A Lattice QCD Study of the ρ Resonance

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Motivation

- Hadrons - composite particles from quarks and gluons
- some: QCD stable (like π)
- most: QCD unstable (like ρ)
- couple to strong decay channels
- couple to radiative decay channels

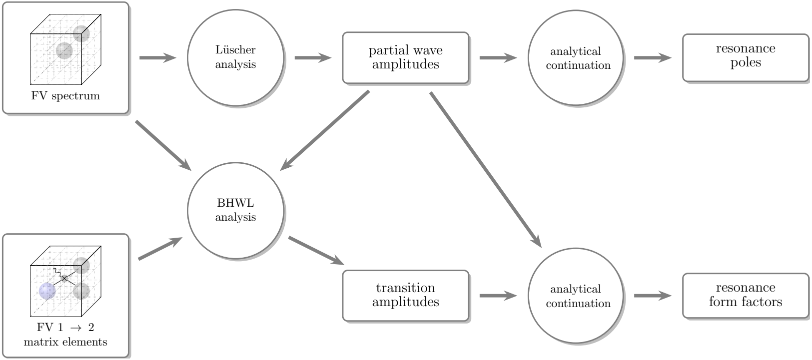
Introduction
Outline

Part I: $\rho \to \pi\pi$

- $I = 1$ P-wave $\pi\pi$ scattering and the ρ
- scattering on the lattice

Part II: $\pi\gamma \to \pi\pi$

- $\pi\gamma \to \rho$
- photoproduction on the lattice



[Briceño,Hansen,Walker-Loud PRD 2015]

The Lattice Ensemble

- $N_f = 2 + 1$ Clover fermions
- isotropic lattice by Orginos et al.
- $m_\pi L > 4$
- m_π low enough: ρ is unstable

Label	$N_L^3 \times N_t$	a (fm)	L (fm)	m_π (MeV)	N_{config}
C13	$32^3 \times 96$	0.11403	3.65	317	1041

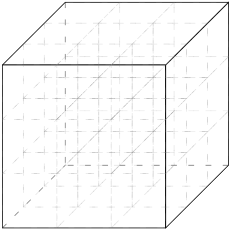
- forward, sequential and stochastic propagators

- thresholds:
  - $\pi\pi$: ≈ 640 MeV
  - 4π: ≈ 1280 MeV
  - $K\bar{K}$: ≈ 1100 MeV

ρ

the strong decay channel ($\rho \to \pi\pi$)

Scattering on the Lattice



- calculate 2-pt correlation functions in box of size L
- $\Psi(x+L) = \Psi(x)$
- discrete spectrum
- finite volume effects! $(\frac{1}{L^n},\, e^{-mL})$

- finite volume states:
  - interaction shifts the energies $\{E_n^{\vec{P},\Lambda}\}$
  - all states look similar
- exploit the finite volume to determine scattering parameters

  [Lüscher NPB1991]

- use multiple moving frames $\vec{P}$ and irreps Λ

  [Gottlieb & Rummukainen NPB1995, . . . , Briceño et al. RMP 2018]

Scattering on the Lattice - the Lüscher Method

lattice multi-hadron spectroscopy: $E_n^{\vec{P},\Lambda}$

- invariant masses:

$$s_n^{\vec{P},\Lambda} = (E_n^{\vec{P},\Lambda})^2 - \vec{P}^2$$

- scattering momentum:

$$\sqrt{s_n^{\vec{P},\Lambda}} = 2\sqrt{m_\pi^2 + (k_n^{\vec{P},\Lambda})}$$

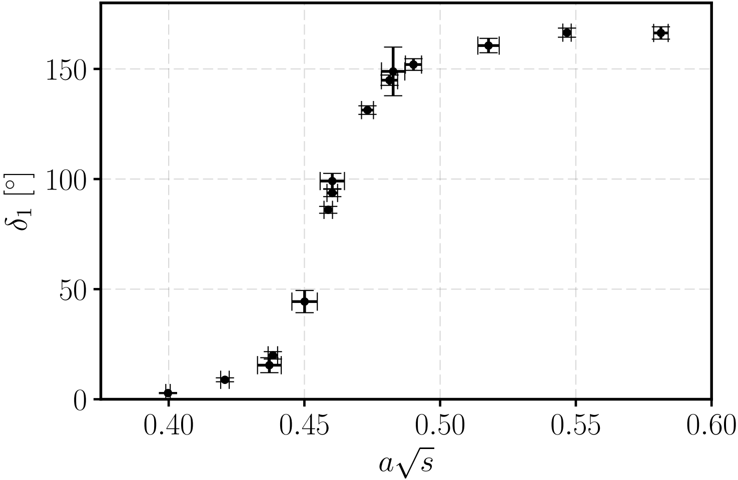
quantization condition [Lüscher NPB1991]:

infinite volume — finite volume

$$\cot\delta(s) + \cot\phi^{\vec{P},\Lambda}(s) = 0$$

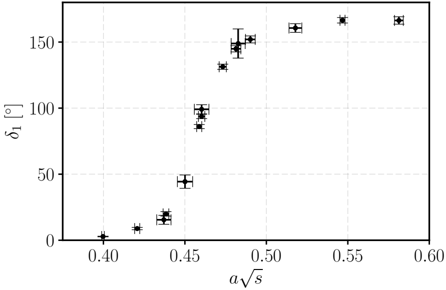
Determine $\delta(s)$ at discrete values of $s_n^{\vec{P},\Lambda}$.

$I = 1$ P-wave $\pi\pi$ Scattering and the ρ Resonance



$I = 1$ P-wave $\pi\pi$ Scattering and the ρ Resonance

the Physics



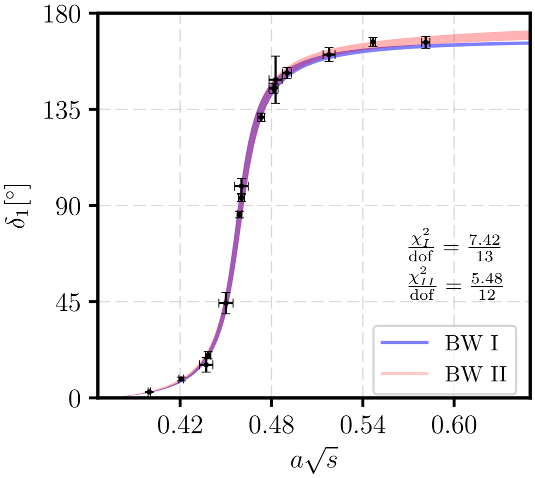
$$\delta(s) = \arctan \frac{\sqrt{s}\Gamma(s)}{m_R^2 - s}$$

- $BW\ I$: $\Gamma = \frac{g^2_{\rho\pi\pi}}{6\pi} \frac{k^3}{s}$
- vanilla P-wave decay width ($k^{2\ell+1}$)
- unitless coupling $g_{\rho\pi\pi}$

- $BW\ II$: $\Gamma = \frac{g^2_{\rho\pi\pi}}{6\pi} \frac{k^3}{s} \frac{1+(k_R r_0)^2}{1+(k r_0)^2}$
- Blatt-Weisskopf barrier factor
- r_0 barrier radius

