

recent progress on hadron spectroscopy from lattice QCD

Jozef Dudek

this presentation is dedicated to the memory of
Mike Pennington,

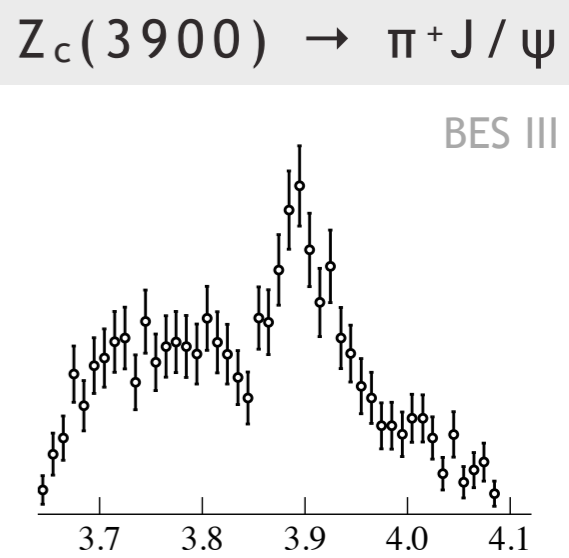
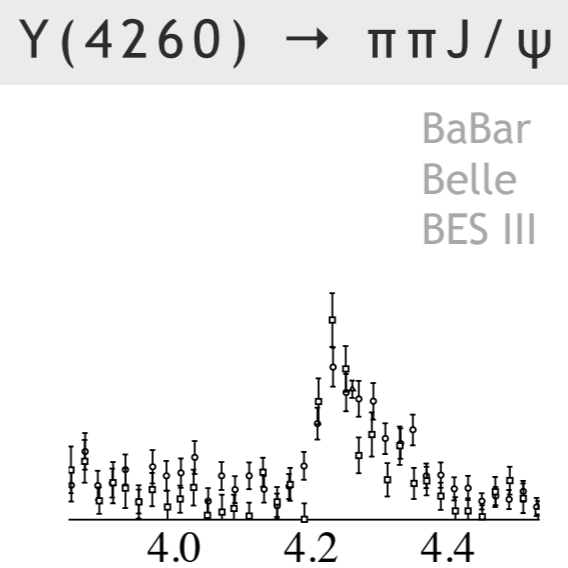
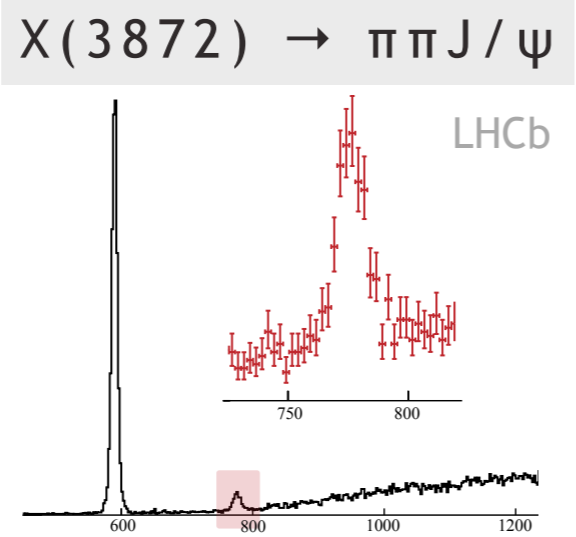
scientist,
mentor,
friend



While **QCD** may be a solid part of the **standard model**, and **hadrons** are ubiquitous in HEP experiments, there remain significant mysteries in how **hadrons** are built from **quarks** and **gluons**

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unexpected ?



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light scalar meson resonances

unexplained ?

$f_0(500)$ or σ [g]
was $f_0(600)$

$$I^G(J^{PC}) = 0^+(0^{++})$$

Mass $m = (400-550)$ MeV
Full width $\Gamma = (400-700)$ MeV

$f_0(980)$ [j]

$$I^G(J^{PC}) = 0^+(0^{++})$$

Mass $m = 990 \pm 20$ MeV
Full width $\Gamma = 10$ to 100 MeV

$K_0^*(800)$
or κ

$$I(J^P) = \frac{1}{2}(0^+)$$

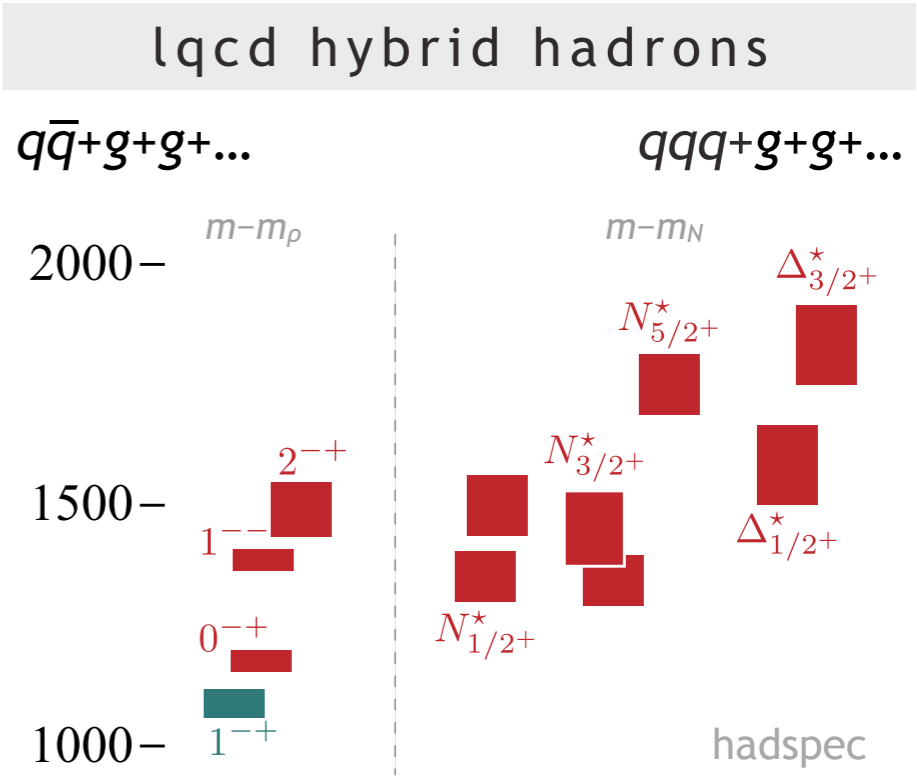
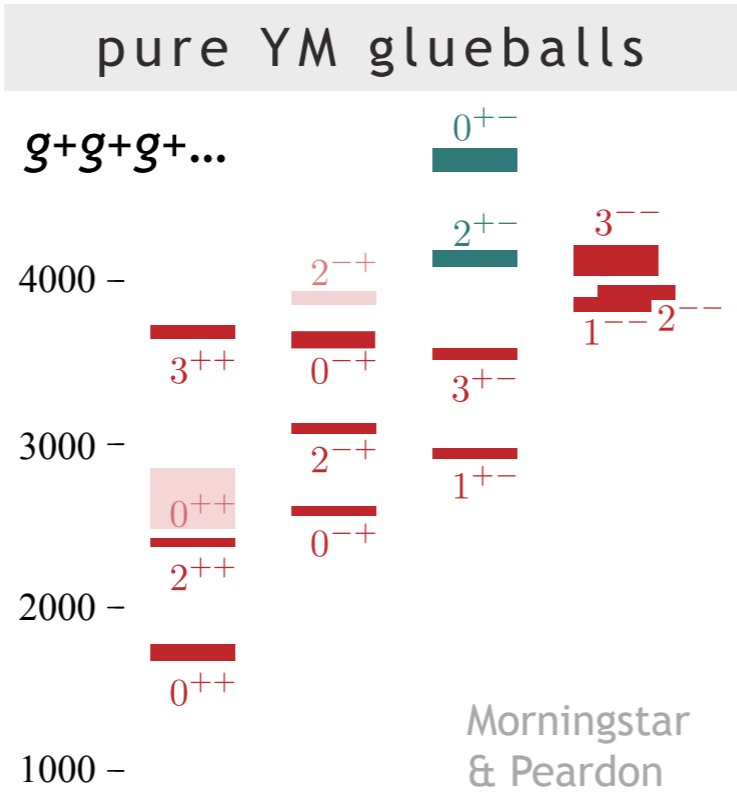
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$$I^G(J^{PC}) = 1^-(0^{++})$$

Mass $m = 980 \pm 20$ MeV
Full width $\Gamma = 50$ to 100 MeV

While **QCD** may be a solid part of the **standard model**, and **hadrons** are ubiquitous in HEP experiments, there remain significant mysteries in how **hadrons** are built from **quarks** and **gluons**

unobserved ?



quark & gluon fields on a **finite space-time grid** (in Euclidean time)

introduce: **lattice spacing**, **lattice volume**, often $m_q > m_q^{\text{phys}}$

Monte Carlo sample field configurations

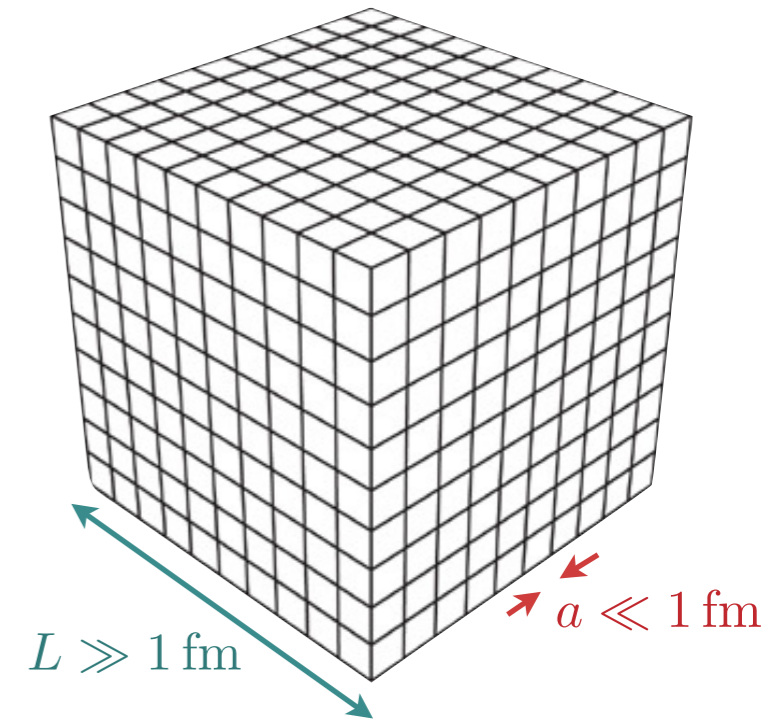
hadron spectrum from two-point correlation functions

$$\langle 0 | \mathcal{O}'(t) \mathcal{O}(0) | 0 \rangle = \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}A \left[\mathcal{O}'(t) \mathcal{O}(0) \right] e^{-S[\psi, \bar{\psi}, A]}$$

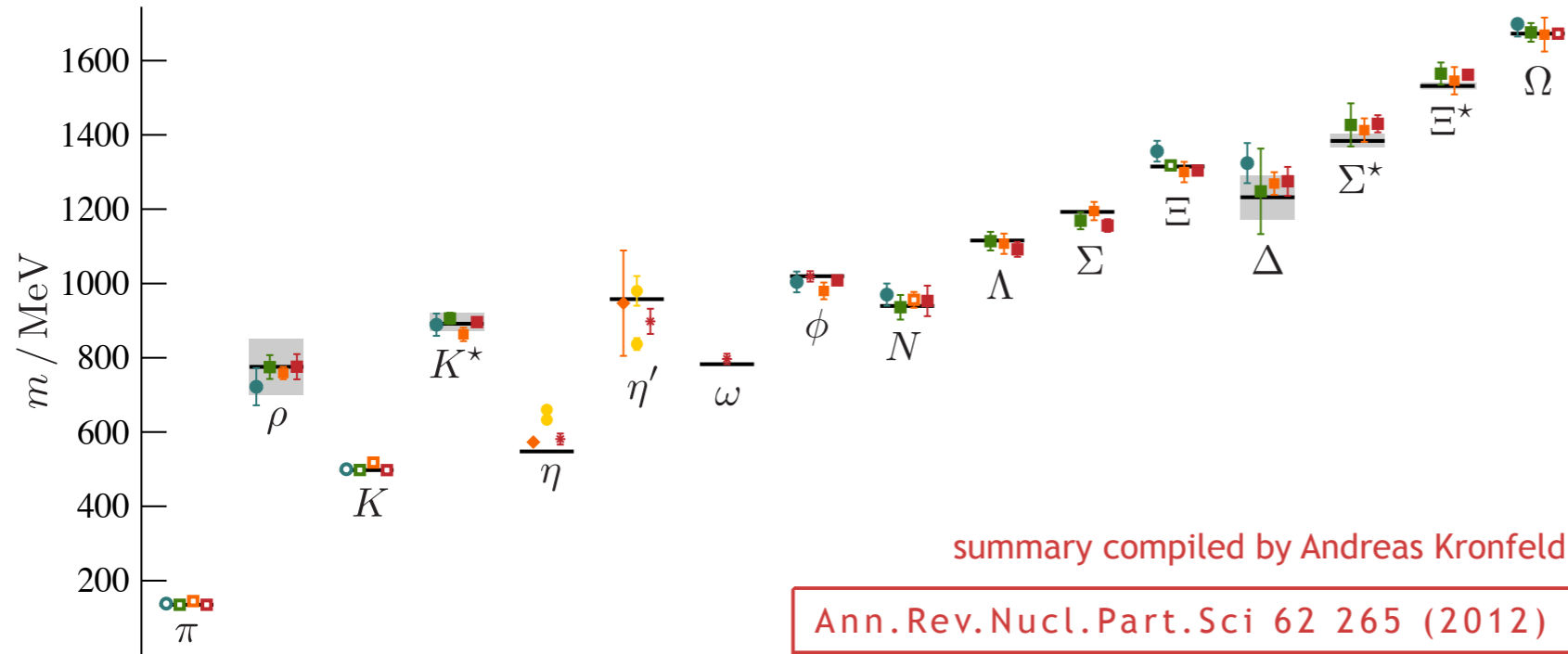
field
configuration
probability

$$\langle 0 | \mathcal{O}'(t) \mathcal{O}(0) | 0 \rangle = \sum_n A'_n A_n e^{-E_n t}$$

