6+19r_1^4+6r_1^2-2)c_{4\beta}+4(r_1^6-15r_1^4-12r_1^2+2)r_1s_{2\beta}+2r_1^8-15r_1^6-59r_1^4-2r_1^2+2]$$
$$+g'^2g^2\left\{\frac{\mathrm{Log}[\frac{M_1^2}{\mu^2}]r_1^3}{256\pi^2(r_1^2-1)^4(r_1-r_2)(r_1+r_2)\mu^2}[-8(r_1+r_2)(r_1^2-5)r_1s_{2\beta}+(r_1^5+2r_2r_1^4-4r_1^3-2r_2r_1^2-7r_1-6r_2)c_{4\beta}\right.$$
$$-7r_1^5-6r_2r_1^4+20r_1^3+22r_2r_1^2+9r_1+10r_2]$$
$$+\frac{\mathrm{Log}[\frac{M_2^2}{\mu^2}]r_2^3}{256\pi^2(r_2^2-1)^4(r_1-r_2)(r_1+r_2)\mu^2}[8(r_1+r_2)(r_2^2-5)r_2s_{2\beta}+(r_1^5+2r_2r_1^4-4r_1^3-2r_2r_1^2-7r_1-6r_2)c_{4\beta}$$
$$+7r_2^5+6r_1r_2^4-20r_2^3-22r_1r_2^2-9r_2-10r_1]$$
$$+\frac{1}{768\pi^2(r_1^2-1)^3(r_2^2-1)^3\mu^2}[c_{4\beta}(3(4r_2^4-9r_2^2-1)r_2r_1^5+(-16r_2^4-19r_2^2+5)r_1^4+3(-9r_2^4+14r_2^2+7)r_2r_1^3$$
$$+(-19r_2^4+110r_2^2-31)r_1^2-3r_2(r_2^4-7r_2^2+12)r_1+5r_2^4-31r_2^2-4)-4s_{2\beta}((r_2^4-32r_2^2+7)r_1^5+(r_2^4-32r_2^2+7)r_2r_1^4$$
$$-4(8r_2^4-25r_2^2+5)r_1^3-4r_2(8r_2^4-25r_2^2+5)r_1^2+(7r_2^4-20r_2^2-11)r_1+(7r_2^4-20r_2^2-11)r_2)$$
$$+(-16r_2^5+101r_2^3-7r_2)r_1^5+(28r_2^4+61r_2^2-23)r_1^4+(101r_2^4-262r_2^2+5)r_2r_1^3+(61r_2^4-290r_2^2+97)r_1^2$$
$$\left.+(-7r_2^5+5r_2^3+80r_2)r_1-23r_2^4+97r_2^2-8]\right\}$$
$$+g^4\left\{\frac{1}{1536\pi^2(r_2^2-1)^4\mu^2}[(30r_2^6-169r_2^4-250r_2^2+53)c_{4\beta}-4(29r_2^4-412r_2^2+47)r_2s_{2\beta}-112r_2^6+681r_2^4+564r_2^2-125]\right.$$
$$\left.-\frac{\mathrm{Log}[\frac{M_2^2}{\mu^2}]}{256\pi^2(r_2^2-1)^5\mu^2}[(4r_2^8-13r_2^6-35r_2^4-18r_2^2+6)c_{4\beta}-4(5r_2^6-35r_2^4-32r_2^2+6)r_2s_{2\beta}-6r_2^8+19r_2^6+155r_2^4+6r_2^2-6]\right\}, \tag{B13}$$

