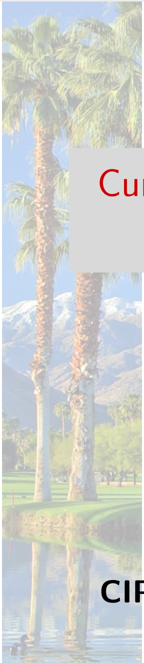

Current Status of Hydrodynamic Modeling, from p+p to Heavy Ions

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Center for Theoretical Physics, MIT

May 31, 2018

CIPANP 2018

Massachusetts
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Overview

quickly review

1. 2+1D hydro+cascade model (superSONIC) calculations of anisotropic flow v_2, v_3, v_4 in $\mathrm{p+p}$, $\mathrm{p+Pb}$, $\mathrm{Pb+Pb}$, plus some

RW, Romatschke, PLB 774 (2017) 351-356

“hydro attractor”

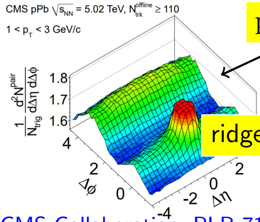
2. The applicability of hydro in $\mathrm{p+p}$ (and $\mathrm{p/d/^3He+A?}$): non-perturbative hydro attractor

Shape of the proton, collectivity in $p{+}p$, $p{+}A$

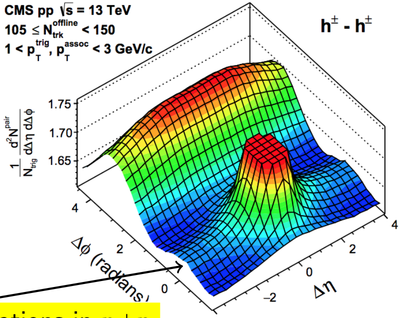
- The proton is not round

  e.g. Miller, PRC 68 (2003) 022201

“protons”



$p{+}Pb$



ridge in $\Delta\phi$ correlations in $p{+}p$

CMS Collaboration, PLB 765 (2017) 193-220

CMS Collaboration, PLB 718 (2013) 795

high-energy/DIS perspective: Mäntysaari, Schenke, PRL 117 (2016) 052301

Small η/s and collectivity in $\mathrm{p/d/^3He{+}Au}$

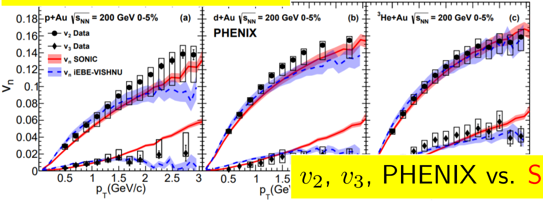
- Response of near-perfect liquid to initial geometry affords natural, quantitative interpretation for ordering of v_2, v_3 in $\mathrm{p/d/^3He{+}Au}$

PHENIX Collaboration (2018) 1805.02973

this session: talk by Sylvia Morrow

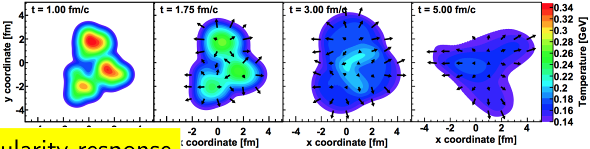
recently challenged?! cf. Mace *et al.* (2018) 1805.09342

$\mathrm{p{+}Au,\ d{+}Au,\ ^3He{+}Au}$



$\eta/s \approx 1/4\pi$ for all 3

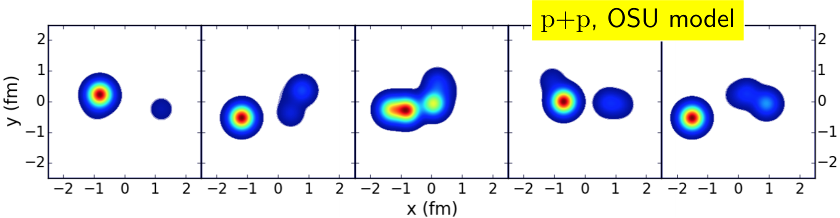
v_2, v_3, PHENIX vs. SONIC, VISHNU (hydro)



$\mathrm{^3He{+}Au}$ triangularity response

Nagle *et al.* PRL 113 (2014) 112301

Monte Carlo Glauber + quarks



- Welsh, Singer, Heinz (2016): 3 transverse constituent quark positions sampled from Gaussian. Low-x gluons contribute entropy around these Welsh, Singer, Heinz, PRC 94 (2016) 024919

$$\frac{d^3S}{dY\,d^2\mathbf{x}_\perp}(\tau_0,\mathbf{x}_\perp) = \kappa(\tau_0) \sum_{\{\text{part } n\}}^{N_\text{part}} \sum_{\{\text{quark } i\in n\}}^{3} \frac{n_i}{2\pi w_\text{q}^2} e^{-\|\mathbf{x}_\perp-\mathbf{x}_i\|^2/2w_\text{q}^2}$$