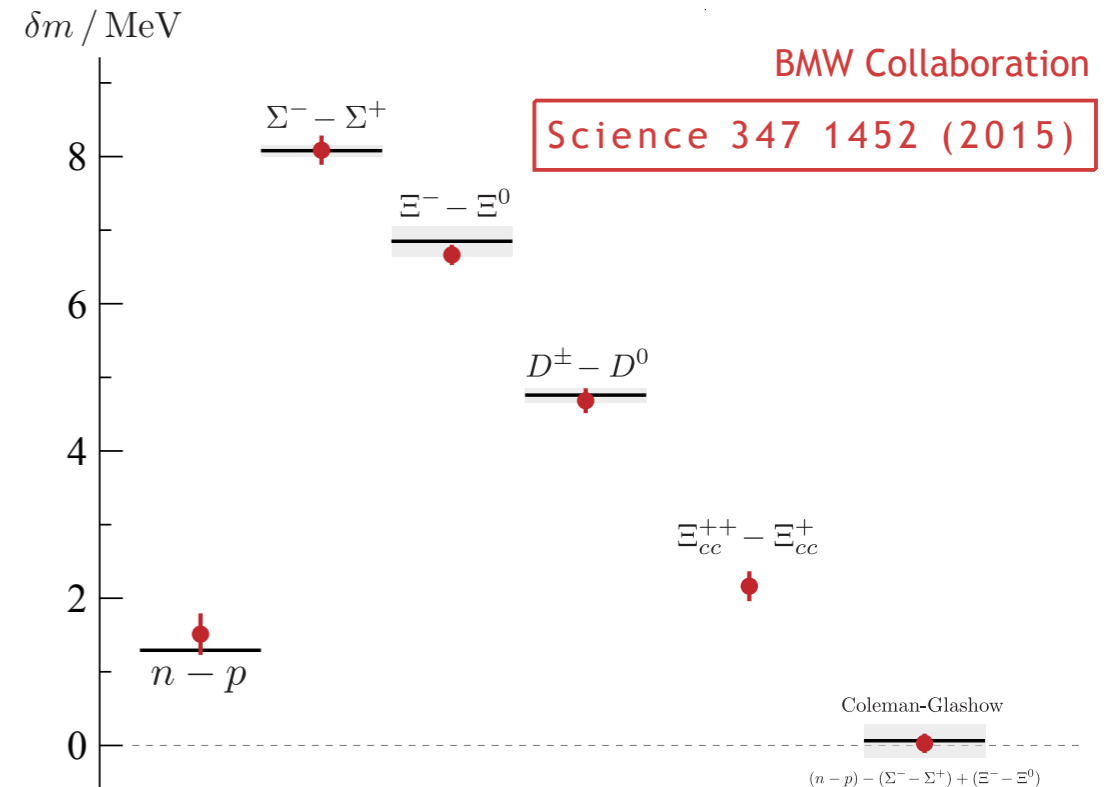
spectrum of
QCD eigenstates



lattice qcd light hadron spectrum



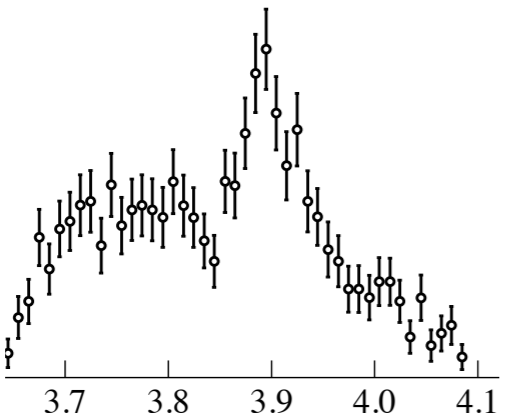
QCD+QED mass shifts



but much of the excitement in hadron spectroscopy is in **heavier states**

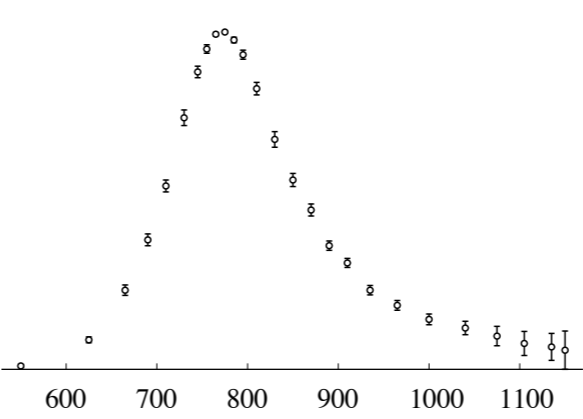
and they are **resonances** observed through their decays

$Z_c(3900) \rightarrow \pi^+ J/\psi$



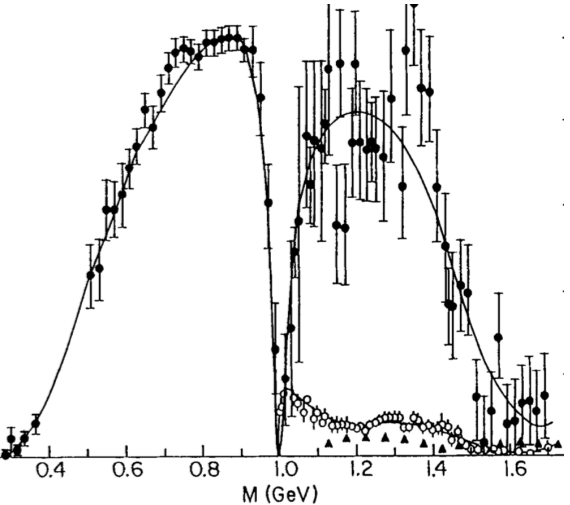
exotic

$\rho \rightarrow \pi\pi$



familiar

$\pi\pi \rightarrow \pi\pi, K\bar{K}, \eta\eta$



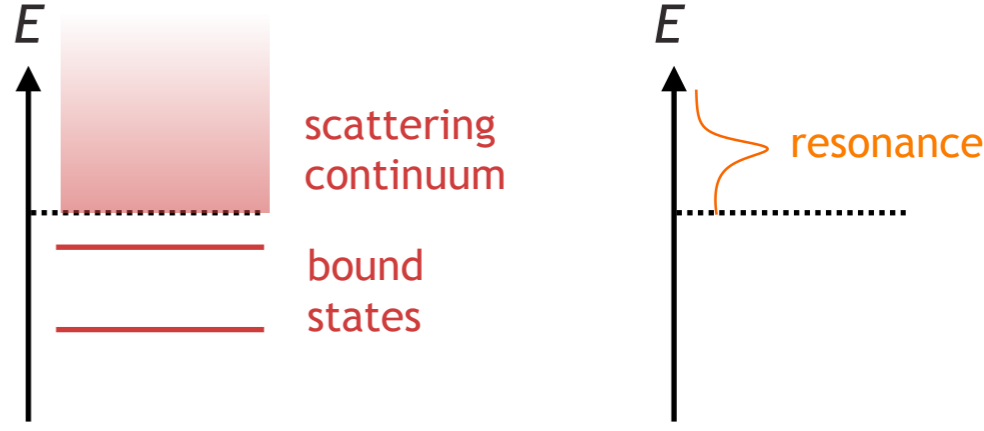
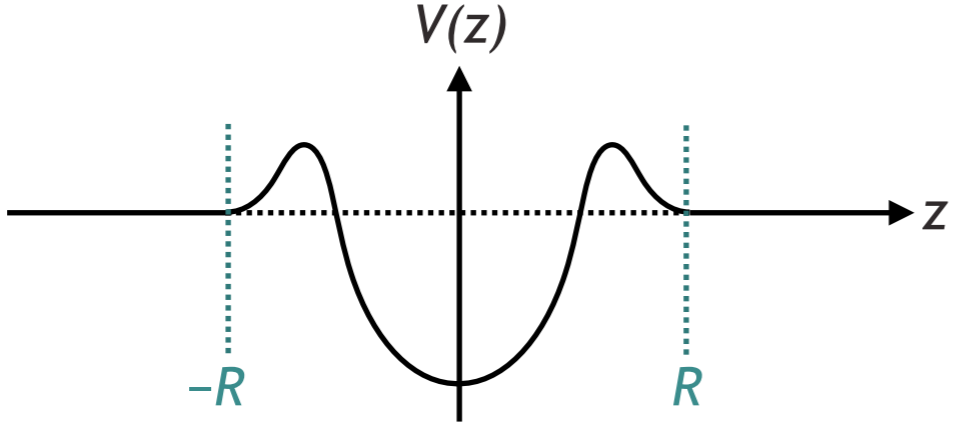
non-trivial

same non-perturbative dynamics **binds** and causes the **decay** – can't be separated within QCD ...

a faithful QCD calculation should give **all the scattering physics** at once ...

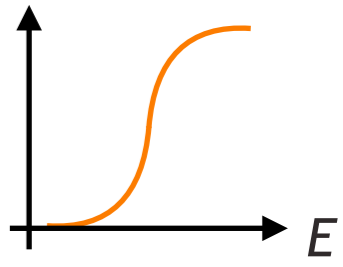
the approach can be illustrated within **one-dimensional quantum mechanics**

imagine two identical bosons separated by a distance z interacting through a finite-range potential $V(z)$



$$\psi(|z| > R) \sim \cos(p|z| + \delta(p))$$

phase-shift



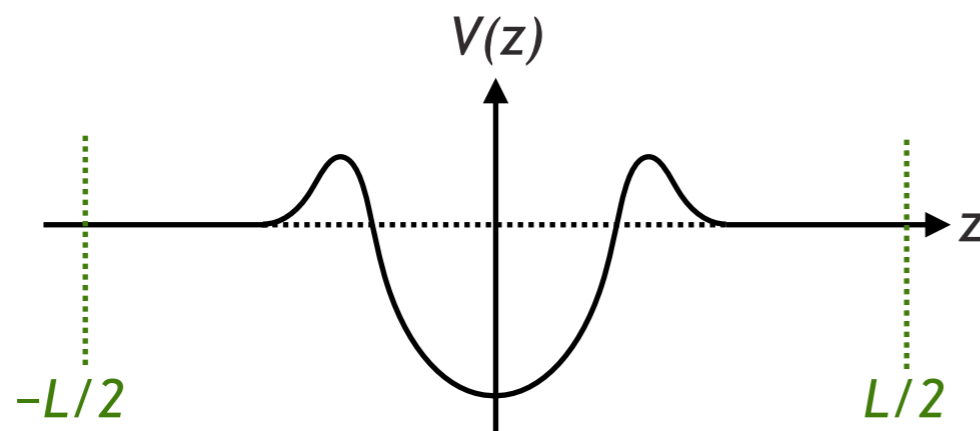
now put the system in a 'box' – periodic boundary condition at $z = \pm L/2$

$$\psi(|z| > R) \sim \cos(p|z| + \delta(p))$$

$$\begin{aligned} \psi(L/2) &= \psi(-L/2) \\ \frac{d\psi}{dz}(L/2) &= \frac{d\psi}{dz}(-L/2) \end{aligned}$$

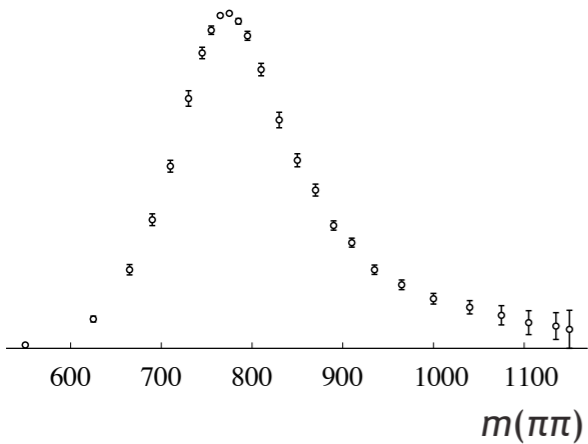
momentum quantization condition

$$p = \frac{2\pi}{L}n - \frac{2}{L}\delta(p)$$



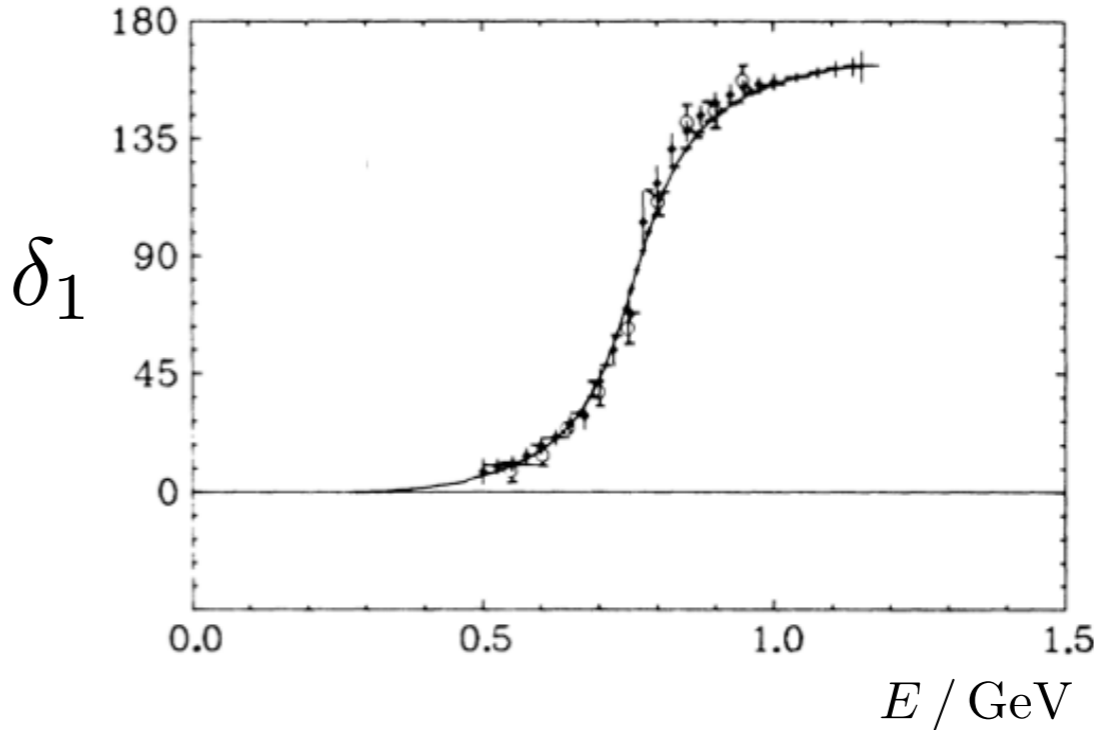
reversing the logic:

if you can compute the **discrete finite-volume spectrum** in a quantum theory, you can find the **scattering amplitude**