$$c_R = g'^4\left\{\frac{1}{384\pi^2(r_1^2-1)^4\mu^2}[(r_1^6-10r_1^4-37r_1^2-2)c_{4\beta}+2(-7r_1^4+92r_1^2+11)r_1s_{2\beta}-18r_1^6+87r_1^4+78r_1^2-3]\right.$$
$$\left.+\frac{\mathrm{Log}[\frac{M_1^2}{\mu^2}]r_1^2}{64\pi^2(r_1^2-1)^5\mu^2}[(5r_1^2+3)c_{4\beta}+2(r_1^4-7r_1^2-10)r_1s_{2\beta}+r_1^6-2r_1^4-20r_1^2-3]\right\}$$
$$+g'^2g^2\left\{\frac{-r_1^3\mathrm{Log}[\frac{M_1^2}{\mu^2}]}{128\pi^2(r_1^2-1)^4(r_1-r_2)^3(r_1+r_2)\mu^2}[-8s_{2\beta}(r_1^6+r_2r_1^5+3r_1^4-5r_2r_1^3-(5r_2^2+1)r_1^2+(3r_2^3+r_2)r_1+2r_2^2)\right.$$
$$+c_{4\beta}(r_1^7-(3r_2^2+2)r_1^5+2(r_2^2+6)r_2r_1^4+(9-6r_2^2)r_1^3-20r_2r_1^2-(3r_2^2+4)r_1+6r_2^3+8r_2)-r_1^7$$
$$-4r_2r_1^6-5r_2^2r_1^5-18r_1^5+2r_2^3r_1^4+16r_2r_1^4+26r_2^2r_1^3-5r_1^3-12r_2^3r_1^2+20r_2r_1^2-9r_2^2r_1+4r_1-6r_2^3-8r_2]$$
$$+\frac{r_2^3\mathrm{Log}[\frac{M_2^2}{\mu^2}]}{128\pi^2(r_2^2-1)^4(r_1-r_2)^3(r_1+r_2)\mu^2}[-8((3r_2r_1^3+(2-5r_2^2)r_1^2+(r_2^5-5r_2^3+r_2)r_1+(r_2^4+3r_2^2-1)r_2^2)s_{2\beta}$$
$$+(2(r_2^4+3)r_1^3-3r_2(r_2^2+1)^2r_1^2+4(3r_2^4-5r_2^2+2)r_1+(r_2^6-2r_2^4+9r_2^2-4)r_2)c_{4\beta}-r_2^7-4r_1r_2^6-5r_1^2r_2^5$$
$$-18r_2^5+2r_1^3r_2^4+16r_1r_2^4+26r_1^2r_2^3-5r_2^3-12r_1^3r_2^2+20r_1r_2^2-9r_1^2r_2+4r_2-6r_1^3-8r_1]$$
$$-\frac{1}{384\pi^2(r_1^2-1)^3(r_2^2-1)^3(r_1-r_2)^2\mu^2}[8(r_2^6-11r_2^4+16r_2^2-3)r_1^3r_2+2(4r_2^6+3r_2^4-12r_2^2+5)r_1r_2$$
$$+c_{4\beta}(-(16r_2^6+27r_2^4-12r_2^2+5)r_1^6+8(r_2^5+r_2^3+r_2)r_1^7+(8r_2^7+26r_2^5-88r_2^3+6r_2)r_1^5-5r_2^6+28r_2^4-11r_2^2$$
$$+(-27r_2^6+184r_2^4-101r_2^2+28)r_1^4+(12r_2^6-101r_2^4+40r_2^2-11)r_1^2)$$
$$-12s_{2\beta}((2r_2^4+5r_2^2-1)r_1^7+(-4r_2^5-5r_2^3+3r_2)r_1^6+(-4r_2^6+8r_2^4-31r_2^2+9)r_1^5+(2r_2^6+8r_2^4+17r_2^2-9)r_2r_1^4$$
$$+3(r_2^2-3)r_1r_2^4+(5r_2^4-31r_2^2+8)r_1^2r_2^3-(r_2^4-9r_2^2+2)r_2^3+(-5r_2^6+17r_2^4+8r_2^2-2)r_1^3)$$
$$+2(r_2^5-26r_2^3+r_2)r_1^7+(20r_2^6+9r_2^4+24r_2^2-17)r_1^6+2(r_2^6+r_2^4+97r_2^2-15)r_2r_1^5$$
$$+(9r_2^6-344r_2^4+271r_2^2-68)r_1^4-17r_2^6-68r_2^4+25r_2^2-2(26r_2^6-97r_2^4+236r_2^2-69)r_1^3r_2$$
$$\left.+2(r_2^6-15r_2^4+69r_2^2-19)r_1r_2+(24r_2^6+271r_2^4-164r_2^2+25)r_1^2]\right\}$$
$$+g^4\left\{-\frac{1}{384\pi^2(r_2^2-1)^4\mu^2}[(5r_2^6+76r_2^4+103r_2^2+8)c_{4\beta}-2(17r_2^4+332r_2^2+35)r_2s_{2\beta}+36r_2^6-321r_2^4-312r_2^2+21]\right.$$
$$\left.+\frac{\mathrm{Log}[\frac{M_2^2}{\mu^2}]r_2^2}{64\pi^2(r_2^2-1)^5\mu^2}[(6r_2^4+17r_2^2+9)c_{4\beta}+2(r_2^4-31r_2^2-34)r_2s_{2\beta}+r_2^6-8r_2^4-80r_2^2-9]\right\}, \tag{B14}$$