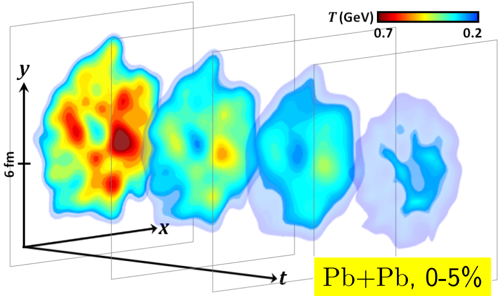
$$n_i \sim \text{Gamma}\left(\tfrac{4}{9},\tfrac{3}{4}\right), \quad w_\text{q} \approx 0.46 \text{ fm}, \quad w_\text{N} \approx 0.52 \text{ fm}$$

p+p, p+Pb, Pb+Pb collisions with superSONIC

- preflow→2+1D viscous hydro→B3D hadron cascade model (superSONIC) van der Schee, Romatschke, Pratt, PRL 111 (2013) 222302

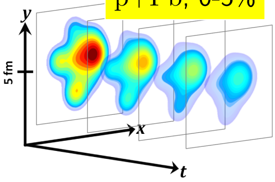
“One fluid to rule them all”

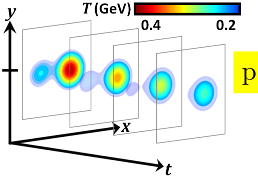
$\eta/s \approx 1/4\pi,\ \zeta/s = 0.01$



Pb+Pb, 0-5%

p+Pb, 0-5%

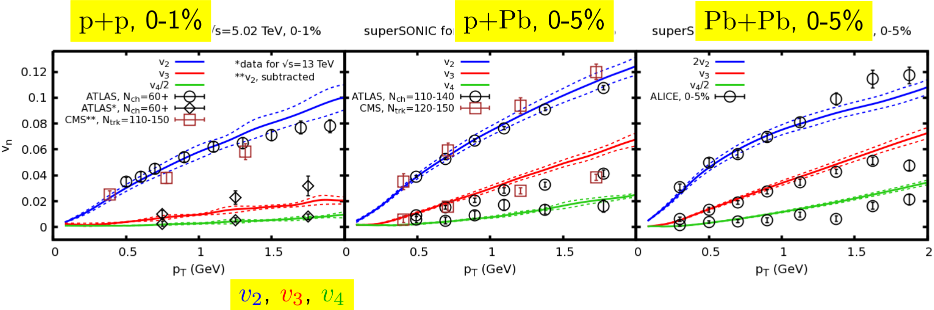




p+p, 0-1%

$\sqrt{s_{\mathrm{NN}}} = 5.02\ \mathrm{TeV}$

Results: v_2, v_3, v_4 in central LHC-energy collisions



RW, Romatschke, PLB 774 (2017) 351-356

Experimental data from:

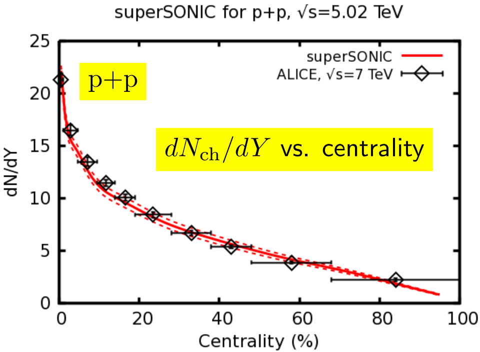
ATLAS: PRC 96 (2017) 024908, PRC 90 (2014) 044906

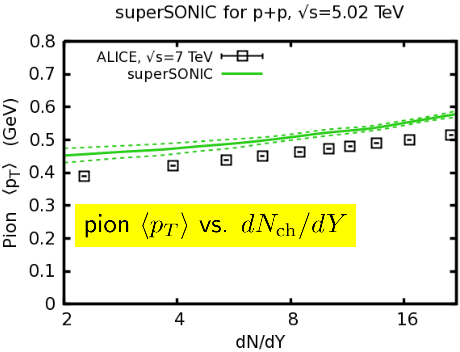
CMS: PLB 765 (2017) 193-220, PLB 724 (2013) 213-240

