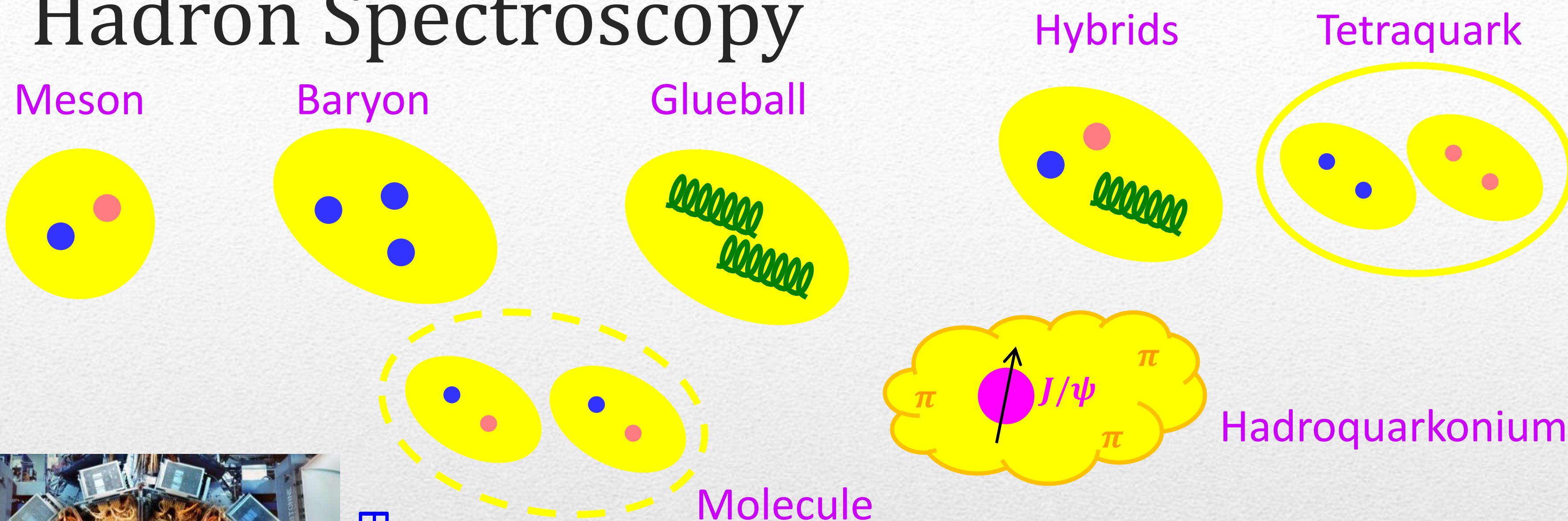
Unitary Reaction models and PWA formalisms

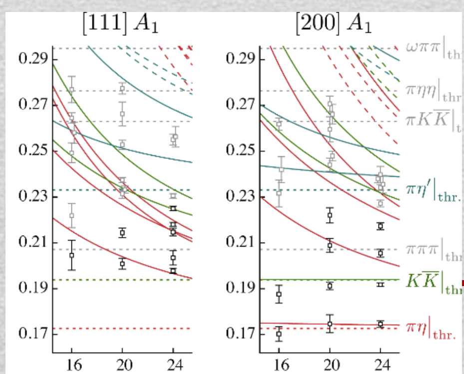
Alessandro Pilloni

CIPANP, Palm Spring, June 1st, 2018

Hadron Spectroscopy



Experiment

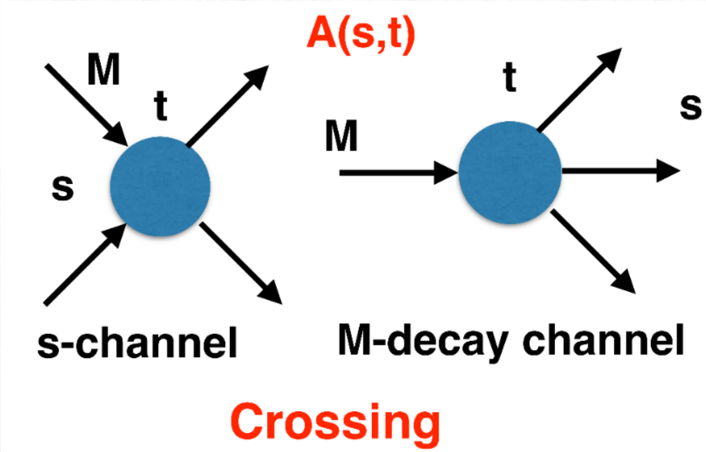


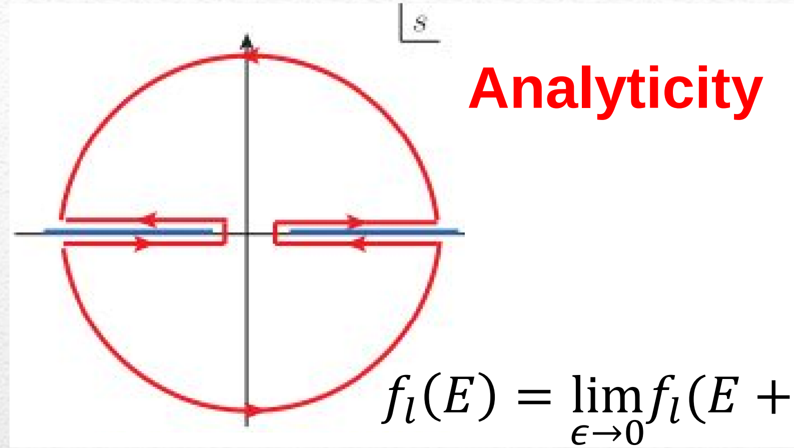
Lattice QCD

Data → Amplitude analysis → Properties, Model building

Interpretations on the spectrum leads to understanding fundamental laws of nature

S-Matrix principles

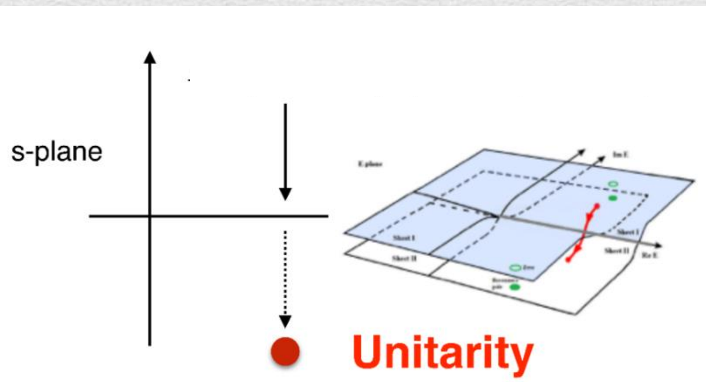




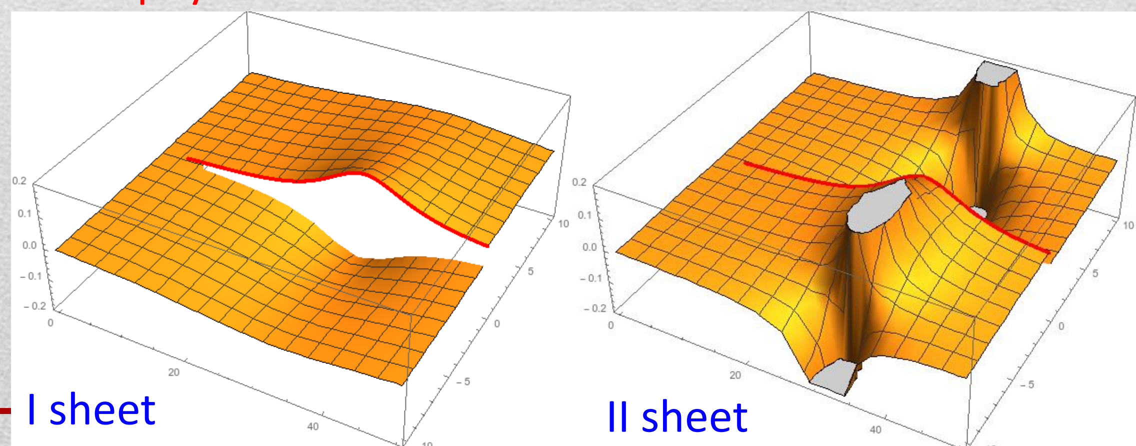
$$f_l(E) = \lim_{\epsilon \to 0} f_l(E + i\epsilon)$$

These are constraints the amplitudes have to satisfy, but do not fix the dynamics

Resonances (QCD states) are poles in the unphysical Riemann sheets



+ Lorentz, discrete & global symmetries



Recipes to build an amplitude

M. Mikhasenko, AP, J. Nys *et al.* (JPAC), EPJC78, 3, 229
AP, J. Nys, M. Mikhasenko *et al.* (JPAC) arXiv:1805.02213

The literature abounds with discussions on the optimal approach to construct the amplitudes for the hadronic reactions

- Helicity formalism

  Jacob, Wick, Annals Phys. 7, 404 (1959)

- Covariant tensor formalisms

  Chung, PRD48, 1225 (1993)
  Chung, Friedrich, PRD78, 074027 (2008)
  Filippini, Fontana, Rotondi, PRD51, 2247 (1995)
  Anisovich, Sarantsev, EPJA30, 427 (2006)

The common lore is that the former one is nonrelativistic, especially when expressed in terms of LS couplings and the latter takes into account the proper relativistic corrections

What do we know?

- Energy dependence is not constrained by symmetry
- Still, there are some known properties one can enforce

$$R_X(m) = B'_{L^X_{\Lambda_b^0}}(p, p_0, d) \left(\frac{p}{M_{\Lambda_b^0}}\right)^{L^X_{\Lambda_b^0}}$$

$$\mathrm{BW}(m|M_{0X}, \Gamma_{0X})\, B'_{L_X}(q, q_0, d) \left(\frac{q}{M_{0X}}\right)^{L_X}$$

- Kinematical singularities: e.g. barrier factors (known)
- Left hand singularities (need model, e.g. Blatt-Weisskopf)
- Right hand singularities = resonant content (Breit Wigner, K-matrix...)

Examples for $B \to \psi\, \pi K$ and $\Lambda_b \to \psi\, pK$

- Kinematical singularities appear because of the spin of the external particles involved
- We can write the most general covariant parameterization of the amplitude as tensor of external polarizations $\otimes$ scalar amplitudes
- Scalar amplitudes are kinematical singularities free, can be matched to helicity amplitudes
- We can get the minimal energy dependent factor, any other would be model-dependent

$$\hat{A}^j_+ = \langle j-1,0;1,1|j,1\rangle g_j(s) + f_j(s)$$

$$\hat{A}^j_0 = \langle j-1,0;1,0|j,0\rangle \frac{s+m_1^2-m_2^2}{2m_1^2} g'_j(s) + f'_j(s)$$

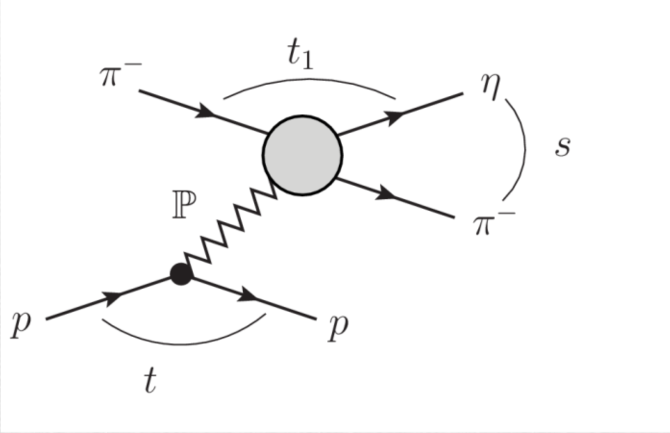
$g_j(s_\pm) = g'_j(s_\pm)$, and $f_j(s), f'_j(s) \sim O(s-s_\pm)$
All these four functions are free of kinematic singularity.

There is no unique recipe to build the right amplitude, but one can ensure the right singularities to be respected

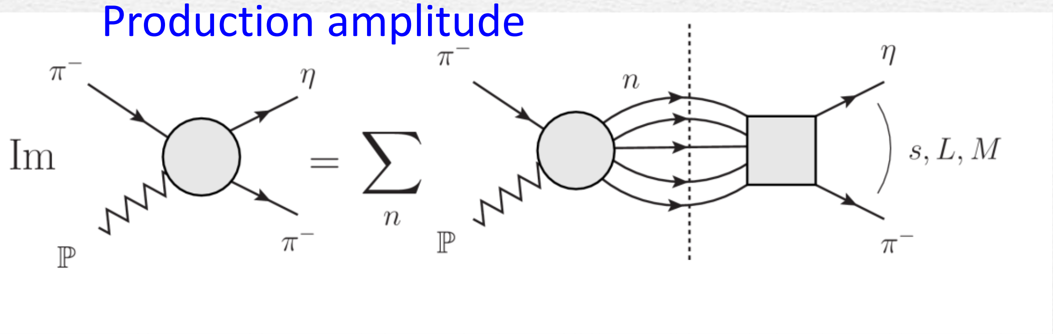
Searching for resonances in $\eta\pi$

- The $\eta\pi$ system is one of the golden modes for hunting hybrid mesons
- We build the partial wave amplitudes according to the N/D method
- We test against the D-wave data, where the a_2 and the a_2' show up

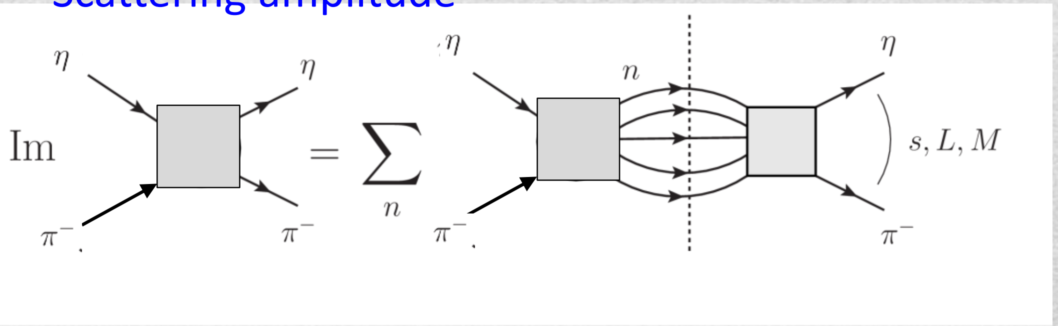
A. Jackura, M. Mikhasenko, AP *et al.* (JPAC & COMPASS), PLB779, 464-472



Production amplitude



Scattering amplitude



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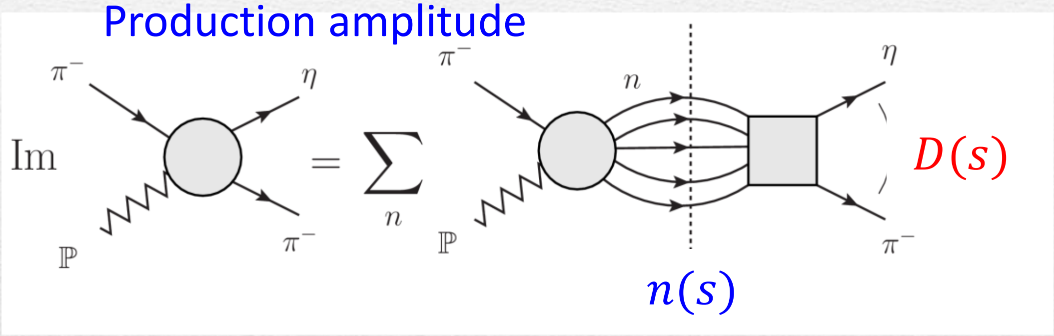
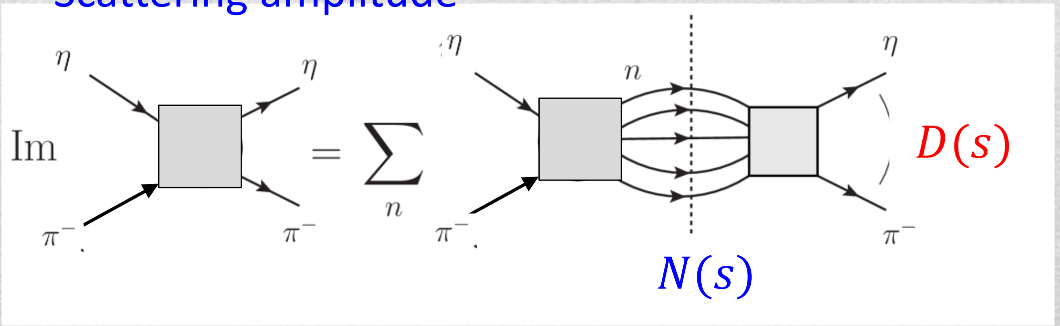
$$t(s) = N(s)D^{-1}(s)$$

