

Influence of the QCD Equation of State by System Size

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CIPANP

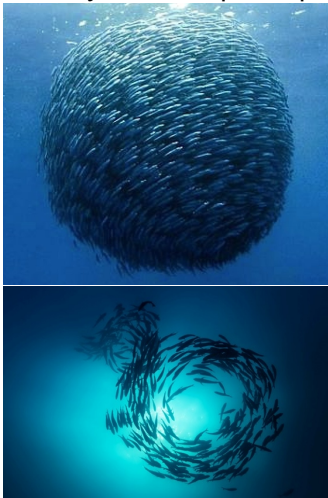
May 31st, 2018



RUTGERS
UNIVERSITY

What is the smallest droplet of liquid?

Collectivity with few participants



+



R~10 protons
 $\sim 10^3\text{-}10^4$ particles
 QGP ✓

proton



+



R~1-3 protons
 $\sim 10^2$ particles
 QGP probably

proton



+

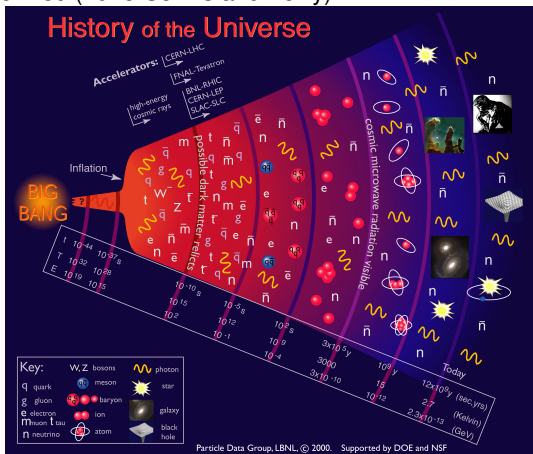


R~1 protons
 ~ 50 particles
 QGP ????

How can we exploit the
 shrinking system size?

Deconfined Quarks and Gluons in the Early Universe

Quark Gluon Plasma: After the Big Bang a plasma of deconfined Quarks and Gluons was formed (1975 Collins and Perry)



This QGP was "slower" and charm quarks were likely thermalized

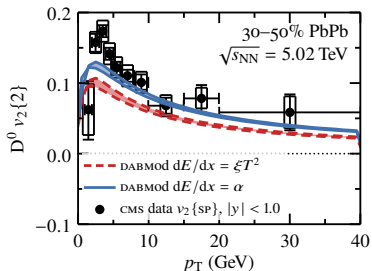
Equilibration times of charm quarks

- Light quarks relaxation time $\tau \sim \frac{\eta}{e+p}$
- Heavy quarks relaxation time $\tau_C \sim \frac{M_C}{T} \frac{\eta}{e+p}$ where the charm quark $M_C \sim 1.3 \text{ GeV}$
- Then $\frac{\tau_C}{\tau} \sim \frac{M}{T}$. Circa 2005, $T_{RHIC} \sim 250 \text{ MeV} \rightarrow \frac{\tau_C}{\tau} \sim 6$.
- Now, with improvements of Equation of State, freeze-out, $\eta/s, \zeta/s...$
- $T_{RHIC}^{max} \sim 400 \text{ MeV}, \frac{\tau_C}{\tau} \sim 3.25$
- $T_{LHC}^{max} \sim 600 \text{ MeV}, \frac{\tau_C}{\tau} \sim 2$
- **Thermalized charm quarks possible at higher temperatures!**

G. D. Moore and D. Teaney, Phys. Rev. C71, 064904 (2005)

Large elliptical flow (and now triangular flow)

See Zhenyu Chen's talk in pPb



Caio Prado, JNH, Katz, Suaide, Noronha, Munhoz, Constenntino, Phys.Rev. C96 (2017) no.6, 064903

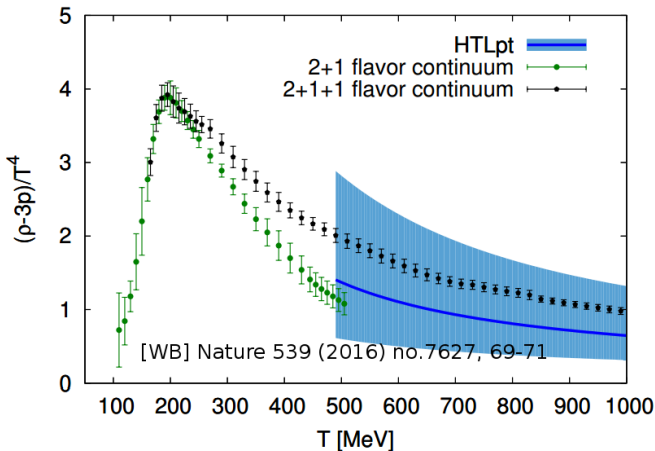
Many other works by TAMU,
 SUBATECH, USP, DUKE,
 CCNU/LBNL, BNL, Catania,
 and Belgrade...

Other approaches

- From Lattice QCD
 S. Mukherjee, P. Petreczky, and S. Sharma, Phys. Rev. D93, 014502 (2016)
- Close in phase space, hard to thermalize
 M. Martinez, M. D. Sievert, and D. E. Wertheim, (2018), arXiv:1801.08986 [hep-ph]
- Experimental Efforts (see X. Dong's Plenary)

Thermalized charm quarks?