ALICE: PRL 116 (2016) 132302

superSONIC results for non-“central” $\mathrm{p+p}$ collisions

Experimental data: ALICE Collaboration, Nature Phys. 13 (2017) 535-539



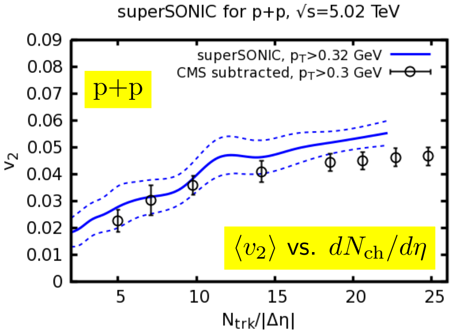


- OSU+superSONIC produces NBD fluctuations of multiplicity
- $\pi^{\pm}$ $\langle p_T \rangle$ too high in $\mathrm{p+p}$ $\implies$ need larger ζ/s near $T_{\mathrm{C}} \approx 170\ \mathrm{MeV}$

Results: $\langle v_2 \rangle$ and geometry response in non-central $p+p$

Experimental data:

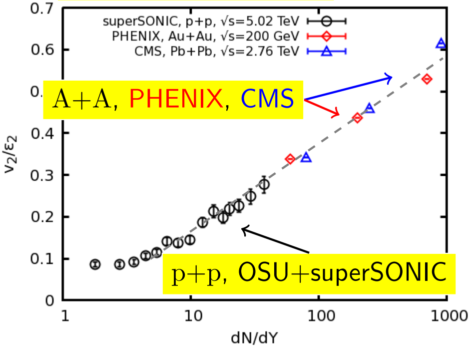
CMS, PLB 765 (2017) 193-220



$p+p$

$\langle v_2 \rangle$ vs. $dN_{\rm ch}/d\eta$

$\langle v_2 \rangle / \langle \epsilon_2 \rangle$ vs. $dN_{\rm ch}/dY$



$A+A$, PHENIX, CMS

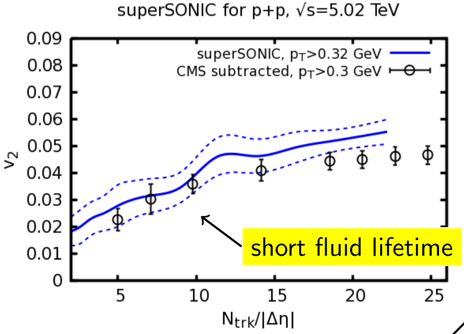
$p+p$, OSU+superSONIC

- $p+p$ response v_2/ϵ_2 to geometry follows same trend as $A+A$ for $dN_{\rm ch}/dY \gtrsim 5$ fm

Comparing non-flow subtraction schemes for v_2

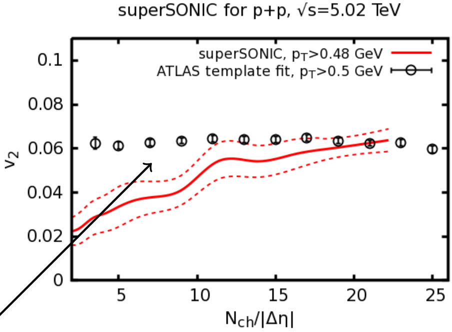
ATLAS experimental data: ATLAS, PRC 96 (2017) 024908

CMS subtracted



short fluid lifetime

ATLAS template fit



- Could increase ζ/s to increase system lifetime
- However, the v_2 at low $N_{\rm ch}$ would then be highly sensitive to non-hydro sector (i.e. dependent on Israel-Stewart τ_π)

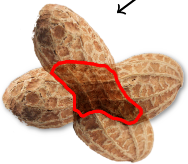
Subtleties in “choice” of hydro initial data

- How does $\langle T^{\mu\nu}\rangle$ at τ_0 *depend* on transverse geometry of proton?

$$\boxed{\frac{d^3E}{d\eta d^2\mathbf{x}_\perp}(\mathbf{x}_\perp, \eta = 0) \propto (\epsilon_L \epsilon_R)^\alpha}$$

p+p

$$\boxed{\frac{d^3E}{d\eta d^2\mathbf{x}_\perp}(\mathbf{x}'_\perp, \eta') \propto (\epsilon_L + \epsilon_R)}$$



vs.

