New Formulation of the γW-box correction to neutron and nuclear β-decay

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In collaboration with

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Outline

- Beta decay in presence of RC
- Dispersive representation of the γW-box
- Physics input to the dispersion integral
- Nuclear effects
- New formulation of RC for V_{ud} extraction
- Can nuclear effects turn the inner correction inside-out?

Neutron β-decay in presence of RC

Beyond RC that enter the Fermi constant:

Sirlin '67, Marciano & Sirlin '86 …

Coulomb distortion - Fermi fn.
$F(\beta) \approx 1 + \alpha\pi/\beta$

"Inner" correction -
depends on hadron structure;
independent of kinematics

$$\frac{d\Gamma}{dE_e d\Omega_e d\Omega_\nu} = \frac{G_\mu^2 V_{ud}^2}{(2\pi)^5}(1+3\lambda^2)|\vec{p}_e|E_e(E_m - E_e)^2 F(\beta)(1+\frac{\alpha}{\pi}\mathrm{Re}\, c)(1+\frac{\alpha}{2\pi}\delta^{(1)})\times$$
$$\left[1+\left(1+\frac{\alpha}{2\pi}\delta^{(2)}\right)a\vec{\beta}\cdot\hat{p}_\nu + \hat{s}\cdot\left(\left(1+\frac{\alpha}{2\pi}\delta^{(2)}\right)A\vec{\beta} + B\hat{p}_\nu\right)\right]$$

"Outer" corrections - IR-sensitive;
depend on kinematics;
independent of hadronic structure
Exactly calculable

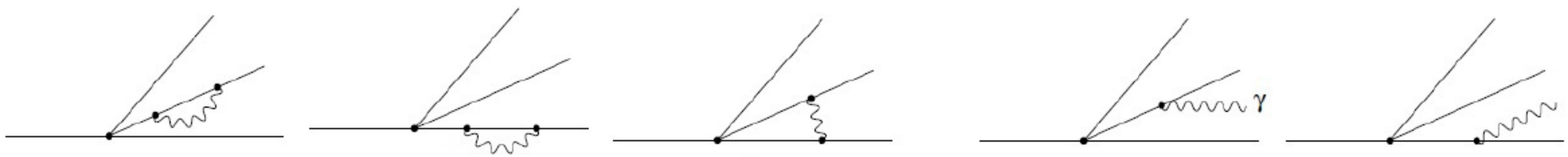
Wilkinson '82, Severijns et al. '17

Separation due to scale hierarchy: Q-values from <1 keV (n) to few MeV (nuclei);
Hadronic scales: at least 140 MeV - on top of $\alpha/2\pi \sim 10^{-3}$ —> 10^{-5} effect <<

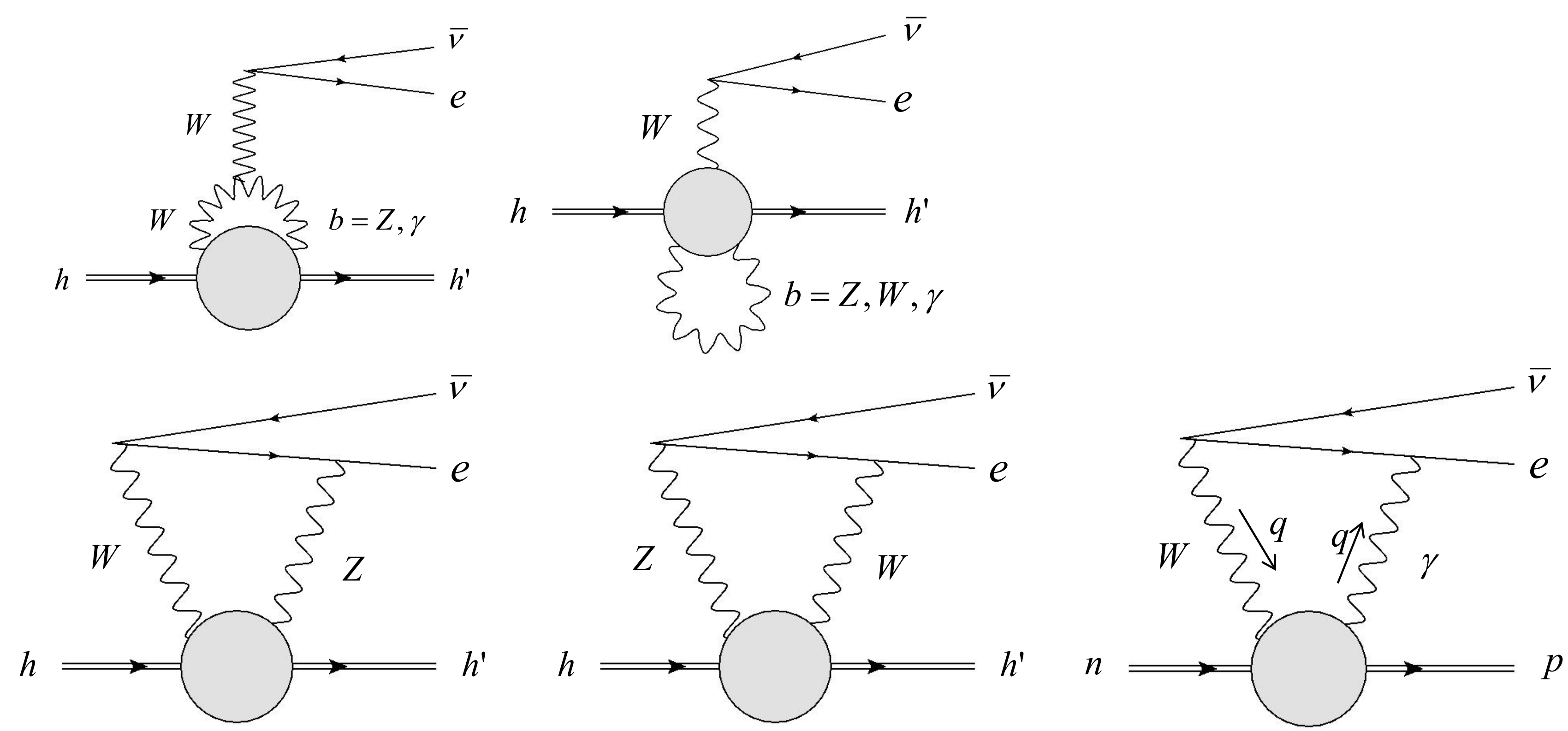
Radiative corrections - In & Out

1-loop RC (specific for a semiletonic process)

Outer: retain only IR divergent pieces



Inner: everything else



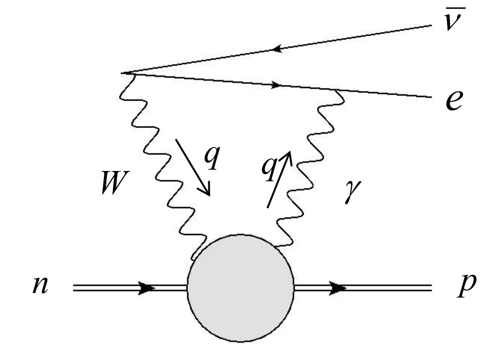
W,Z-exchange: UV-sensitive, pQCD; model-independent

When γ involved - possible sensitivity to long range physics

Model-dependent!

γW-box

Consider the box at zero energy
and zero momentum transfer



$$T_{\gamma W} \;=\; \sqrt{2}e^2 G_F V_{ud} \int \frac{d^4q}{(2\pi)^4} \frac{\bar{u}_e \gamma^\mu (\not{k} - \not{q} + m_e)\gamma^\nu (1-\gamma_5) v_\nu}{q^2[(k-q)^2 - m_e^2]} \frac{M_W^2}{q^2 - M_W^2} T_{\mu\nu}^{\gamma W}$$

Hadronic tensor: two-current correlator $T_{\gamma W}^{\mu\nu} = \int dx e^{iqx} \langle p | T[J_{em}^\mu(x) J_W^\nu(0)] | n \rangle$

General gauge-invariant decomposition (spin-independent)

$$T_{\gamma W}^{\mu\nu} \;=\; \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) T_1 + \frac{1}{(p\cdot q)} \left(p - \frac{(p\cdot q)}{q^2} q\right)^\mu \left(p - \frac{(p\cdot q)}{q^2} q\right)^\nu T_2 + \frac{i\epsilon^{\mu\nu\alpha\beta} p_\alpha q_\beta}{2(p\cdot q)} T_3$$

γW-box

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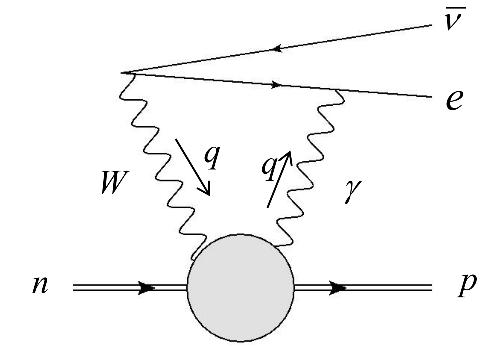

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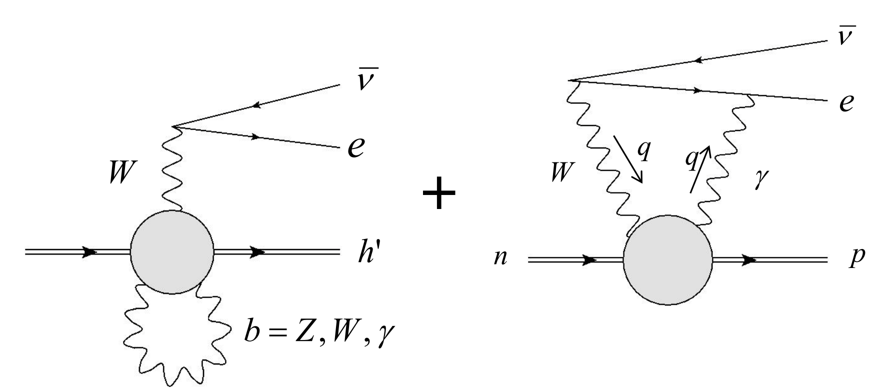
V-V correlator is cancelled exactly
By the 3-current correlator - Sirlin '67
Reason: conserved vector-isovector current