Production amplitude



Scattering amplitude



The $D(s)$ has only right hand cuts;
it contains all the Final State Interactions constrained by unitarity → universal

$$\mathrm{Im}\, D(s) = -\rho\, N(s)$$

Searching for resonances in $\eta\pi$

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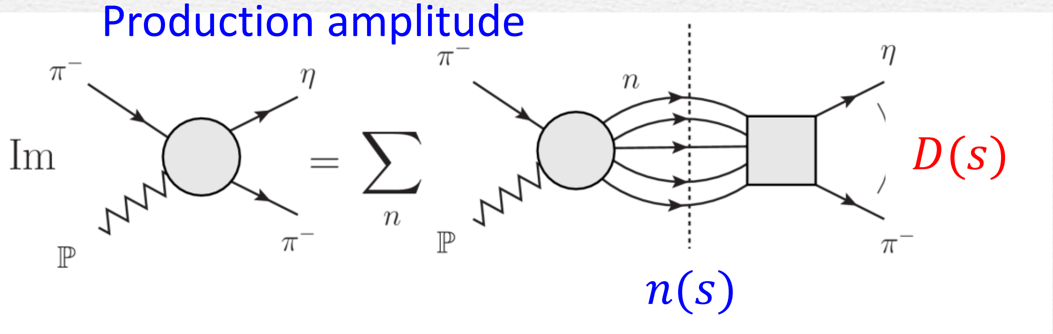
A. Jackura, M. Mikhasenko, AP *et al.* (JPAC & COMPASS), PLB779, 464-472

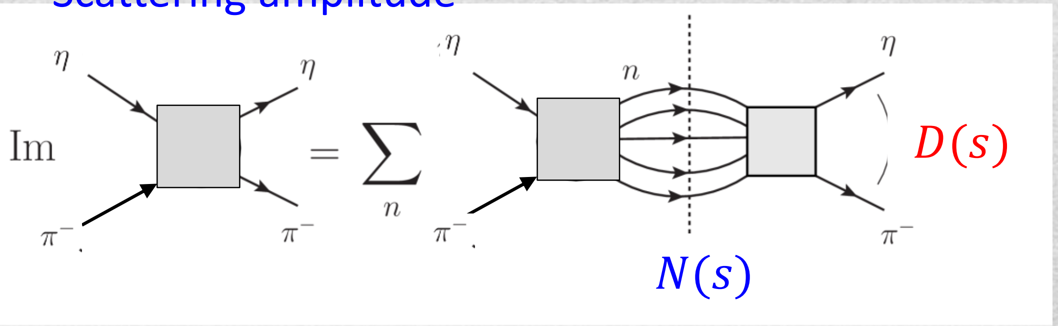
$$t(s) = N(s)D^{-1}(s)$$

The $n(s), N(s)$ have left hand cuts only, they depend on the exchanges → process-dependent, smooth

Production amplitude



Scattering amplitude



Searching for resonances in $\eta\pi$

The denominator $D(s)$ contains all the FSI constrained by unitarity → universal

$$D(s) = (K^{-1})(s) - \frac{s}{\pi}\int_{s_i}^{\infty} \frac{\rho(s')N(s')}{s'\,(s'-s)} ds'$$

$$K(s) = \sum_R \frac{g_R^2}{M_R^2 - s}$$

OR

$$K^{-1}(s) = c_0 - c_1 s + \sum_i \frac{c_i}{M_i^2 - s}$$

$$\rho(s)N(s) = g\frac{\lambda^{(2l+1)/2}\left(s, m_\pi^2, m_\eta^2\right)}{(s+s_R)^{2l+3}}$$

Searching for resonances in $\eta\pi$

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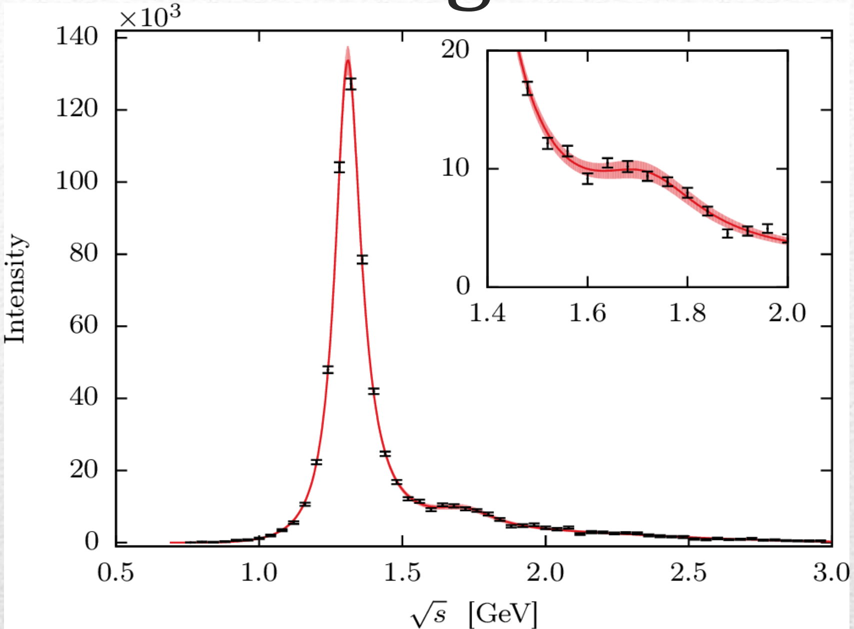
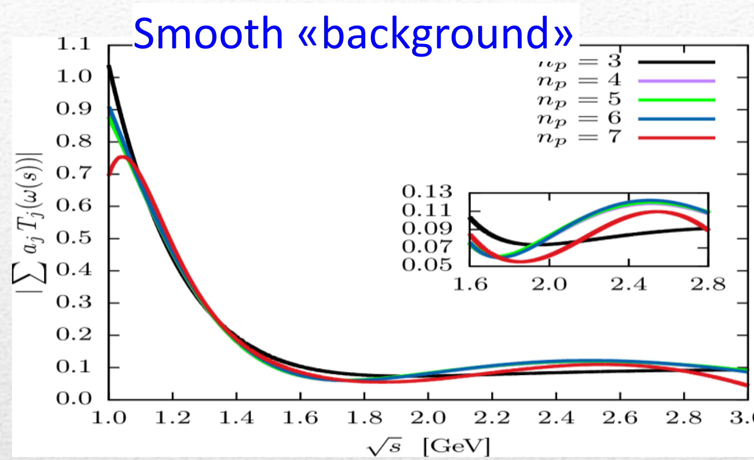
$$K(s) = \sum_R \frac{g_R^2}{M_R^2 - s}$$ **OR** $$K^{-1}(s) = c_0 - c_1 s + \sum_i \frac{c_i}{M_i^2 - s}$$

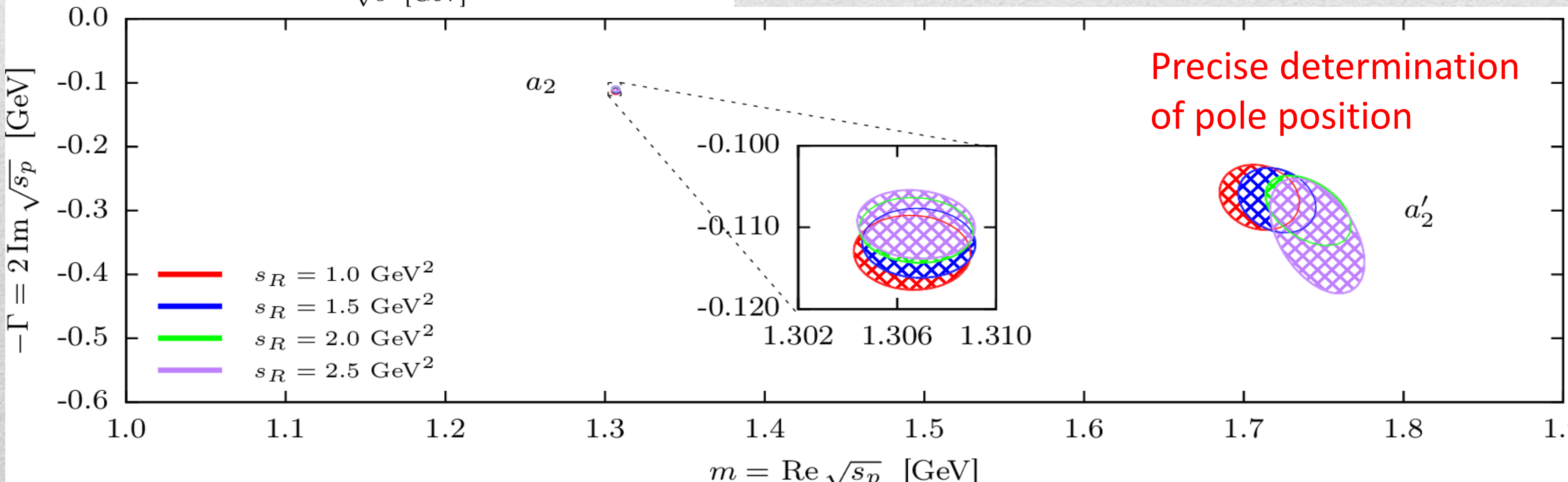
The $n(s)$ is process-dependent, smooth

$$n(s) = \sum_j a_j T_j\left(\omega(s)\right) \qquad \omega(s) = \frac{s}{s + s_0}$$

Searching for resonances in $\eta\pi$



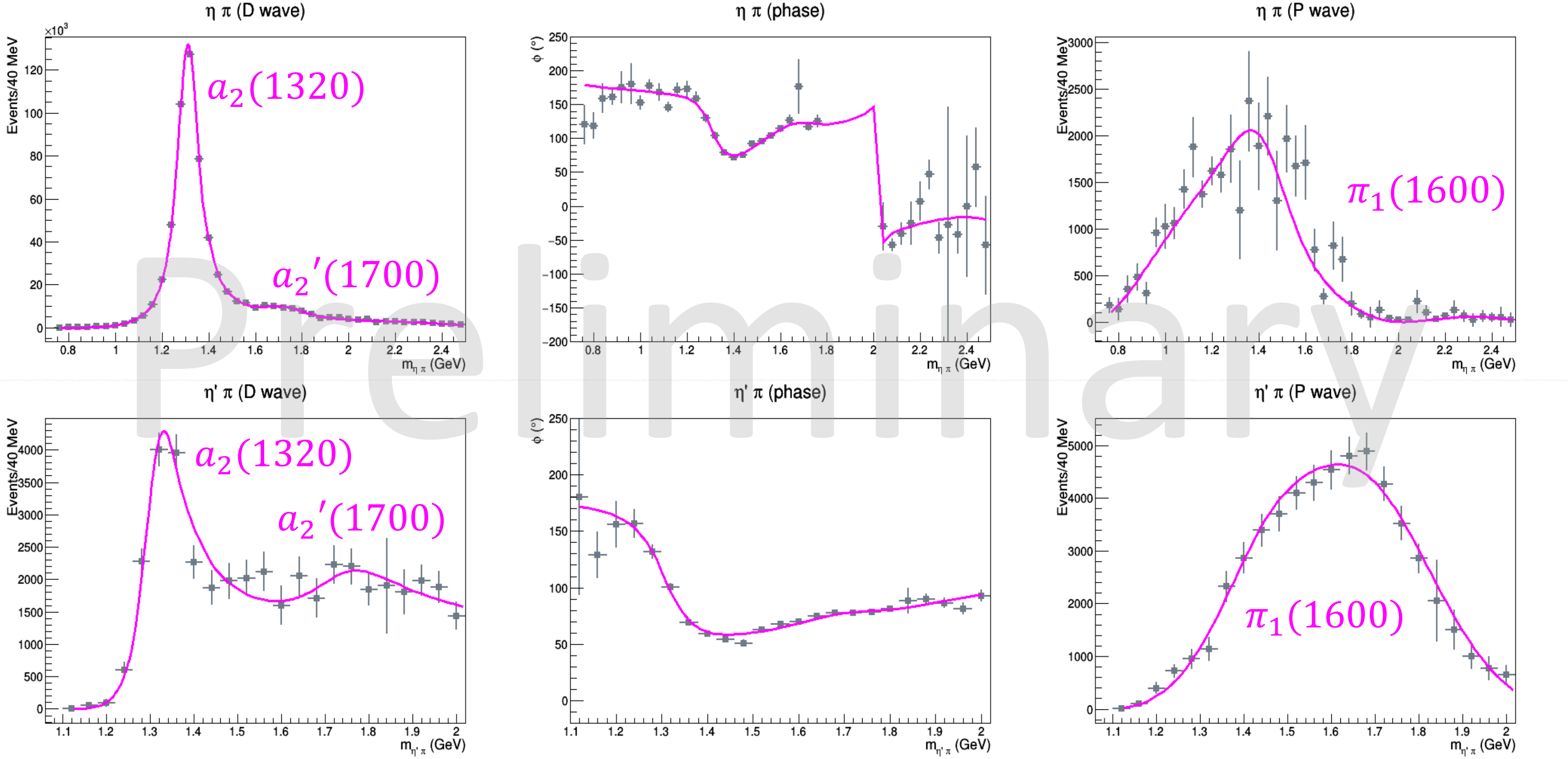




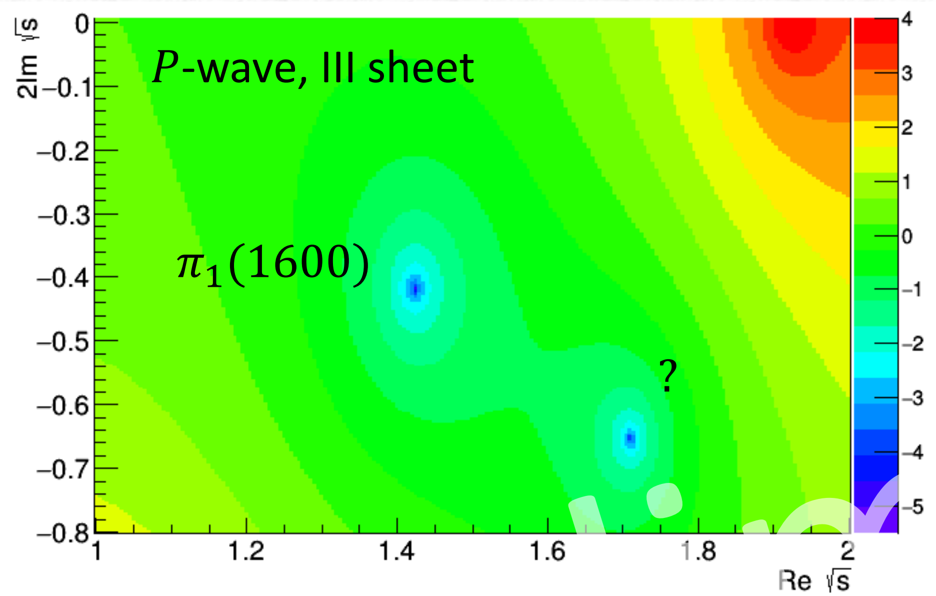
Searching for resonances in $\eta\pi$ and $\eta'\pi$