$$c_T = g'^4\left\{\frac{(17r_1^4-7r_1^2-4)c_{2\beta}^2}{768\pi^2(r_1^2-1)^3\mu^2}-\frac{r_1^2(r_1^4+2r_1^2-2)c_{2\beta}^2\mathrm{Log}[\frac{M_1^2}{\mu^2}]}{128\pi^2(r_1^2-1)^4\mu^2}\right\}+g^4\left\{\frac{(29r_2^4-7r_2^2-16)c_{2\beta}^2}{768\pi^2(r_2^2-1)^3\mu^2}-\frac{r_2^2(r_2^4+6r_2^2-6)c_{2\beta}^2\mathrm{Log}[\frac{M_2^2}{\mu^2}]}{128\pi^2(r_2^2-1)^4\mu^2}\right\}$$
$$+g'^2g^2\left\{\frac{r_1^3\mathrm{Log}[\frac{M_1^2}{\mu^2}]}{128\pi^2(r_1^2-1)^4(r_1-r_2)^3(r_1+r_2)\mu^2}[-4c_{4\beta}(2(r_2^2-1)r_1+r_1^5+3r_2r_1^4+(2-3r_2^2)r_1^3+(r_2^2-8)r_2r_1^2+4r_2)\right.$$
$$+2(-(r_2^2-1)r_1^4+(r_2^2-5)r_2r_1^3+3r_1^6+r_2r_1^5-(5r_2^2+2)r_1^2+(r_2^2+2)r_2r_1+4r_2^2)s_{2\beta}+3r_1^7-r_2^2r_1^5+r_1^5$$
$$+2r_2^3r_1^4+r_2r_1^4-7r_2^2r_1^3+r_1^3-r_2^3r_1^2-8r_2r_1^2+5r_2^2r_1-2r_1+2r_2^3+4r_2]$$
$$-\frac{r_2^3\mathrm{Log}[\frac{M_2^2}{\mu^2}]}{128\pi^2(r_2^2-1)^4(r_1-r_2)^3(r_1+r_2)\mu^2}[-c_{4\beta}((r_2^2r_1^3+(2r_2-3r_2^3)r_1^2+(3r_2^4-8r_2^2+4)r_1+(r_2^4+2r_2^2-2)r_2)$$
$$+2((r_2^3+r_2)r_1^3-(r_2^4+5r_2^2-4)r_1^2+(r_2^4-5r_2^2+2)r_2r_1+(3r_2^4+r_2^2-2)r_2^2)s_{2\beta}+3r_2^7-r_1^2r_2^5+r_2^5+2r_1^3r_2^4+r_1r_2^4$$
$$-7r_1^2r_2^3+r_2^3-r_1^3r_2^2-8r_1r_2^2+5r_1^2r_2-2r_2+2r_1^3+4r_1]$$
$$+\frac{1}{768\pi^2(r_1^2-1)^3(r_2^2-1)^3(r_1-r_2)^2\mu^2}[((-2r_2^5-5r_2^3+r_2)r_1^7+(4r_2^6+17r_2^4-19r_2^2+4)r_1^6-r_2(2r_2^6+12r_2^4-57r_2^2+25)r_1^5$$
$$+(17r_2^6-126r_2^4+120r_2^2-29)r_1^4+(-5r_2^7+57r_2^5-126r_2^3+56r_2)r_1^3+(-19r_2^6+120r_2^4-102r_2^2+19)r_1^2$$
$$+(r_2^6-25r_2^4+56r_2^2-26)r_2r_1+(4r_2^4-29r_2^2+19)r_2^2)c_{4\beta}+2((13r_2^4-2r_2^2+1)r_1^7+(-25r_2^5+2r_2^3+11r_2)r_1^6$$
$$+(-25r_2^6+48r_2^4-93r_2^2+34)r_1^5+(13r_2^6+48r_2^4+9r_2^2-34)r_2r_1^4+(2r_2^6+9r_2^4+48r_2^2-23)r_1^3$$
$$+(-2r_2^7-93r_2^5+48r_2^3+11r_2)r_1^2+(11r_2^4-34r_2^2+11)r_2^2r_1+(r_2^4+34r_2^2-23)r_2^3)s_{2\beta}$$
$$+3(3r_2r_1^5((r_2^2-1)+2r_2^6-8r_2^4)+r_2^2((r_2^2-5)+10r_2^4))+3(2r_2^5-r_2^3+r_2)r_1^7+(-20r_2^6+29r_2^4-25r_2^2+10)r_1^6$$
$$\left.+(29r_2^6+6r_2^4-18r_2^2+1)r_1^4-3r_2(r_2^6-r_2^4-10r_2^2+4)r_1^3-(25r_2^6+18r_2^4-30r_2^2+5)r_1^2+3(r_2^6-r_2^4-4r_2^2+2)r_2r_1)]\right\}. \tag{B15}$$