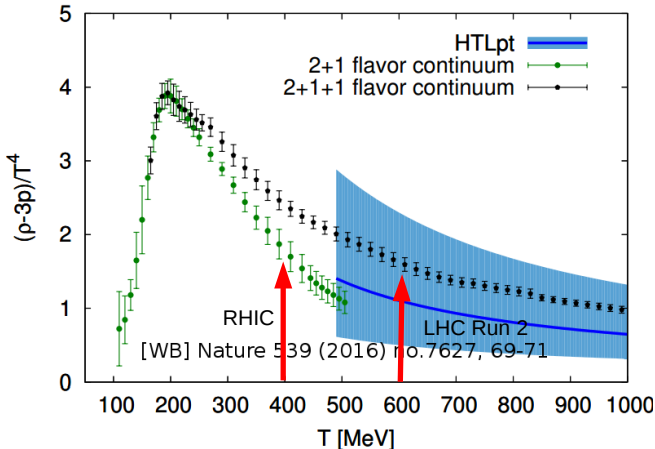
Still questions at $\mu_B = 0$. Sensitive Observables?



WB Collaboration Nature 539 (2016) no.7627, 69-71

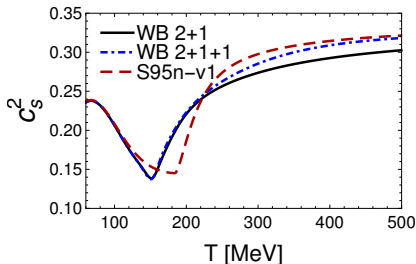
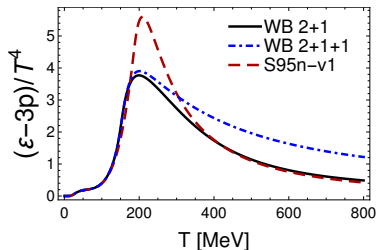
LHC Run 2 good probe of thermalized charm quarks

Still questions at $\mu_B = 0$. Sensitive Observables?



WB Collaboration Nature 539 (2016) no.7627, 69-71

State-of-the-art Equation of State at $\mu_B = 0$



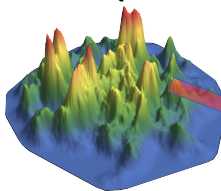
- 2+1 vs. 2+1+1 Lattice QCD based fits combined with PDG 2016+
- PDG16+ needed according to Lattice QCD
[WB] PRD 96, no. 3, 034517 (2017)
- tanh used for smooth fit at $T=150$ MeV
- Resonances decays from PDG or extrapolated in the same family (Paolo Parotto)
- Radiation important!
- S95n-v1 with PDG 2005
Huovinen, Petreczky NPA837 (2010) 26-53

See backup for other recent EOS studies

EOS a key ingredient of relativistic hydrodynamics

Initial Conditions

Quantum fluctuations in the position of nucleons/QCD fields



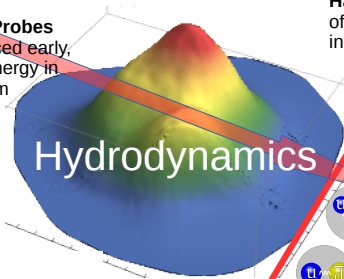
τ_0 initial time to switch on hydro

Hydrodynamics
(for heavy-ions collisions)
in a nutshell

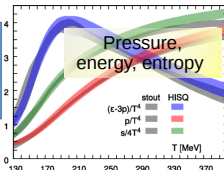
Hydrodynamics viscosity and thermodynamics

Hard Probes

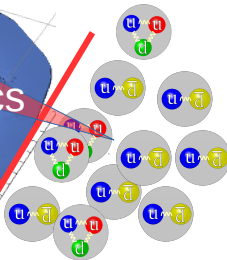
Produced early, lose energy in medium



T_{sw} temperature at which the Quark Gluon Plasma switches to hadrons

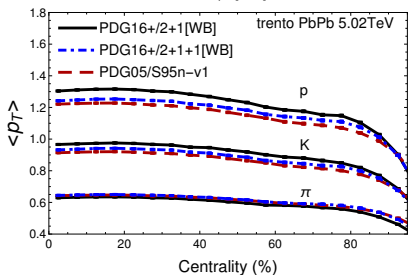
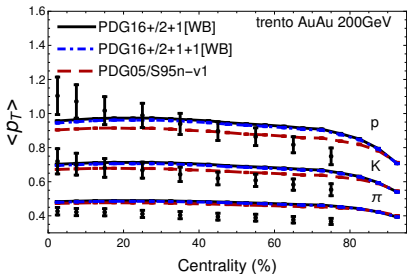
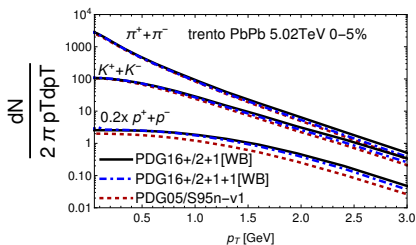
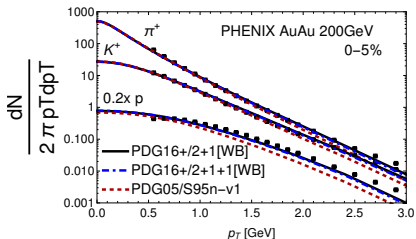


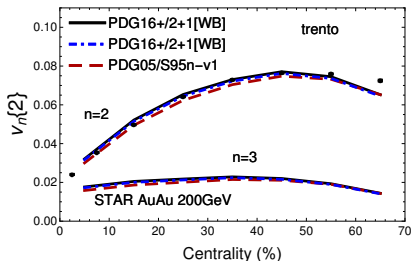
Hadron Gas: number of hadrons, decays, interactions etc