“optical” model: collisions in classical Yang-Mills, holographic $\mathcal{N} = 4$ SYM

“participant” model: collisions with tilted event planes $\hat{\mathbf{n}} \cdot \hat{\mathbf{z}} \neq 0$

valid only near $\eta = 0$ (*sans* boost-invariance)

$\sim$ MC Glauber variants

Holographic SYM: Romatschke, Hogg, JHEP 04 (2013) 048

and van der Schee, Schenke, PRC 92 (2015) 064907

Glasma: Schenke, Venugopalan, PRL 113 (2014) 102301

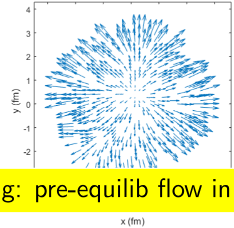
Next steps

Be more rigorous (proton is a well-studied object)

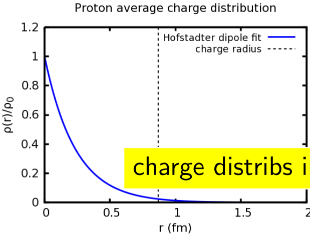
- Nucleon form factors, (G)PDFs, spin content, etc.
- HERA constraints on $SU(3)$ glue content

yesterday's session: talk by Prithwish Tribedy

Initial stage dynamics??



(e.g: pre-equilib flow in $\mathrm{p+Pb}$)



charge distribs in hadrons

cf. Mitchell, Perepelitsa, Tannenbaum, Stankus, PRC 93 (2016) 054910

and Habich, Miller, Romatschke, Xiang, EPJC 76 (2016) 408

Question

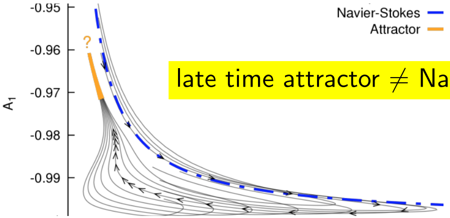
How far can we push hydrodynamics?

Puzzle: Hydrodynamics predicts its own demise

- First-order Navier-Stokes is non-causal ← ??? need non-hydro modes to stabilize Spaliński, PRD 94 (2016) 085002

$$\underbrace{\delta\langle T^{\mu\nu}\rangle \sim e^{-i(\omega t-\mathbf{k}\cdot\mathbf{x})}}_{\text{linear response}}: \quad \text{modes with} \quad \lim_{\mathbf{k}\to 0}\underset{\substack{\uparrow\\ \text{hydro modes}}}{\omega(\mathbf{k})} = 0 \qquad \lim_{\mathbf{k}\to 0}\underset{\substack{\uparrow\\ \text{non-hydro modes}}}{\omega(\mathbf{k})} \neq 0$$

- Non-hydro modes can relax slower than expansion of fluid, causing the system to jump out of local near-equilibrium



late time attractor ≠ Navier-Stokes

Kurkela, Wiedemann (2017) 1712.04376

cf. Romatschke, JHEP 1712 (2017) 079

Divergence of "perturbative" hydrodynamic series

- Standard hydro is an EFT: e.g. Baier *et al.* JHEP 0804 (2008) 100

$$
\begin{aligned}
\langle T^{\mu\nu}\rangle_{\text{QCD}} &= \overbrace{\epsilon u^{\mu}u^{\nu} + p(\epsilon) g_{\perp}^{\mu\nu}}^{\text{ideal, } \mathcal{O}(\partial^0)} + \Pi^{\mu\nu}, \\
\Pi^{\mu\nu} &= \underbrace{-\eta(\epsilon)\nabla^{\langle\mu}u^{\nu\rangle} - \zeta(\epsilon) g_{\perp}^{\mu\nu}\nabla_{\alpha}u^{\alpha}}_{\text{viscous, } \mathcal{O}(\partial^1)} + \mathcal{O}(\partial^2).
\end{aligned}
$$

- Hydro gradient expansion diverges asymptotically

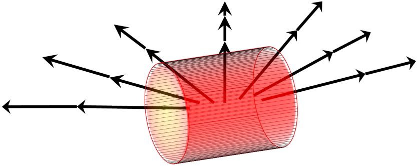
0+1D Bjorken flow
$$\epsilon(\tau) = \frac{1}{\tau^{4/3}} \sum_{n=0}^{\infty} \frac{a_n}{\tau^{2n/3}}, \quad a_n \sim n! \text{ as } n \gg 1 \qquad (1)$$

Heller, Janik, Witaszczyk, PRL 110 (2013) 211602

Borel Resummation: 0+1D Bjorken Expansion

a_n for $n > 0$ come from viscous + other transport corrections

0+1D Bjorken flow



$$\epsilon(\tau) = \frac{1}{\tau^{4/3}} \sum_{n=0}^{\infty} \frac{a_n}{\tau^{2n/3}}, \quad a_n \sim n! \text{ as } n \gg 1$$

Use a Borel resummation of the series:

$$\left(\frac{1}{\tau^{2/3}}\right)^n \longleftrightarrow \frac{1}{n!} \int_0^{\infty} d\xi \, e^{-\xi} \left(\frac{\xi}{\tau^{2/3}}\right)^n$$

Heller, Spaliński, PRL 115 (2015) 072501

Convergent series:

$$\tilde{\epsilon}(\tau, \xi) = \frac{1}{\tau^{4/3}} \sum_{n=0}^{\infty} \frac{a_n}{n!} \frac{\xi^n}{\tau^{2n/3}} \tag{2}$$

Oh no!