And we don't do c_6 which is even more complicated but less useful.

MSSM Neutralino Chargino 1

Gaugino Mass Unification $M_1/M_2 = 0.5$ & $\tan\beta = 2$

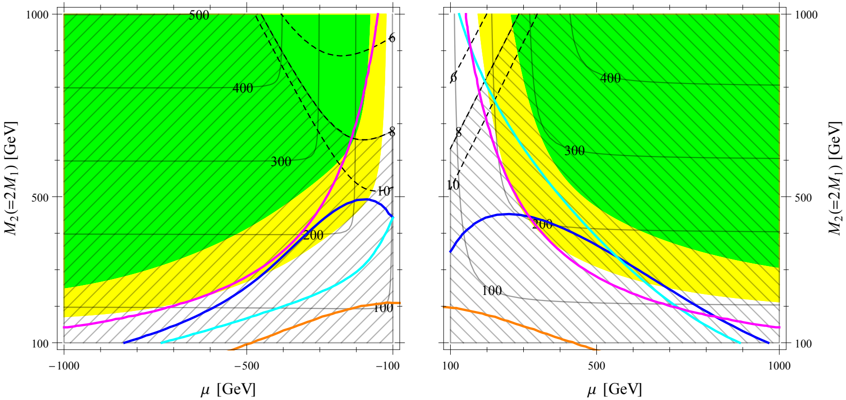

Figure: Hatched: $m_{\tilde{\chi}_2^0/\tilde{\chi}_1^\pm} - m_{\tilde{\chi}_1^0} > 8$ GeV, covered by soft lepton plus missing E_T search. Individual 1σ constraint curves: Blue: FCC-ee T, Cyan: FCC-ee S, Magenta: FCC-ee $h \to \gamma\gamma$, Orange: CEPC Δg_1^Z.