Spectra and $\langle p_T \rangle$

Alba, Mantovani, Noronha, JNH, Parotto, Portillo, Ratti, arXiv:1711.05207

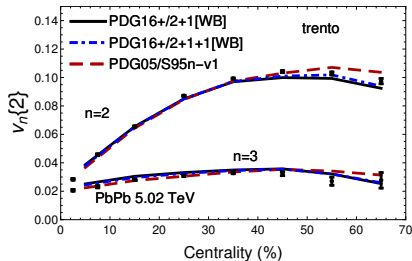


Equation of State connection to η/s 

$$\eta/s = 0.05$$

Consistent with Moreland and Soltz

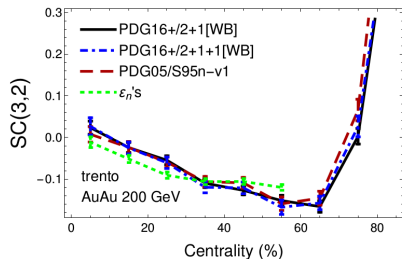
Phys.Rev. C93 (2016) no.4, 044913



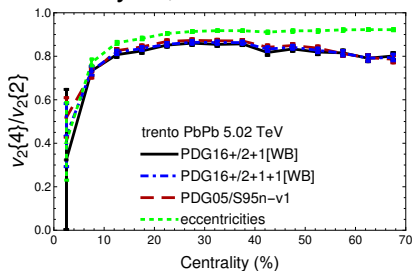
- EOS2+1 roughly 2x η/s compared to S95n-v1
- Thermalized charm quarks $\sim 15\%$ lower than 2+1

Multiparticle cumulants not dependent on EOS

Symmetric Cumulants



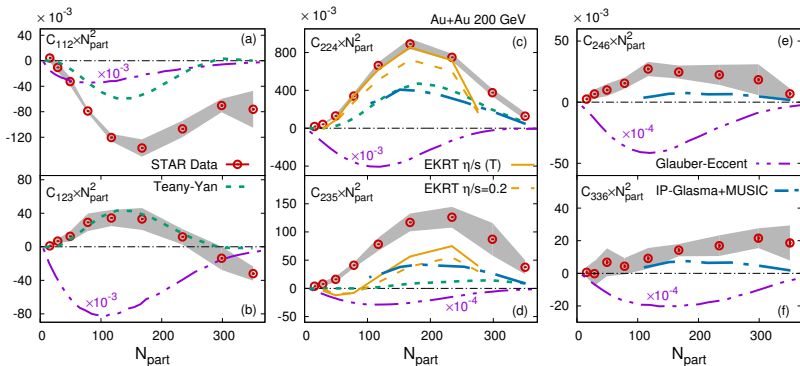
Multiparticle cumulants Driven by IC, see G. Giacalone



Alba, Mantovani, Noronha, JNH, Parotto, Portillo, Ratti, arXiv:1711.05207

Event Plane Correlations- a solution?

$$C_{m,n,m+n} = \langle v_n v_m v_{m+n} \cos(m\phi_m + n\phi_n - (m+n)\phi_{m+n}) \rangle$$

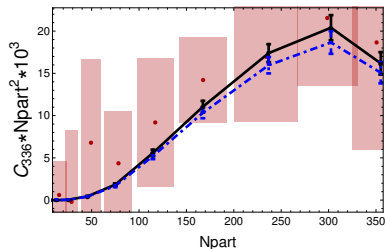
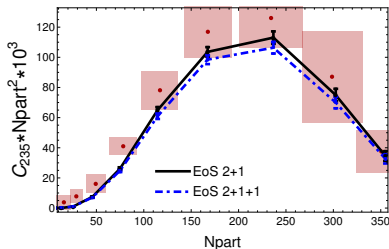
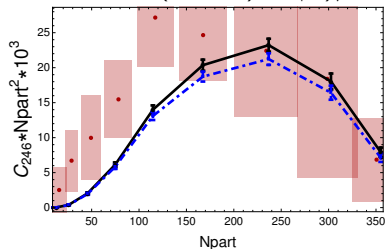
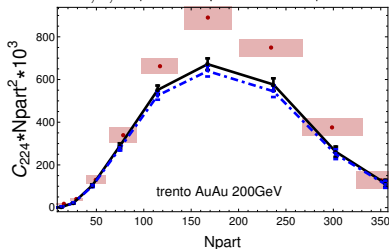


[STAR] arXiv:1701.06497

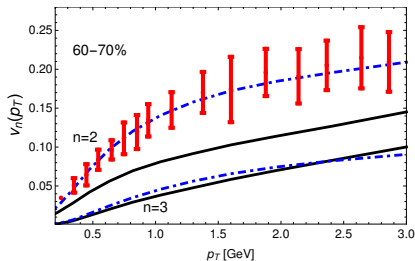
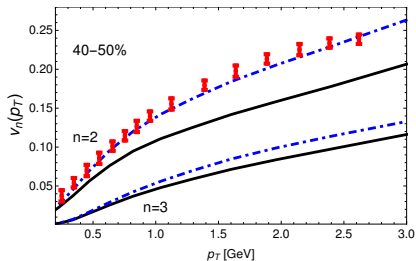
Need to understand $v_1, v_5, v_6 \rightarrow \varepsilon_n$'s or $\eta/s(T), \zeta/s(T)$?