Axial current not conserved -> only A-V correlator contributes

γW-box

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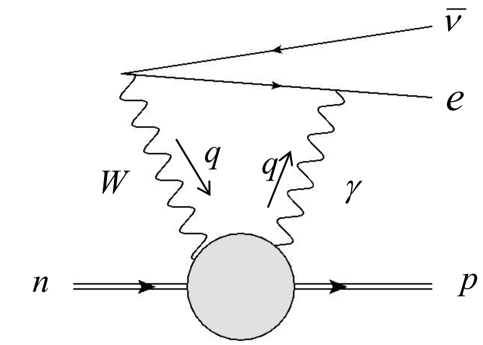


Define the box contribution as

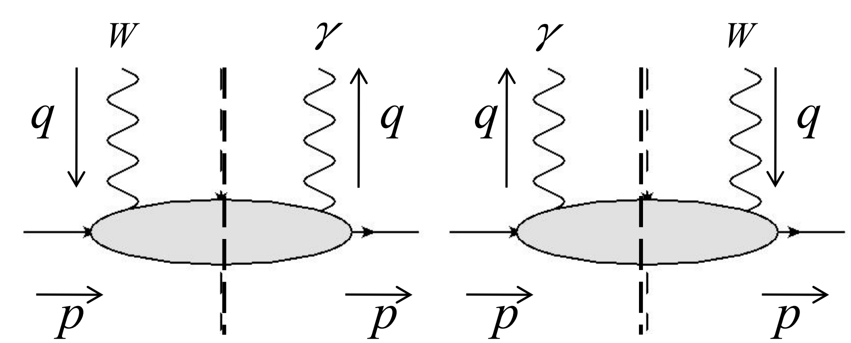
$\nu = (pq)/M$

$$T_W + T_{\gamma W}^{VA} = -\sqrt{2} G_F V_{ud} \left(1 + \Box_{\gamma W}^{VA}\right) \bar{u}_e \not{p} (1-\gamma_5) v_\nu$$

Loop integral with T_3

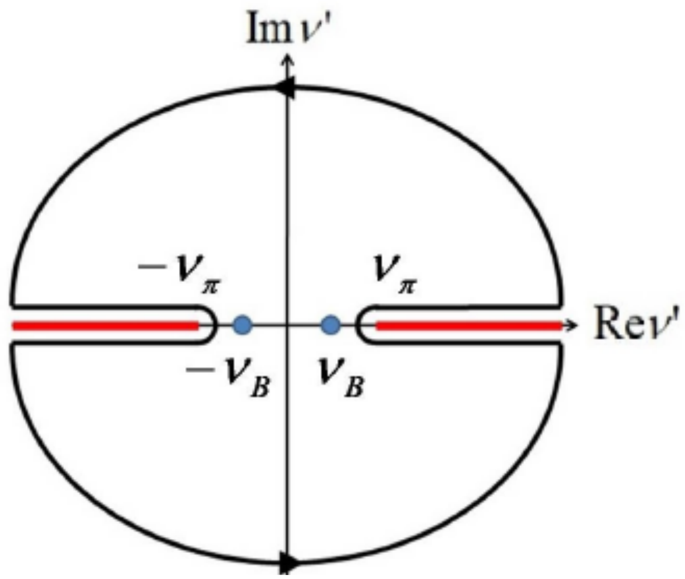
$$\Box_{\gamma W}^{VA} = 4\pi\alpha \mathrm{Re} \int \frac{d^4 q}{(2\pi)^4} \frac{M_W^2}{M_W^2 + Q^2} \frac{Q^2 + \nu^2}{Q^4} \frac{T_3(\nu, Q^2)}{M\nu}$$

Forward amplitude T_3 - unknown;
Its absorptive part could be related to
production of on-shell intermediate states
a γW-analog of the SF F_3



$$\mathrm{Im}\, T_3^{\gamma W}(\nu, Q^2) = 2\pi F_3^{\gamma W}(\nu, Q^2)$$

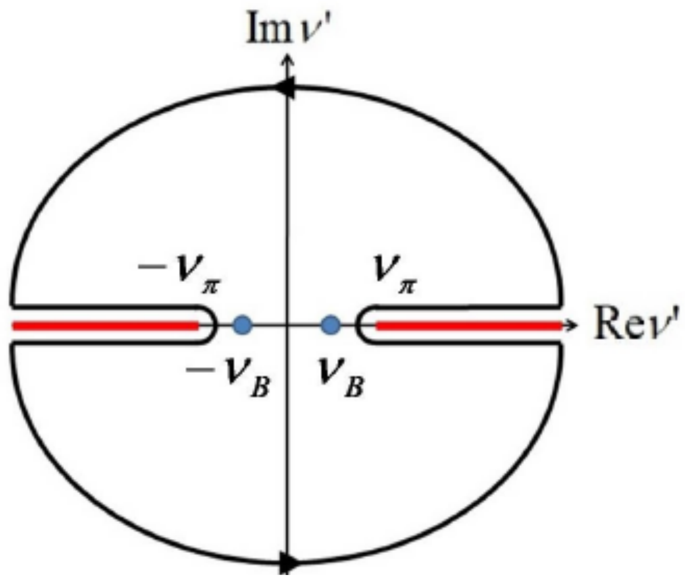
γW-box in dispersion representation



T_3 - analytic function inside the contour C in the complex ν-plane determined by its singularities on the real axis - poles + cuts

$$T_3(\nu, Q^2) = \frac{1}{2\pi i} \oint_C \frac{T_3(z, Q^2) dz}{z - \nu} \quad \nu \in C$$

γW-box in dispersion representation



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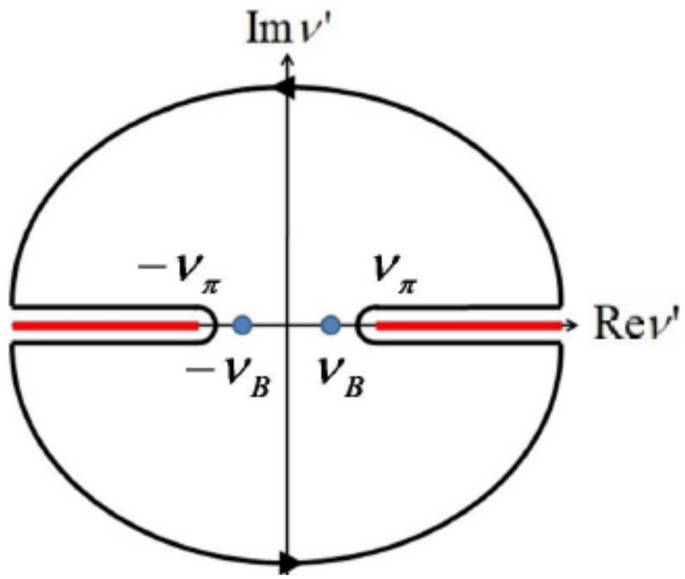
Crossing behavior: photon is isoscalar or isovector $\qquad T_3^{\gamma W} = T_3^{(0)} + T_3^{(3)}$

Different isospin channels behave differently under crossing

$$T_3^{(0)}(-\nu, Q^2) = -T_3^{(0)}(\nu, Q^2), \quad T_3^{(3)}(-\nu, Q^2) = +T_3^{(3)}(\nu, Q^2)$$

γW-box in dispersion representation



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Dispersion representation of the γW-box correction at zero energy

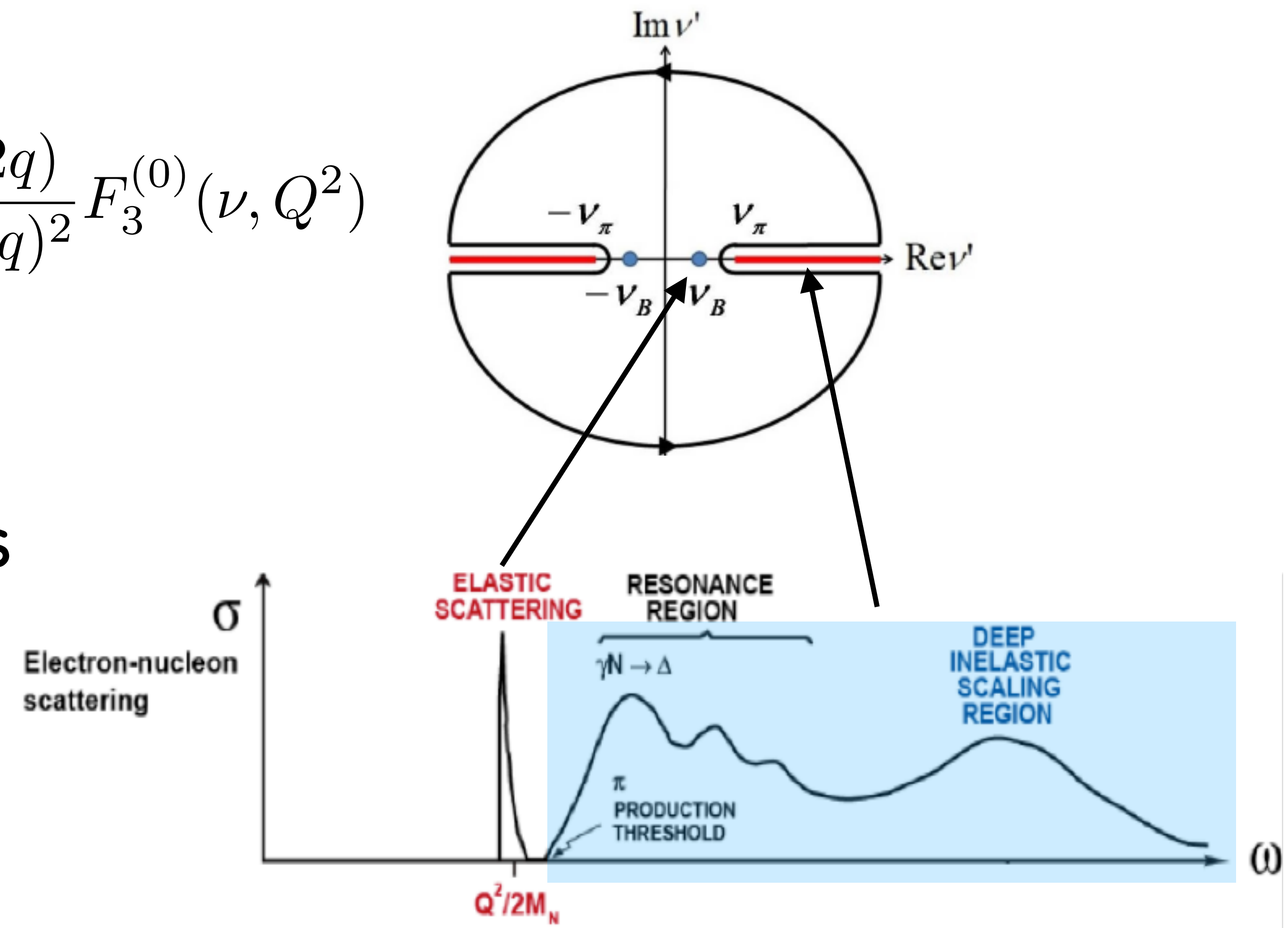
$$\Box_{\gamma W}^{VA\,(0)} = \frac{\alpha}{\pi M} \int_0^\infty \frac{dQ^2 M_W^2}{M_W^2 + Q^2} \int_0^\infty d\nu \frac{(\nu + 2q)}{\nu(\nu+q)^2} F_3^{(0)}(\nu, Q^2),$$

$$\Box_{\gamma W}^{VA\,(3)} = 0,$$

$$q = \sqrt{\nu^2 + Q^2}$$

Input into dispersion integral

$$\Box_{\gamma W}^{VA\,(0)} = \frac{\alpha}{\pi M}\int_0^\infty \frac{dQ^2 M_W^2}{M_W^2+Q^2}\int_0^\infty d\nu \frac{(\nu+2q)}{\nu(\nu+q)^2} F_3^{(0)}(\nu,Q^2)$$