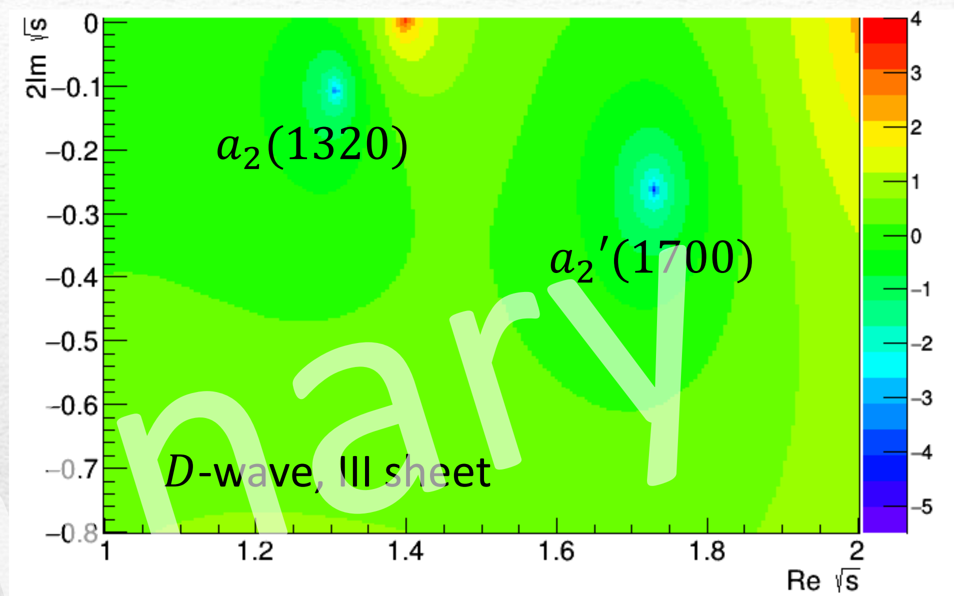
Coupled channel analysis ongoing for the P- and D-wave, same model

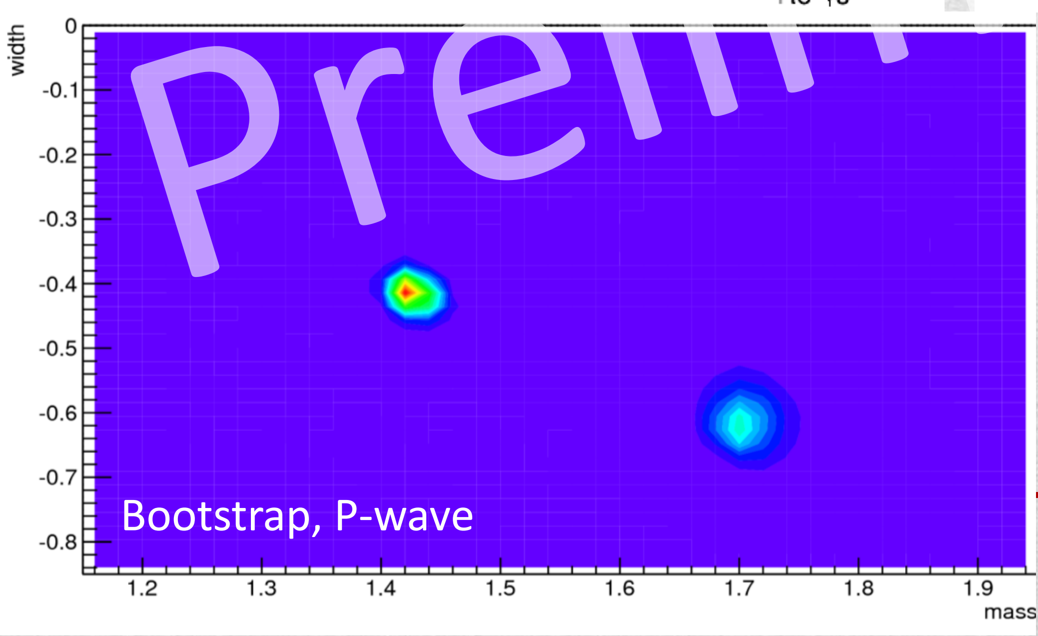
A. Rodas, AP *et al.* (JPAC), in preparation



Searching for resonances in $\eta\pi$ and $\eta'\pi$





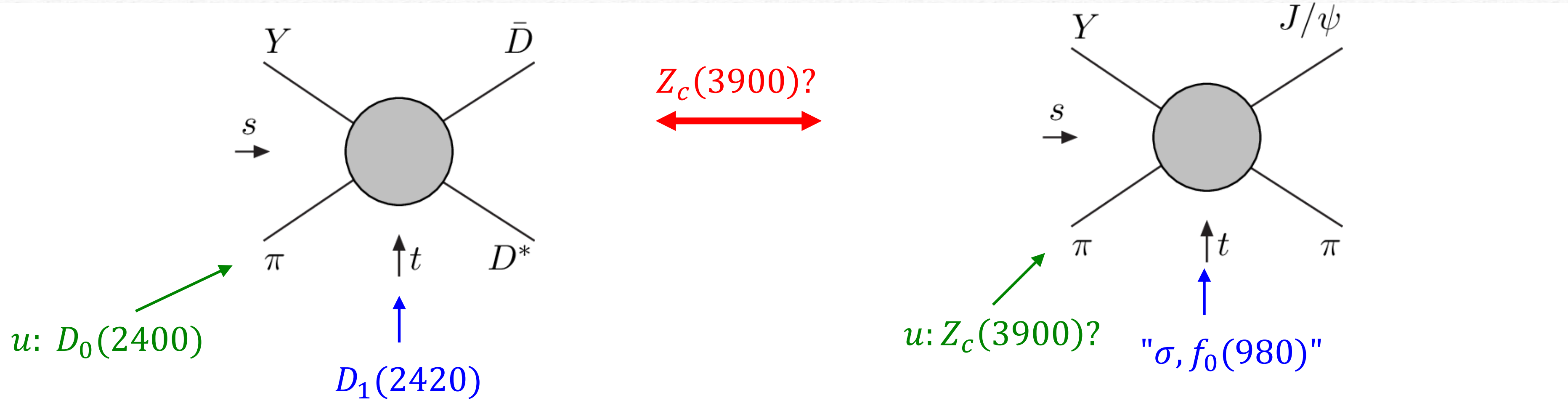


A Statistical Boostrap analysis
will establish the sensitivity
to a second pole,
likely to be systematically dominated

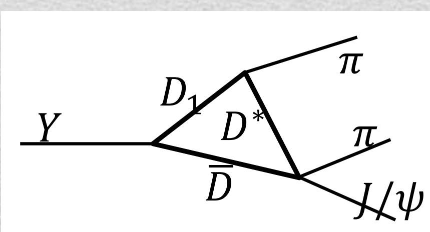
Amplitude analysis for $Z_c(3900)$

One can test different parametrizations of the amplitude, which correspond to
different singularities → different natures

AP *et al.* (JPAC), PLB772, 200

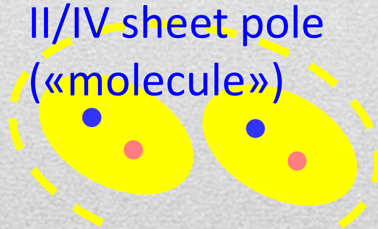


Triangle rescattering,
logarithmic branching point



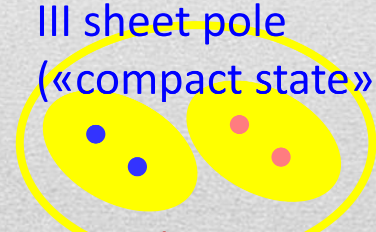
Szczepaniak, PLB747, 410

(anti)bound state,
II/IV sheet pole
(«molecule»)



Tornqvist, Z.Phys. C61, 525
Swanson, Phys.Rept. 429
Hanhart *et al.* PRL111, 132003

Resonance,
III sheet pole
(«compact state»)



Maiani *et al.*, PRD71, 014028
Faccini *et al.*, PRD87, 111102
Esposito *et al.*, Phys.Rept. 668

Testing scenarios

- We approximate all the particles to be scalar – this affects the value of couplings, which are not normalized anyway – but not the position of singularities.
  This also limits the number of free parameters

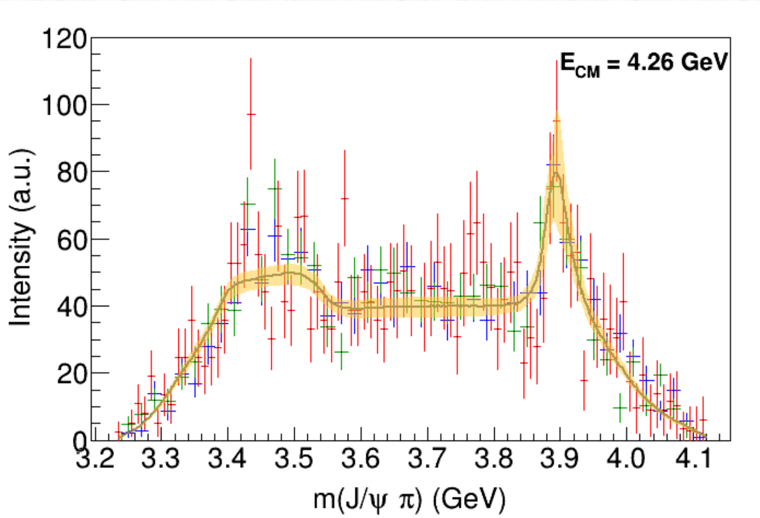
$$f_i(s,t,u) = 16\pi \left[a_{0,i}^{(t)}(t) + a_{0,i}^{(u)}(u) + \sum_j t_{ij}(s) \left(c_j + \frac{s}{\pi} \int_{s_j}^{\infty} ds' \frac{\rho_j(s') b_{0,j}(s')}{s'(s'-s)} \right) \right],$$

The scattering matrix is parametrized as $(t^{-1})_{ij} = K_{ij} - i\,\rho_i\,\delta_{ij}$

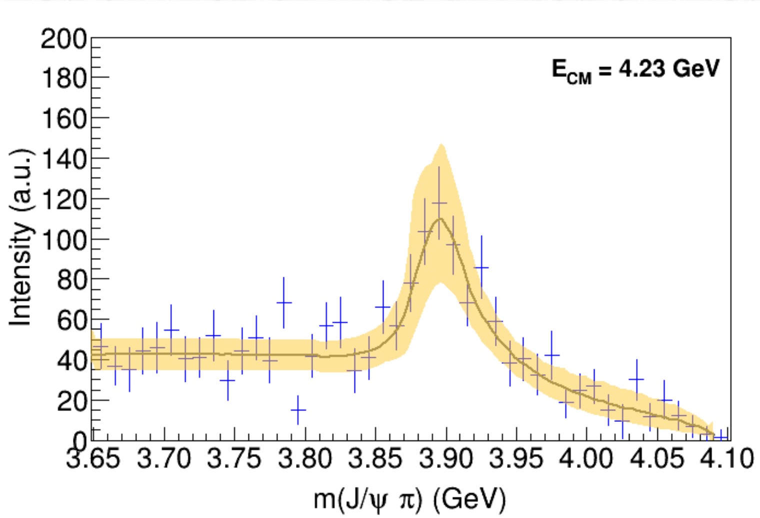
Four different scenarios considered:

- «III»: the K matrix is $\frac{g_i\, g_j}{M^2 - s}$, this generates a pole in the closest unphysical sheet the rescattering integral is set to zero
- «III+tr.»: same, but with the correct value of the rescattering integral
- «IV+tr.»: the K matrix is constant, this generates a pole in the IV sheet
- «tr.»: same, but the pole is pushed far away by adding a penalty in the χ^2

Fit: III



E$_{CM}$ = 4.26 GeV
Intensity (a.u.)
m(J/ψ π) (GeV)



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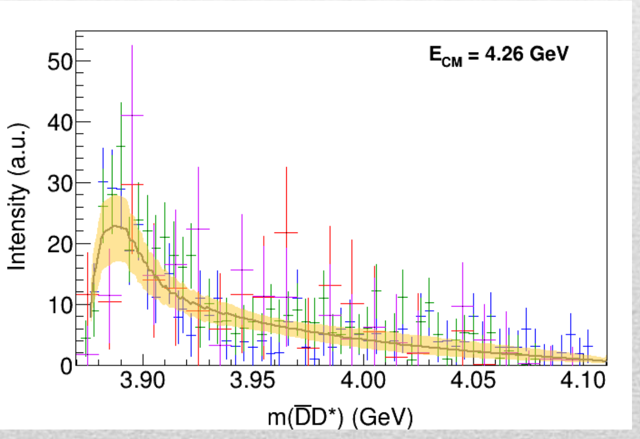

E$_{CM}$ = 4.26 GeV
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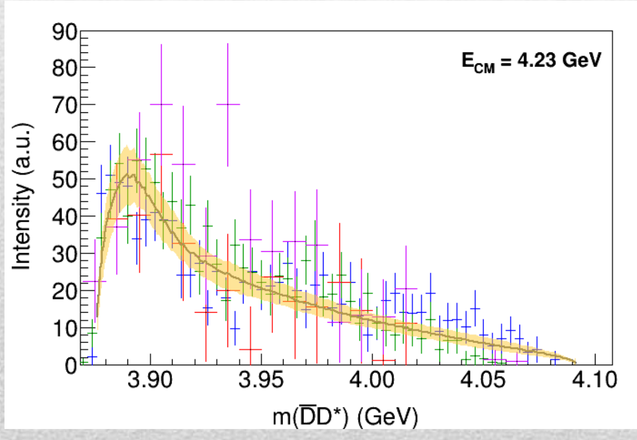
m($\bar{D}$D*) (GeV)



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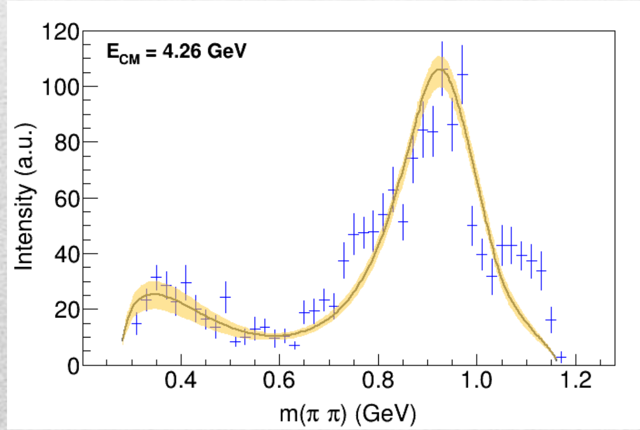

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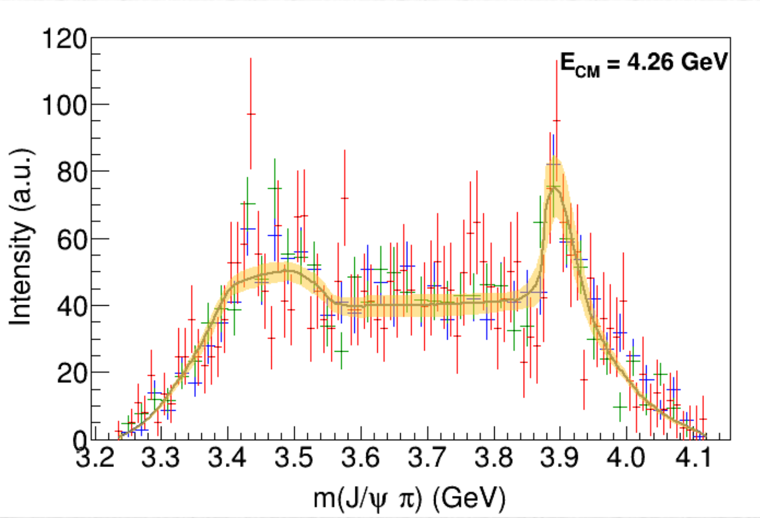
Intensity (a.u.)
m(π π) (GeV)