"Moving target" but with "fixed splitting" to fill

MSSM Neutralino Chargino 2

Gaugino Mass Unification $M_1/M_2 = 0.5$ & $\tan\beta = 50$

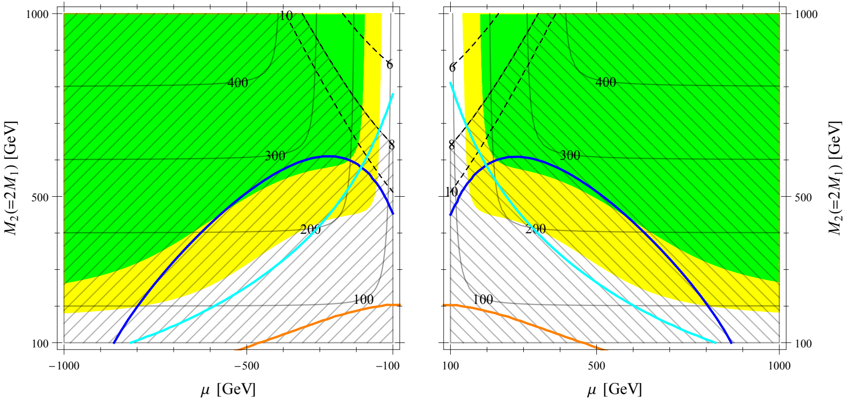

Figure: Hatched: $m_{\tilde{\chi}_2^0/\tilde{\chi}_1^\pm} - m_{\tilde{\chi}_1^0} > 8$ GeV, covered by soft lepton plus missing E_T search. Individual 1σ constraint curves: Blue: FCC-ee T, Cyan: FCC-ee S, Magenta: FCC-ee $h \to \gamma\gamma$, Orange: CEPC Δg_1^Z.

"Moving target" but with "fixed splitting" to fill

Anomaly Mediation $M_1/M_2 = 3.3$ & $\tan\beta = 2$

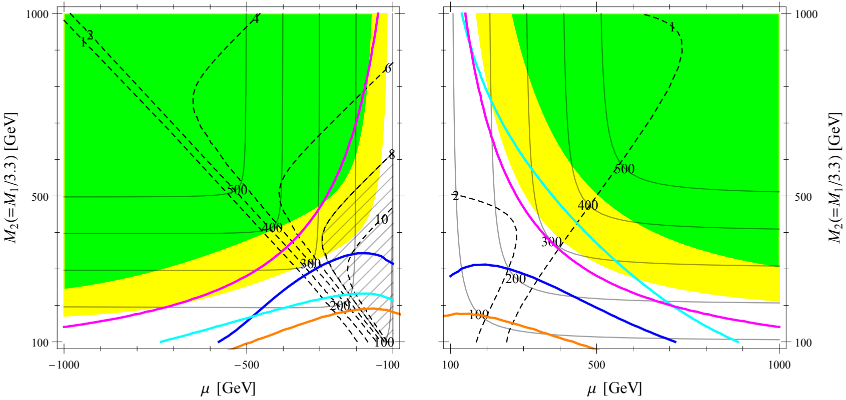

Figure: Hatched: $m_{\tilde{\chi}_2^0/\tilde{\chi}_1^\pm} - m_{\tilde{\chi}_1^0} > 8$ GeV, covered by soft lepton plus missing E_T search. Individual 1σ constraint curves: Blue: FCC-ee T, Cyan: FCC-ee S, Magenta: FCC-ee $h \to \gamma\gamma$, Orange: CEPC Δg_1^Z.