canonical resonance ‘bump’
described by a rapidly rising phase-shift

scattering phase-shift

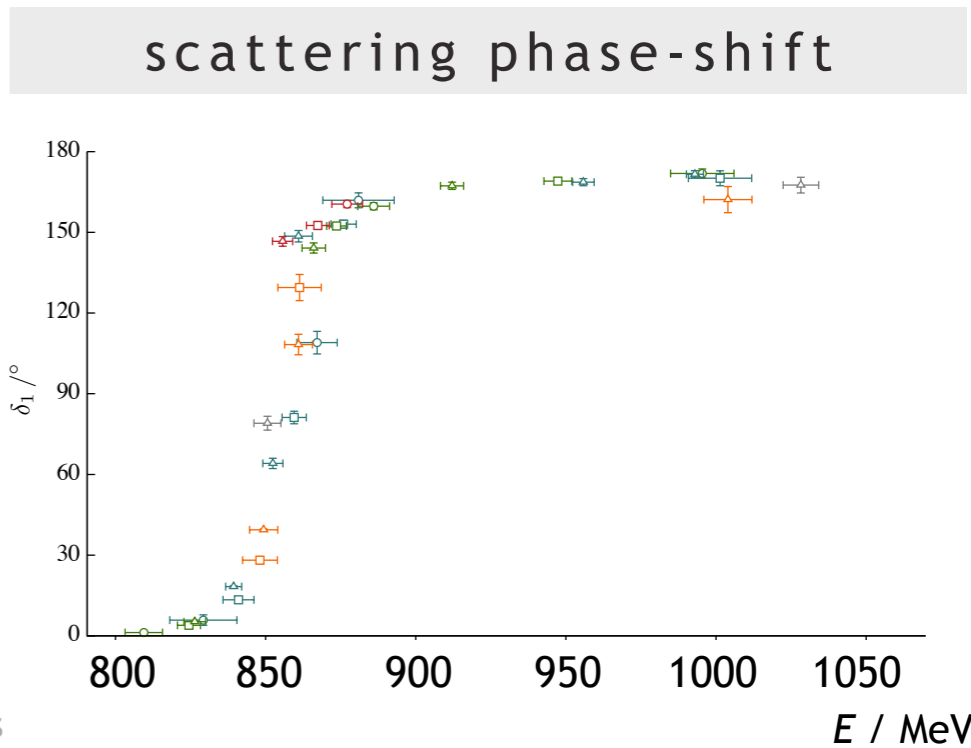
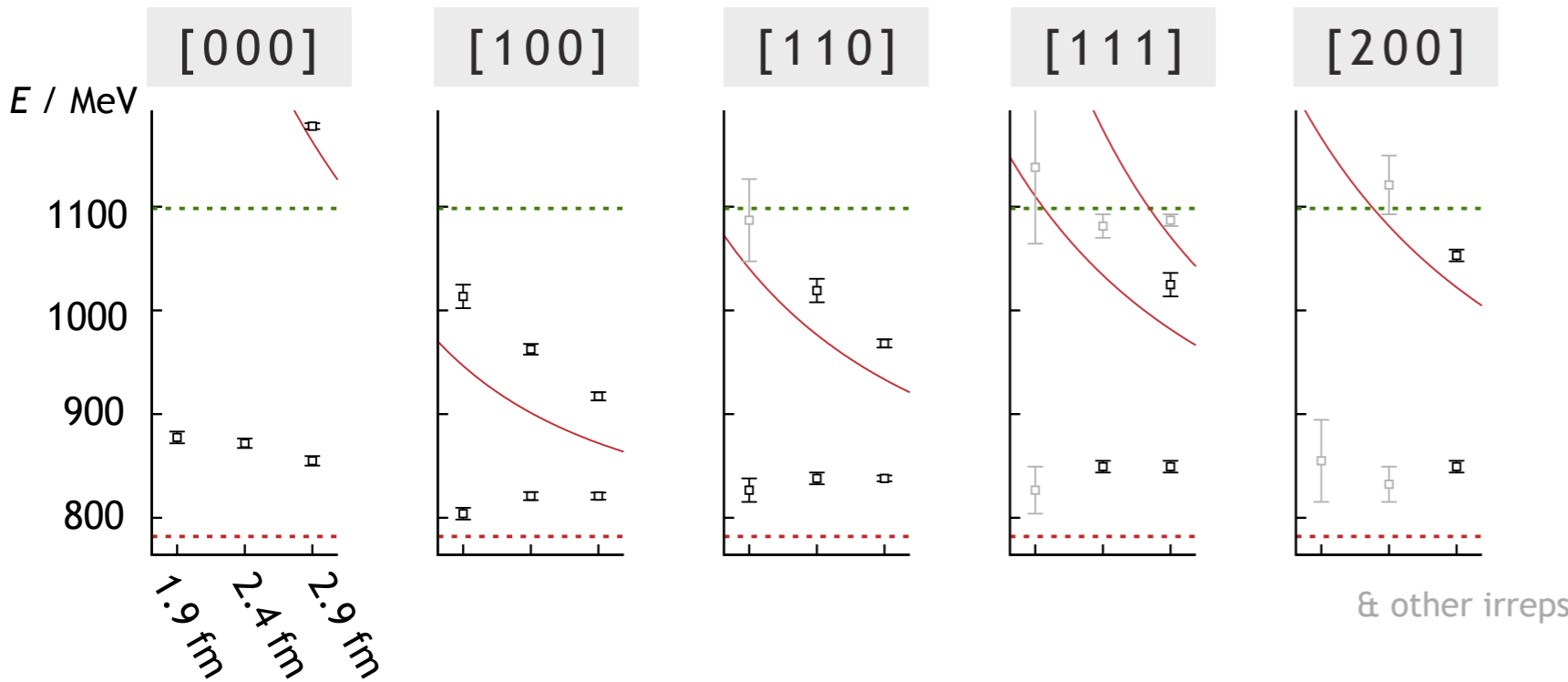


PHYSICAL REVIEW D VOLUME 7, NUMBER 5 1 MARCH 1973

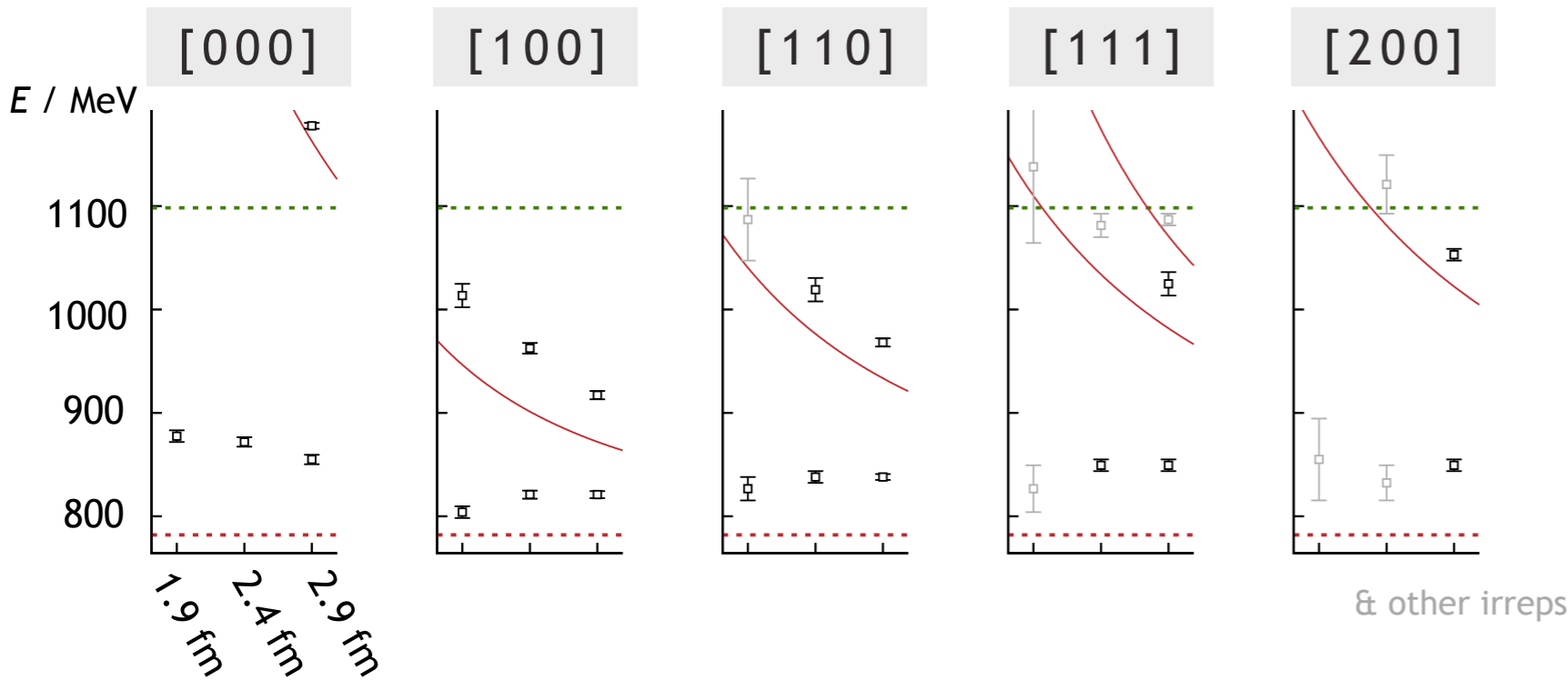
$\pi\pi$ Partial-Wave Analysis from Reactions $\pi^+p \rightarrow \pi^+\pi^-\Delta^{++}$ and $\pi^+p \rightarrow K^+K^-\Delta^{++}$ at 7.1 GeV/c[†]

S. D. Protopopescu,* M. Alston-Garnjost, A. Barbaro-Galtieri, S. M. Flatté,†
J. H. Friedman,§ T. A. Lasinski, G. R. Lynch, M. S. Rabin, || and F. T. Solmitz
Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720
(Received 25 September 1972)

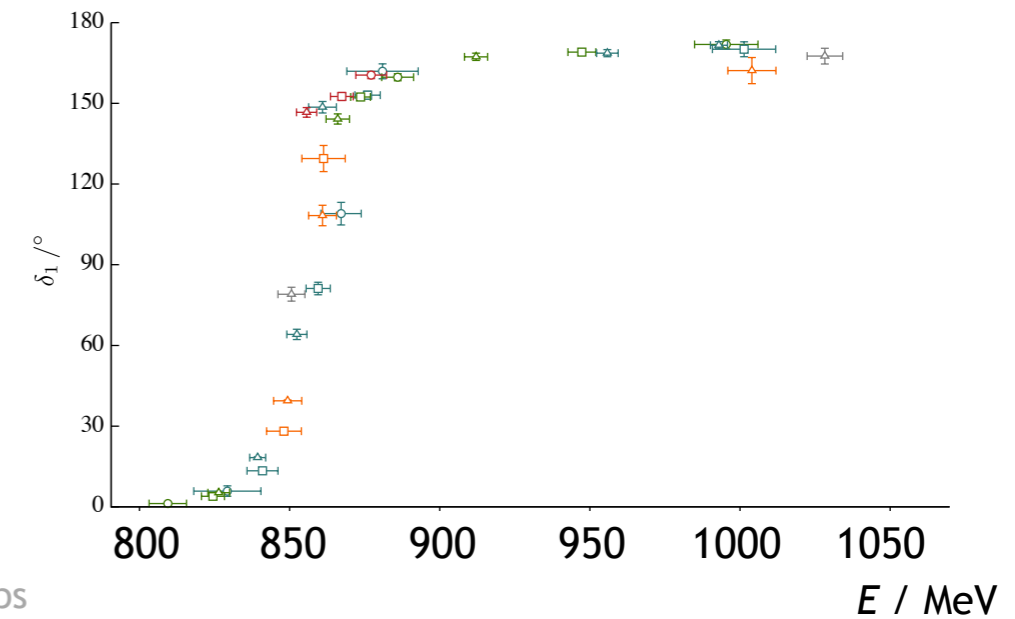
PRD87 034505 (2013) $m_\pi \sim 391$ MeV



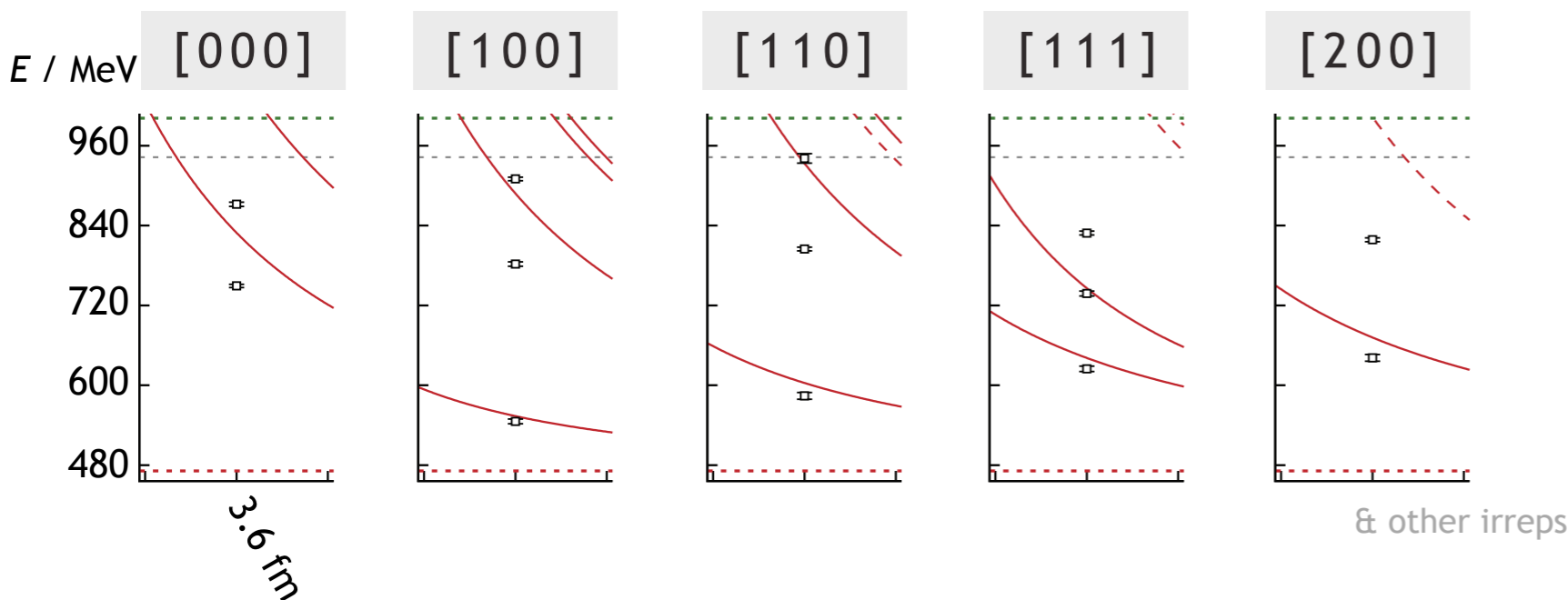
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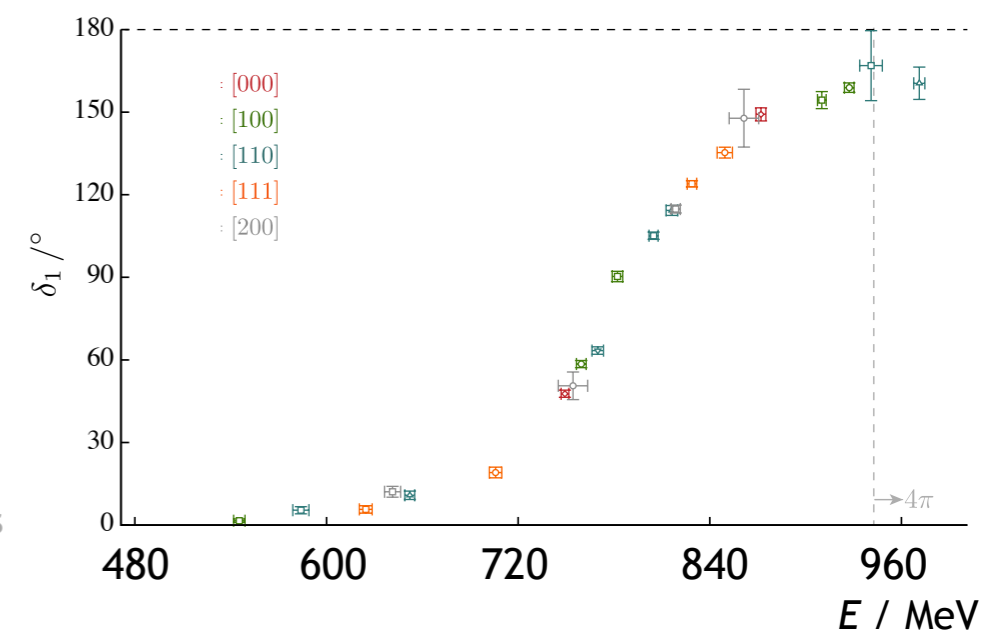
scattering phase-shift