Fit: III+tr.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Intensity (a.u.)
m(J/ψ π) (GeV)
0
20
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3.2
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3.6
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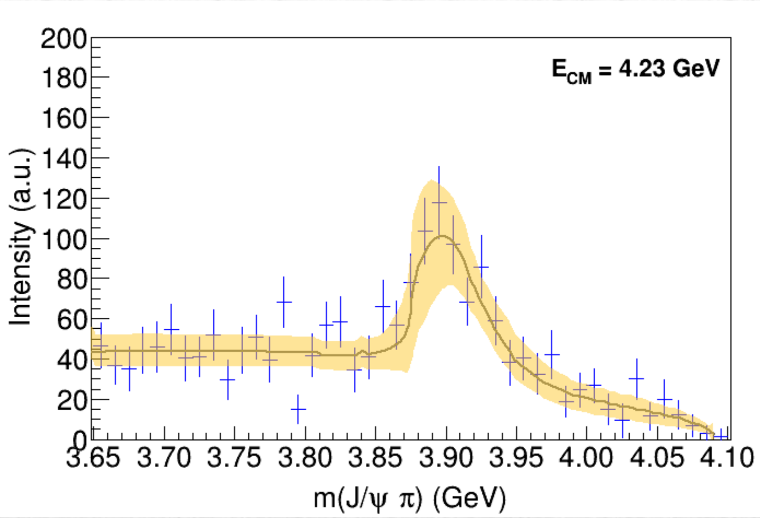
3.9
4.0
4.1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Intensity (a.u.)
m(J/ψ π) (GeV)
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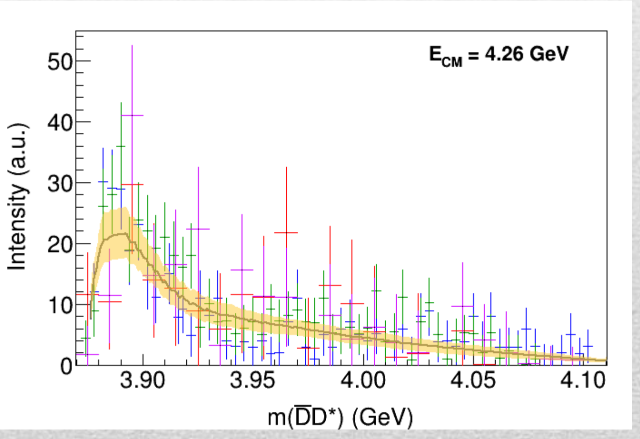
4.05
4.10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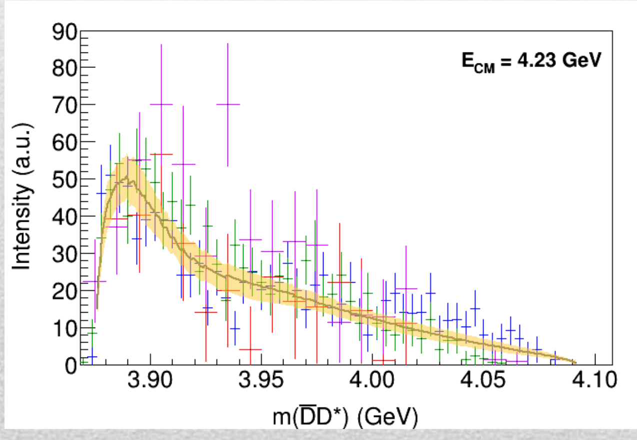
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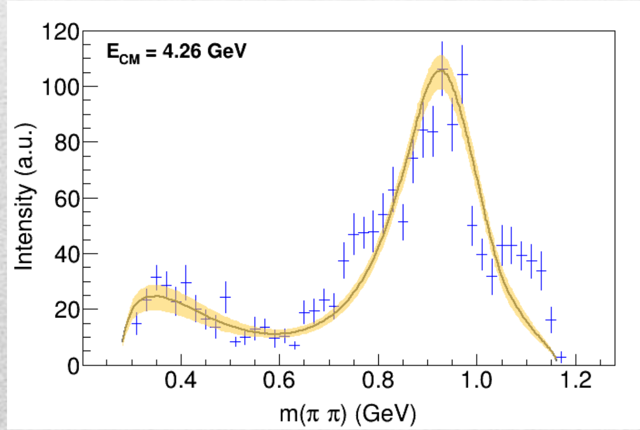
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Intensity (a.u.)
m(π π) (GeV)
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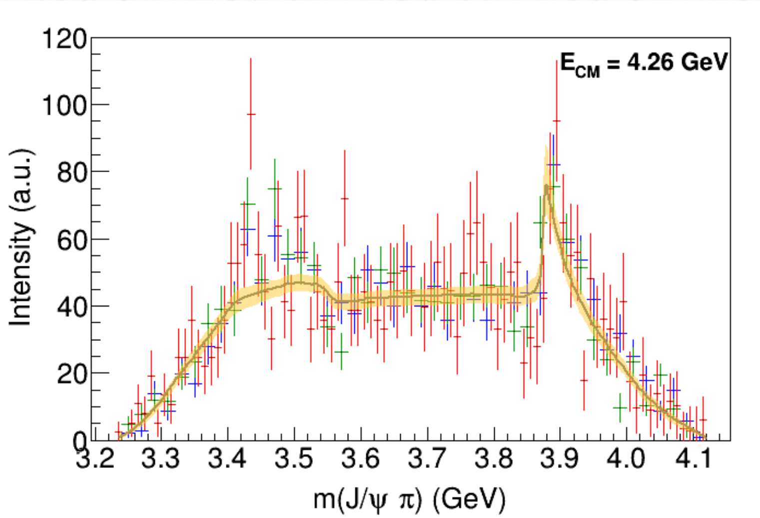
1.0
1.2

Fit: IV+tr.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Intensity (a.u.)
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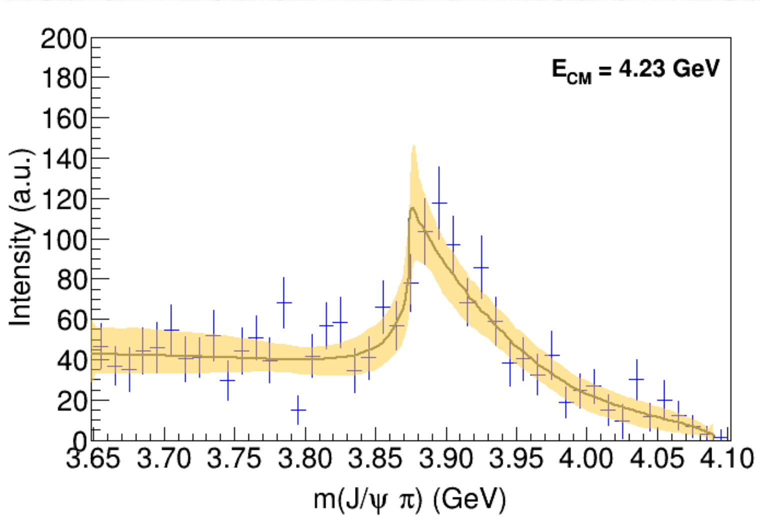
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Intensity (a.u.)
m(J/ψ π) (GeV)
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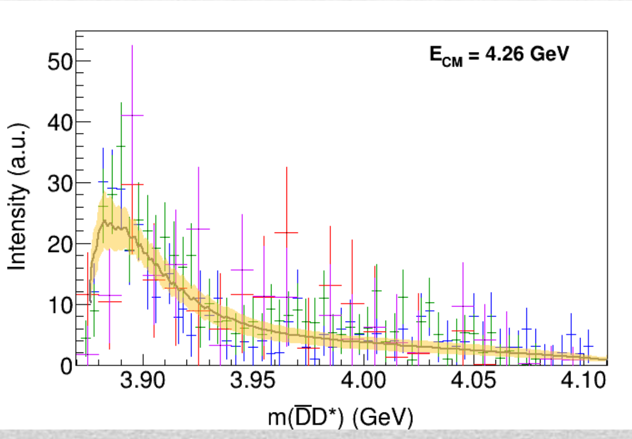
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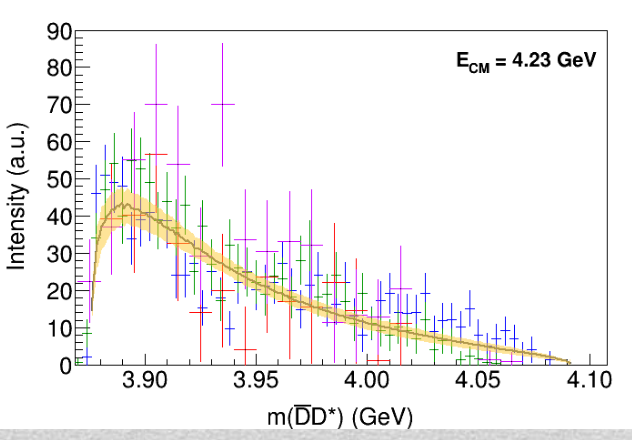
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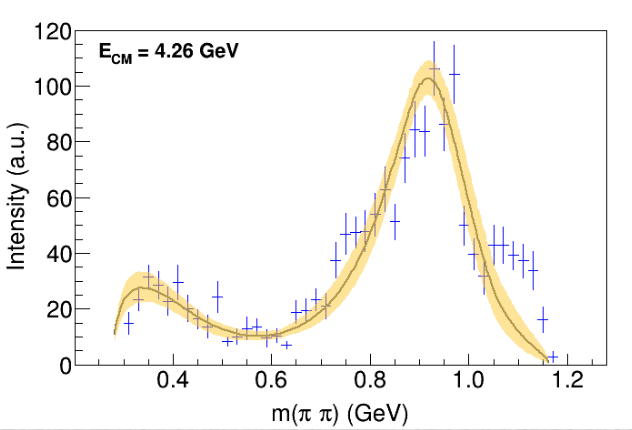
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Intensity (a.u.)
m(π π) (GeV)
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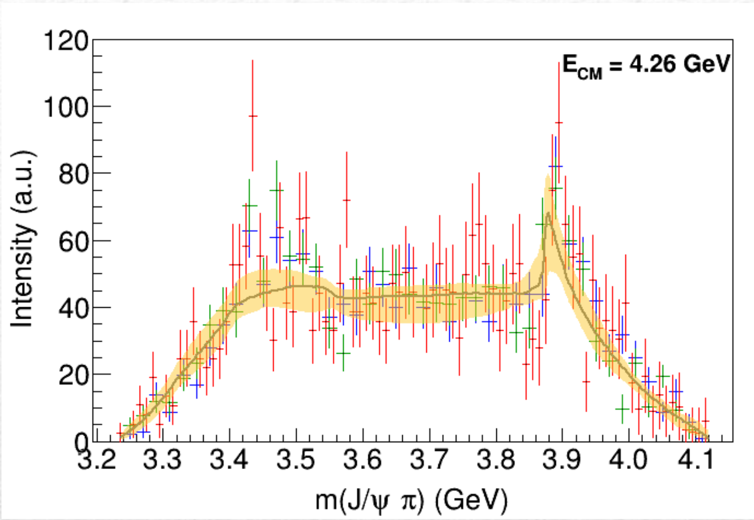
0.6
0.8
1.0
1.2

Fit: tr.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Intensity (a.u.)
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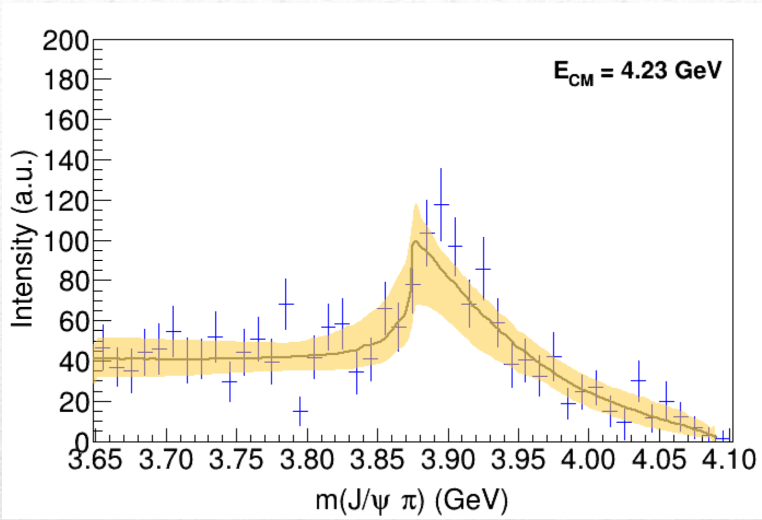
m(J/ψ π) (GeV)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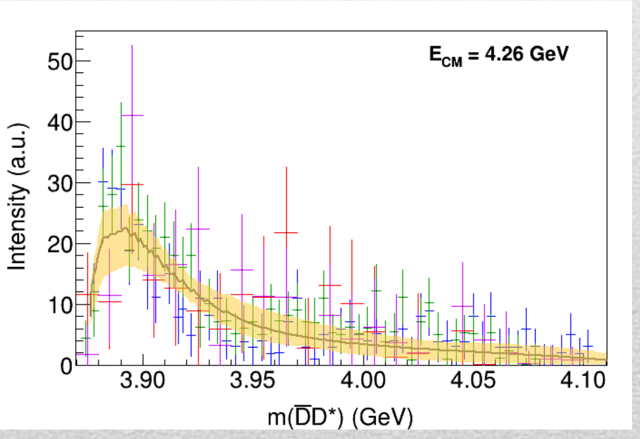
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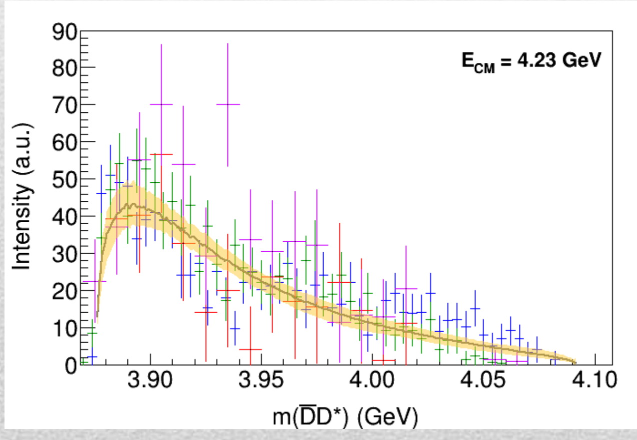
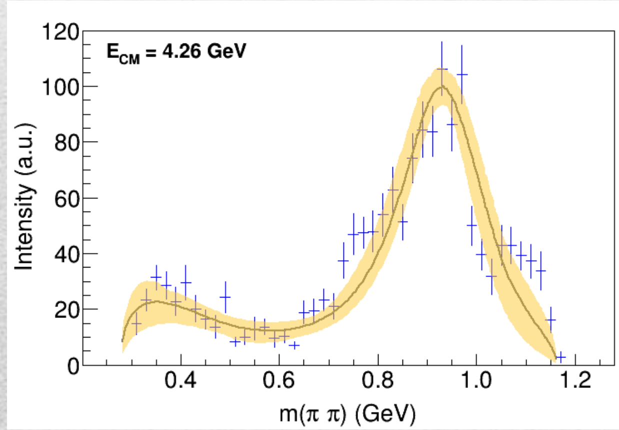
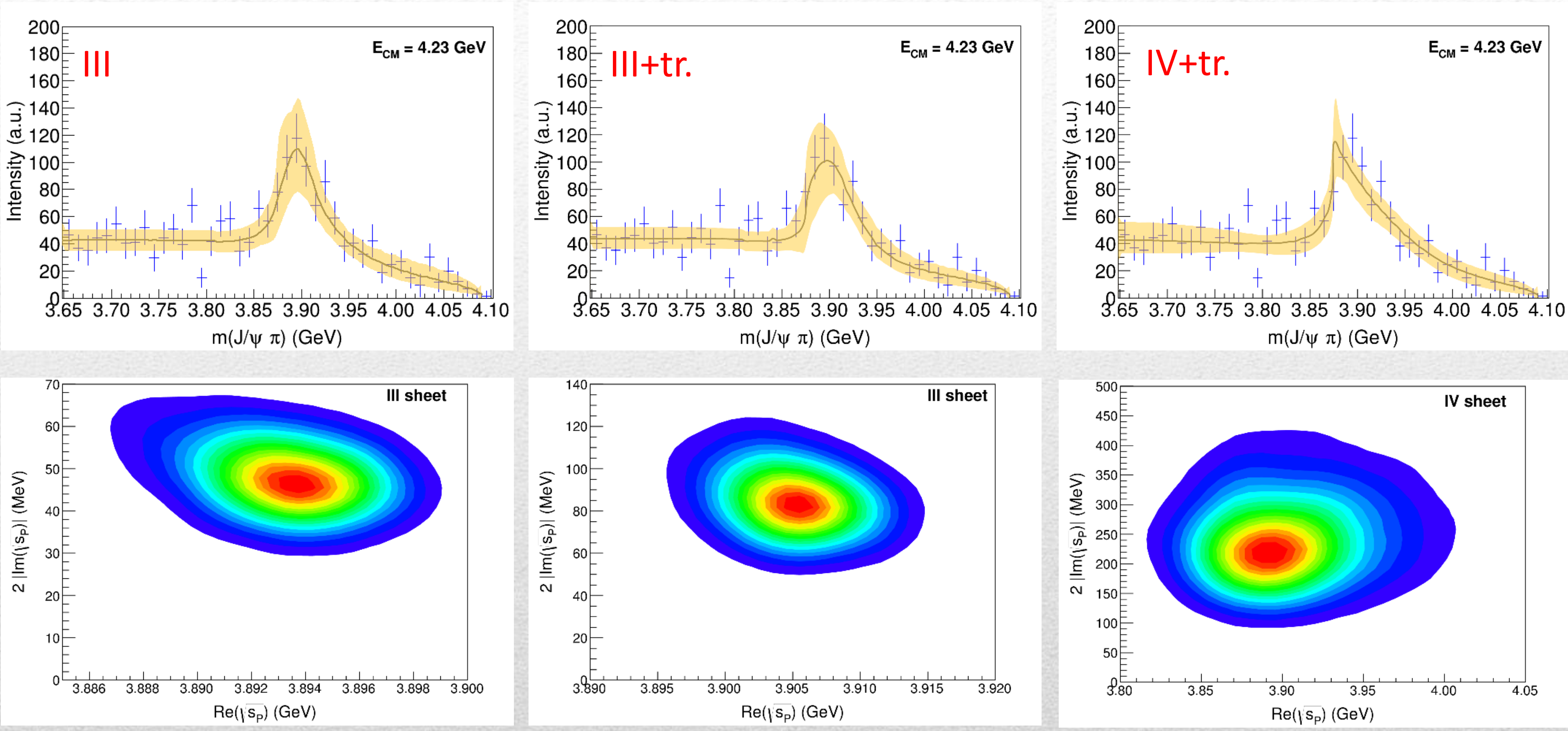
4.10
m(D̄D*) (GeV)