"Moving target" but with "fixed splitting" to fill

Anomaly Mediation $M_1/M_2 = 3.3$ & $\tan\beta = 50$

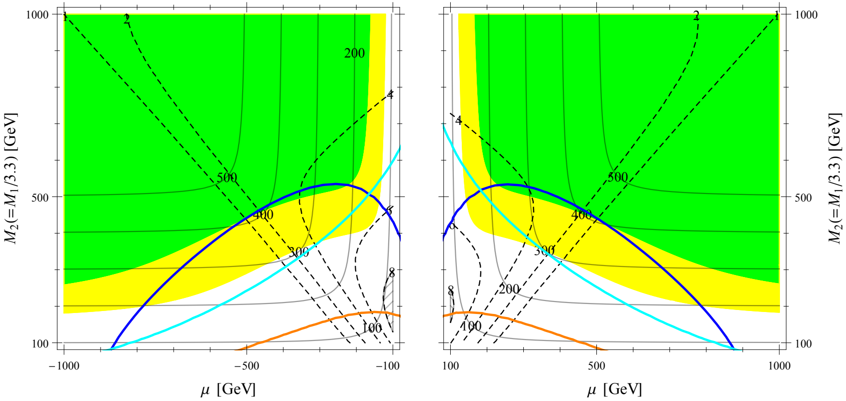

Figure: Hatched: $m_{\tilde{\chi}_2^0/\tilde{\chi}_1^\pm} - m_{\tilde{\chi}_1^0} > 8$ GeV, covered by soft lepton plus missing E_T search. Individual 1σ constraint curves: Blue: FCC-ee T, Cyan: FCC-ee S, Magenta: FCC-ee $h \to \gamma\gamma$, Orange: CEPC Δg_1^Z.

"Moving target" but with "fixed splitting" to fill

$\tan\beta = 10$ and $M_1/M_2 = 1.3$.

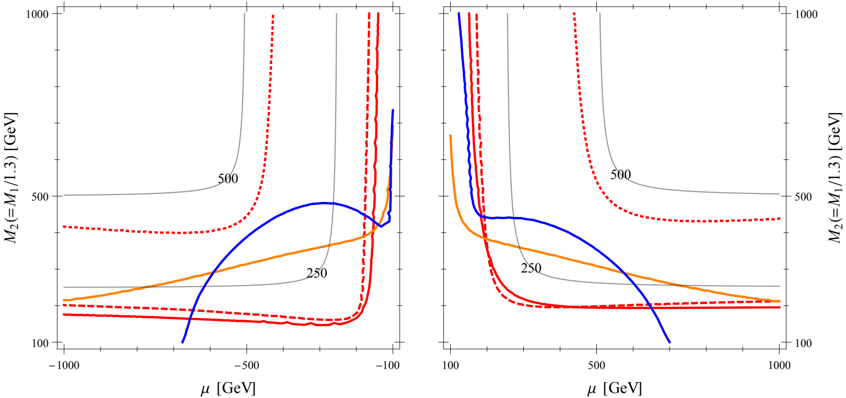

Figure: 1σ constraint curves for each set: Blue: FCC-ee EWPT, Orange: CEPC TGC, Red Solid: FCC-ee Higgs, Red Dashed: ILC Higgs at $\sqrt{s} = 500$ GeV run, Red Dotted: ILC Higgs at $\sqrt{s} = 1$ TeV run.

- We have developed the CDE technique to calculate the dimension-6 operators of integrating out the MSSM electroweakino sector.
- We show the indirect way can probe the very degenerated electroweakino spectrum which is difficult for direct collider experiment.