Need to invert Borel transform: Heller, Spaliński, PRL 115 (2015) 072501

$$\epsilon_B(\tau) = \int_0^\infty d\xi e^{-\xi} \tilde{\epsilon}(\tau, \xi) \tag{3}$$

But $\tilde{\epsilon}(\tau, \xi)$ has singularities on real axis that must be subverted via:

1. analytic continuation (e.g. via Padé approximation)
2. plus some choice of deformation of the contour $[0, \infty)$

choice of deformation is not unique!!

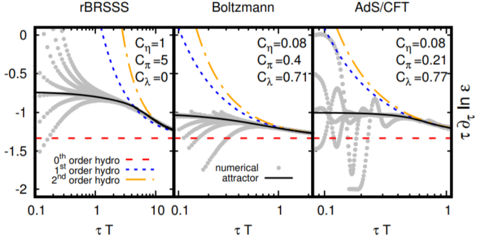
- MEANING: missing UV physics that is not captured at any order in perturbative gradient expansion

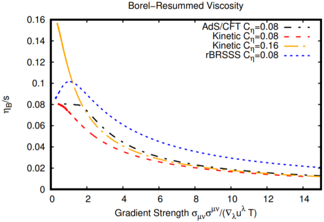
The hydro attractor

1. But ambiguities from all singularities must cancel to give finite, real result Heller, Spaliński, PRL 115 (2015) 072501

Trans-series solution (Écalle, 1980)

$$\epsilon_B(\tau) = \frac{1}{\tau^{4/3}}\left(\underbrace{\sum_{n=0}^{\infty} \frac{a_n}{\tau^{2n/3}}}_{\text{local equilib sector}} + \underbrace{\sum_{k,n=0,\beta}^{\infty} \frac{a_{k,n,\beta}}{\tau^{2n/3}} (\tau T)^{-\beta} e^{-S_k(\tau T)}}_{\text{contribs from non-perturb sectors}} \right) \tag{4}$$



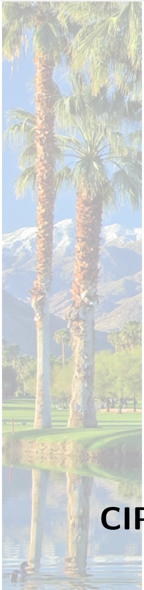


Romatschke, PRL 120 (2018) 012301

Comments

- Lower effective viscosity far from equilibrium may explain why $\eta/s \approx 1/4\pi$ works so well for describing small systems, whereas $\eta/s \approx 0.12 \sim 0.2$ for $\mathrm{Pb+Pb}$ data at LHC
- Non-perturbative corrections $e^{-S_k(\tau T)}$ look like instanton corrections e^{-S_k/g^2} in non-perturbative QFT... origin from hydro path integrals?

The End



Thanks!

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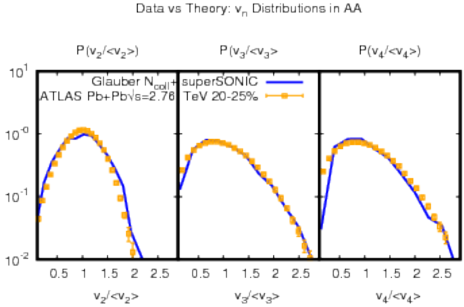
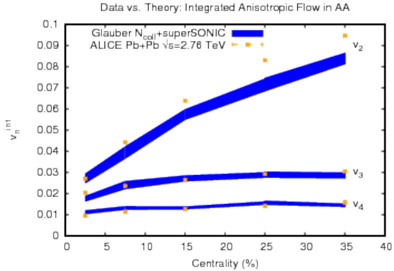
Extras

Backup slides

Are MC Glauber models justified?

$P(v_n)$'s are reproduced by Monte Carlo Glauber in $\mathrm{Pb+Pb}$ collisions with N_{coll} scaling of $\frac{d^3E}{d\eta d^2\mathbf{x}_\perp}$:





Romatschke, Romatschke (2017) 1712.05815