Dispersion in energy:
scanning hadronic intermediate states

Relation to the old Marciano & Sirlin's result:

$$\Box_{\gamma W}^{VA} = \frac{\alpha}{8\pi}\int_0^\infty \frac{dQ^2 M_W^2}{M_W^2+Q^2} F(Q^2)$$

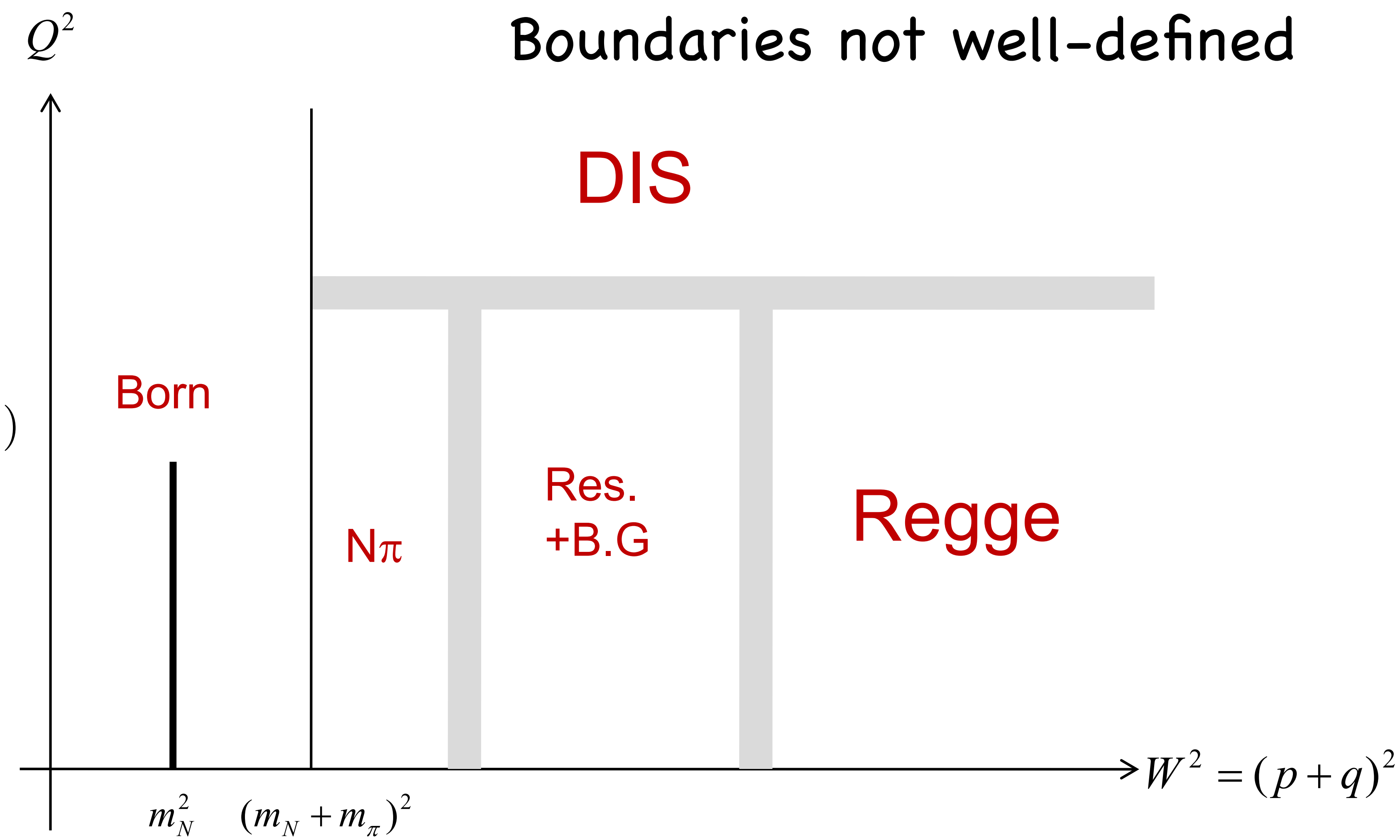
Dispersion representation of M&S loop function F:

$$F(Q^2) = \int_0^\infty d\nu \frac{8(\nu+2q)}{M\nu(\nu+q)^2} F_3^{(0)}(\nu,Q^2)$$

Input into dispersion integral

Dispersion in Q²: scanning dominant physics pictures

$$F(Q^2) = \int_0^\infty d\nu \frac{8(\nu+2q)}{M\nu(\nu+q)^2} F_3^{(0)}(\nu, Q^2)$$



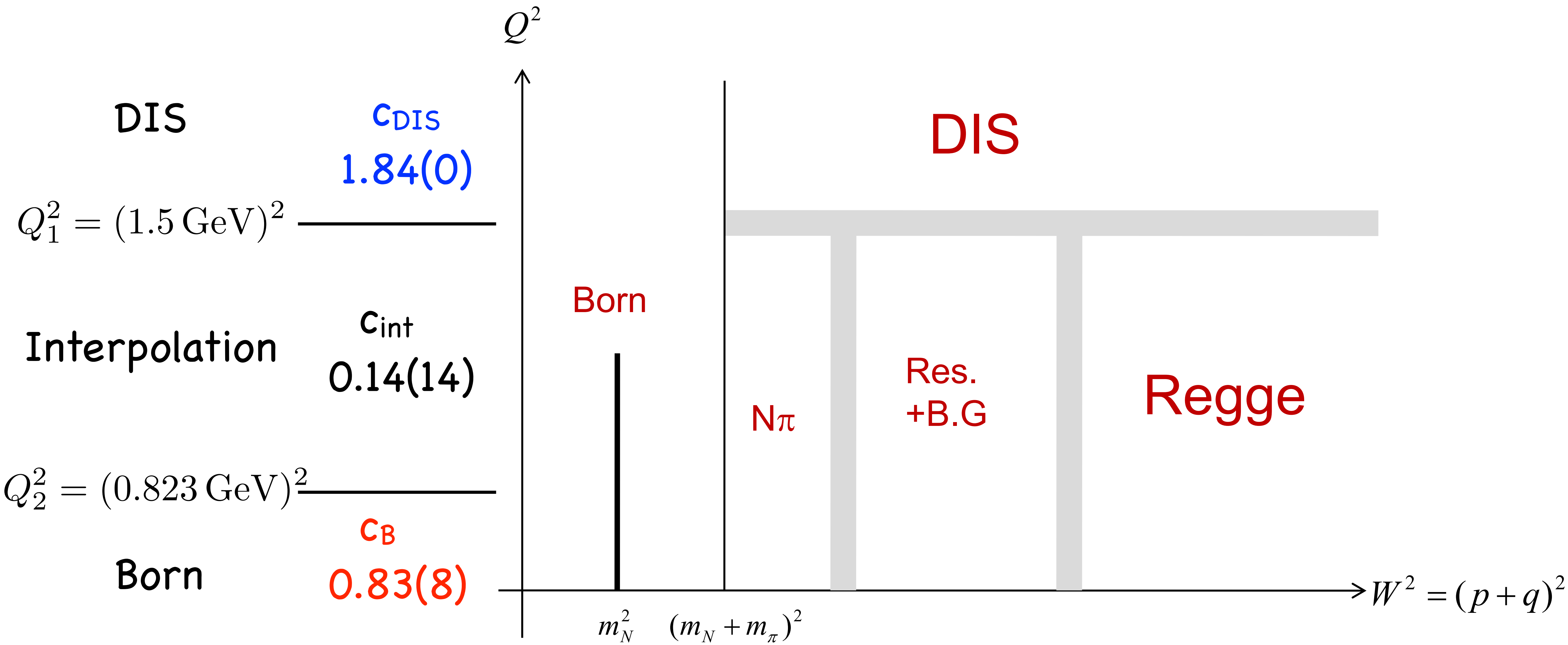
For each value of Q² we can relate F to particular hadronic processes

F can be related to exp. accessible cross sections

$$\mathrm{Im}\,[W^{+*} + n \to \gamma^* + p] \leftrightarrow \begin{cases} \sigma(\gamma^* + p \to X) \\ \sigma(W^{+*} + n \to X) \end{cases}$$

M&S treatment

M&S notation of the γW-box correction: $\Box^{VA}_{\gamma W} = \frac{\alpha}{2\pi}[c_B + c_{int} + c_{DIS}]$



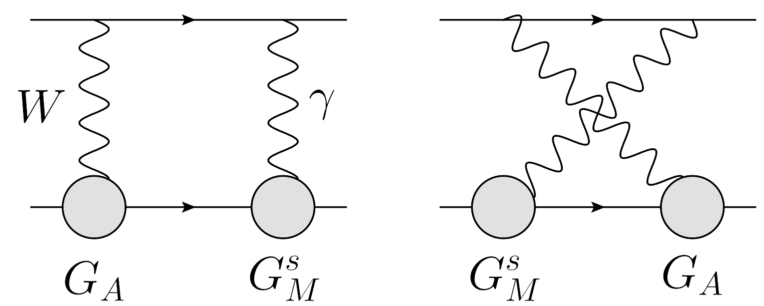
What can be improved?