Event Plane Correlations- EOS effects

$$C_{m,n,m+n} = \langle v_n v_m v_{m+n} \cos(m\phi_m + n\phi_n - (m+n)\phi_{m+n}) \rangle$$



Differential Flow in PbPb shows preference for thermalized charm



ALICE data arXiv:1804.02944

Hydro JNH, Ratti arXiv:1804.10661

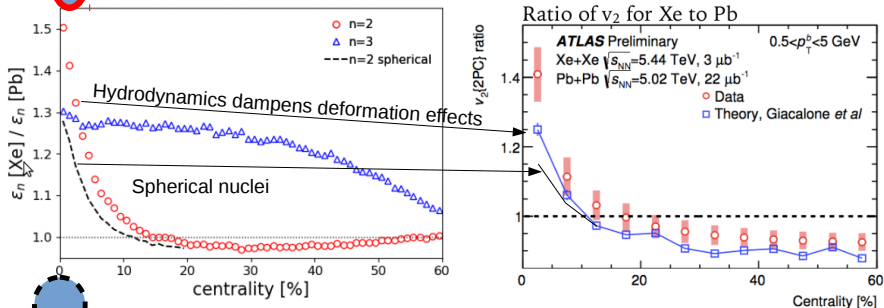
Xe¹²⁹Xe¹²⁹ collisions at $\sqrt{s_{NN}} = 5.44$ TeV



- Same energy as $Pb^{208}Pb^{208}$ 5.02 TeV collisions but smaller system size
- First noble gas collisions
- $v_2\{2\}$ in central collisions indicates deformed Xe
 Giacalone, JNH, Luzum, Ollitrault, Phys. Rev. C 97, no. 3, 034904 (2018)

Deformed Xe^{129} nucleus

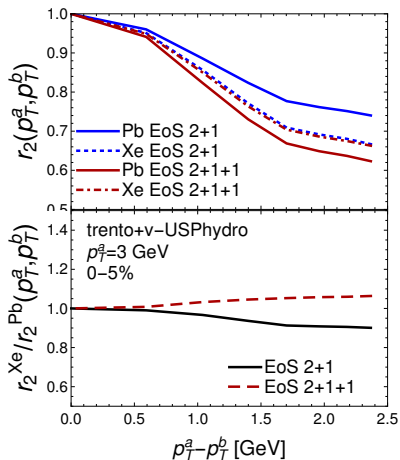
v-USPhydro sensitive to deformed nucleus
 Giacalone, JNH et al. Phys. Rev. C 97,034904 (2018)



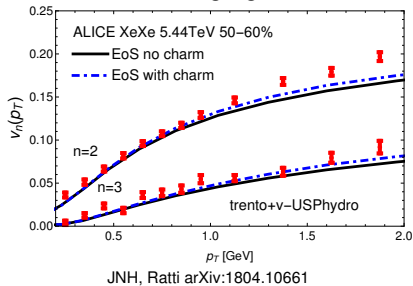
Initial conditions

Questions still remain in central collisions, possible insights into nuclear structure?

Factorization Break Ion Dependence vs. EOS



$v_2(p_T)$ less connected to EOS
in XeXe



Conclusions

- At high energies EoS can affect the extraction of η/s
- Event plane correlation *may* be used to constrain the EoS at RHIC
- Xenon deformation possible to measure in central collisions
- Possibility for thermalized charm quarks in PbPb systems
- Other observables possible to investigate small systems?
Effects on hard probes?

BACKUP

Pressure by Baryon Number, Strangeness, Charge

Pressure comes from: $p^{HRG} / T^4 = \frac{1}{VT^3} \sum_i \ln Z_i(T, \mu)$

such that

$$\ln Z_i^{M/B} \simeq \frac{d_i}{2\pi^2} \left(\frac{m_i}{T}\right)^2 \sum_{k=1}^{\infty} \frac{(\pm 1)^{k+1}}{k^2} K_2\left(\frac{km_i}{T}\right) \cosh[k(B_i\mu_B + S_i\mu_S + Q_i\mu_Q)/T]$$

E.g. for strange hadrons (can separate by any BSQ, though)

$$\begin{aligned}
 P_S(\hat{\mu}_B, \hat{\mu}_S) &= P_{0|1|} \cosh(\hat{\mu}_S) & P_{0|1|} &= \chi_2^S - \chi_{22}^{BS} \\
 &+ P_{1|1|} \cosh(\hat{\mu}_B - \hat{\mu}_S) & P_{1|1|} &= \frac{1}{2} (\chi_4^S - \chi_2^S + 5\chi_{13}^{BS} + 7\chi_{22}^{BS}) \\
 &+ P_{1|2|} \cosh(\hat{\mu}_B - 2\hat{\mu}_S) & P_{1|2|} &= -\frac{1}{4} (\chi_4^S - \chi_2^S + 4\chi_{13}^{BS} + 4\chi_{22}^{BS}) \\
 &+ P_{1|3|} \cosh(\hat{\mu}_B - 3\hat{\mu}_S) & P_{1|3|} &= \frac{1}{18} (\chi_4^S - \chi_2^S + 3\chi_{13}^{BS} + 3\chi_{22}^{BS})
 \end{aligned}$$

Note all $P_{B|S|}$ taken at the limit of $\mu_B = 0$

Fluctuations of Conserved Charges

Susceptibilities

$$\chi_{lmn}^{BSQ} = \frac{\delta^{l+m+n} p / T^4}{\delta (\mu_B / T)^l \delta (\mu_S / T)^m \delta (\mu_Q / T)^n}$$

where the chemical potentials are related via:

$$\begin{aligned}\mu_u &= \frac{1}{3}\mu_B + \frac{2}{3}\mu_Q \\ \mu_d &= \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q \\ \mu_s &= \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q - \mu_S\end{aligned}$$