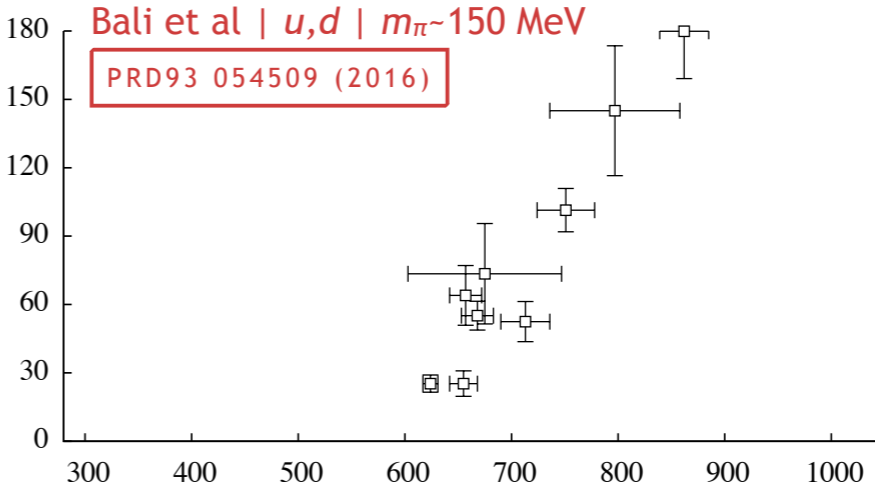
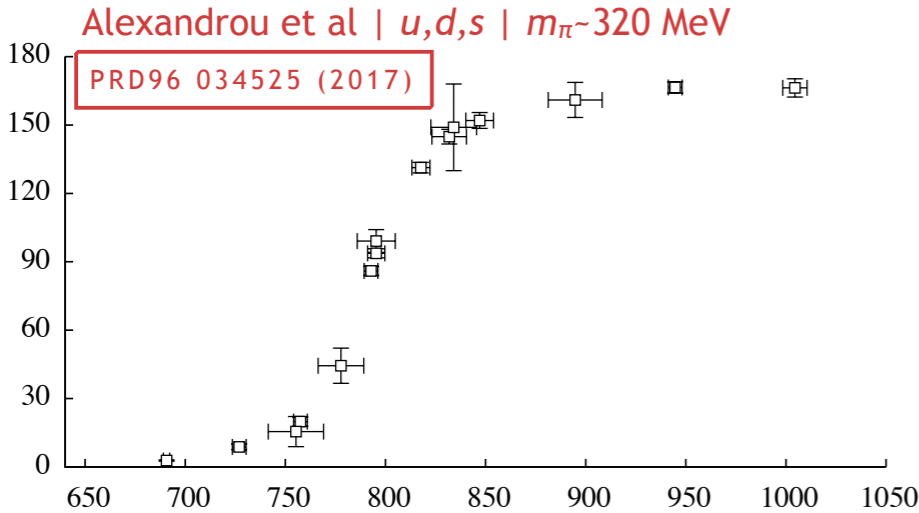
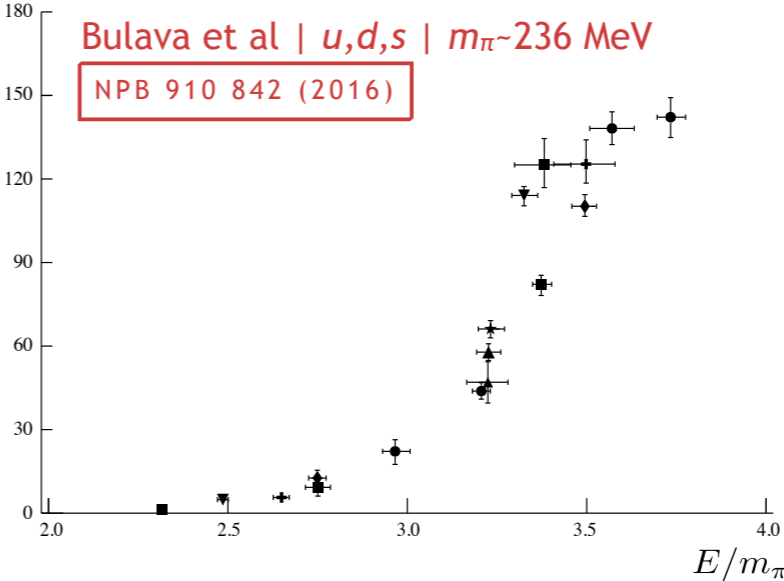
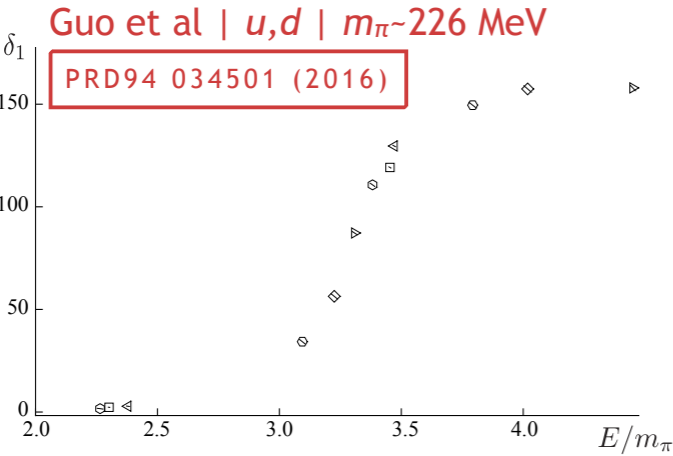
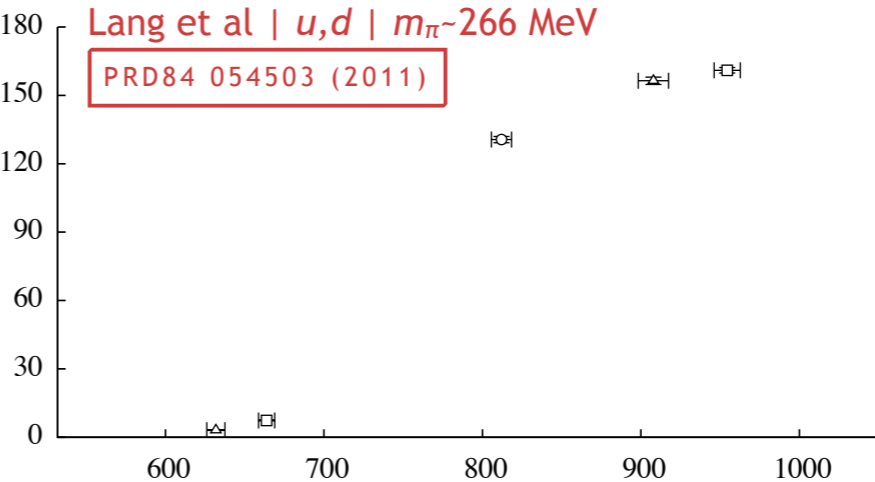
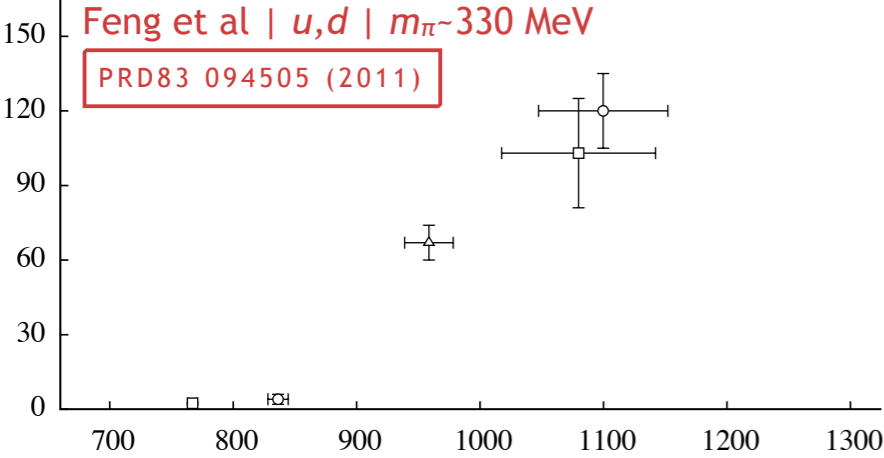


PRD92 094502 (2015)

 $m_\pi \sim 236$ MeV


scattering phase-shift





most resonances decay into more than one final state

e.g. two-channel scattering described by a \mathbf{t} -matrix

$$\mathbf{t}(E) = \begin{pmatrix} t_{11}(E) & t_{12}(E) \\ t_{21}(E) & t_{22}(E) \end{pmatrix}$$

finite-volume spectrum as a function of scattering becomes more complicated

coupled-channel spectrum

solutions $E_n(L)$ of

$$\det \left[\mathbf{t}^{-1}(E) - \widetilde{\mathcal{M}}(E, L) \right] = 0$$

matrix of
known kinematic
functions

no longer a one-to-one mapping from energy to scattering ...

... can parameterize the energy dependence of the scattering \mathbf{t} -matrix

first lattice QCD calculations of **coupled meson-meson scattering** have appeared in the last four years ...

a narrow resonance seen in the $\pi\eta$ final state

$a_0(980)$ [1]

$I^G(J^{PC}) = 1^-(0^{++})$

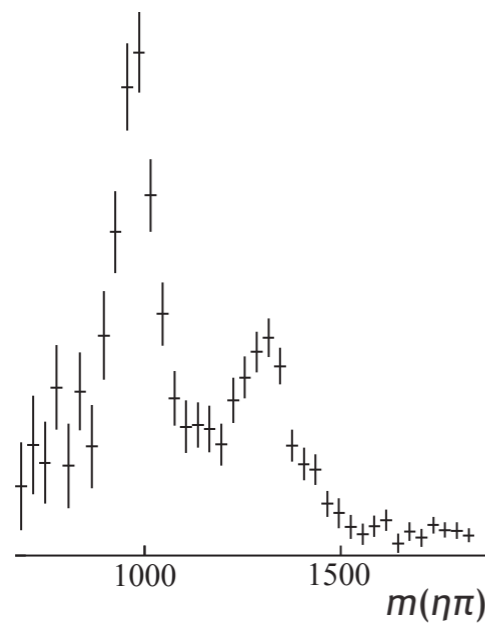
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Full width $\Gamma = 50$ to 100 MeV

e.g.

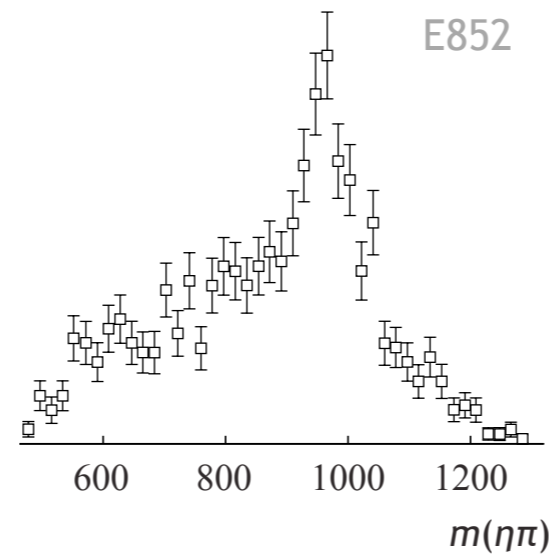
$p p \rightarrow p \eta \pi^0 p$

WA102



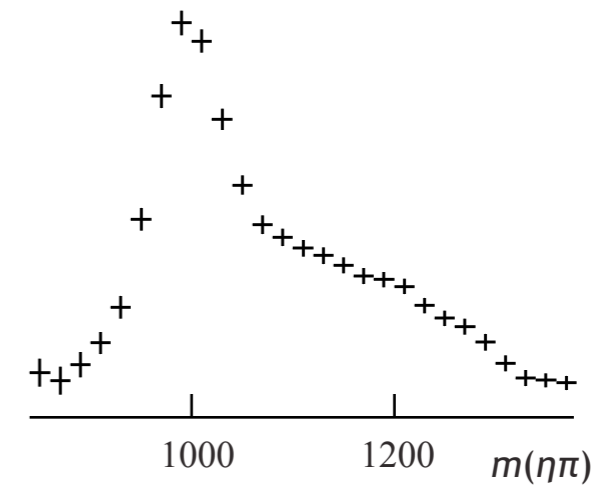
$\pi^- p \rightarrow \eta \pi^+ \pi^- n$

E852



$\gamma\gamma \rightarrow \eta \pi^0$

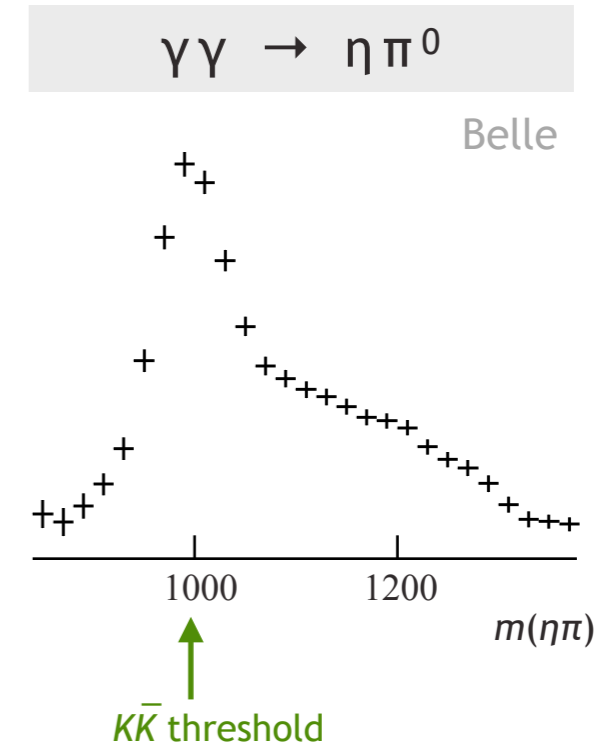
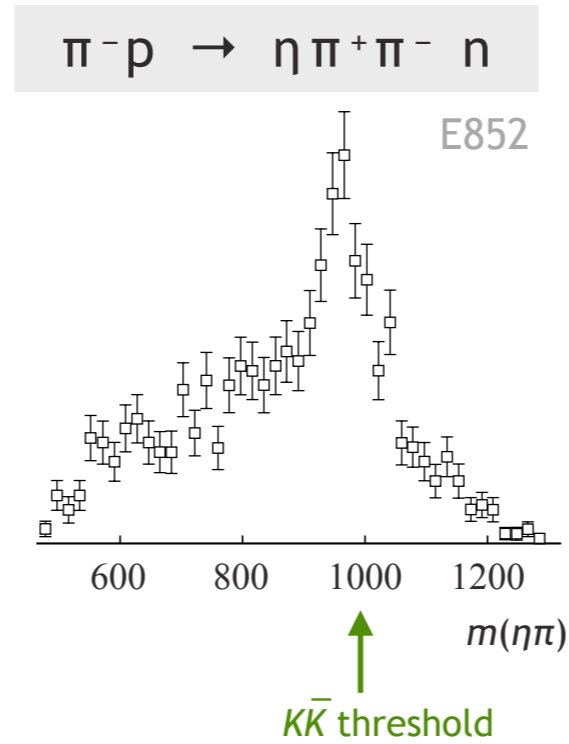
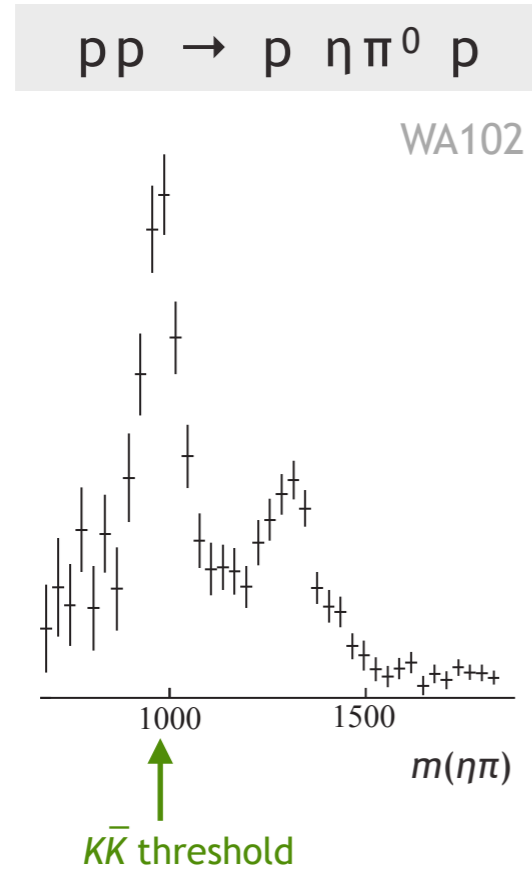
Belle