* what is the physics content of the interpolating function?
* are the M&S constraints on F_{int} justified?

Physics input

Elastic (Born) contribution

$$\Box^{VA,\mathrm{Born}}_{\gamma W} = -\frac{\alpha}{\pi}\int_0^\infty dQ \frac{2\sqrt{4M^2+Q^2}+Q}{\left(\sqrt{4M^2+Q^2}+Q\right)^2} G_A(Q^2) G_M^S(Q^2)$$



M & S

$$\Box^{VA,\mathrm{Born}}_{\gamma W}\Big|_{\mathrm{MS}} = \frac{\alpha}{2\pi}(0.829 \pm 0.083)$$

Central value:
two dipoles integrated to (0.823 GeV)^2

Uncertainty: vary axial dipole masses between 1 and 1.4 GeV

New evaluation

$$\Box^{VA,\mathrm{Born}}_{\gamma W} = \frac{\alpha}{2\pi}(0.908 \pm 0.049)$$

Central value: full Q^2 integral with most recent FF parametrization

Uncertainty: from recent analyses

Magnetic FF: Ye, Arrington, Hill, Lee '18

Axial FF: Bhachatarya, Hill, Paz '11

Physics input

Inelastic contributions $\Box_{\gamma W}^{Inel.} = \frac{\alpha}{\pi}\int_0^\infty \frac{dQ^2}{1+\frac{Q^2}{M_W^2}}\int_{\nu_\pi}^\infty \frac{d\nu}{(\nu+q)^2}\frac{\nu+2q}{M\nu}F_3^{(0),inel.}$

Split the Q^2 integral: above $(1.5\ \mathrm{GeV})^2$ - DIS; below - hadronic stuff

Physics input

Inelastic contributions $$\Box_{\gamma W}^{Inel.} = \frac{\alpha}{\pi}\int_0^\infty \frac{dQ^2}{1+\frac{Q^2}{M_W^2}}\int_{\nu_\pi}^\infty \frac{d\nu}{(\nu+q)^2}\frac{\nu+2q}{M\nu}F_3^{(0),inel.}$$

Split the Q² integral: above (1.5 GeV)² - DIS; below - hadronic stuff

DIS contribution $$\Box_{\gamma W}^{\rm DIS} = \frac{2\alpha}{\pi}\int_{\Lambda^2}^\infty \frac{dQ^2 M_W^2}{Q^2(Q^2+M_W^2)}\int_0^{x_\pi} dx \frac{1+2\sqrt{1+4M^2x^2/Q^2}}{(1+\sqrt{1+4M^2x^2/Q^2})^2}F_3^{(0)}(x,Q^2)$$

Physics input

Inelastic contributions $\Box_{\gamma W}^{Inel.} = \frac{\alpha}{\pi}\int_0^\infty \frac{dQ^2}{1+\frac{Q^2}{M_W^2}}\int_{\nu_\pi}^\infty \frac{d\nu}{(\nu+q)^2}\frac{\nu+2q}{M\nu}F_3^{(0),inel.}$

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Parton model: $F_3^{(0)}(x) = \frac{e_u+e_d}{8}(d(x)-\bar{u}(x))$ $\int_0^1 dx d_v(x) = 2$

M/Q —> 0; loop function becomes $F^{\rm DIS}(Q^2) = \frac{1}{Q^2}$

Large log: $\Box_{\gamma W}^{\rm DIS} = \frac{\alpha}{8\pi}\int_{\Lambda^2}^\infty \frac{dQ^2 M_W^2}{M_W^2+Q^2}F^{\rm DIS}(Q^2) = \frac{\alpha}{4\pi}\ln\frac{M_W}{\Lambda}$

Physics input

Inelastic contributions $\Box^{Inel.}_{\gamma W} = \frac{\alpha}{\pi}\int_0^\infty \frac{dQ^2}{1+\frac{Q^2}{M_W^2}}\int_{\nu_\pi}^\infty \frac{d\nu}{(\nu+q)^2}\frac{\nu+2q}{M\nu}F_3^{(0),inel.}$

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pQCD corrections:
cf. GLS and Bjorken SR

$$F^{\rm DIS} = \frac{1}{Q^2} \to \frac{1}{Q^2}\left[1-\frac{\alpha_s^{\overline{MS}}}{\pi} - C_2\left(\frac{\alpha_s^{\overline{MS}}}{\pi}\right)^2 - C_3\left(\frac{\alpha_s^{\overline{MS}}}{\pi}\right)^3\right]$$

M&S '06; Larin, Vermaseren '97

$\Box^{\rm DIS}_{\gamma W} = \frac{\alpha}{4\pi}[4.11-0.34] = \frac{\alpha}{2\pi}1.84(0)$ Uncertainty: virtually zero

Physics input

The DIS contribution can be validated by data

Use the Gross-Llewellyn-Smith sum rule (GLS SR) in nu/anti-nu scattering

$$\frac{d^2\sigma^{\nu(\bar{\nu})}}{dxdy} = \frac{G_F^2 ME}{\pi}\left[xy^2F_1 + \left(1 - y - \frac{Mxy}{2E}\right)F_2 \pm x\left(y - \frac{y^2}{2}\right)F_3\right]$$

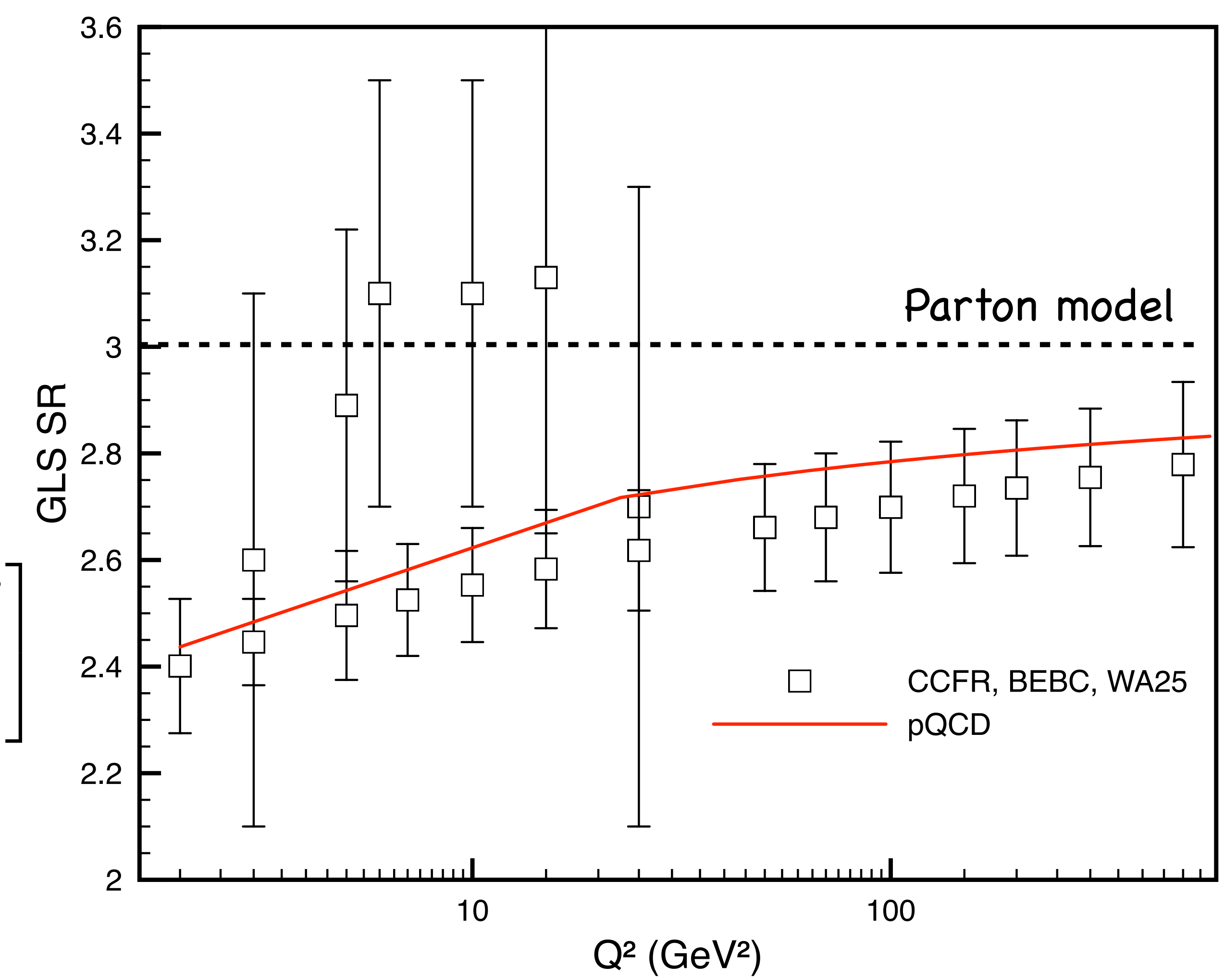
$$\sigma^{\nu p} - \sigma^{\bar{\nu}p} \sim F_3^{\nu p} + F_3^{\bar{\nu}p} = u_v^p(x) + d_v^p(x)$$

$$\int_0^1 dx(u_v^p(x) + d_v^p(x)) = 3$$

Plot vs. data derived from nu-DIS

Including pQCD corrections

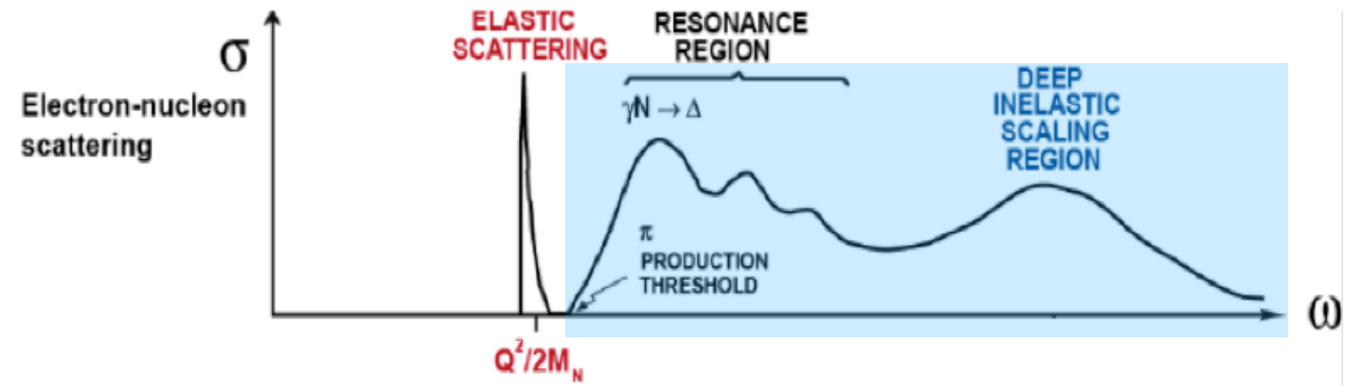
$$\mathrm{GLS\,SR} = 3\left[1 - \frac{\alpha_s^{\overline{MS}}}{\pi} - C_2\left(\frac{\alpha_s^{\overline{MS}}}{\pi}\right)^2 - C_3\left(\frac{\alpha_s^{\overline{MS}}}{\pi}\right)^3\right]$$