Thermodynamic quantities arXiv:1711.05207

π gas $T \leq 33.5$ MeV HRG $T \leq 153$ MeV Lattice QCD

$$\left(\frac{\varepsilon - 3p}{T^4}\right)_{all} = \left(\frac{\varepsilon - 3p}{T^4}\right)_{\pi} - \frac{1 + \tanh[b(T - T_{HRG+Latt}/\pi)]}{2} \left[\left(\frac{\varepsilon - 3p}{T^4}\right)_{HRG+Latt} - \left(\frac{\varepsilon - 3p}{T^4}\right)_{\pi} \right]$$

$$\left(\frac{\varepsilon - 3p}{T^4}\right)_{HRG+Latt} = \left(\frac{\varepsilon - 3p}{T^4}\right)_{HRG} - \frac{1 + \tanh[a(T - T_{HRG/Latt})]}{2} \left[\left(\frac{\varepsilon - 3p}{T^4}\right)_{Latt} - \left(\frac{\varepsilon - 3p}{T^4}\right)_{HRG} \right]$$

$$\frac{\varepsilon}{T^4} = \left(\frac{\varepsilon - 3p}{T^4}\right)_{all} + \frac{3p}{T^4}$$

$$\frac{s}{T^3} = \left(\frac{\varepsilon - 3p}{T^4}\right)_{all} + \frac{4p}{T^4}$$

$$c_s^2 = \frac{s}{T} \frac{dT}{ds} = \frac{dp}{d\varepsilon}$$

$$\frac{p}{T^4} = \int_0^T dT \frac{1}{T} \left(\frac{\varepsilon - 3p}{T^4}\right)_{all}$$

$$a = 0.1 \text{ MeV}^{-1}, T_{HRG/Latt} = 153 \text{ MeV}$$

$$b = 1 \text{ MeV}^{-1}, T_{HRG+Latt}/\pi = 33.5 \text{ MeV}$$

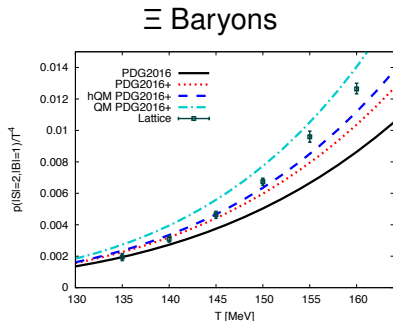
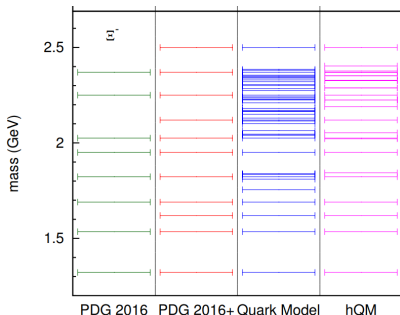
Missing hadron resonances

[WB] Phys.Rev. D96 (2017) no.3, 034517

see also [HotQCD] Phys.Rev.Lett. 113 (2014) no.7, 072001

Partial pressure can find missing strange states.

$$P_{1|2} = -\frac{1}{4} (\chi_4^S - \chi_2^S + 4\chi_{13}^{BS} + 4\chi_{22}^{BS}) \cosh(\hat{\mu}_B - 2\hat{\mu}_S)$$



Uncertainty in decays of heavier resonances! Extrapolated from same family.

Alba, Mantovani Sarti, JNH, P. Parotto, Portillo Vazquez, Ratti. To appear soon.

TRENTO

TRENTO Moreland, Bernhard, Bass PRC92(2015)no.1,011901

Total initial entropy profile

$$S(p; S_A, S_B) = \left(\frac{S_A^p + S_B^p}{2} \right)^{\frac{1}{p}}, \quad (1)$$

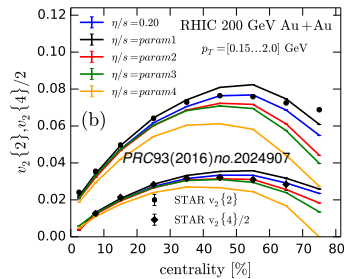
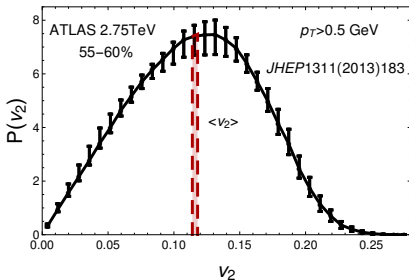
where

$$S_{A,B} = w_{A,B} \frac{1}{2\pi\sigma^2} \exp \left[-\frac{(x - x_{A,B})^2 + (y - y_{A,B})^2}{2\sigma^2} \right]. \quad (2)$$

normalization, w , is a random number which is assigned to each participant nucleon, Γ probability distribution with the width k .