a narrow resonance seen in the $\pi\eta$ final state

right at the $K\bar{K}$ threshold

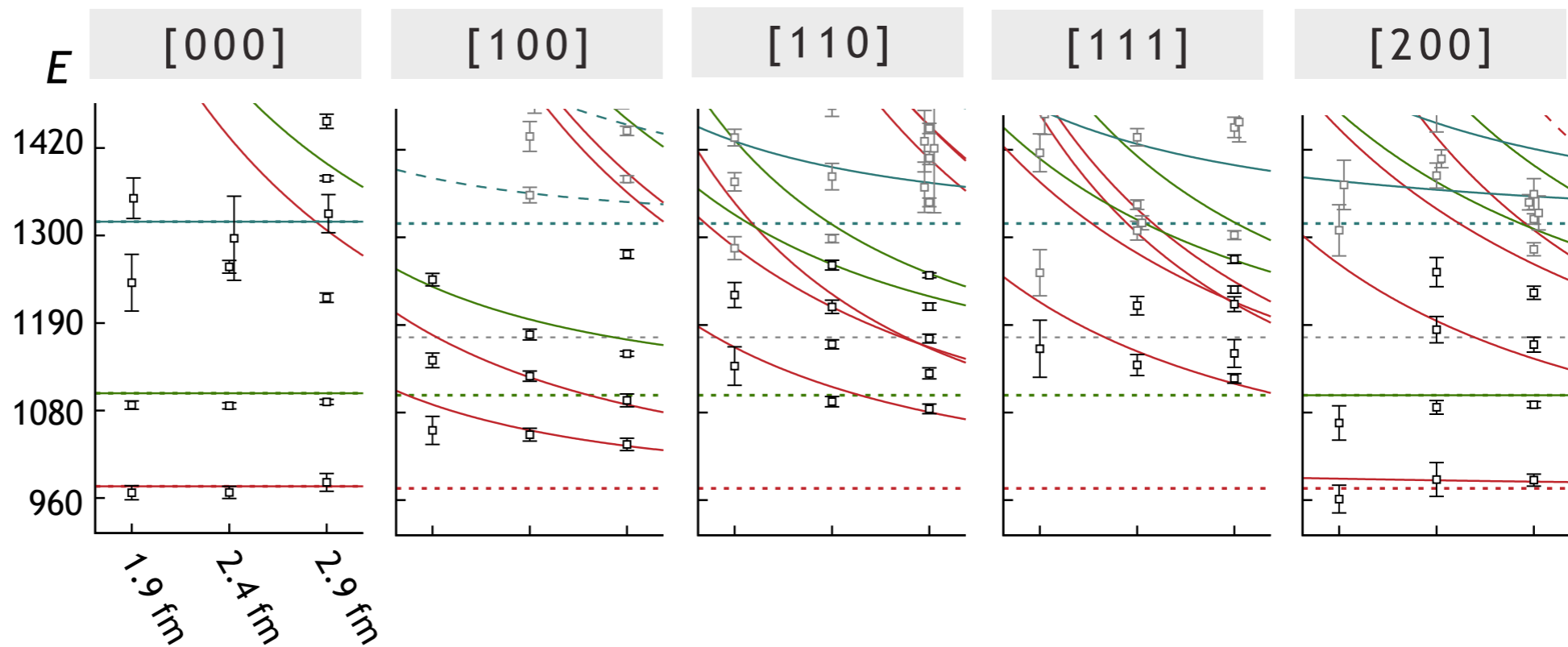
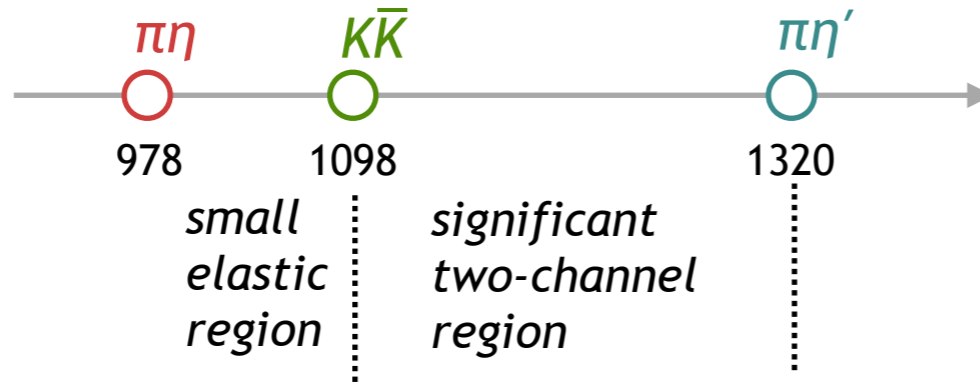
e.g.



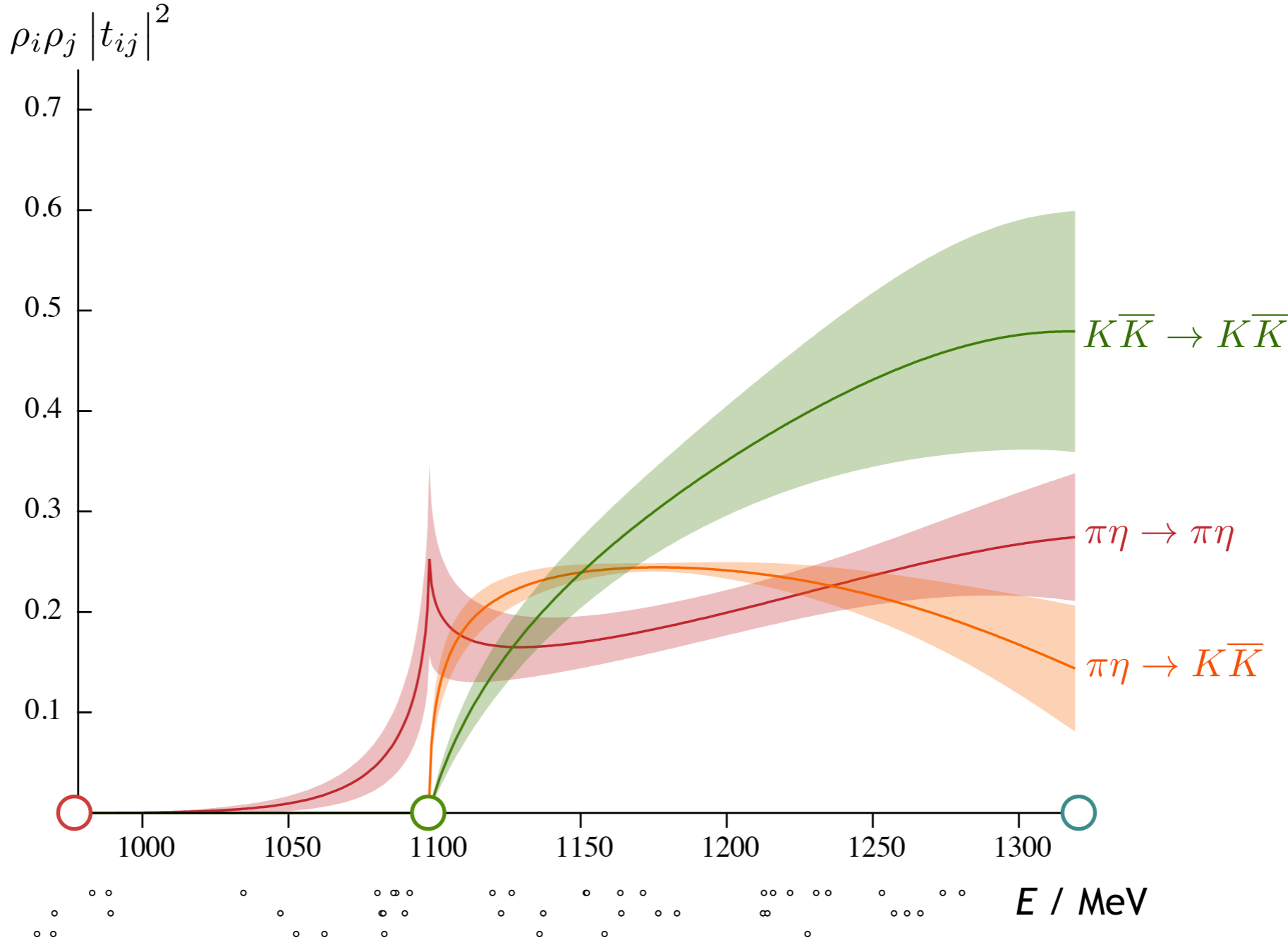
will need a coupled $\pi\eta$, $K\bar{K}$ approach ...

first calculation – unphysically heavy u,d quarks

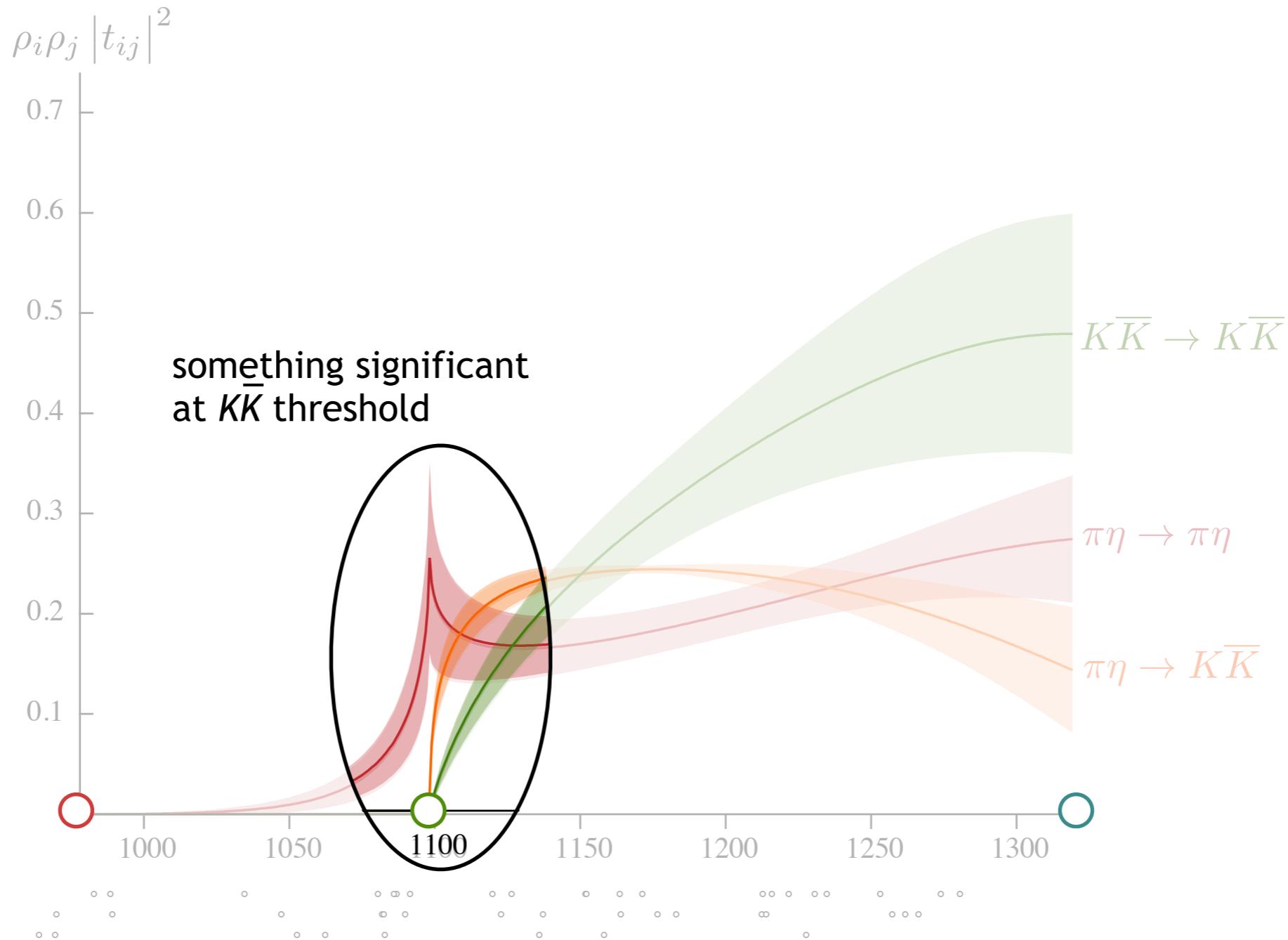
$m_\pi \sim 391$ MeV
 $m_K \sim 549$ MeV
 $m_\eta \sim 587$ MeV



$\pi\eta/K\bar{K}$ coupled-channel scattering amplitudes



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is there a resonance causing this ?

a **resonance** can be rigorously defined to be a **pole singularity** at a **complex energy**