Physics input

Inelastic contributions beyond DIS

$$\Box_{\gamma W}^{low\,Q^2} = \frac{\alpha}{\pi}\int_0^{\Lambda^2} dQ^2 \int_{\nu_\pi}^{\infty} \frac{d\nu}{(\nu+q)^2}\frac{\nu+2q}{M\nu}F_3^{(0)}$$



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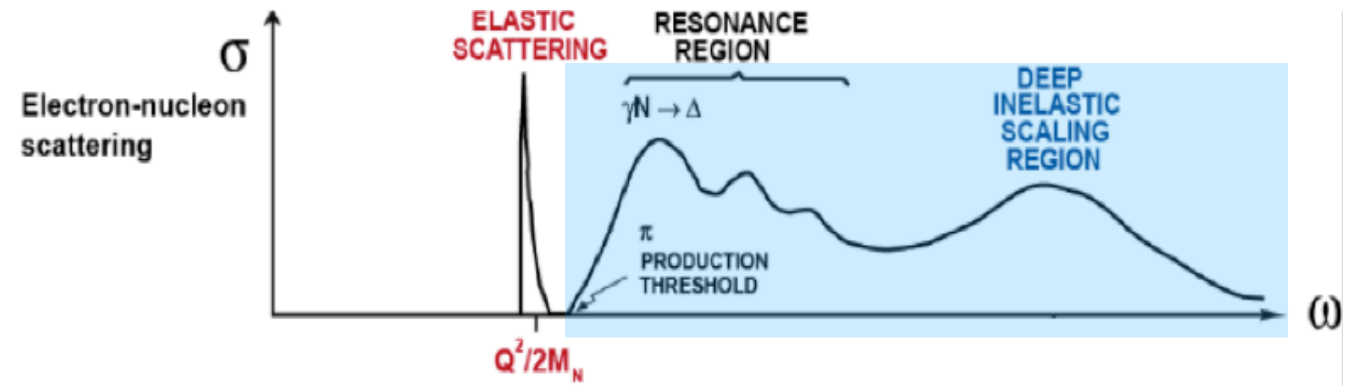

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M&S interpolating contribution

$$\Box_{\gamma W}^{VA\,(0)} = \frac{\alpha}{8\pi}\int_{Q_2^2}^{\Lambda^2} dQ^2 F^{\mathrm{INT}}(Q^2)$$

Constraints:

I $F^{\mathrm{INT}}(\Lambda^2) = F^{\mathrm{DIS}}(\Lambda^2)$

II $F^{\mathrm{INT}}((0.823\,\mathrm{GeV})^2) = F^{\mathrm{Born}}((0.823\,\mathrm{GeV})^2)$

III $F^{\mathrm{INT}}(0) = 0.$

3 Eqs. -> 3 free parameters

$$F^{\mathrm{INT}}(Q^2) = -\frac{1.490}{Q^2+m_\rho^2} + \frac{6.855}{Q^2+m_A^2} - \frac{4.414}{Q^2+m_{\rho'}^2}$$

Physics input

Inelastic contributions beyond DIS

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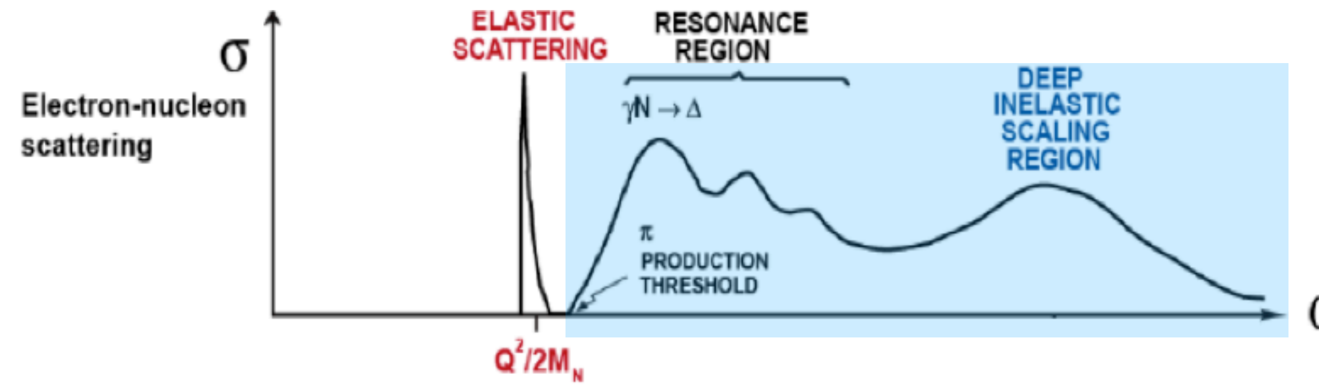

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Only constraint I is justified!

II: no reason Born is the whole story below some arbitrary Q^2

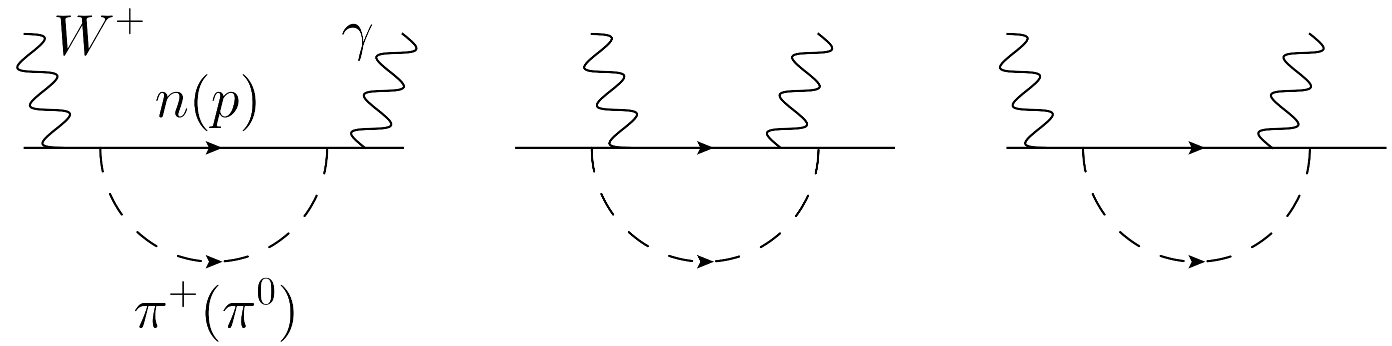
III: M&S claim it is required by chiral symmetry

Check in ChPT at one-loop

$$\int_{\nu_\pi}^{\infty}\frac{d\nu}{\nu^2}F_3^{(0)}(\nu,Q^2=0) = \frac{2M}{Q^2}\int_0^{x_\pi} F_3^{(0)}(x,Q^2)\Big|_{Q^2\to 0} = 0$$