Backup: Operators Basis

Notation	Operator	Notation	Operator	Notation	Operator
$\mathcal{O}_6$	$(H^\dagger H)^3$	$\mathcal{O}_{GG}$	$g_s^2 H^\dagger H G^a_{\mu\nu} G^{a\mu\nu}$	$\mathcal{O}_W$	$ig(H^\dagger \overleftrightarrow{D}_\mu t^a H) D_\nu W^{a\mu\nu}$
$\mathcal{O}_H$	$\frac{1}{2}(\partial_\mu (H^\dagger H))^2$	$\mathcal{O}_{WW}$	$g^2 H^\dagger H W^a_{\mu\nu} W^{a\mu\nu}$	$\mathcal{O}_B$	$ig'(H^\dagger \overleftrightarrow{D}_\mu H) \partial_\nu B^{\mu\nu}$
$\mathcal{O}_T$	$\frac{1}{2}(H^\dagger \overleftrightarrow{D}_\mu H)^2$	$\mathcal{O}_{BB}$	$g'^2 H^\dagger H B_{\mu\nu} B^{\mu\nu}$	$\mathcal{O}_{HW}$	$2ig(D_\mu H)^\dagger t^a (D_\nu H) W^{a\mu\nu}$
$\mathcal{O}_R$	$(H^\dagger H)(D_\mu H^\dagger D^\mu H)$	$\mathcal{O}_{WB}$	$2gg' H^\dagger t^a H W^a_{\mu\nu} B^{\mu\nu}$	$\mathcal{O}_{HB}$	$2ig' Y_H (D_\mu H)^\dagger (D_\nu H) B^{\mu\nu}$
				$\mathcal{O}_D$	$(D_\mu D^\mu H^\dagger)(D_\nu D^\nu H)$

Backup: Operators Basis

Notation	Operator	Notation	Operator	Notation	Operator
$\mathcal{O}_6$	$(H^\dagger H)^3$	$\mathcal{O}_{GG}$	$g_s^2 H^\dagger H G^a_{\mu\nu} G^{a\mu\nu}$	$\mathcal{O}_W$	$ig(H^\dagger \overleftrightarrow{D}_\mu t^a H) D_\nu W^{a\mu\nu}$
$\mathcal{O}_H$	$\frac{1}{2}(\partial_\mu (H^\dagger H))^2$	$\mathcal{O}_{WW}$	$g^2 H^\dagger H W^a_{\mu\nu} W^{a\mu\nu}$	$\mathcal{O}_B$	$ig'(H^\dagger \overleftrightarrow{D}_\mu H) \partial_\nu B^{\mu\nu}$
$\mathcal{O}_T$	$\frac{1}{2}(H^\dagger \overleftrightarrow{D}_\mu H)^2$	$\mathcal{O}_{BB}$	$g'^2 H^\dagger H B_{\mu\nu} B^{\mu\nu}$	$\mathcal{O}_{HW}$	$2ig(D_\mu H)^\dagger t^a (D_\nu H) W^{a\mu\nu}$
$\mathcal{O}_R$	$(H^\dagger H)(D_\mu H^\dagger D^\mu H)$	$\mathcal{O}_{WB}$	$2gg' H^\dagger t^a H W^a_{\mu\nu} B^{\mu\nu}$	$\mathcal{O}_{HB}$	$2ig' Y_H (D_\mu H)^\dagger (D_\nu H) B^{\mu\nu}$
				$\mathcal{O}_D$	$(D_\mu D^\mu H^\dagger)(D_\nu D^\nu H)$

Table: The uncertainties expected at each experiments, for the EWPT experiments (T and S), the TGC experiments (Δg_1^Z, $\Delta\kappa_\gamma$ and λ_γ) and two representative channels of eight Higgs experiment channels (the others refer to the references therein).

Observable	T	S	$10^4\Delta g_1^Z$	$10^4\Delta\kappa_\gamma$	$10^4\lambda_\gamma$	$\frac{\Delta(\sigma_{Zh}\mathrm{Br}_{bb})}{\sigma_{Zh}^{\mathrm{SM}}\mathrm{Br}_{bb}^{\mathrm{SM}}}$	$\frac{\Delta(\sigma_{Zh}\mathrm{Br}_{\gamma\gamma})}{\sigma_{Zh}^{\mathrm{SM}}\mathrm{Br}_{\gamma\gamma}^{\mathrm{SM}}}$
ILC (250 GeV)	0.022	0.017				1.1%	35%
ILC (500 GeV)	0.022	0.017	2.8	3.1	4.3	0.66%	23%
ILC (1 TeV)	0.022	0.017	1.8	1.9	2.6	0.47%	8.5%
CEPC	0.009	0.014	1.59	2.30	1.67	0.32%	9.1%
FCC-ee	0.004	0.007				0.2%	3.0%