E_CM = 4.23 GeV
Intensity (a.u.)
0
10
20
30
40
50
60
70
80
90
3.90
3.95
4.00
4.05
4.10
m(D̄D*) (GeV)


E_CM = 4.26 GeV
Intensity (a.u.)
0
20
40
60
80
100
120
0.4
0.6
0.8
1.0
1.2
m(π π) (GeV)

Pole extraction



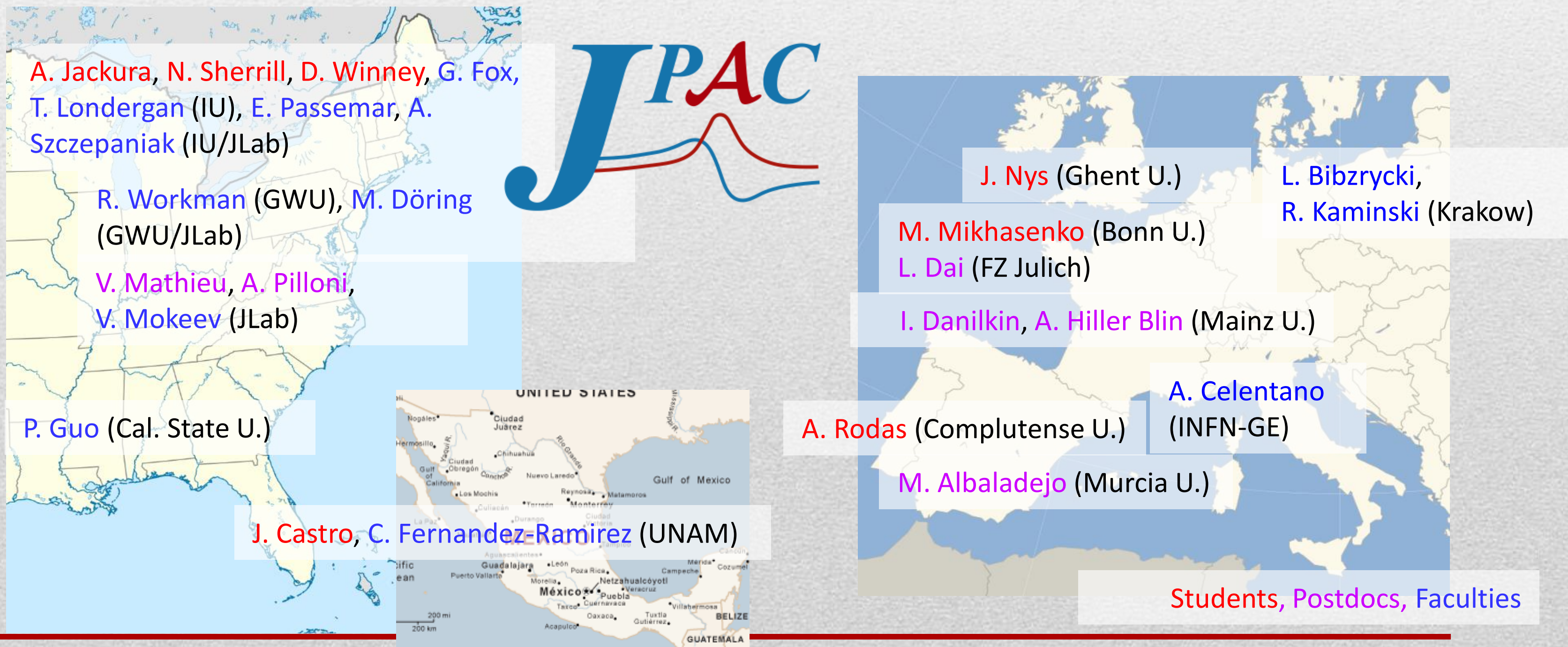
Scenario	III+tr.	IV+tr.	tr.
III	1.5σ (1.5σ)	1.5σ (2.7σ)	"2.4σ" ("1.4σ")
III+tr.	–	1.5σ (3.1σ)	"2.6σ" ("1.3σ")
IV+tr.	–	–	"2.1σ" ("0.9σ")

	III	III+tr.	IV+tr.
M (MeV)	$3893.2^{+5.5}_{-7.7}$	3905^{+11}_{-9}	3900^{+140}_{-90}
Γ (MeV)	48^{+19}_{-14}	85^{+45}_{-26}	240^{+230}_{-130}

Not conclusive at this stage

Conclusions & prospects

- We aim at developing new theoretical tools, to get insight on QCD using first principles of QFT (unitarity, analyticity, crossing symmetry, low and high energy constraints,…) to extract the physics out of the data
- Many other ongoing projects (both for meson and baryon spectroscopy, and for high energy observables), with a particular attention to producing complete reaction models for the golden channels in exotic meson searches



BACKUP

Crossing symmetry in tensor formalisms

- The process $B \to \bar{D}\pi\pi$ is composed of scalar particles only

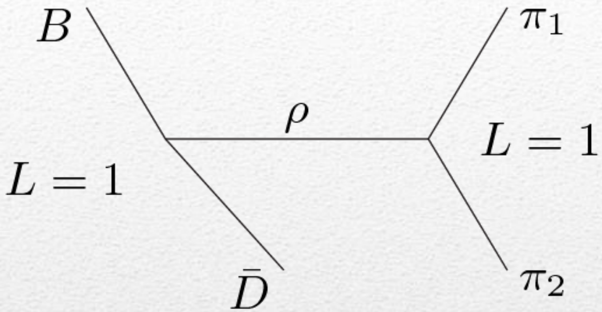
  LHCb, PRD92, 032002 (2015)

- One defines the helicity angle in the isobar rest frame, then the amplitude is Lorentz Invariant
- Let's consider the ρ intermediate state, $B \to \bar{D}\rho(\to \pi\pi)$

$$A = \frac{m_B^2 + s - m_D^2}{2m_B^2} \cos\theta \times qp = \frac{E_\rho^{(B)}}{m_B} \cos\theta \times qp$$

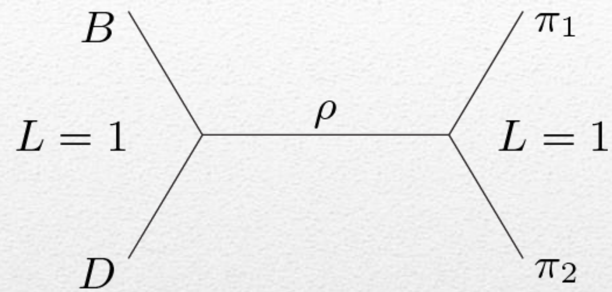
- The factors p and q are the $L = 1$ expected barrier factors. The additional factor is analytical in s, not a kinematical singularity. Why is it there?

Crossing symmetry in tensor formalisms



The tensor amplitude is given by $p_D^{(B)} \cdot p_\pi^{(\rho)}$, where $p_D^{(B)}$ is the breakup momentum in the B frame, and $p_\pi^{(\rho)}$ the decay momentum in the isobar frame

$$A = \frac{m_{B^0}^2 + s - m_{h_3}^2}{2m_{B^0}^2} pq \cos\theta$$



However, one can consider the scattering process just in the isobar rest frame.

$$A = pq \cos\theta$$

By crossing symmetry the amplitudes must be the same.

The usual implementation fails crossing symmetry

How helicity formalism works

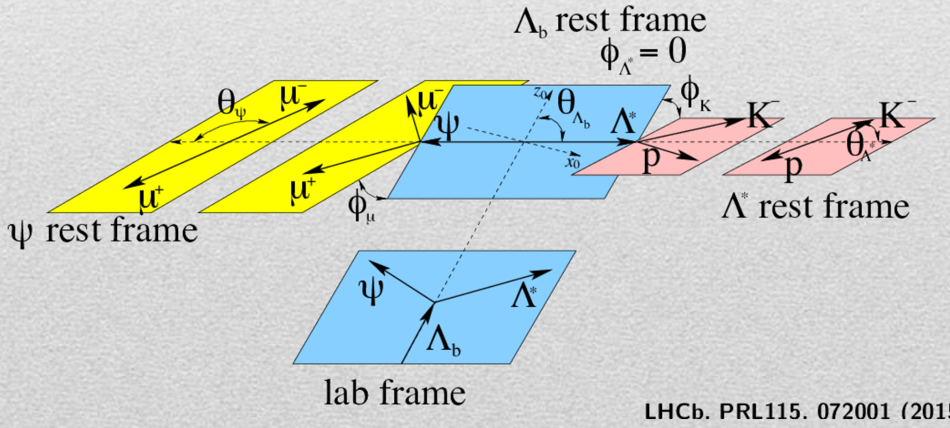
- Helicity formalism enforces the constraints about rotational invariance
- It allows us to fix the angular dependence of the amplitude
- What about energy dependence?

Example: $B \to \psi K^* \to \pi K$

$$\mathcal{M}^{K^*}_{\Delta\lambda_\mu} \equiv \sum_n \sum_{\lambda_{K^*}} \sum_{\lambda_\psi} \mathcal{H}^{B\to K^*_n \psi}_{\lambda_{K^*},\,\lambda_\psi} \delta_{\lambda_{K^*},\lambda\psi}$$

$$\mathcal{H}^{K^*_n \to K\pi} D^{J_{K^*_n}}_{\lambda_{K^*},0}(\phi_K, \theta_{K^*}, 0)^*$$

$$R_{K^*{}_n}(m_{K\pi}) D^1_{\lambda_\psi,\,\Delta\lambda_\mu}(\phi_\mu, \theta_\psi, 0)^*,$$



LHCb. PRL115. 072001 (2015)

Each set of angles is defined in a different reference frame

How tensor formalism works

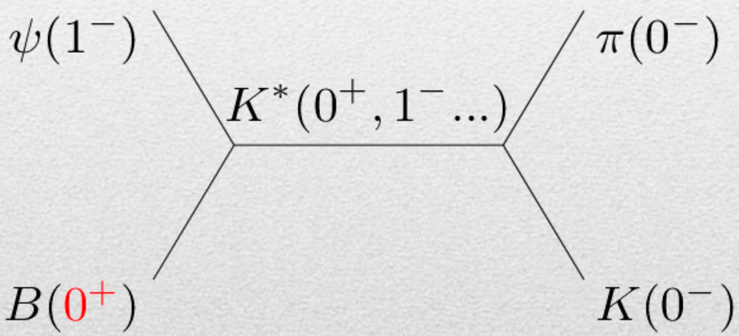
The method is based on the construction of explicitly covariant expressions.

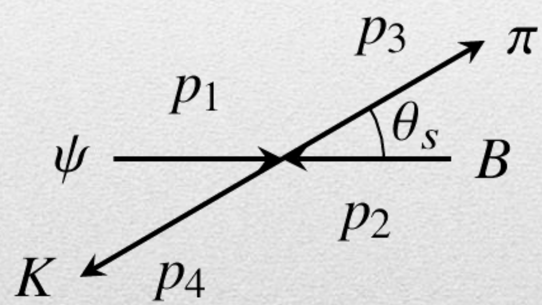
- To describe the decay $a \to bc$, we first consider the polarization tensor of each particle, $\varepsilon^i_{\mu_1 \ldots \mu_{j_i}}(p_i)$
- We combine the polarizations of b and c into a "total spin" tensor $S_{\mu_1 \ldots \mu_S}(\varepsilon_b, \varepsilon_c)$
- Using the decay momentum, we build a tensor $L_{\mu_1 \ldots \mu_L}(p_{bc})$ to represent the orbital angular momentum of the bc system, orthogonal to the total momentum of p_a
- We contract S and L with the polarization of a

Tensor $\times R_X(m)$ which contain resonances and form factors

$B \to \psi\, \pi K$

To consider the effect of spin, let's consider $B \to \psi\pi K$
We focus on the parity violating amplitude for the K^* isobars, scattering kinematics





$$p = \text{incoming 3-momentum in the COM} = \frac{\lambda_{12}^{1/2}}{2\sqrt{s}}$$

$$= \frac{\sqrt{[s-(m_1+m_2)^2]\,[s-(m_1-m_2)^2]}}{2\sqrt{s}}$$

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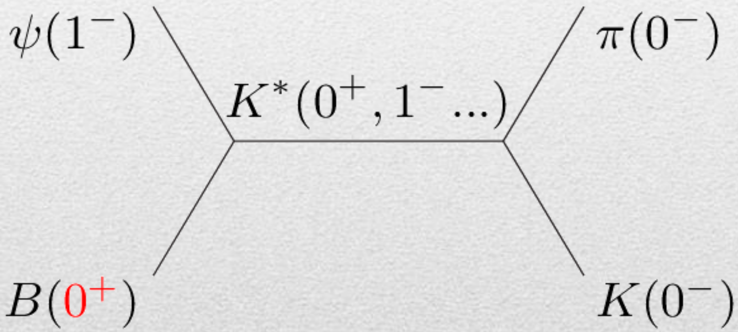

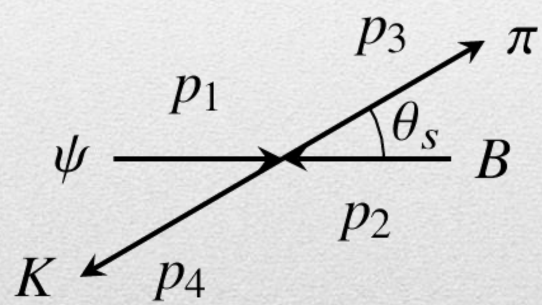

$$q = \text{outgoing 3-momentum in the COM} = \frac{\lambda_{34}^{1/2}}{2\sqrt{s}}$$

$$= \frac{\sqrt{[s-(m_3+m_4)^2]\,[s-(m_3-m_4)^2]}}{2\sqrt{s}}$$

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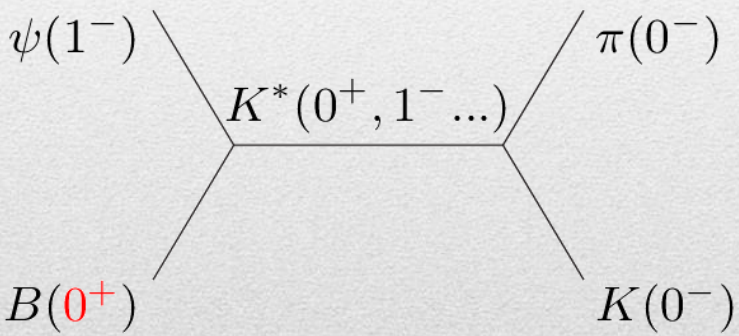
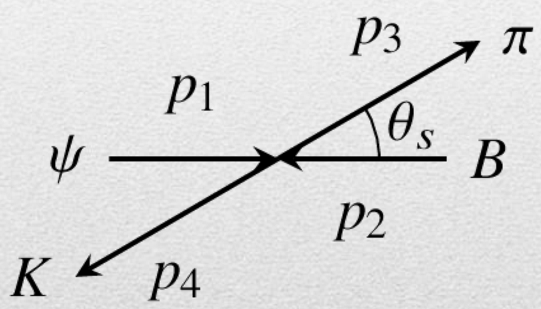




$$z_s = \text{cosine of the scatt. angle in the COM}$$
$$= \frac{s(t-u) + (m_1^2 - m_2^2)(m_3^2 - m_4^2)}{\lambda_{12}^{1/2}\lambda_{34}^{1/2}} = \frac{\text{polynomial}}{\lambda_{12}^{1/2}\lambda_{34}^{1/2}}$$