ChPT - check if F(0)=0

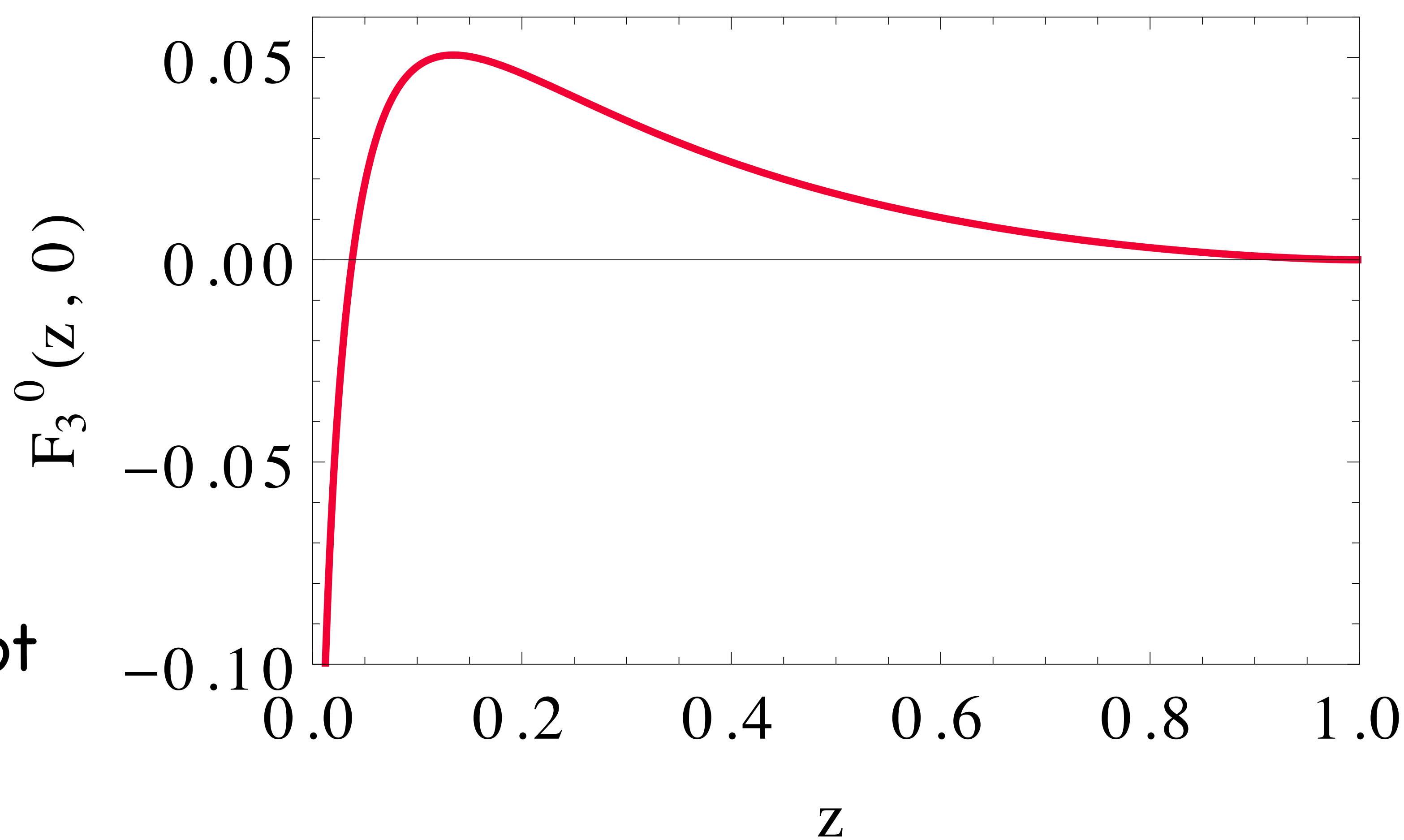
Representative Feynman graphs



To visualize: change integration variable to $z = \nu_\pi/\nu$

$$\int_{\nu_\pi}^{\infty} \frac{d\nu\nu_\pi}{\nu^2} F_3^{(0)}(\nu, Q^2=0) = \int_0^1 dz F_3^{(0)}(\nu_\pi/z, Q^2=0)$$

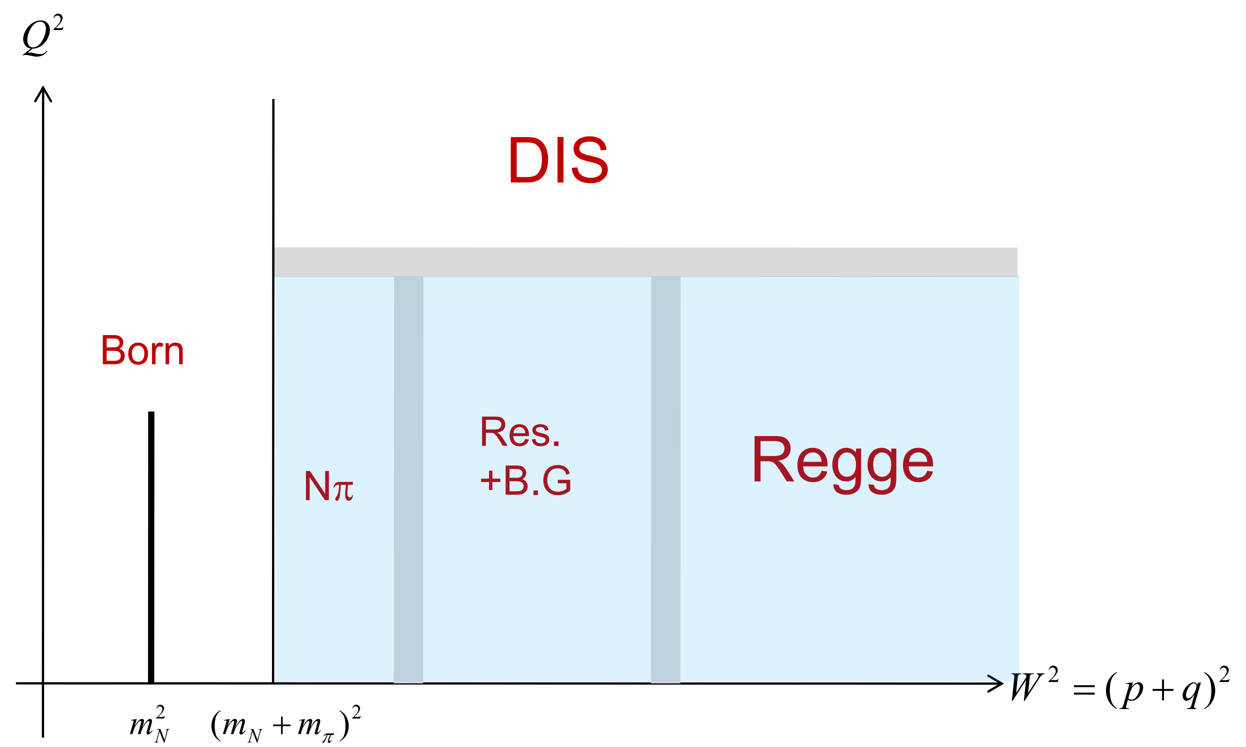
The surface below and above the curve is not the same



The claim of M&S about the superconvergence relation is not supported by ChPT

Inelastic states beyond DIS

Our approach
- saturate F^{INT} by hadronic states:
low-energy pi-N continuum
I=1/2 resonances
dominant Regge exchange (ρ)



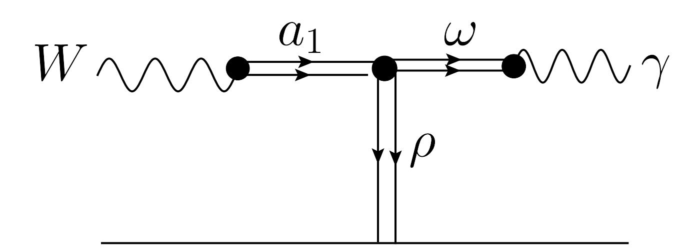
πN contribution to the box: $\Box^{\pi N}_{\gamma W} = \frac{\alpha}{2\pi} 0.044(4)$

I=1/2 resonances: tiny contribution! $\Box^{Res}_{\gamma W} \leq \frac{\alpha}{2\pi} 0.01$

Inelastic states beyond DIS

Regge exchange

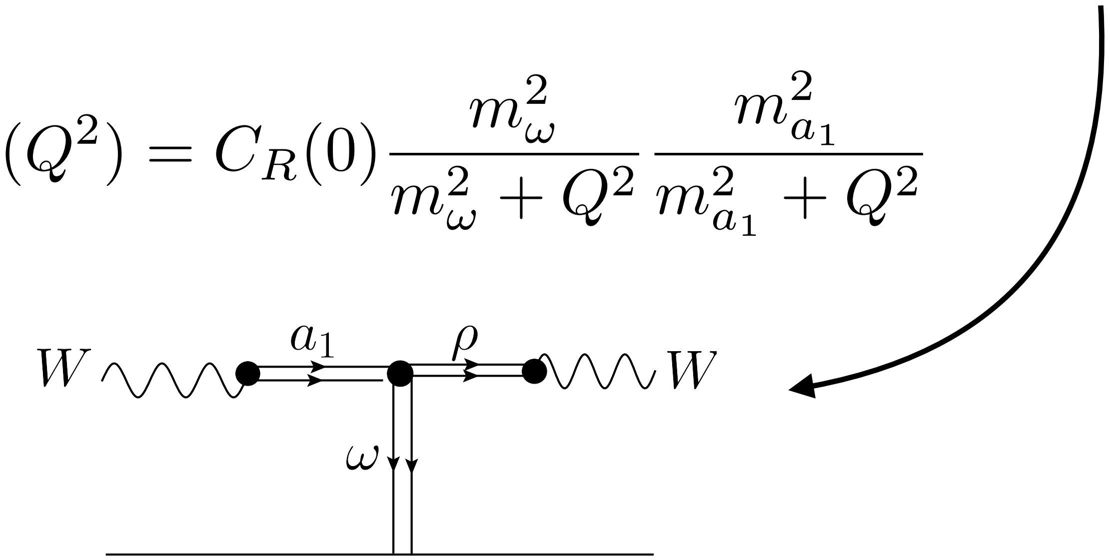
$$F_3^{(0),\mathrm{Regge}}(\nu, Q^2) = C_R(Q^2)\left(\frac{\nu}{\nu_0}\right)^{\alpha_\rho}$$



Vector/axial vector dominance:

Stodolsky, Piketty '70

$$C_R^{\mathrm{VDM}}(Q^2) = C_R(0)\frac{m_\omega^2}{m_\omega^2 + Q^2}\frac{m_{a_1}^2}{m_{a_1}^2 + Q^2}$$



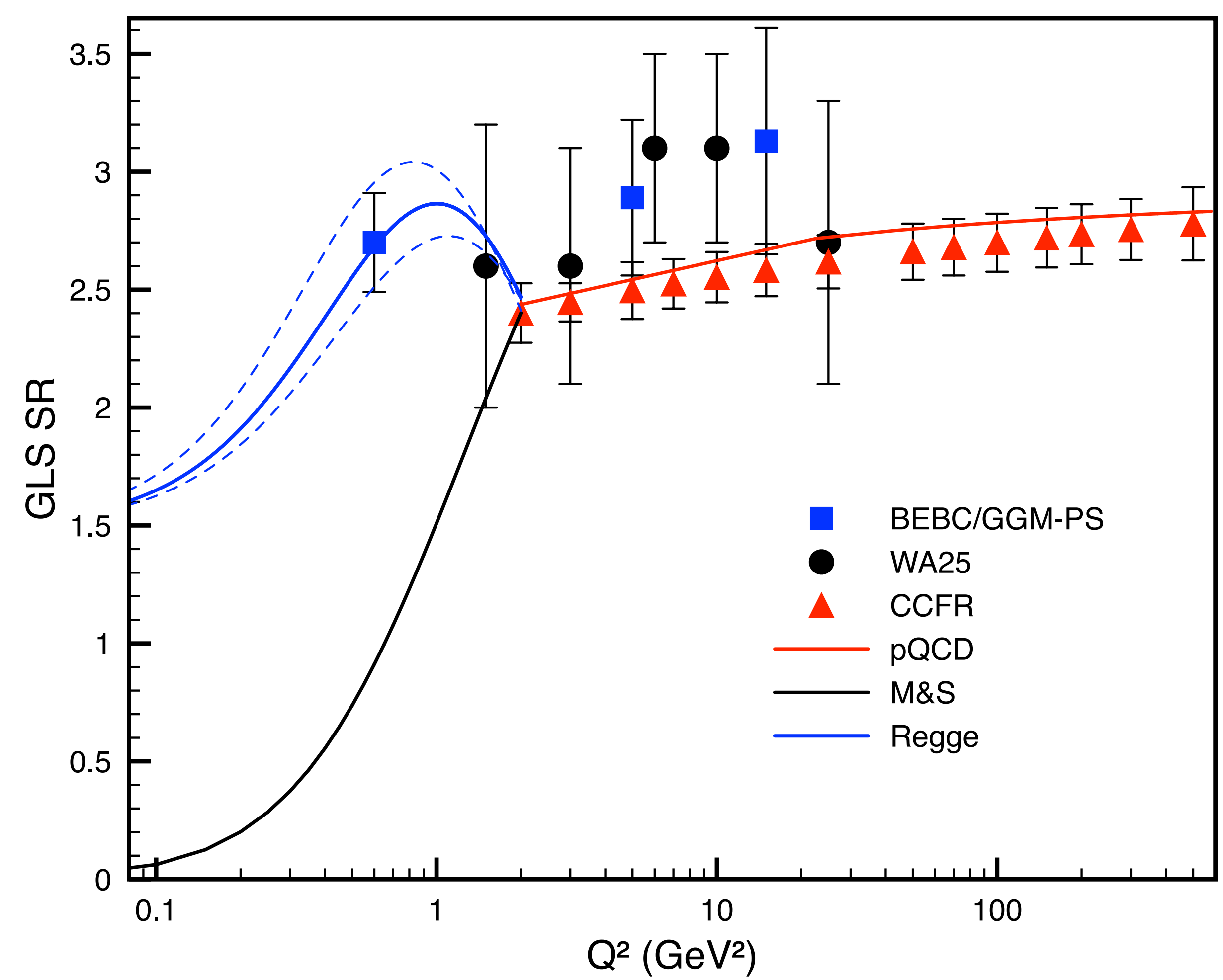
Guidance from GLS sum rule:
CC process, A-V interference