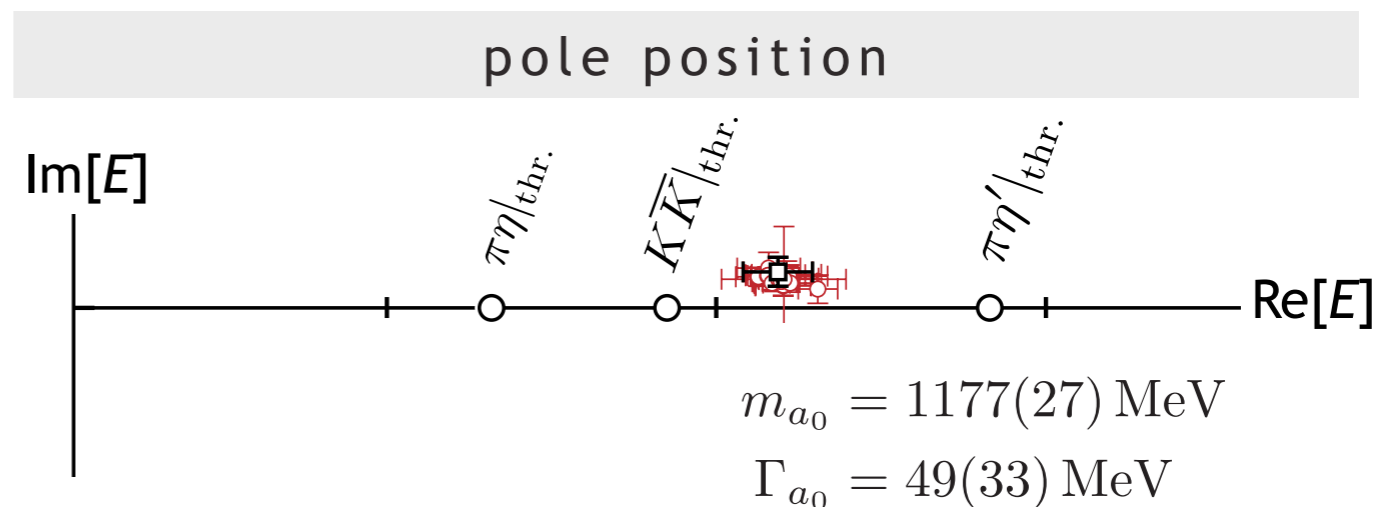
$$t_{ij}(E) \sim \frac{c_i c_j}{E_0^2 - E^2}$$

with pole position $E_0 = m_R \pm i\frac{1}{2}\Gamma_R$

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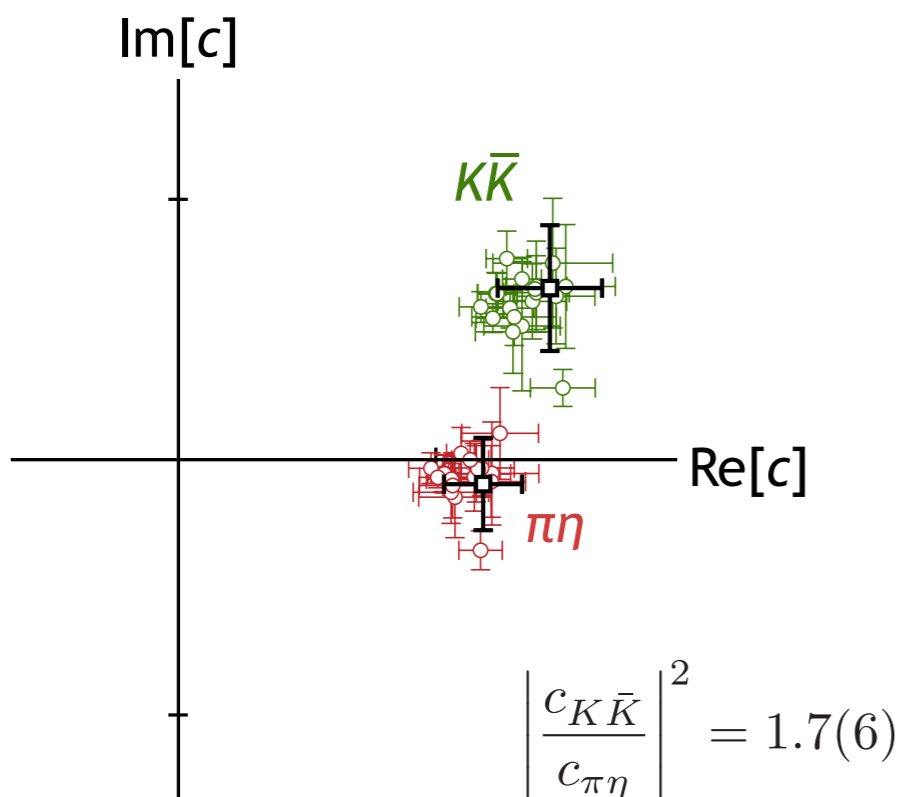


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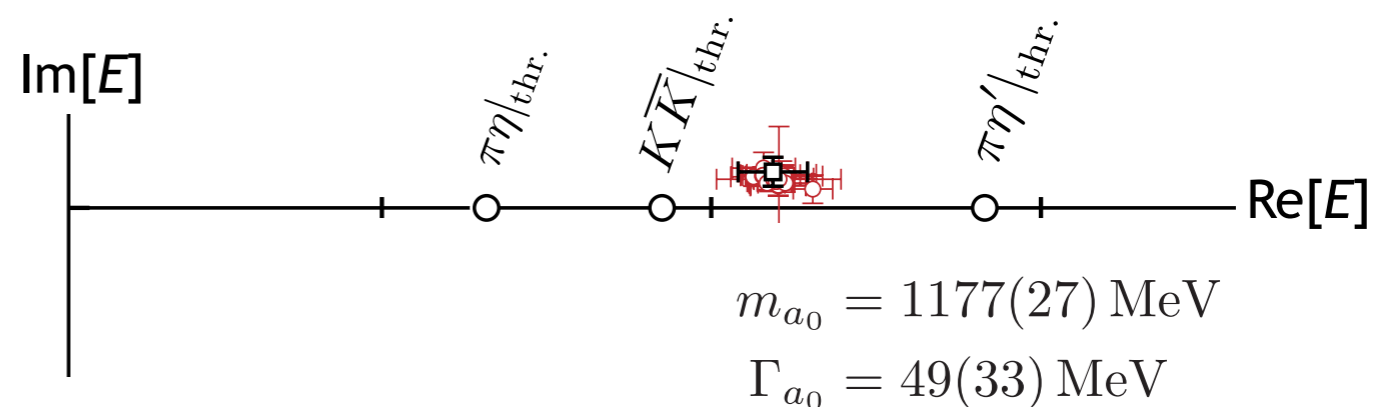
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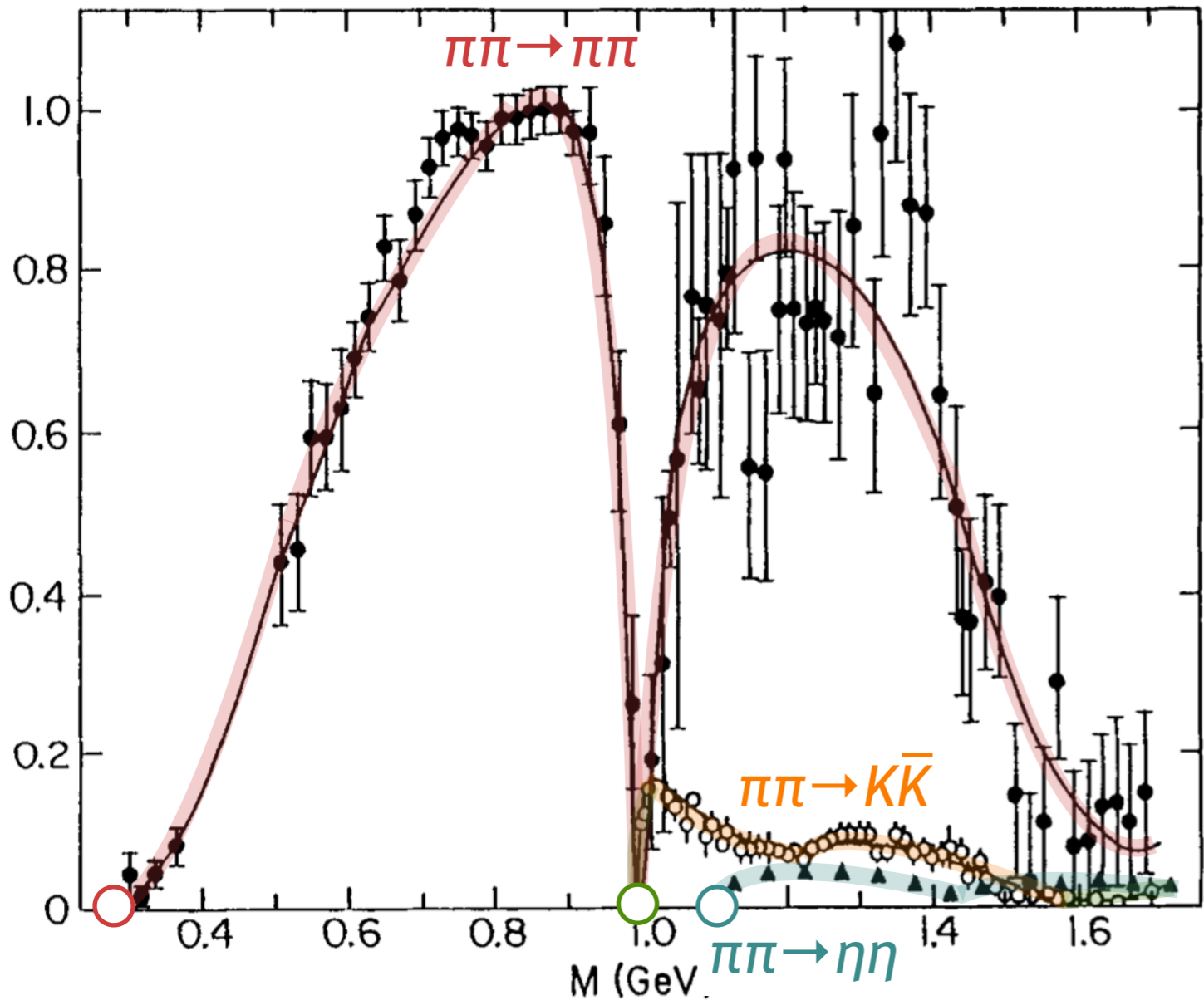
resonance couplings



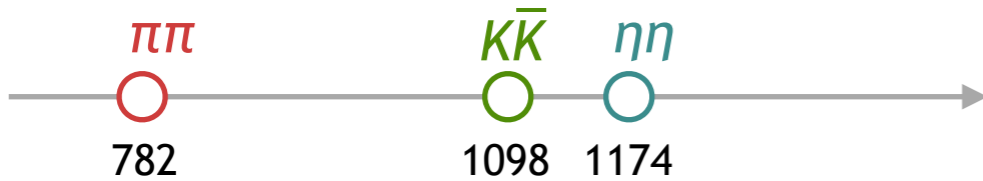
pole position



experimental amplitudes



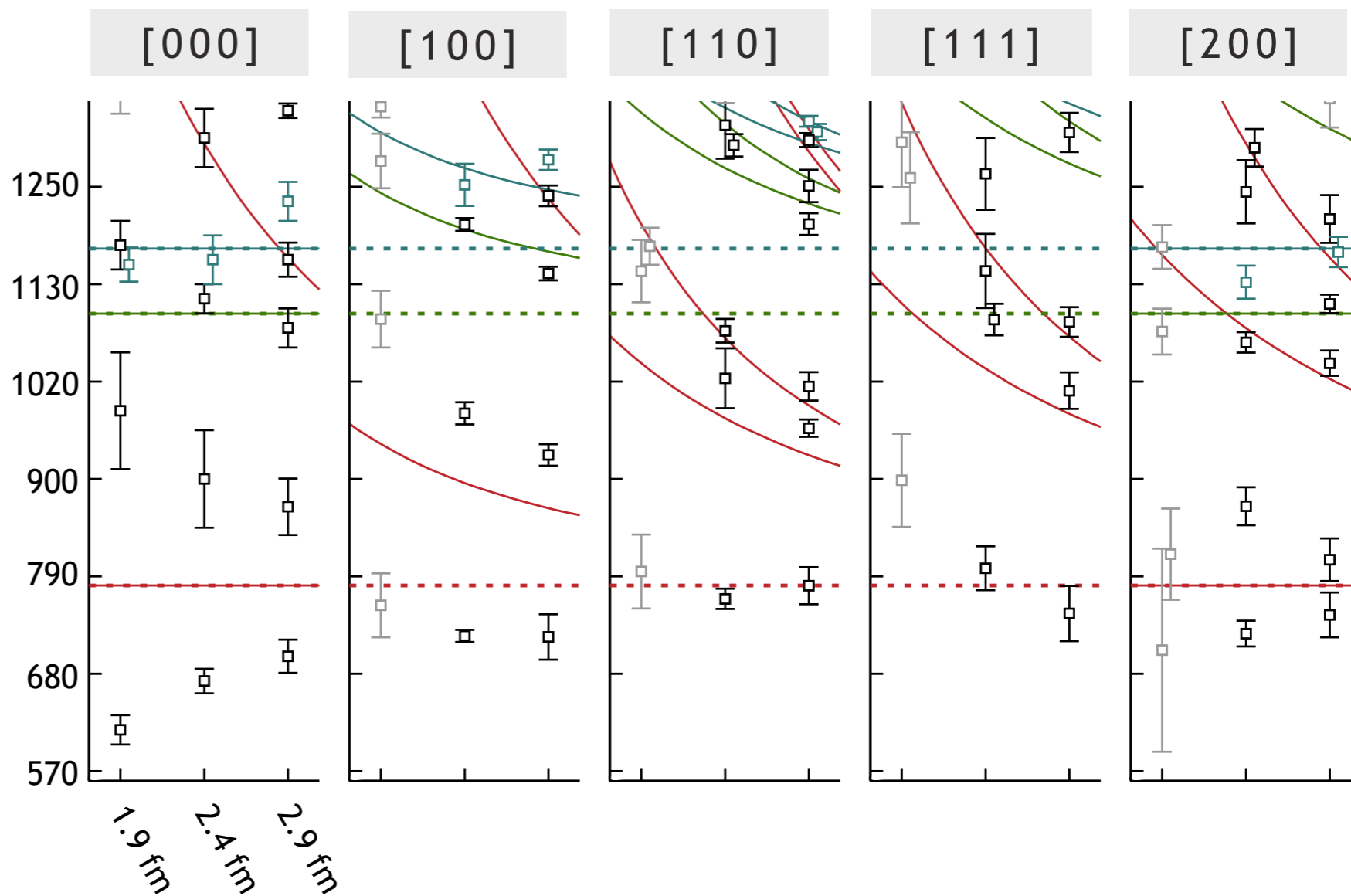
combination of broad σ resonance and narrow $f_0(980)$ at $K\bar{K}$ threshold