Helicity amplitudes

$$A_\lambda = \frac{1}{4\pi} \sum_{j=|\lambda|}^{\infty} (2j+1) A_\lambda^j(s)\, d_{\lambda 0}^j(z_s)$$

$$d_{\lambda 0}^j(z_s) = \hat{d}_{\lambda 0}^j(z_s) \xi_{\lambda 0}(z_s), \qquad \xi_{\lambda 0}(z_s) = \left(\sqrt{1-z_s^2}\right)^\lambda$$

$\hat{d}_{\lambda 0}^j(z_s)$ is a polynomial of order $j-|\lambda|$ in z_s,
The kinematical singularities of $A_\lambda^j(s)$ can be isolated by writing

$$A_0^j = \frac{m_1}{p\sqrt{s}} (pq)^j\ \hat{A}_0^j \qquad \text{for } j \geq 1,$$

$$A_\pm^j = q\ (pq)^{j-1}\ \hat{A}_\pm^j \qquad \text{for } j \geq 1,$$

$$A_0^0 = \frac{p\sqrt{s}}{m_1} \hat{A}_0^0 \qquad \text{for } j = 0,$$

Identify covariants

Two helicity couplings → two independent covariant structures

Important: we are not imposing any intermediate isobar

$$A_\lambda(s,t) = \varepsilon_\mu(\lambda, p_1)\left[(p_3 - p_4)^\mu - \frac{m_3^2 - m_4^2}{s}(p_3+p_4)^\mu\right] C(s,t) + \varepsilon_\mu(\lambda, p_1)(p_3+p_4)^\mu B(s,t)$$

$$C(s,t) = \frac{1}{4\pi\sqrt{2}}\sum_{j>0}(2j+1)(pq)^{j-1}\hat{A}^j_+(s)\,\hat{d}^j_{10}(z_s)$$

$$B(s,t) = \frac{1}{4\pi}\hat{A}^0_0 + \frac{1}{4\pi}\frac{4m_1^2}{\lambda_{12}}\sum_{j>0}(2j+1)(pq)^j\left[\hat{A}^j_0(s)\hat{d}^j_{00}(z_s) + \frac{s+m_1^2-m_2^2}{\sqrt{2}m_1^2}\hat{A}^j_+(s)\,z_s\hat{d}^j_{10}(z_s)\right]$$

Everything looks fine but the λ_{12} in the denominator

The brackets must vanish at $\lambda_{12} = 0 \Rightarrow s = s_\pm = (m_1 \pm m_2)^2$,

$\hat{A}^j_+$ and $\hat{A}^j_0$ cannot be independent

General expression and comparison

$$\hat{A}^j_+ = \langle j-1,0;1,1|j,1\rangle g_j(s) + f_j(s)$$

$$\hat{A}^j_0 = \langle j-1,0;1,0|j,0\rangle \frac{s+m_1^2-m_2^2}{2m_1^2} g'_j(s) + f'_j(s)$$

$g_j(s_\pm) = g'_j(s_\pm)$, and $f_j(s), f'_j(s) \sim O(s-s_\pm)$
All these four functions are free of kinematic singularity.

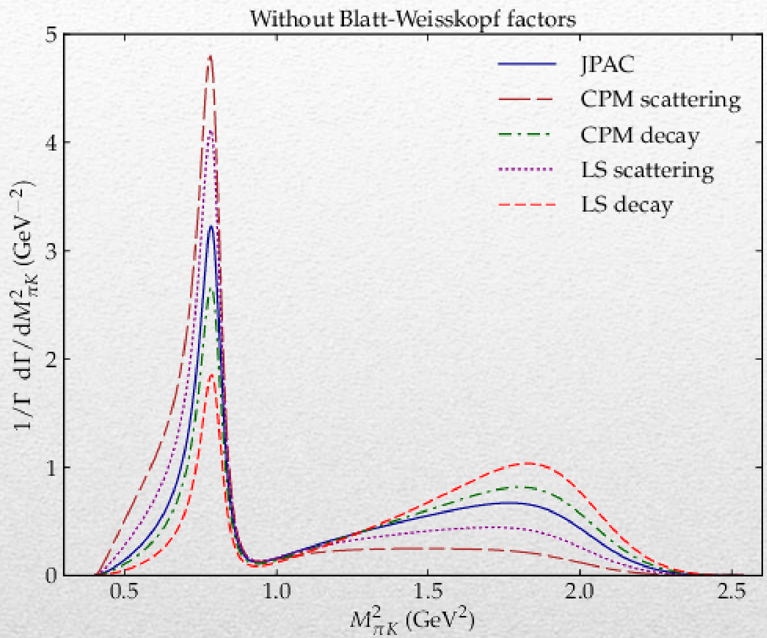
Comparison with tensor formalisms ($j=1$)

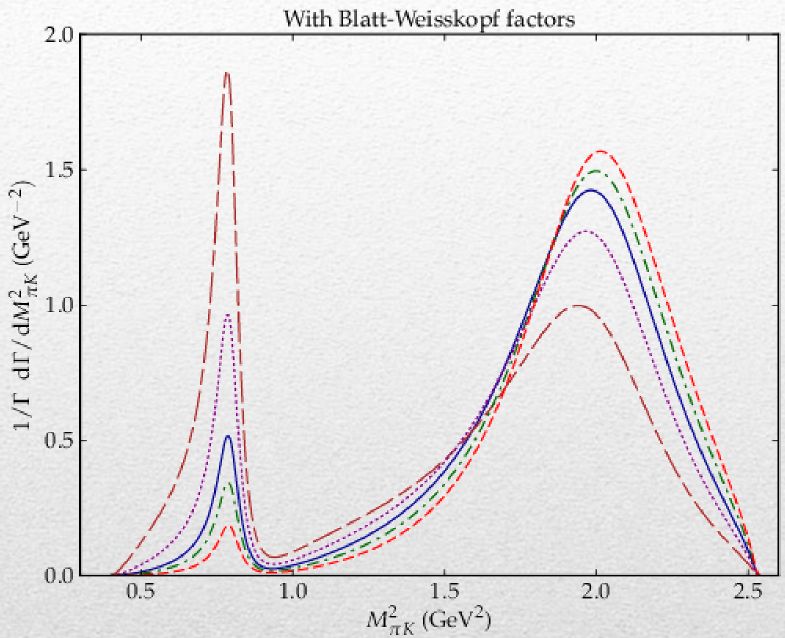
$$g_1 = g'_1 = \frac{4\pi}{3} g_S, \quad f_1 = \frac{2\pi\lambda_{12}}{3s} g_D, \quad f'_1 = -\frac{4\pi\lambda_{12}}{3s}\frac{s+m_1^2-m_2^2}{m_1^2} g_D.$$

If the g_S, g_D are the usual Breit-Wigner, the g', f' are fine

There is no unique recipe to build the right amplitude, but one can ensure the right singularities to be respected

General expression and comparison





We consider the example of two intermediate $K^*(892)$ and $K^*(1410)$

We set $g_S(s) = 0$ and $g_D(s) =$ sum of Breit-Wigner

For the plot on the right we multiply the amplitudes by the Blatt-Weisskopf barrier factors

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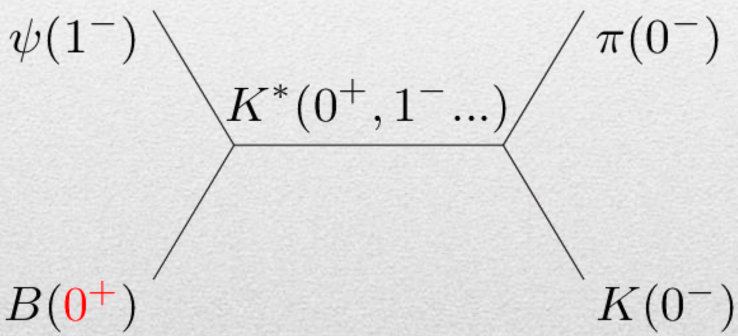

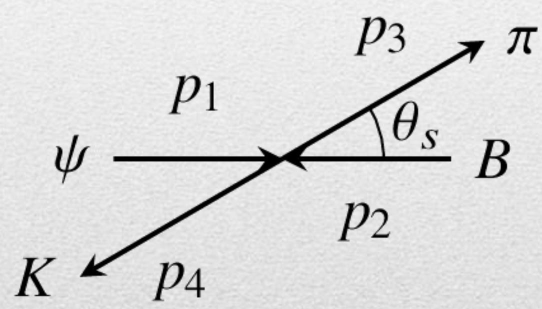

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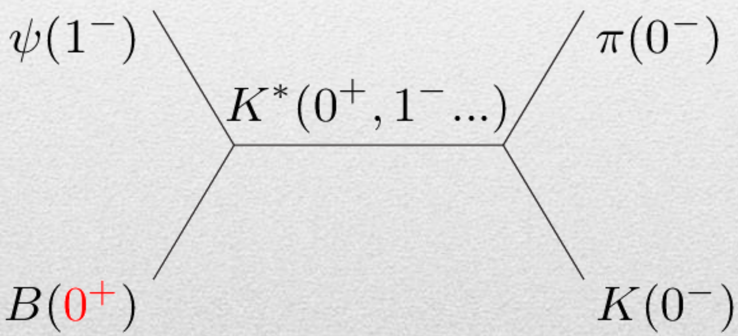

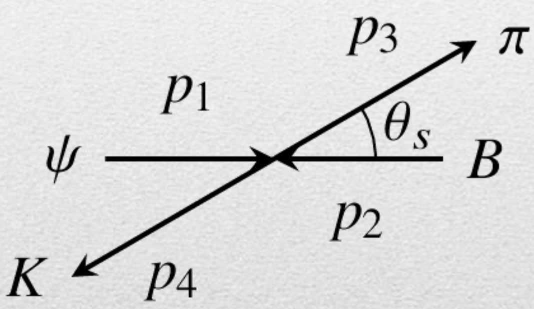

$z_s =$ cosine of the scatt. angle in the COM

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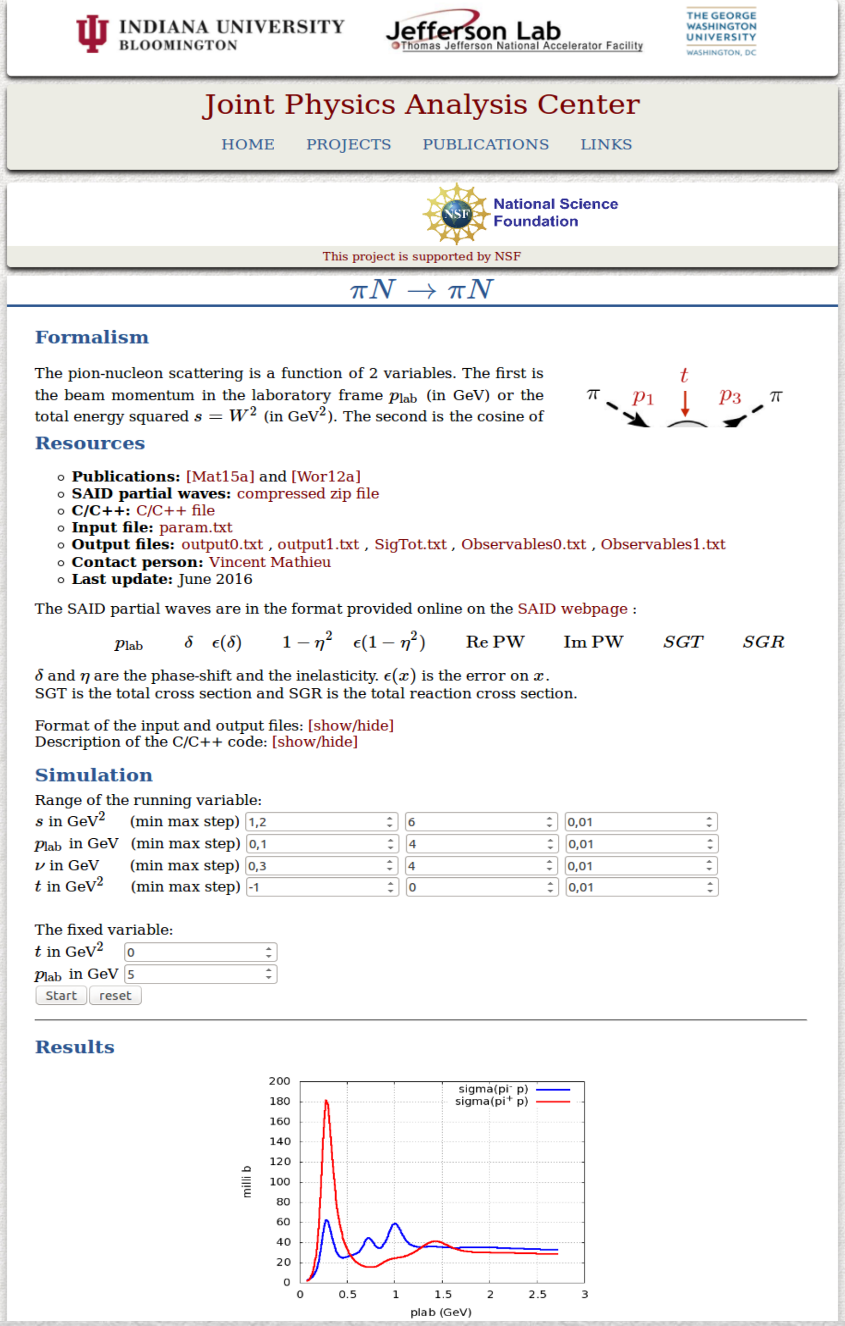
Everything looks fine but the λ_{12} in the denominator
The brackets must vanish at $\lambda_{12}=0 \Rightarrow s = s_\pm = (m_1 \pm m_2)^2$,
$\hat{A}^j_+$ and $\hat{A}^j_0$ cannot be independent