Only overall normalization is changed!
Match to pQCD at Q^2=2GeV^2
VDM + fit to (few) low-Q^2 data below

$$C_R(Q^2) = C_R^{\mathrm{VDM}}(Q^2) \times h(Q^2)$$

Deduce $h(Q^2)$, $\Delta h(Q^2)$ from data!

$$\Box_{\gamma W}^{\mathrm{Regge}} = \frac{\alpha}{2\pi} 0.238(14)$$



γW-box on free neutron

Marciano & Sirlin '06

$$\Box_{\gamma W}^{VA} = \frac{\alpha}{2\pi}[c_B + c_{int} + c_{DIS}] = \frac{\alpha}{2\pi}[0.83(8) + 0.14(14) + 1.84(0)]$$

$$\Box_{\gamma W}^{MS} = \frac{\alpha}{2\pi} 2.79(17) = 3.24(20) \times 10^{-3}$$

New evaluation

$$\Box_{\gamma W}^{VA} = \frac{\alpha}{2\pi}[c_B + c_{piN} + c_{\mathrm{Res}} + c_{\mathrm{Regge}} + c_{DIS}] = \frac{\alpha}{2\pi}[0.91(5) + 0.044(5) + 0.01(1) + 0.238(14) + 1.84(0)]$$

$$\Box_{\gamma W}^{\mathrm{New}} = \frac{\alpha}{2\pi} 3.03(5) = 3.51(6) \times 10^{-3}$$

V_{ud} from free n: about 1 sigma smaller

Numbers are preliminary but all crucial ingredients are in place.
Central value shifted by 1 sigma; uncertainty is likely to be reduced by factor 3

Currently the uncertainty for neutron decay is dominated by the experiment

Nuclear β-decay

General structure of RC for nuclear decay (see John's talk)

$$1 + \mathrm{RC} = (1 + \delta_R)(1 - \delta_C)(1 + \Delta).$$

Nuclear β-decay

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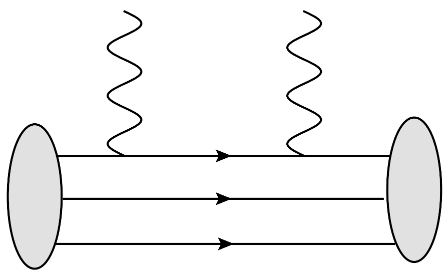

$$1 + \mathrm{RC} = (1+\delta_R)(1-\delta_C)(1+\Delta).$$

Universal (no NS)

Nuclear β-decay

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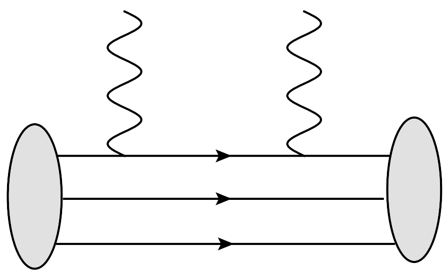

Isospin breaking

$$1 + \mathrm{RC} = (1 + \delta_R)(1 - \delta_C)(1 + \Delta).$$

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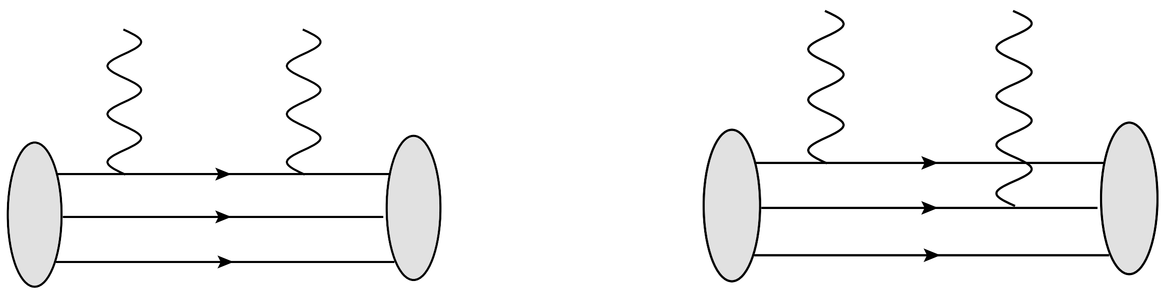

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Nuclear structure (NS)

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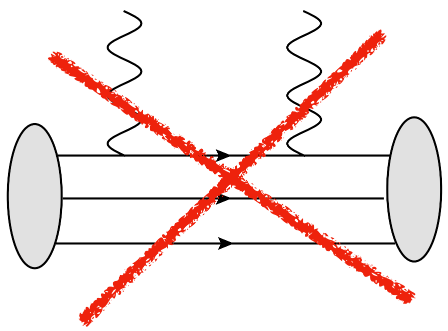
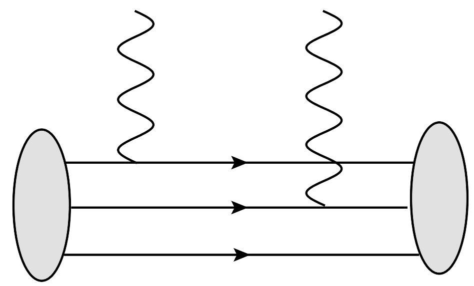




Isospin breaking

$$1 + \mathrm{RC} = (1 + \delta_R)(1 - \delta_C)(1 + \Delta).$$

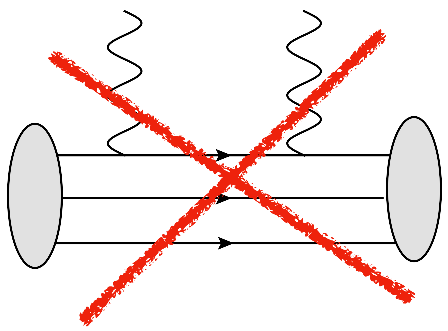
Nuclear structure (NS)

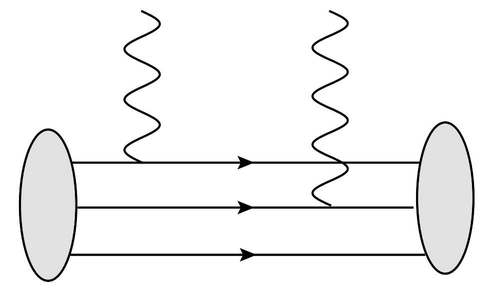
Universal (no NS)

Nuclear Green fn: only with 2 active N

Nuclear β-decay

General structure of RC for nuclear decay (see John's talk)





Isospin breaking

$$1 + \mathrm{RC} = (1 + \delta_R)(1 - \delta_C)(1 + \Delta).$$

Nuclear structure (NS)

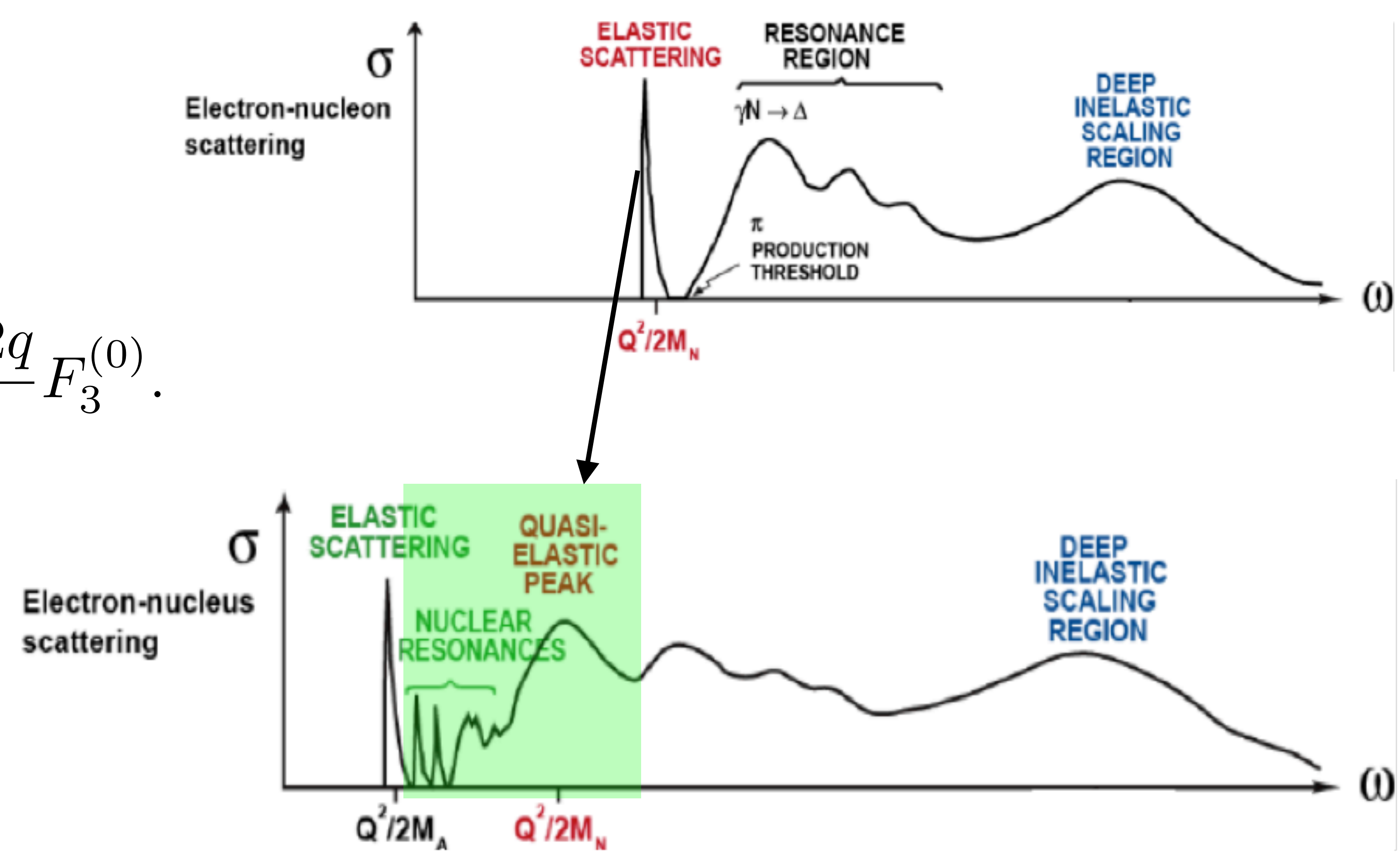
Universal (no NS)