Constraining initial condition models

- Mean shape $\langle \varepsilon_n \rangle \rightarrow \eta/s, \text{ EOS etc..}$



- Size of event-by-event fluctuations $\varepsilon_n\{4\}/\varepsilon_n\{2\}$



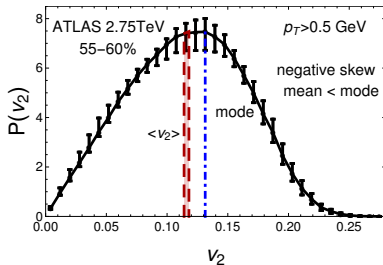
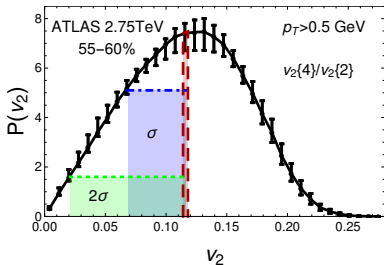
- Correlation different harmonics $SC(3,2)$



Constraining initial condition models

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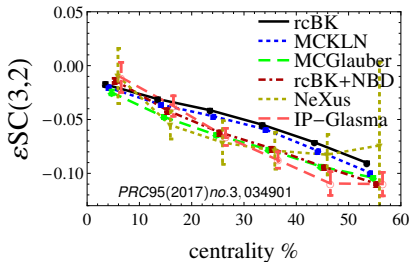
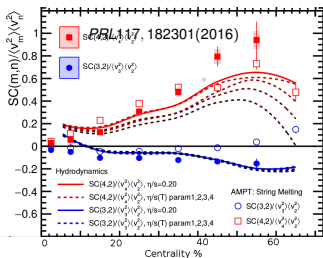
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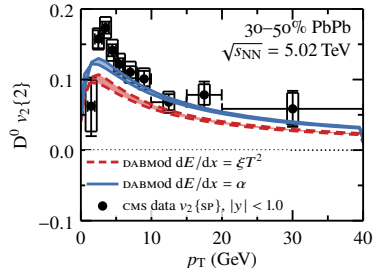
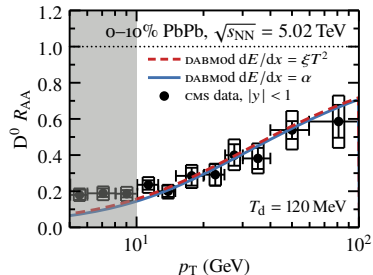
- Correlation different harmonics $SC(3,2)$



DABMOD- parameterized energy loss model

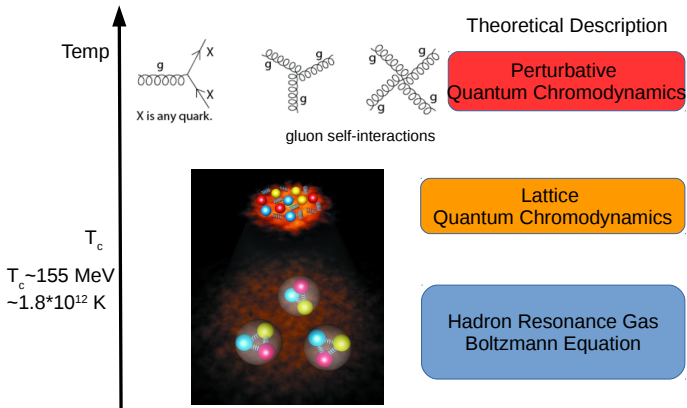
- Sample charm quarks inside medium with initial momentum distribution from pQCD fonll calculations
- Energy loss motivated by:
S. K. Das, F. Scardina, S. Plumari, and V. Greco, Phys. Lett. B747, 260 (2015)
- Decoupling temperature
 $T_d = 120 - 160$ MeV
- Hadronization: Peterson fragmentation function
- Quark Coalescence being implemented (Roland Katz).

Caio Prado, JNH, Katz, Suaide, Noronha, Munhoz, Constantino, Phys.Rev. C96 (2017) no.6, 064903



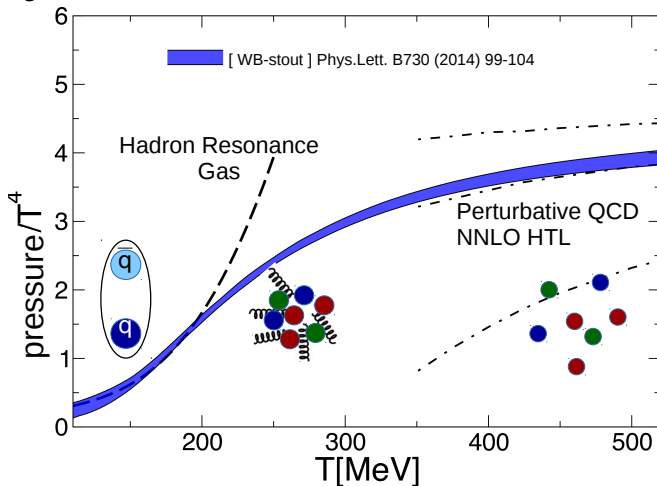
Solving Quantum Chromodynamics

$$L_{QCD} = \underbrace{-\frac{1}{4} F_{\mu\nu}^a F_a^{\mu\nu}}_{\text{Gluon Interactions}} + \underbrace{\bar{\psi}^q (i\gamma_\mu D^\mu - m_q) \psi^q}_{\text{Quark Interactions}} \text{ where } D^\mu = \partial^\mu - ig \underbrace{A^\mu(x)}_{\text{Gluons}}$$

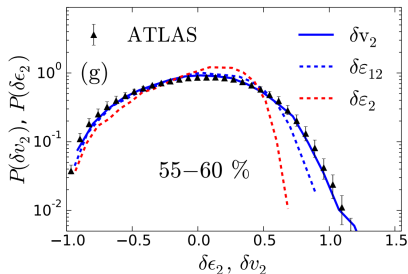
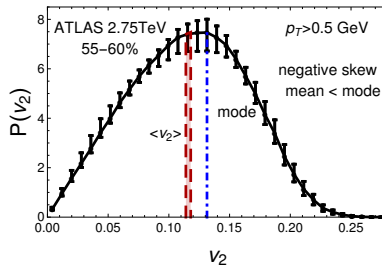
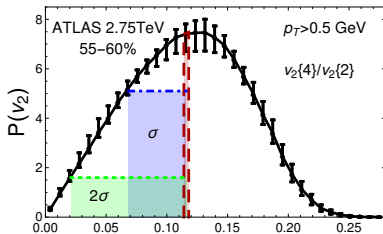


Lattice QCD: Phase Transition

- Cross-over phase transition $T_c \sim 155$ MeV
- Degrees of freedom increase x10!