$I = 1$ P-wave $\pi\pi$ Scattering and the ρ Resonance

comparing $BW\ I$ and $BW\ II$



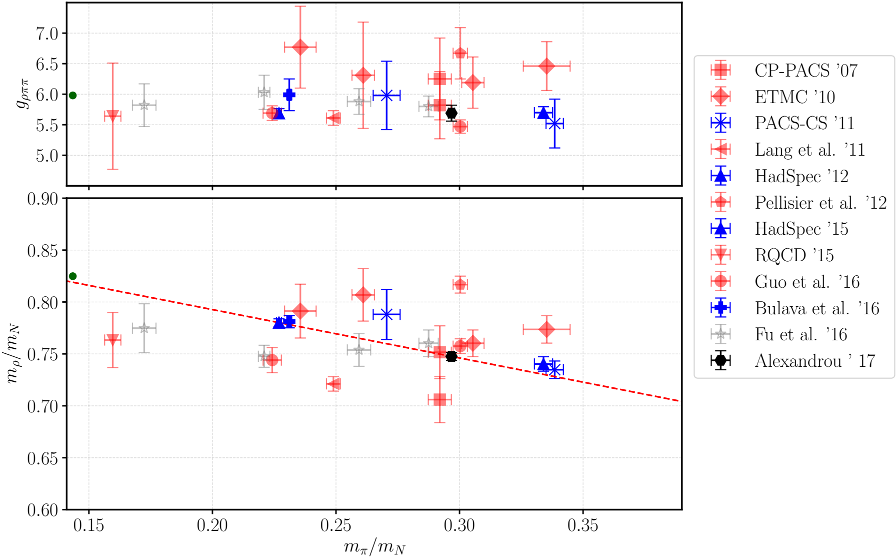
[Alexandrou et al.,]

- $BW\ I$:
  $am_\rho = 0.4609(16)$
  $g_{\rho\pi\pi} = 5.69(13)$

- $BW\ II$:
  $am_\rho = 0.4603(16)$
  $g_{\rho\pi\pi} = 5.77(13)$
  $(r_0/a)^2 = 9.6(5.9)$

$I = 1$ P-wave $\pi\pi$ Scattering and the ρ Resonance

placing the results in perspective

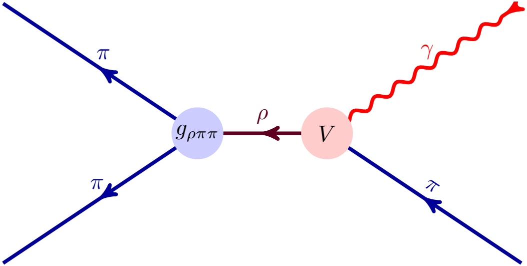


ρ

photoproduction channel $\pi\gamma \to \rho$

Transition Matrix Elements

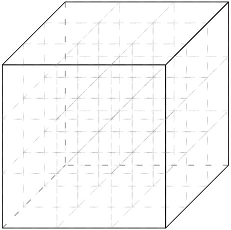
resonant contribution



$$q^\mu = p_\pi^\mu - P^\mu$$

$$A(\pi\gamma \to \pi\pi) = \frac{i\,2\,V(q^2, s)}{m_\pi} \varepsilon^{\mu\nu\alpha\beta} \epsilon^{*\nu} P^\alpha p_\pi^\beta$$

Photoproduction on the Lattice
adding on top of spectroscopy



- calculate 3-pt correlation functions in box of size L
- use information from the 2-pt functions in 3-pt
- determine matrix elements
- finite volume effects! $(\frac{1}{L^n}, e^{-mL})$

- finite volume states:
  - interaction modifies the matrix elements
  - all states look similar

- Take into account the finite volume effects in the transition matrix elements

[Lellouch & Lüshcer CMP2001]

Transition Matrix Elements

Briceño-Hansen-Walker-Loud Formalism

lattice multi-hadron matrix elements:
$\{\langle n, \vec{P}, \Lambda, \nu | J^\mu(\vec{q}) | \pi, \vec{p}_\pi \rangle\}$ at set of $\{s_n^{\vec{P},\Lambda}\}$ at several $s_n^{\vec{P},\Lambda}$, $\vec{q}$ and $\vec{p}_\pi$