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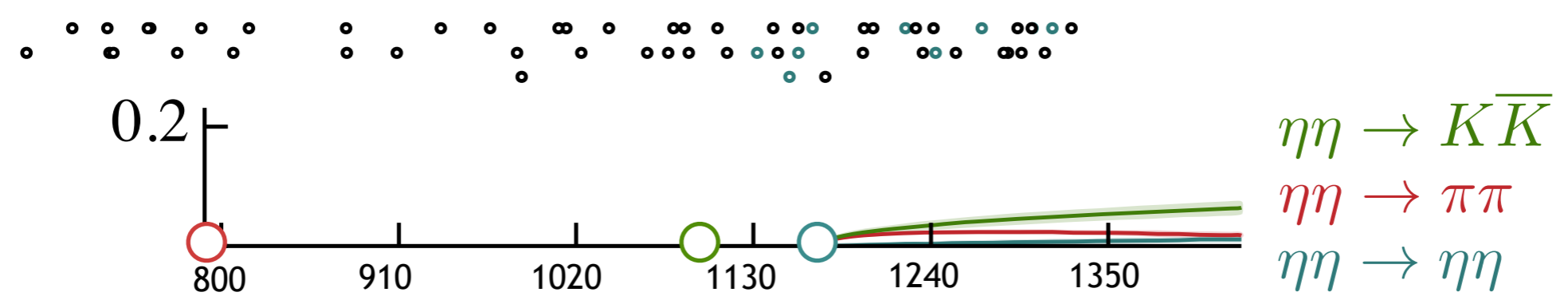
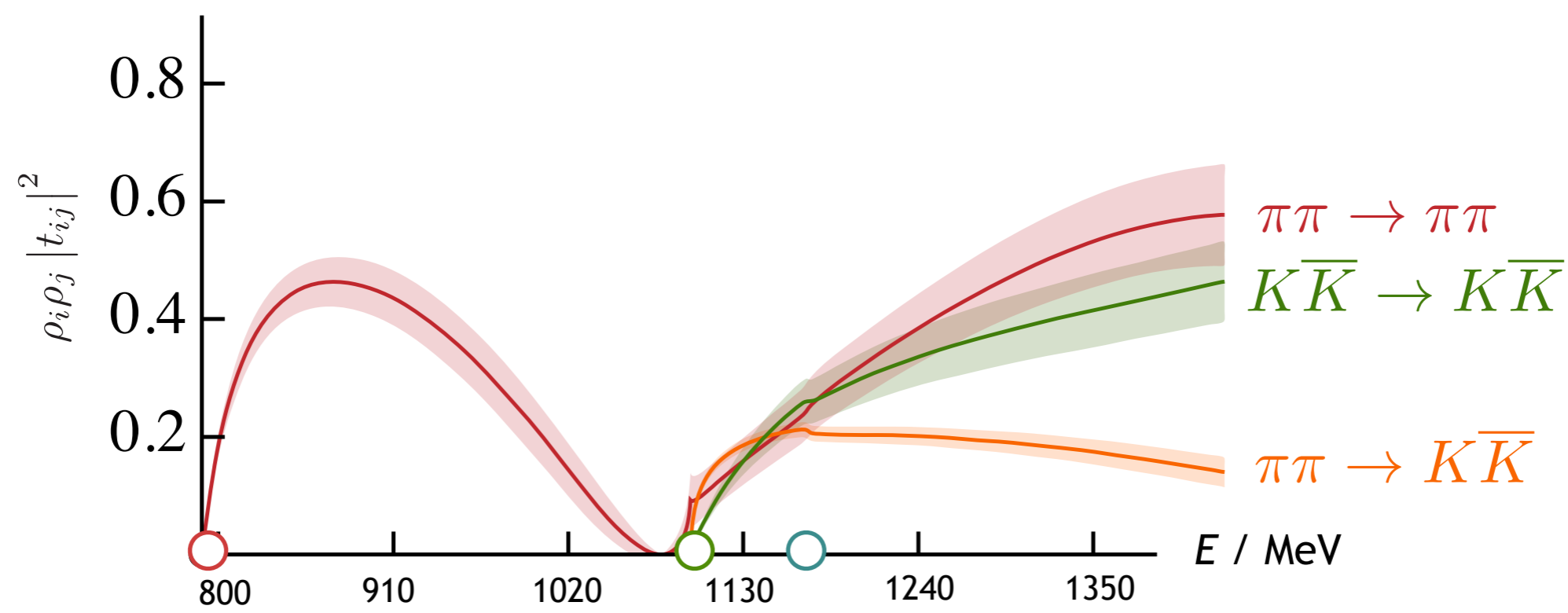
considerable elastic region

three-channel almost immediate



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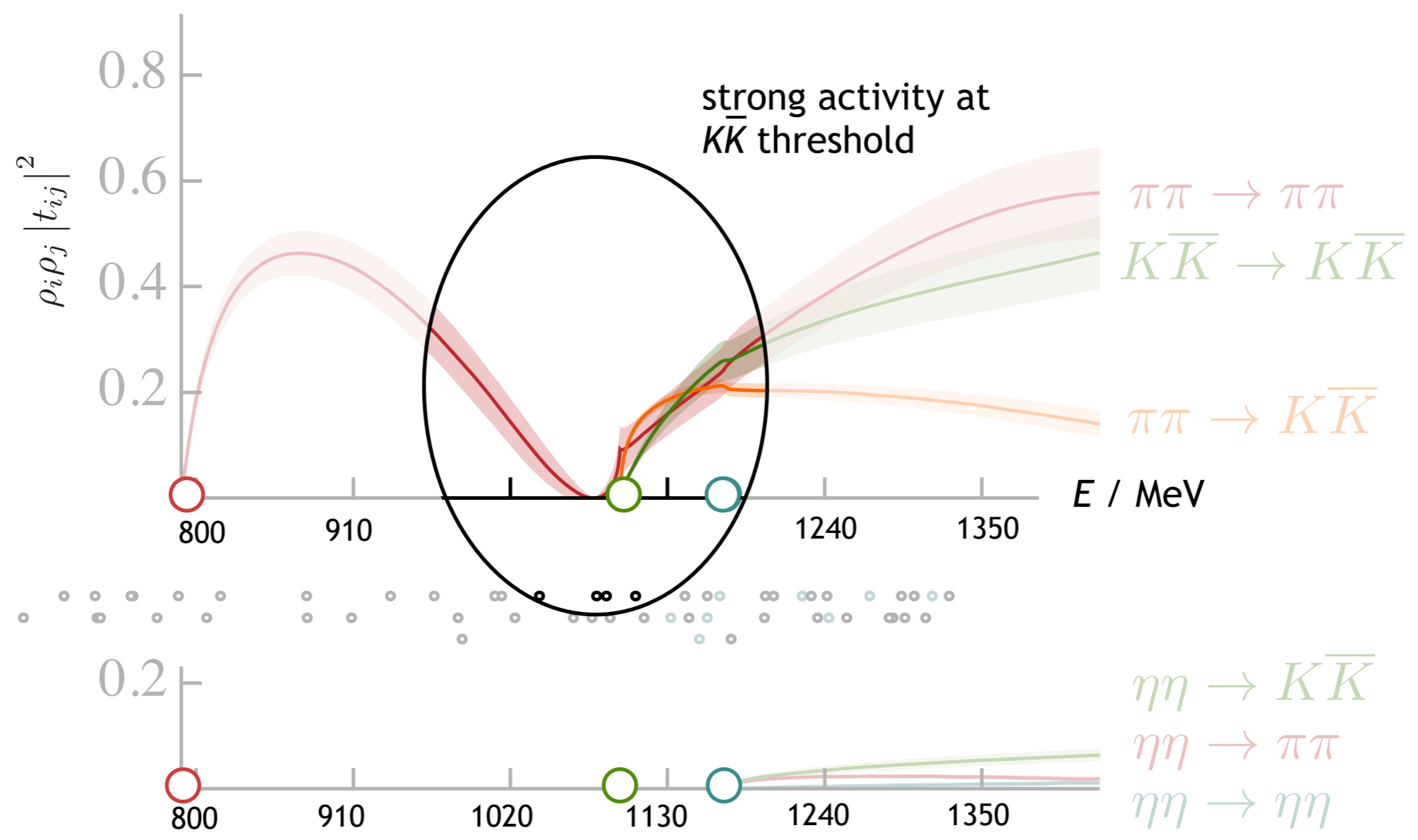
$\pi\pi / K\bar{K} / \eta\eta$ coupled-channel scattering amplitudes



nothing much happening in $\eta\eta$

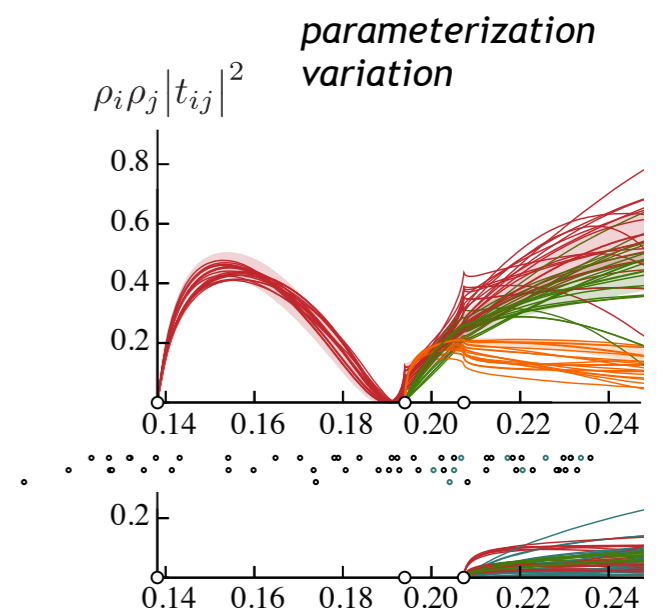
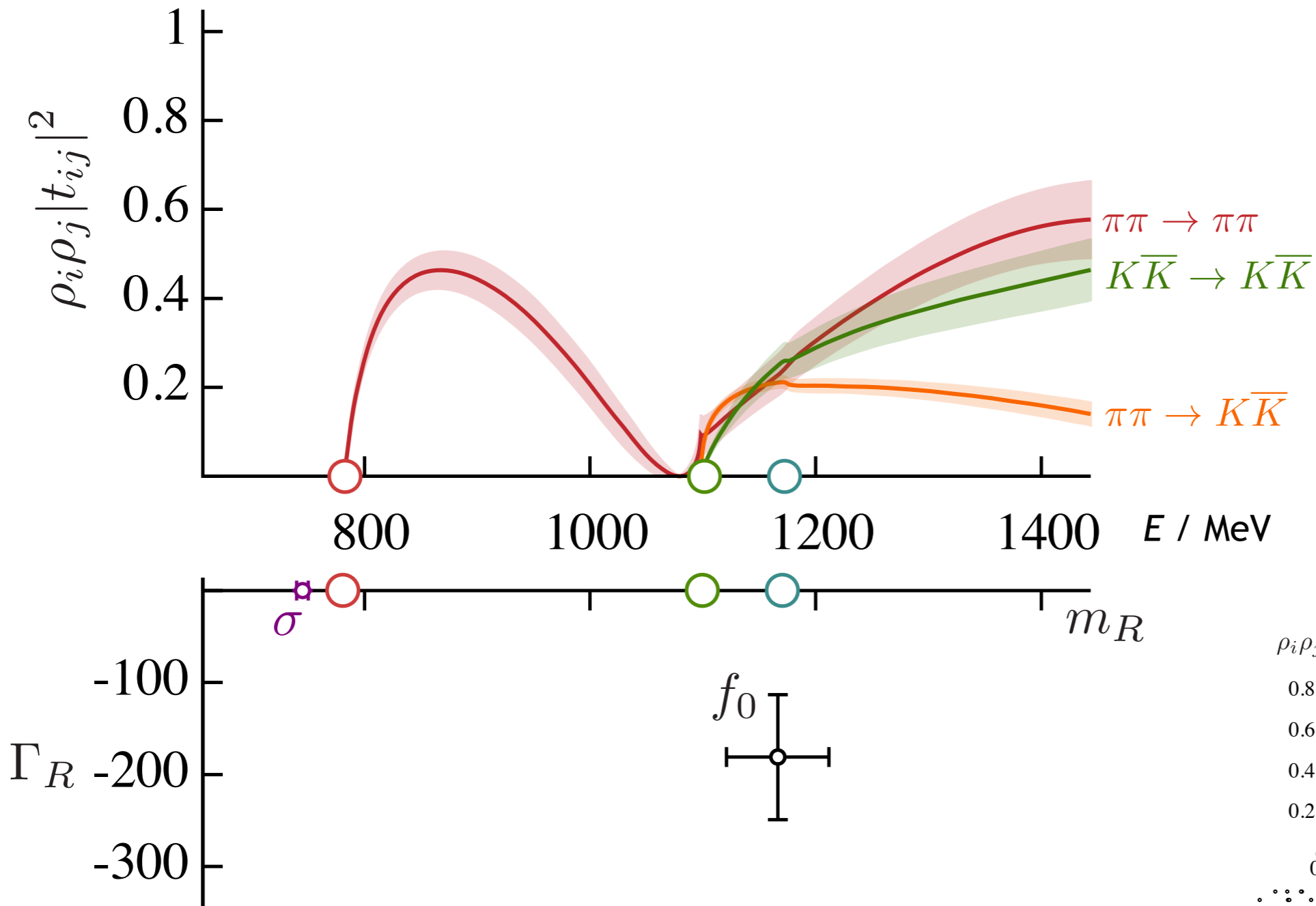
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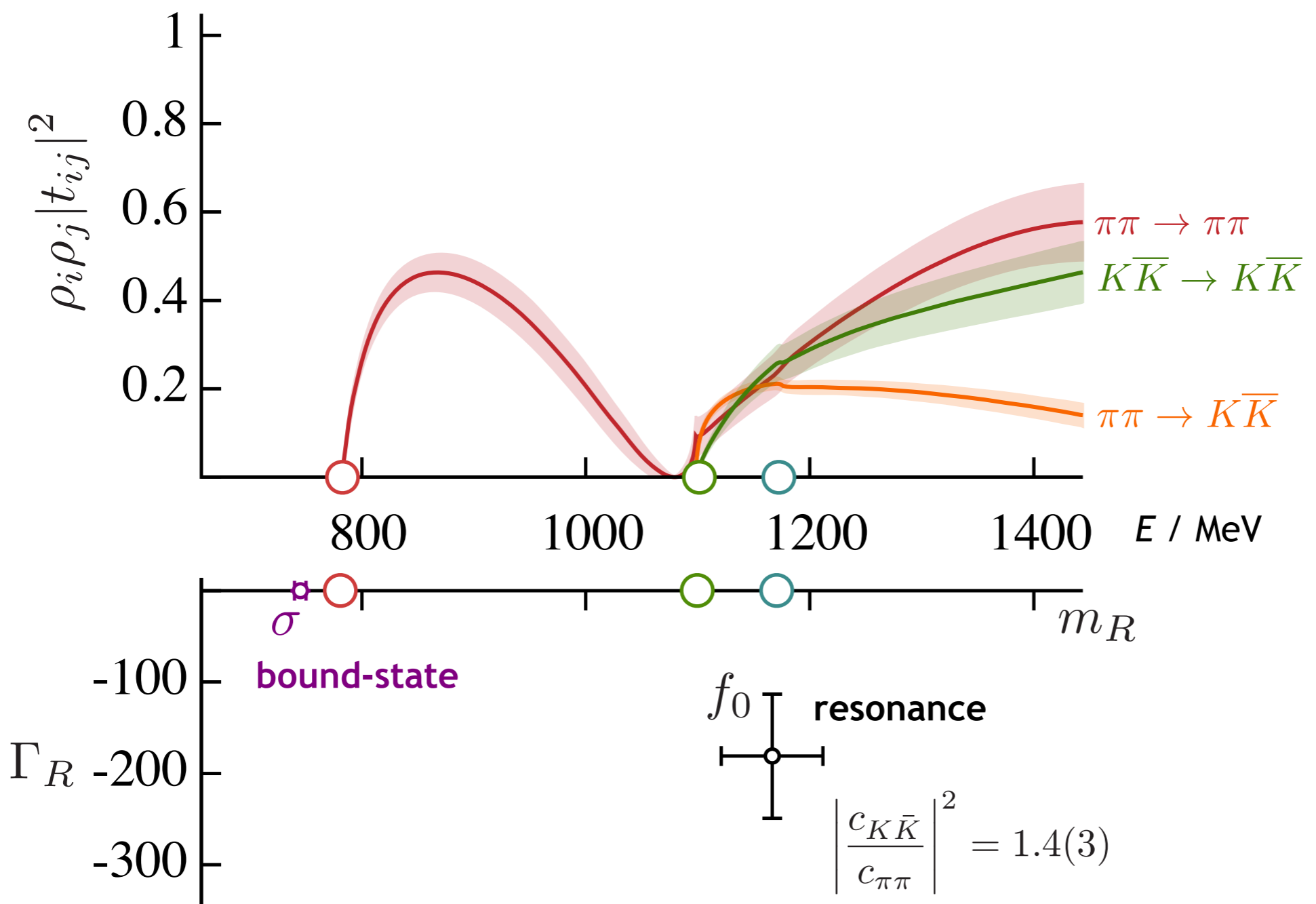
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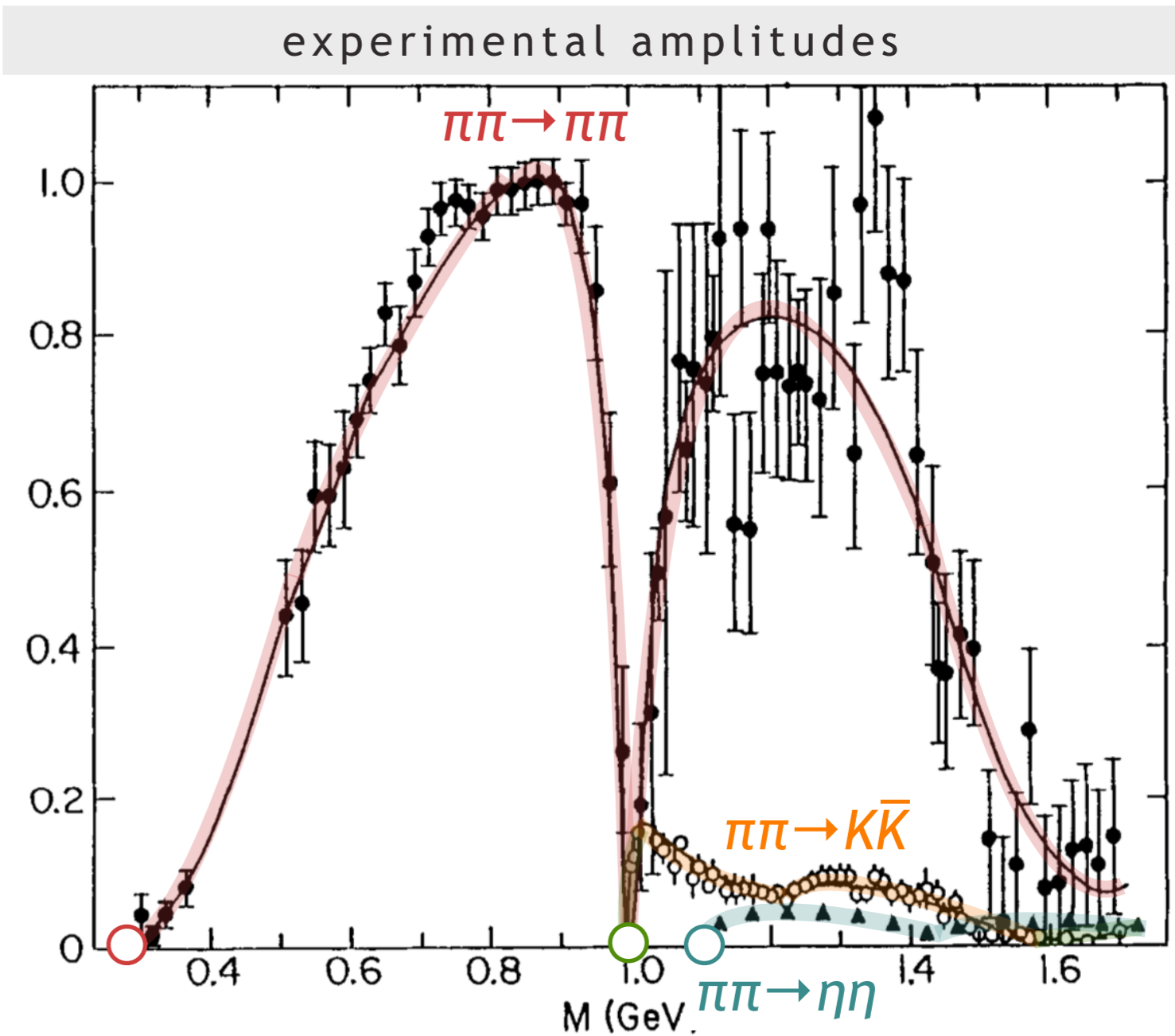
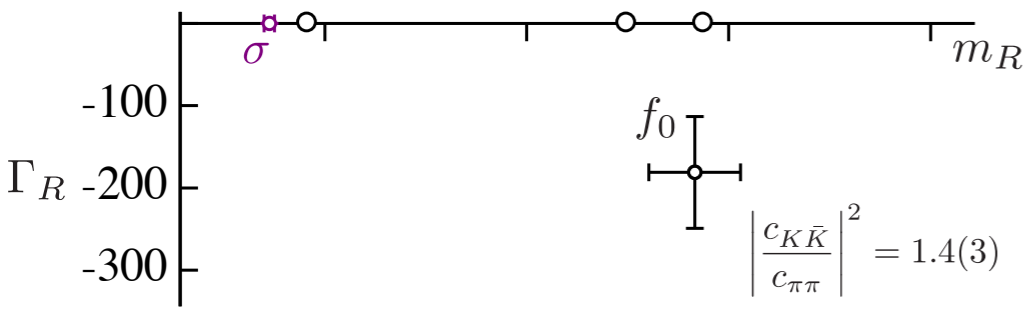
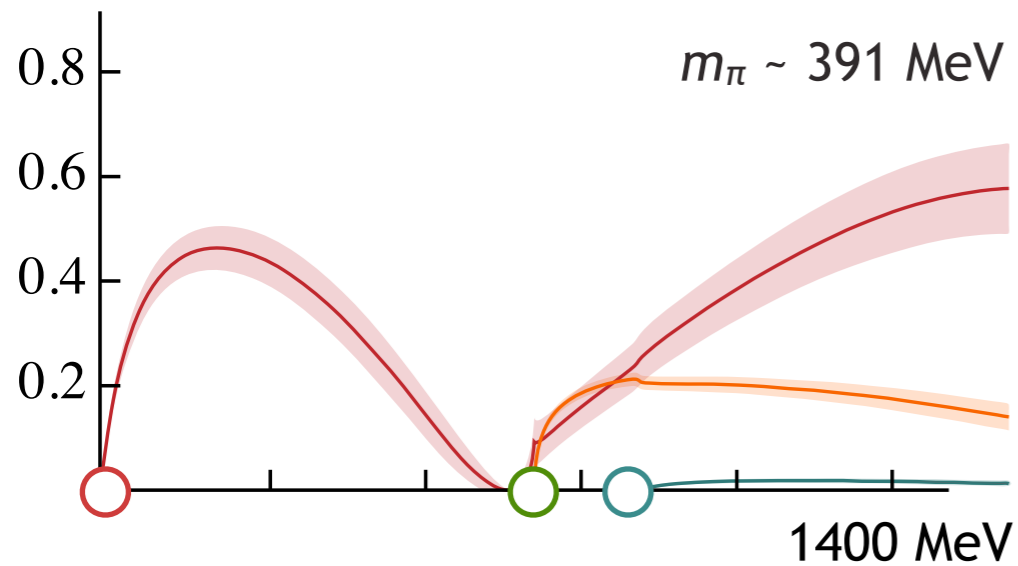
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Full width $\Gamma = 10$ to 100 MeV