Elliptical Flow distribution



In peripheral collisions, cubic response affects $P(v_2)$, otherwise $P(\epsilon_2) \sim P(v_2)$

JNH, Yan, Gardim, Ollitrault Phys.Rev. C93 (2016) no.1, 014909

Multi-particle cumulants

Reconstructing the v_n distribution with cumulants

$$v_n\{2\}^2 = \langle v_n^2 \rangle,$$

$$v_n\{4\}^4 = 2\langle v_n^2 \rangle^2 - \langle v_n^4 \rangle,$$

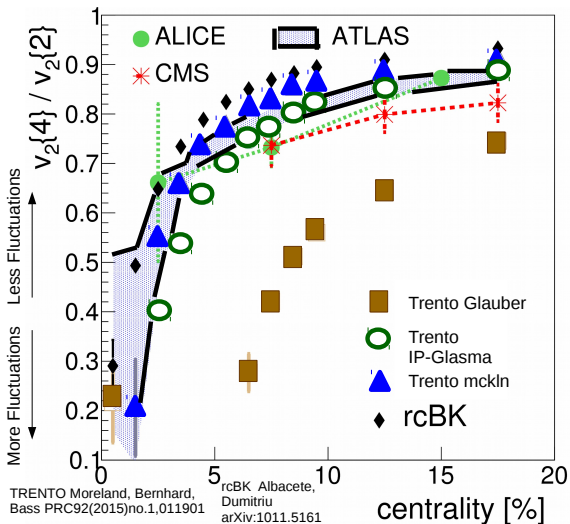
$$v_n\{6\}^6 = \frac{1}{4} \left[\langle v_n^6 \rangle - 9\langle v_n^2 \rangle \langle v_n^4 \rangle + 12\langle v_n^2 \rangle^3 \right],$$

$$v_n\{8\}^8 = \frac{1}{33} \left[144\langle v_n^2 \rangle^4 - 144\langle v_n^2 \rangle^2 \langle v_n^4 \rangle + 18\langle v_n^4 \rangle^2 \right. \\ \left. + 16\langle v_n^2 \rangle \langle v_n^6 \rangle - \langle v_n^8 \rangle \right],$$

where collectivity $\rightarrow v_n\{2\} > v_n\{4\} \sim v_n\{6\} \sim v_n\{8\}$ but there are differences between higher order cumulants!

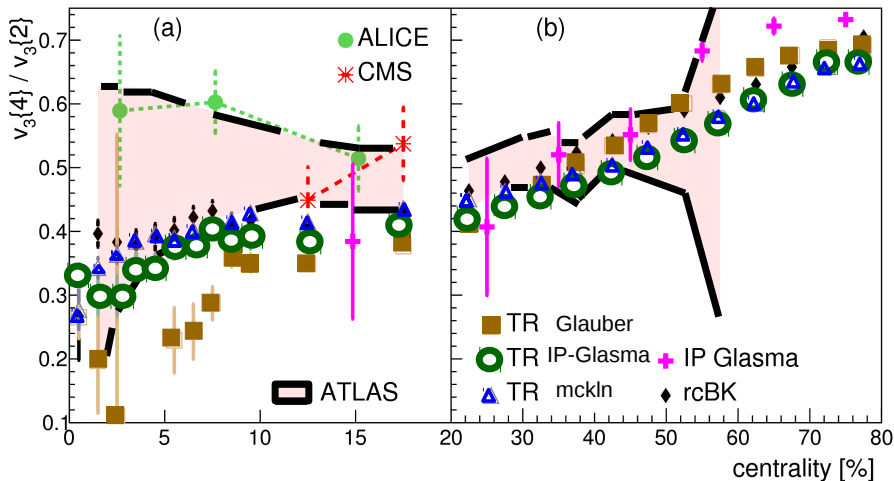
$v_2\{4\}/v_2\{2\}$ vs initial conditions in PbPb

Giacalone, JNH, Ollitrault Phys.Rev. C95 (2017) no.5, 054910



All initial conditions miss $v_3\{4\}/v_3\{2\}$ in PbPb

Giacalone, JNH, Ollitrault Phys.Rev. C95 (2017) no.5, 054910



Azimuthal anisotropies

The distribution of particles can be written as a Fourier series

$$E \frac{d^3 N}{d^3 p} = \frac{1}{2\pi} \frac{d^2 N}{p_T dp_T dy} \left[1 + \sum_n 2v_n \cos [n(\phi - \psi_n)] \right]$$

- Flow Harmonics at mid-rapidity

$$v_n(p_T) = \frac{\int_0^{2\pi} d\phi \frac{dN}{p_T dp_T d\phi} \cos [n(\phi - \Psi_n)]}{\int_0^{2\pi} d\phi \frac{dN}{p_T dp_T d\phi}}$$

where $\Psi_n = \frac{1}{n} \arctan \frac{\langle \sin[(n\phi)] \rangle}{\langle \cos[(n\phi)] \rangle}$