infinte volume — finite volume spectroscopy — finite volume

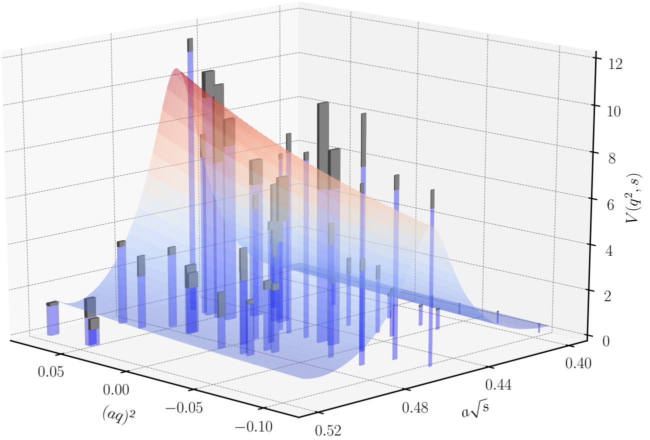
$$|A(\nu;\mu;q^2,s_n^{\vec{P},\Lambda})|^2 = \frac{32\pi E_\pi \sqrt{s_n^{\vec{P},\Lambda}}}{k_n}\left[\frac{\partial\delta(s)}{\partial E} + \frac{\partial\phi^{\vec{P},\Lambda}(k)}{\partial E}\right]_{E=E_n^{\vec{P},\Lambda}} |\langle n, \vec{P}, \Lambda, \nu | J^\mu(\vec{q}) | \pi, \vec{p}_\pi \rangle|^2$$

[Lellouch & Lüscher CMP 2001; Briceño,Hansen,Walker-Loud PRD 2015, Rusetsky et al NPB 2014]

Determine A at several values of momentum transfer q and invariant mass s.

[HadSpec, PRL 2015, PRD 2016]

$\pi\gamma \to \pi\pi$ Infinite Volume Transition Amplitude
PRELIMINARY



$\pi\gamma \to \pi\pi$ Infinite Volume Transition Amplitude
PRELIMINARY

$$V(q^2, s) = \frac{F(q^2, s)}{m_R^2 - s - i\sqrt{s}\,\Gamma_{II}(s)} \sqrt{\frac{16\pi s \Gamma_{II}(s)}{k}},$$

$$F(q^2, s) = \frac{\sum_n \sum_m A_{nm} z^n \mathcal{S}^m}{1 - \frac{q^2}{m_P^2}}$$

- z-expansion [BGL PRD1997, BCL PRD2009]

$$z = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$$

$$t_+ = (2m_\pi)^2, \quad t_0 = 0$$

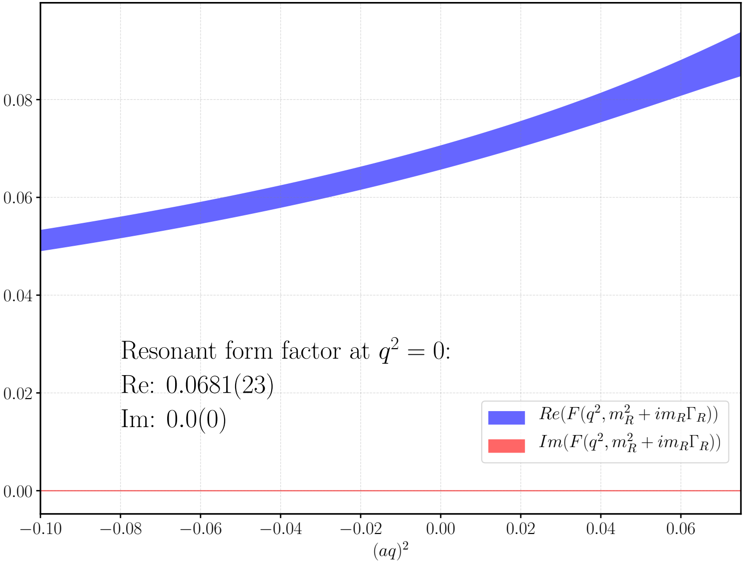
- s dependance:

$$\mathcal{S} = (s - m_R^2)$$

- non-zero A_{nm}: $A_{00}, A_{02}, A_{04}, A_{10}, A_{12}$
- $\frac{\chi^2}{dof} = \frac{40.4}{43}$

$\pi\gamma \to \rho$ Resonant form factor

PRELIMINARY; statistical uncertainty only



Findings

- lattice QCD can provide theory values for:
  - resonance masses
  - decay widths to strong decay channels
  - transition amplitudes
  - photocouplings
  - resonant form factors

Thank you :)