Nuclear Green fn: only with 2 active N

But data tell us differently:
prominent broad QE peak -
mostly 1N knock-out

$$\Box_{\gamma W}^{\mathrm{Nucl.}} = \frac{\alpha}{\pi}\int_0^{\Lambda^2} dQ^2 \int_{\nu_N}^{\infty} \frac{d\nu}{(\nu+q)^2}\frac{\nu+2q}{M\nu}F_3^{(0)}.$$

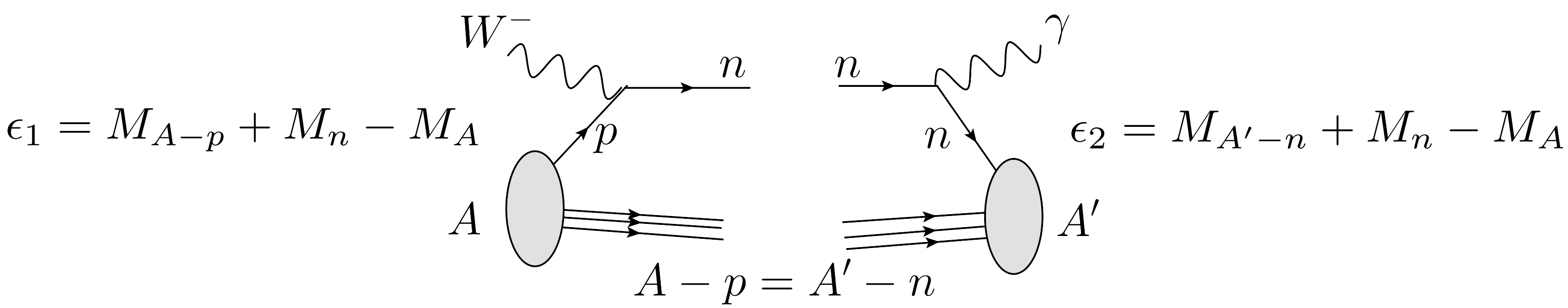
QE peak is common for all nuclei
Modify the universal correction
to account for bulk QE effect



QE contribution to γW-box

Bulk nuclear properties: Fermi momentum and break-up threshold

20 decays: ^{10}C -> ^{10}B through ^{74}Rb -> ^{74}Kr (Towner&Hardy '14 review)



$$\bar{\epsilon} = \sqrt{\epsilon_1 \epsilon_2}$$

Decay	ϵ_2 (MeV)	ϵ_1 (MeV)	$\bar{\epsilon}$ (MeV)
$^{10}C \to ^{10}B$	8.44	4.79	6.36
$^{14}O \to ^{14}N$	10.55	5.41	7.55
$^{18}Ne \to ^{18}F$	9.15	4.71	6.56
$^{22}Mg \to ^{22}Na$	11.07	6.28	8.34
$^{26}Si \to ^{26}Al$	11.36	6.30	8.46
$^{30}S \to ^{30}P$	11.32	5.18	7.66
$^{34}Ar \to ^{34}Cl$	11.51	5.44	7.91
$^{38}Ca \to ^{38}K$	12.07	5.33	8.02
$^{42}Ti \to ^{42}Sc$	11.55	4.55	7.25
$^{26m}Al \to ^{26}Mg$	11.09	6.86	8.72
$^{34}Cl \to ^{34}S$	11.42	5.92	8.22
$^{38m}K \to ^{38}Ar$	11.84	5.79	8.28
$^{42}Sc \to ^{42}Ca$	11.48	5.05	7.61
$^{46}Va \to ^{46}Ti$	13.19	6.14	9.00
$^{50}Mn \to ^{50}Cr$	13.00	5.37	8.35
$^{54}Co \to ^{54}Fe$	13.38	5.13	8.28
$^{62}Ga \to ^{62}Zn$	12.90	3.72	6.94
$^{66}As \to ^{66}Ge$	13.29	3.16	6.48
$^{70}Br \to ^{70}Se$	13.82	3.20	6.65
$^{74}Rb \to ^{74}Kr$	13.85	3.44	6.90

Effective removal energies - all in a small range

$$\bar{\epsilon} = 7.68 \pm 1.32 \text{ MeV}$$

Fermi momentum also not too different for all A

$$k_F(A=10) = 228\,\text{MeV}, \quad k_F(A=74) = 245\,\text{MeV}$$

Can define a universal correction that correctly represents bulk nuclear effect!

Further ingredients:
Free Fermi gas model (or superscaling)
+ Pauli blocking

QE contribution to γW-box

γW-box for bound neutron: $\Box_{\gamma W}^{\text{free n}} = \frac{\alpha}{2\pi} 0.91(5) \;\to\; \Box_{\gamma W}^{\text{QE}} = \frac{\alpha}{2\pi} 0.44(4)$

Reduction: finite breakup threshold $\int\limits_{\frac{Q^2}{2M}} \frac{d\nu}{\nu^2} F_3^n \;\to\; \int\limits_{\frac{Q^2}{2M_A}+\bar\epsilon} \frac{d\nu}{\nu^2} F_3^{\text{Nucl}}$

New formulation of the γW-box:

$$\Box_{\gamma W}^{MS} = \frac{\alpha}{2\pi} 2.79(17) = 3.24(20) \times 10^{-3}$$

$$\Box_{\gamma W}^{\text{Nucl. New}} = \frac{\alpha}{2\pi} 2.56(4) = 2.97(5) \times 10^{-3}$$

A mere shift by 1 sigma; uncertainty significantly reduced.
Nuclear Structure corrections should be revisited and possibly redefined

V_{ud} from superallowed β: 1 sigma larger

Summary

- New dispersive representation of the γW-box
- Data driven uncertainties
- Crucial input: GLS sum rule
- New formulation of RC for V_{ud} extraction:
  overall small effect; uncertainty significantly reduced
- Nuclear Structure corrections may need to be reformulated
- Backup: can nuclear structure effects lead to additional energy dependence?

Turn “inner” correction inside-out?

γW-box correction at zero energy

$$\Box_{\gamma W}^{VA\,(0)} = \frac{\alpha}{\pi M}\int_0^\infty \frac{dQ^2 M_W^2}{M_W^2+Q^2}\int_0^\infty d\nu \frac{(\nu+2q)}{\nu(\nu+q)^2} F_3^{(0)}(\nu,Q^2),$$
$$\Box_{\gamma W}^{VA\,(3)} = 0,$$

γW-box correction with linear E-dependece

$$E_{min} = (\nu + \sqrt{\nu^2+Q^2})/2$$

$$\mathrm{Re}\,\overline{\Box_{\gamma W}^{even}} = \frac{\alpha_{em}}{\pi}\int_{\nu_{thr}}^\infty d\nu \int_0^\infty dQ^2 \frac{F_3^{(0)}}{2M\nu}\left(\frac{1}{E_{min}} - \frac{\nu}{4E_{min}^2}\right),$$

$$\mathrm{Re}\,\Box_{\gamma W}^{odd} = \frac{\alpha_{em}}{\pi}E\int_{\nu_{thr}}^\infty d\nu \int_0^\infty dQ^2 \left[\frac{F_1^{(0)}}{6ME_{min}^3} + \left(\frac{\sqrt{\nu^2+Q^2}}{2E_{min}^2\nu Q^2} - \frac{1}{12E_{min}^3\nu}\right)F_2^{(0)} + \frac{F_3^{(-)}}{2M\nu}\left(\frac{1}{2E_{min}^2} - \frac{\nu}{6E_{min}^3}\right)\right]$$

Common wisdom: E-dep. negligible because should come as $(\alpha/2\pi)\, E/m_\pi < 10^{-5}$

But nuclear excitations live at few MeV —> large nuclear polarizabilities

$$\alpha_E + \beta_M = \frac{2\alpha_{em}}{M}\int \frac{d\omega}{\omega^3} F_1(\omega, Q^2=0) = 2\alpha_{em}\int \frac{d\omega}{\omega^2}\left.\frac{F_2(\omega,Q^2)}{Q^2}\right|_{Q^2=0}$$

New energy scale: polarizability/radius^2

$$\mathrm{Re}\,\Box_{\gamma W}^{odd} \sim \frac{2}{\pi}E\frac{\alpha_E+\beta_M}{R_{Ch}^2}$$

$$R_{\mathrm{Ch}} \sim 1.2\mathrm{fm} A^{1/3}$$

$$\alpha_E \sim (2.2\times 10^{-3}\,\mathrm{fm})A^{5/3}$$

Expect $$\mathrm{Re}\,\Box_{\gamma W}^{odd} \sim 1\times 10^{-3}\left(\frac{E}{5\,\mathrm{MeV}}\right)\left(\frac{A}{30}\right)$$