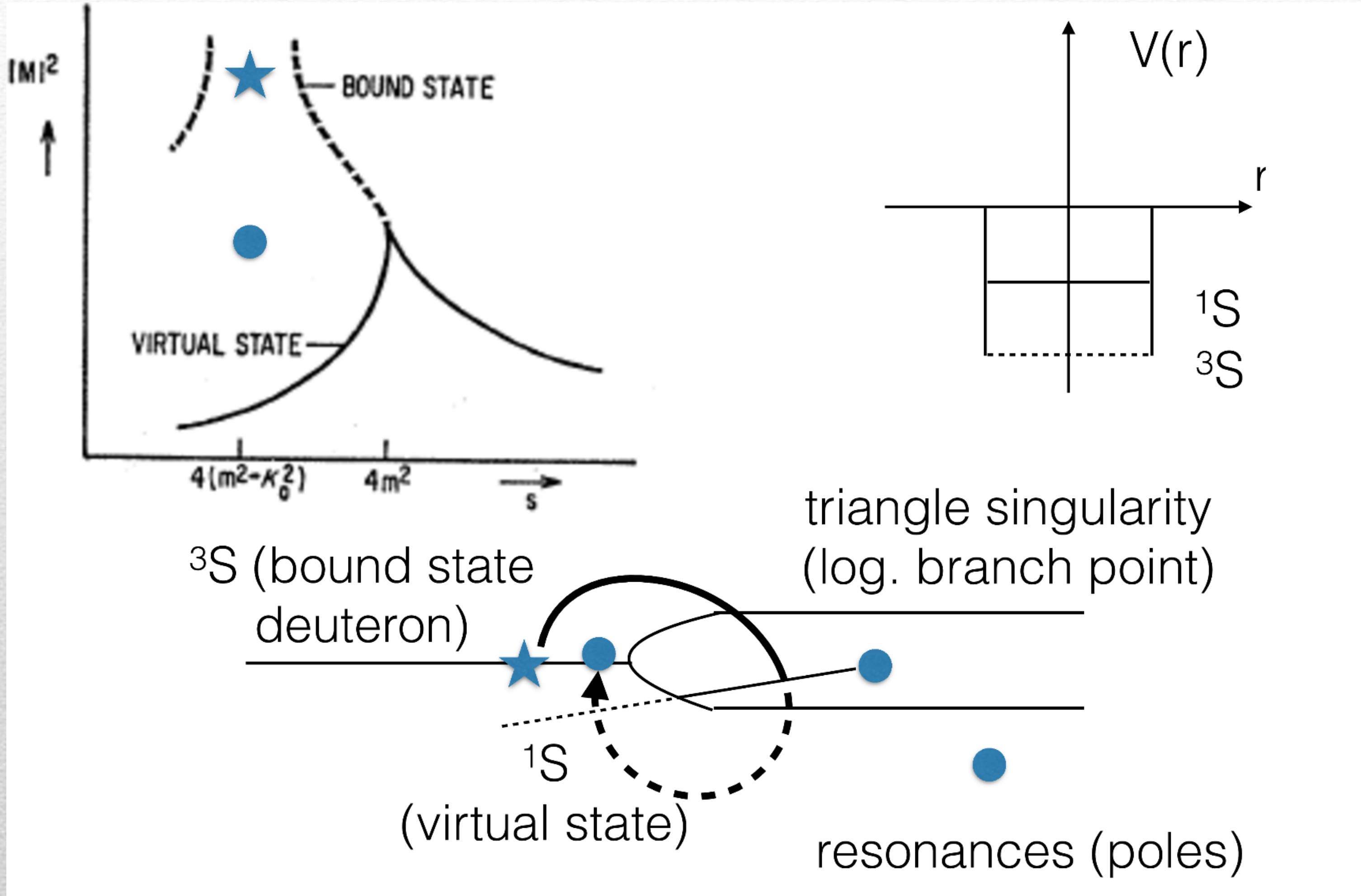
Interactive tools

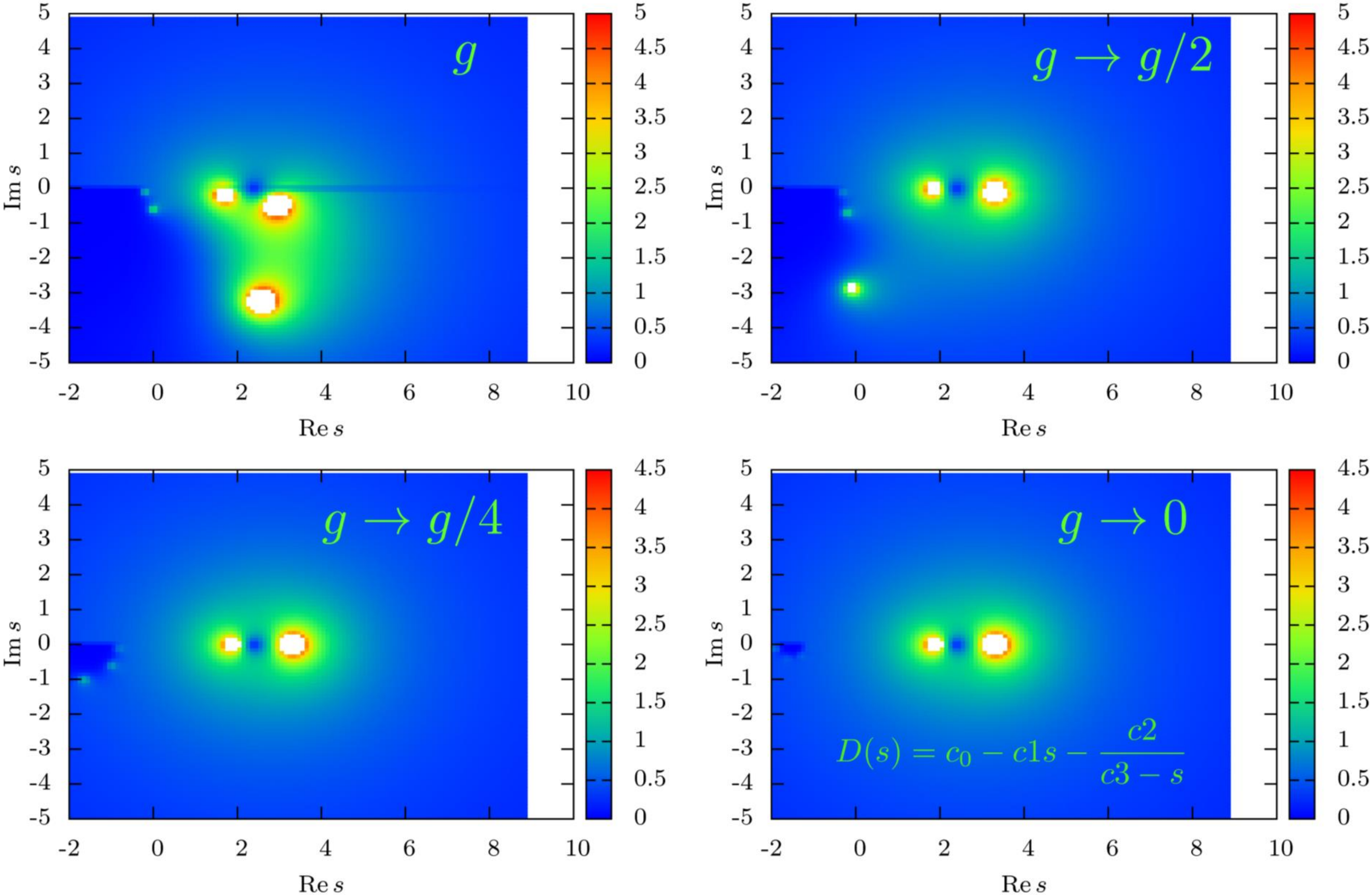
- Completed projects are fully documented on interactive portals
- These include description on physics, conventions, formalism, etc.
- The web pages contain source codes with detailed explanation how to use them. Users can run codes online, change parameters, display results.



http://www.indiana.edu/~jpac/

Pole hunting



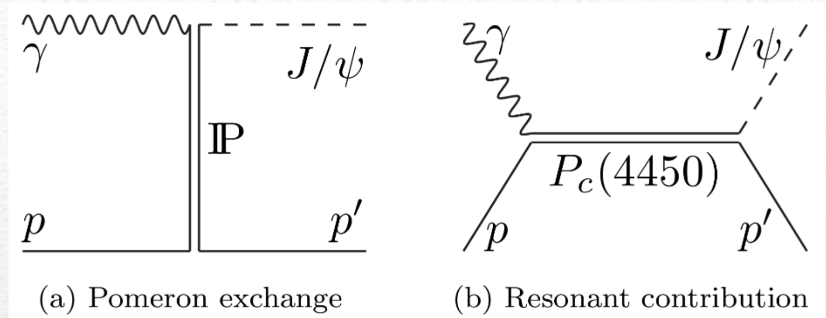


Im s
Re s
g
g → g/2
g → g/4
g → 0
D(s) = c0 − c1s − c2/(c3 − s)

Pentaquark photoproduction

We propose to search the $P_c(4450)$ state in photoproduction

Q. Wang *et al.* PRD92, 034022
M. Karliner *et al.* PLB752, 329-332
Kubarovsky *et al.* PRD92, 031502

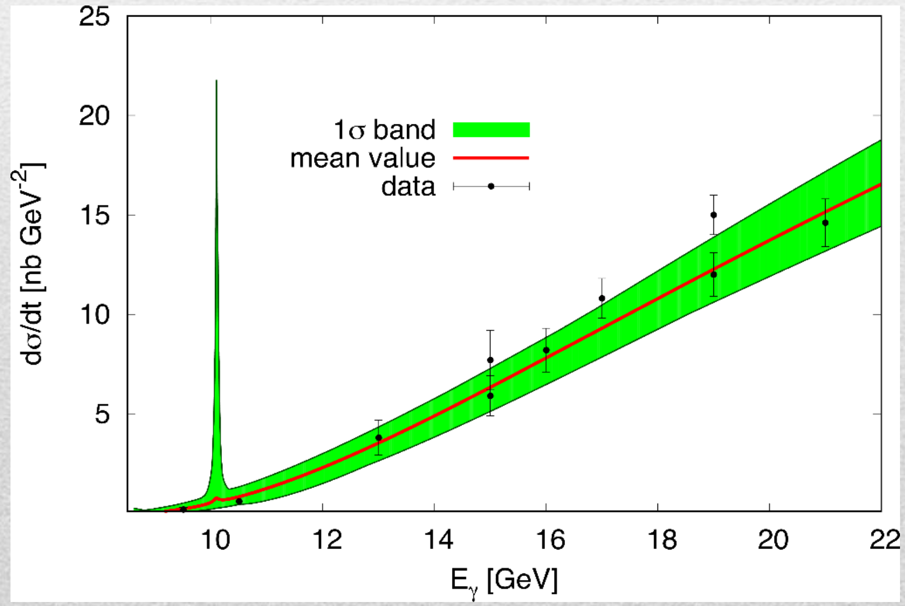


(a) Pomeron exchange (b) Resonant contribution

We use the (few) existing data and VMD + pomeron inspired bkg to estimate the cross section

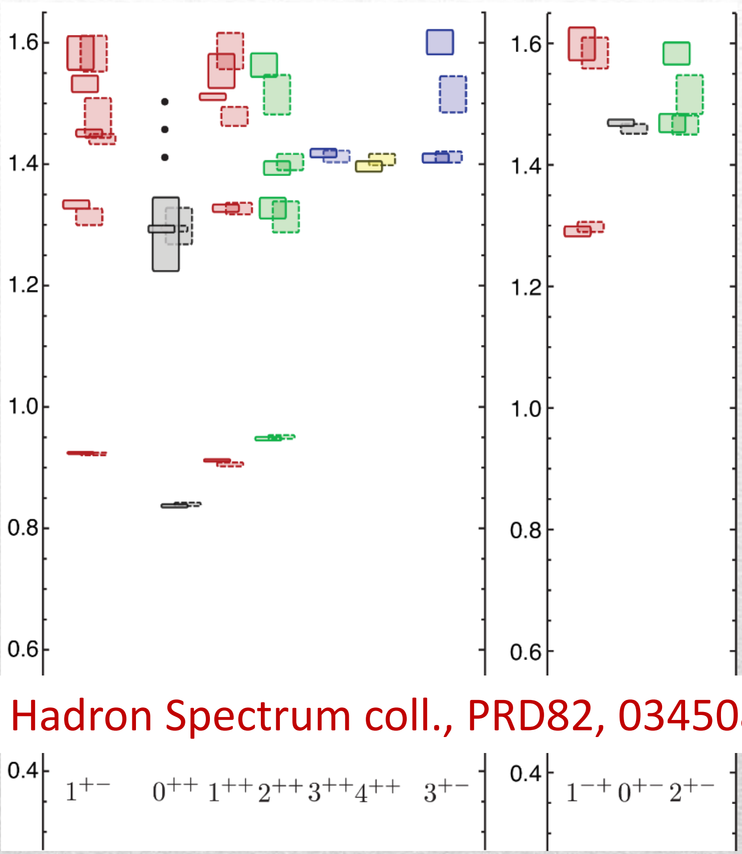
$J^P = (3/2)^-$

σ_s (MeV)	0	60	120
A	$0.156^{+0.029}_{-0.020}$	$0.157^{+0.039}_{-0.021}$	$0.157^{+0.037}_{-0.022}$
α_0	$1.151^{+0.018}_{-0.020}$	$1.150^{+0.018}_{-0.026}$	$1.150^{+0.015}_{-0.023}$
α' (GeV^{-2})	$0.112^{+0.033}_{-0.054}$	$0.111^{+0.037}_{-0.064}$	$0.111^{+0.038}_{-0.054}$
s_t (GeV^2)	$16.8^{+1.7}_{-0.9}$	$16.9^{+2.0}_{-1.6}$	$16.9^{+2.0}_{-1.1}$
b_0 (GeV^{-2})	$1.01^{+0.47}_{-0.29}$	$1.02^{+0.61}_{-0.32}$	$1.03^{+0.49}_{-0.31}$
$\mathcal{B}_{\psi p}$ (95% CL)	≤ 29 %	≤ 30 %	≤ 23 %

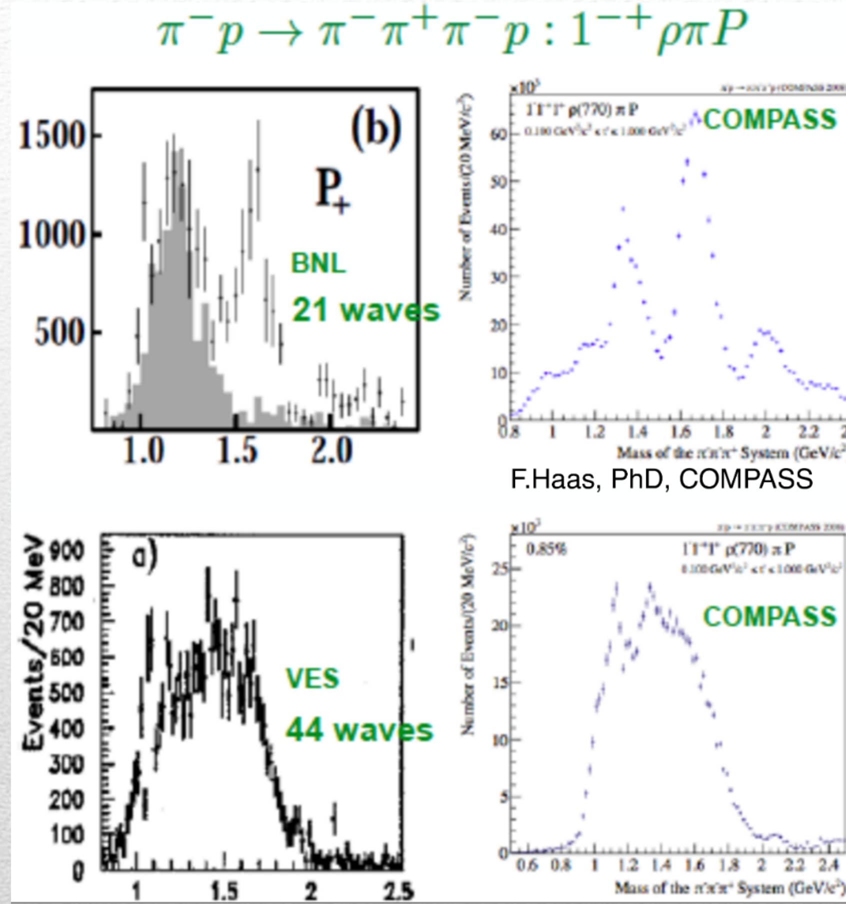


A. Blin *et al.* (JPAC), PRD94, 034002

Hybrids



Hadron Spectrum coll., PRD82, 034508



Signatures as $J^{PC} = 1^{-+}$ are not allowed in the quark model, Coulomb gauge QCD and flux tube predict glue excitation to be a quasi-particle with $J^{PC} = 1^{+-}$, $q\bar{q}g$ states expected
Need some constraint to draw robust conclusions about the existence of exotic states