$n = 2$



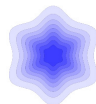
$n = 3$



$n = 4$



$n = 5$



$n = 6$

Initial Conditions and Hydrodynamics

TRENTO parameters

Moreland, Bernhard, Bass PRC92(2015)no.1,011901

- $p = 0$ entropy deposition parameter
- $k = 1.6$ nucleon-nucleon fluctuation shape parameter
- $\sigma = 0.51$ nucleon width

Hydro Parameters

- $\tau_0 = 0.6$ fm
- $T_{SW} = T_{CE} = 150$ MeV
- v-USPhydro
JNH et al, PRC88(2013)no.4,044916;
PRC90(2014)no.3,034907
- Hadronic feed-down only

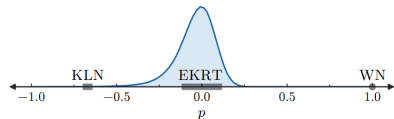


FIG. 9. Posterior distribution of the TRENTO entropy deposition parameter p introduced in Eq. (14). Approximate p -values are annotated for the KLN ($p \approx 0.67 \pm 0.01$), EKRT ($p \approx 0.0 \pm 0.1$), and wounded nucleon ($p = 1$) models. J.

S. Moreland, J. E. Bernhard, and S. A. Bass, Phys. Rev. C92, 011901 (2015)
J. E. Bernhard, J. S. Moreland, S. A. Bass, J. Liu, and U. Heinz, Phys. Rev. C94, 024907 (2016),

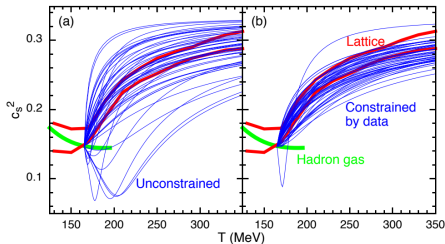
Bulk viscosity

$\zeta/s = 0$ to avoid δf uncertainty.

Constraining the EoS

Bayesian analysis

RHIC/LHC, smoothed IC



Pratt, Sangaline, Sorensen, Wang Phys.Rev.Lett. 114 (2015) 202301; Sangaline and Pratt Phys.Rev. C93 (2016) no.2, 024908

RHIC, event-by-event 1 – 10% effect

Moreland and Soltz Phys.Rev. C93 (2016) no.4, 044913

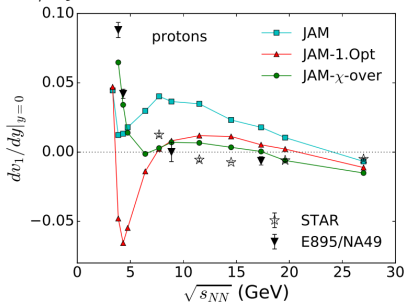
$\langle m_T \rangle$ vs. dN/dy of central collisions

Monnai and Ollitrault Phys.Rev. C96 (2017) no.4, 044902

Deep Learning confirms Lattice QCD EoS

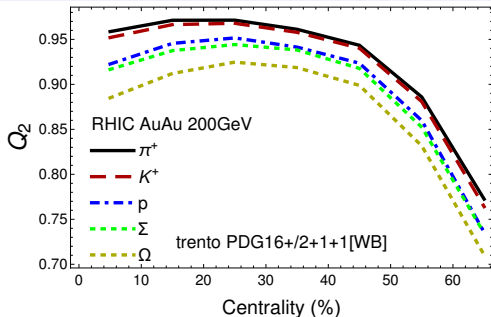
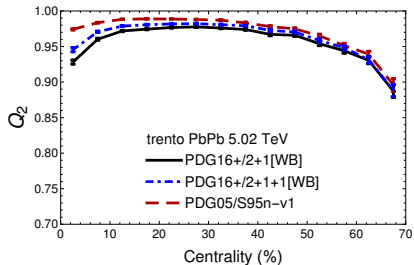
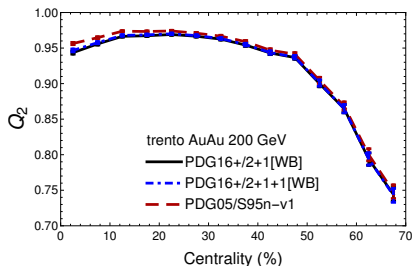
Pang et al, arXiv:1612.04262

dv_1/dy sensitive to EoS



Nara et al Phys.Lett. B769 (2017) 543-548

Correlation between Initial State and Flow Harmonics



Pearson Coefficient ($\varepsilon_n \rightarrow v_n$)

$$Q_n = \frac{\langle v_n \varepsilon_n \cos(n[\psi_n - \phi_n]) \rangle}{\sqrt{\langle |\varepsilon_n|^2 \rangle \langle |v_n|^2 \rangle}}$$

$Q_n = (-)1$ (anti-)correlated

$Q_n = 0$ no correlation

Gardim et al, PRC85 (2012) 024908; PRC91 (2015) no.3, 034902

Normalized Event Plane Correlations

$$\text{Normalizing } C_{m,n,m+n} = \frac{\langle v_n v_m v_{m+n} \cos(m\phi_m + n\phi_n - (m+n)\phi_{m+n}) \rangle}{\sqrt{\langle v_n^2 \rangle \langle v_m^2 \rangle \langle v_{m+n}^2 \rangle}}$$