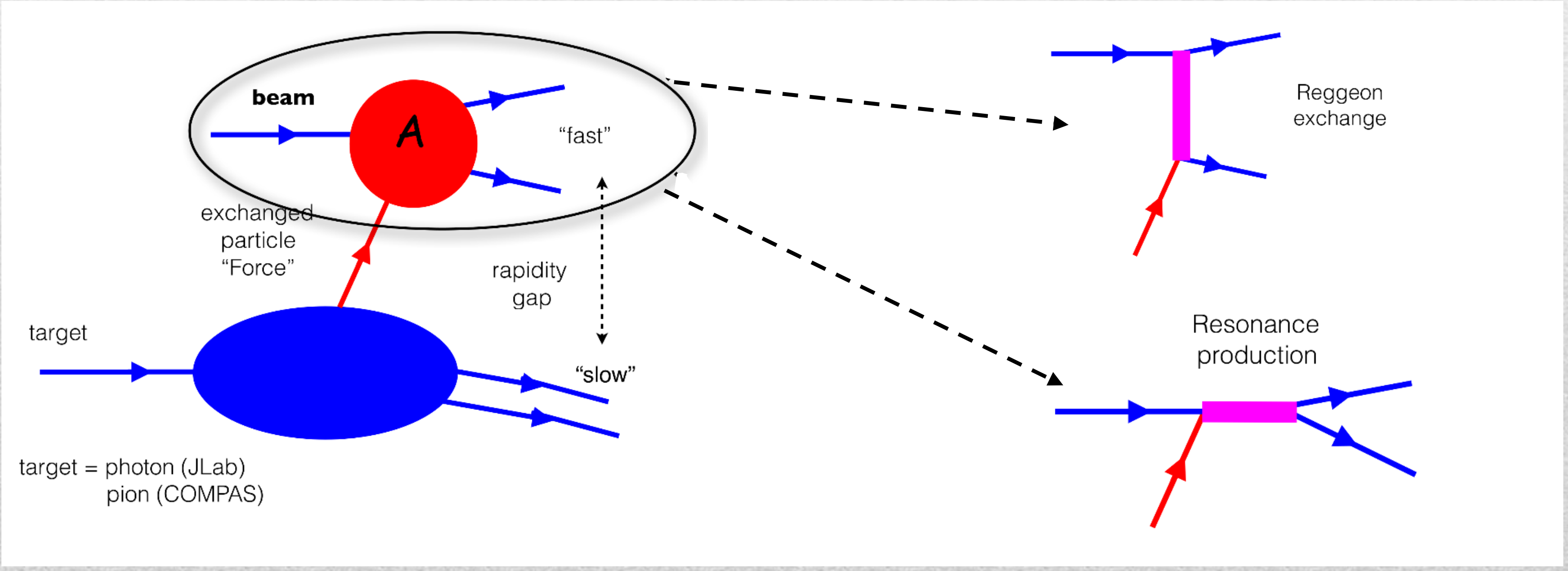
Regge exchange

Resonances are poles in s for fixed l
dominate low energy region

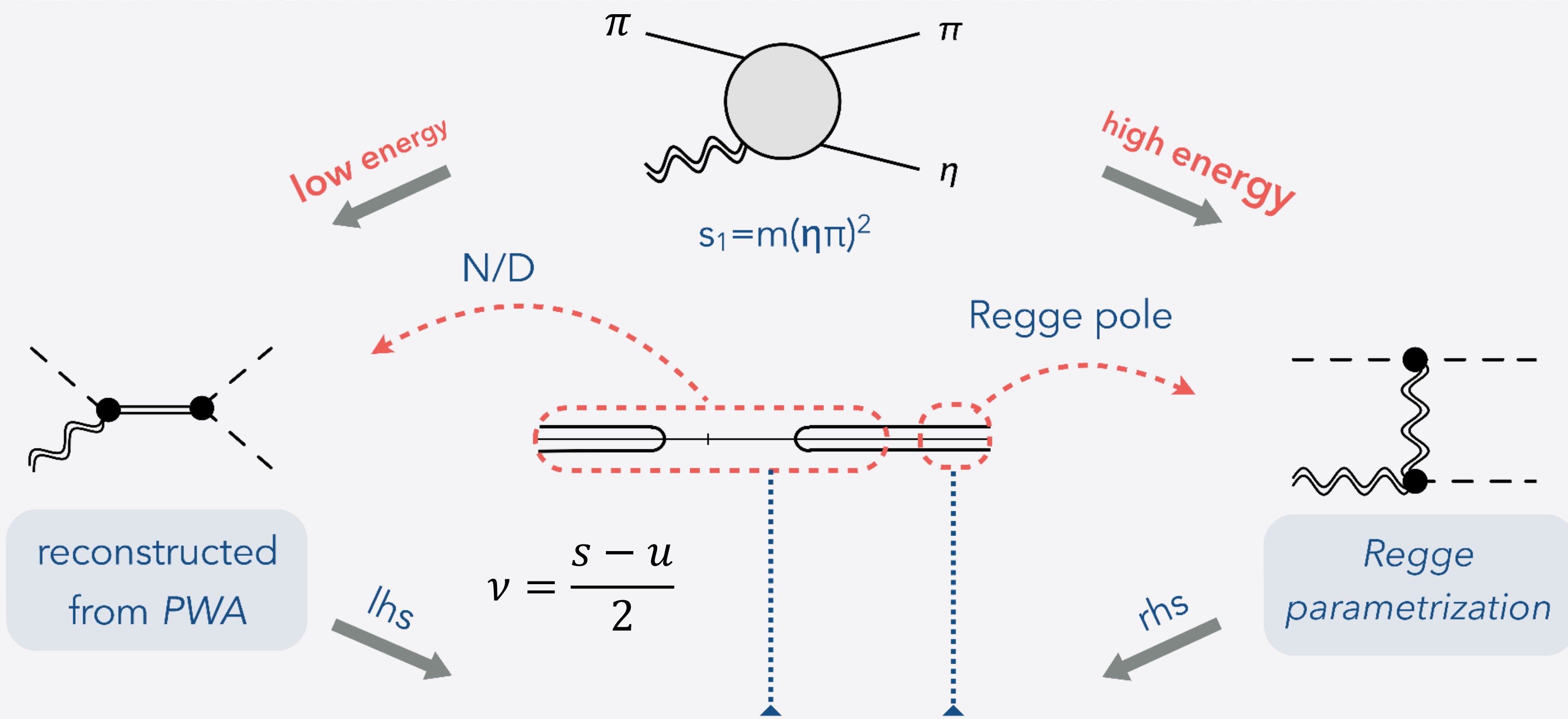
$$A_l \sim \frac{g_1 g_2}{s_p - s}$$

Reggeons are poles in l for fixed s
dominate high energy region

$$A \sim \sum s^l \sim g_1(t) g_2(t) s^{\alpha_\pm(t)} \left(- \frac{e^{i\pi\alpha^\pm(t)} \pm 1}{\sin \pi\, \alpha^\pm(t)} \right)$$



Finite energy sum rules

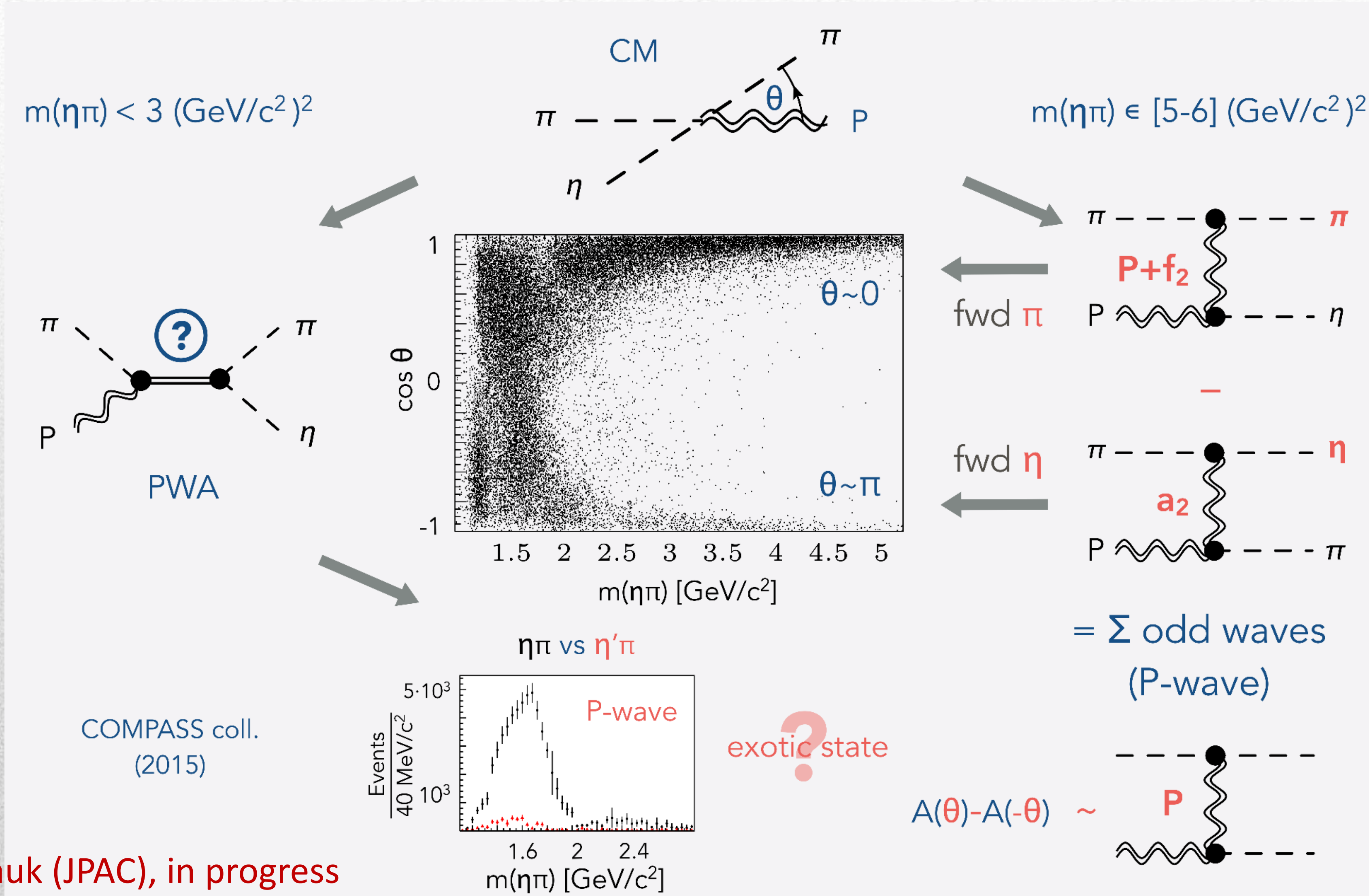


$$\nu = \frac{s-u}{2}$$

$$\frac{1}{\Lambda^n}\int_{\nu_{th}}^{\Lambda} Im\, A(\nu,t)\, \nu^n d\nu = \frac{\beta(t)\Lambda^{\alpha(t)+1}}{\alpha(t)+n+1}$$

aim: *first systematic analysis of* ***peripheral production*** *using* ***FESR***

$\eta\pi$ production



$\eta\pi$ production

K matrix:

$$A_l(s) = \frac{1}{K^{-1} - i\,\rho_l}$$

$$K = \sum \frac{g_i^2}{m_i^2 - s} + bkg$$

- No direct meaning of m_i, g_i
- Can have poles in the Ist sheet

M matrix:

$$A_l(s) = \frac{N(s)}{M - i\,\rho_l}$$

$$M = c_0 + c_1 s + \sum \frac{a_i}{b_i - s}$$

CDD poles

- No poles in the Ist sheet
- Numerator depending on the process

V. Pauk (JPAC), in progress

Hadron Spectroscopy

$\rho(770)$ $I^G(J^{PC}) = 1^+(1^{--})$

Review: The $\rho(770)$

$\rho(770)$ MASS

NEUTRAL ONLY, e^+e^-	775.26 ± 0.25 MeV
CHARGED ONLY, τ DECAYS and e^+e^-	775.11 ± 0.34 MeV
MIXED CHARGES, OTHER REACTIONS	763.0 ± 1.2 MeV
Mass m	
CHARGED ONLY, HADROPRODUCED	766.5 ± 1.1 MeV
NEUTRAL ONLY, PHOTOPRODUCED	769.0 ± 1.0 MeV
NEUTRAL ONLY, OTHER REACTIONS	769.0 ± 0.9 MeV (S = 1.4)
$m_{\rho(770)^0} - m_{\rho(770)^\pm}$	-0.7 ± 0.8 MeV (S = 1.5)
$m_{\rho(770)^+} - m_{\rho(770)^-}$	
$\rho(770)$ RANGE PARAMETER	$5.3^{+0.9}_{-0.7}$ GeV^{-1}

$\rho(770)$ WIDTH

NEUTRAL ONLY, e^+e^-	147.8 ± 0.9 MeV (S = 2.0)
CHARGED ONLY, τ DECAYS and e^+e^-	149.1 ± 0.8 MeV
MIXED CHARGES, OTHER REACTIONS	149.5 ± 1.3 MeV
CHARGED ONLY, HADROPRODUCED	150.2 ± 2.4 MeV
NEUTRAL ONLY, PHOTOPRODUCED	151.7 ± 2.6 MeV
NEUTRAL ONLY, OTHER REACTIONS	150.9 ± 1.7 MeV (S = 1.1)
$\Gamma_{\rho(770)^0} - \Gamma_{\rho(770)^\pm}$	0.3 ± 1.3 (S = 1.4)
$\Gamma_{\rho(770)^+} - \Gamma_{\rho(770)^-}$	1.8 ± 2.1

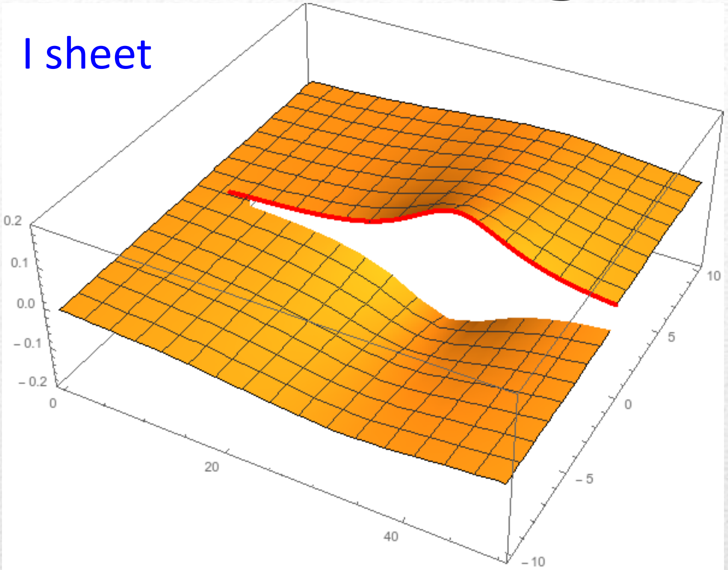
Hadron Spectroscopy

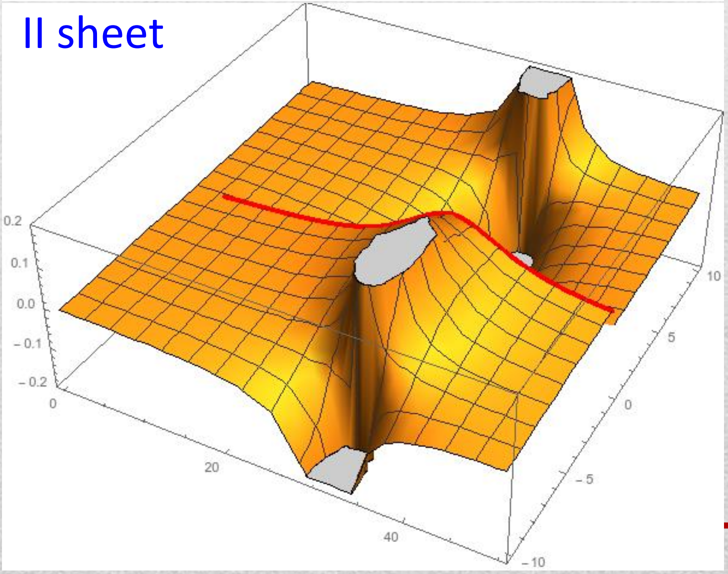
$a_1(1260)$ WIDTH

INSPIRE search

VALUE (MeV)	EVTS	DOCUMENT ID		TECN	COMMENT
250 to 600	**OUR ESTIMATE**				
$367 \pm 9 ^{+28}_{-25}$	420k	ALEKSEEV	2010	COMP	$190\ \pi^- \to \pi^-\pi^-\pi^+ Pb'$
• • • We do not use the following data for averages, fits, limits, etc. • • •					
$410 \pm 31 \pm 30$		[1] AUBERT	2007AU	BABR	$10.6\ e^+ e^- \to \rho^0 \rho^\pm \pi^\mp \gamma$
520 - 680	6360	[2] LINK	2007A	FOCS	$D^0 \to \pi^-\pi^+\pi^-\pi^+$
480 ± 20		[3] GOMEZ-DUMM	2004	RVUE	$\tau^+ \to \pi^+\pi^+\pi^-\nu_\tau$
580 ± 41	90k	SALVINI	2004	OBLX	$\overline{p}\, p \to 2\pi^+ 2\pi^-$
460 ± 85	205	[4] DRUTSKOY	2002	BELL	$B^{(*)} K^- K^{*0}$
$814 \pm 36 \pm 13$	37k	[5] ASNER	2000	CLE2	$10.6\ e^+ e^- \to \tau^+\tau^-,\ \tau^- \to \pi^-\pi^0\pi^0\nu_\tau$

Pole hunting





More complicated structure when more thresholds arise:
two sheets for each new threshold

III sheet: usual resonances
IV sheet: cusps (virtual states)