have demonstrated presence of coupled-channel resonances in (lattice) QCD at unphysical quark masses initially

can determine pole positions (mass, width) and couplings to decay channels

would like to know if there're simple ways to 'understand' them

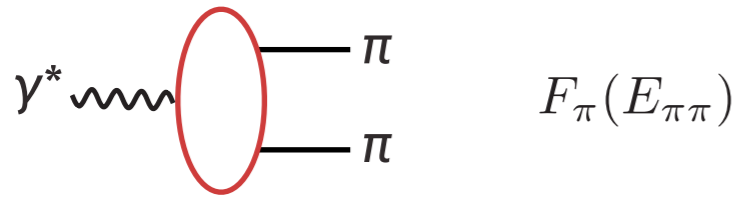
e.g. big differences between **scalar**, **vector**, **tensor** mesons

long-standing ideas of $q\bar{q}$ versus $qq\bar{q}\bar{q}$ versus **meson-meson molecules**

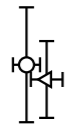
one possible approach to this

— consider their couplings to external currents ...

$\gamma^* \rightarrow \pi\pi$

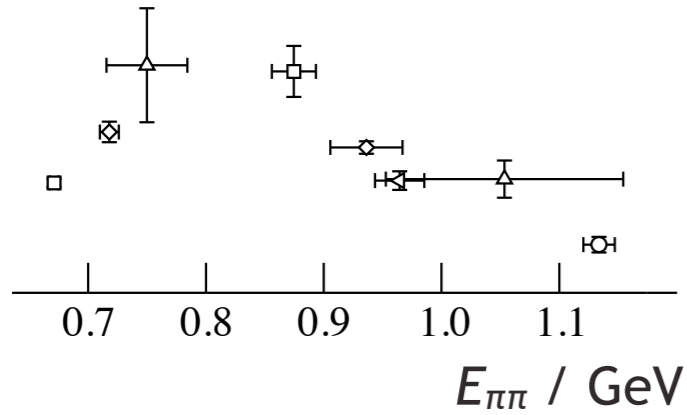


$$F_\pi(E_{\pi\pi})$$



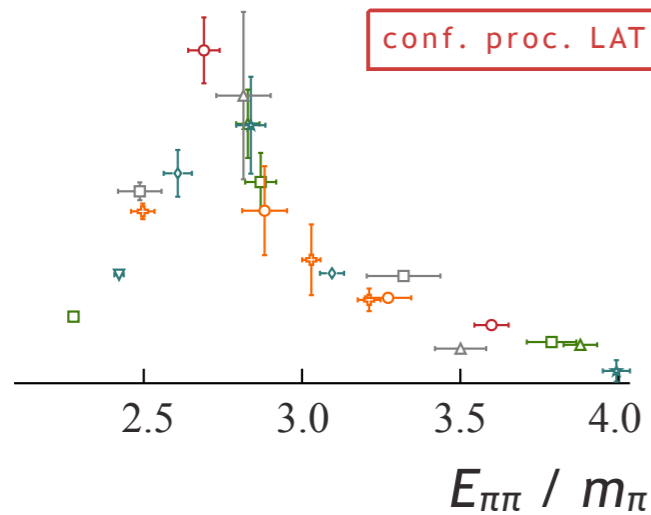
Feng et al | *u,d,s* | $m_\pi \sim 290$ MeV

PRD97 054513 (2018)

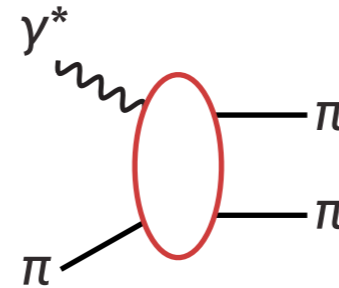


Bulava et al | *u,d,s* | $m_\pi \sim 280$ MeV

conf. proc. LAT '15



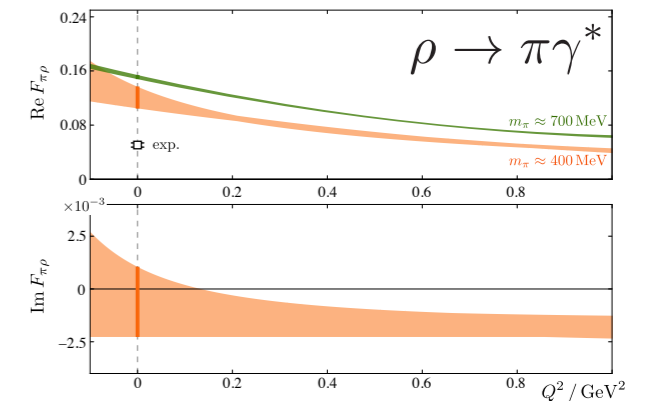
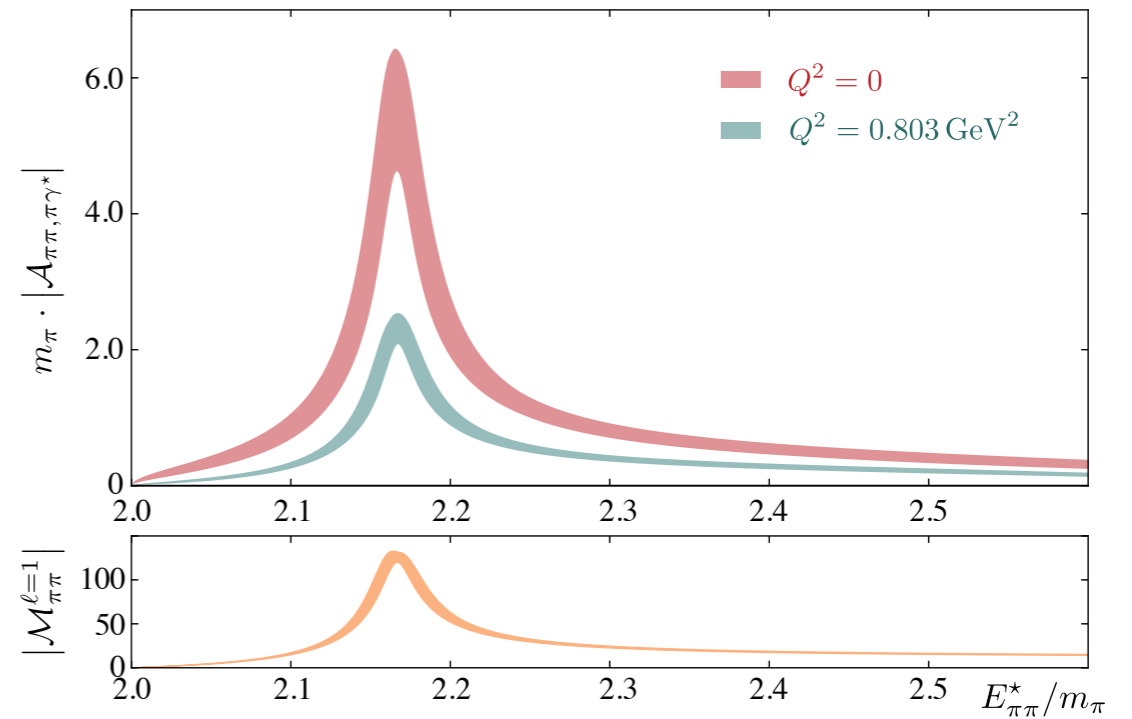
$\gamma^* \pi \rightarrow \pi\pi$



$$\mathcal{A}(E_{\pi\pi}, Q^2)$$

Briceno et al | *u,d,s* | $m_\pi \sim 391$ MeV

PRL115 242001 (2015)



addressing the new observations in charmonium (XYZ)

- challenging, often lie above several thresholds \Rightarrow multiple coupled channels

predicting resonant properties of hybrid hadrons

- preferred decay modes, couplings to photons (relevant to GlueX, see next talk)

(finally) understanding the scalar mesons ?

- studying their behaviour with changing quark mass, evaluating their form-factors ...

-
-
-

a big current challenge is the importance of **three-body final states**

- lack of a complete finite-volume formalism so far

recent pedagogic review

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Scattering processes and resonances from lattice QCDRaúl A. Briceño^{*}

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The vast majority of hadrons observed in nature are not stable under the strong interaction; rather they are resonances whose existence is deduced from enhancements in the energy dependence of scattering amplitudes. The study of hadron resonances offers a window into the workings of quantum chromodynamics (QCD) in the low-energy nonperturbative region, and in addition many probes of the limits of the electroweak sector of the standard model consider processes which feature hadron resonances. From a theoretical standpoint, this is a challenging field: the same dynamics that binds quarks and gluons into hadron resonances also controls their decay into lighter hadrons, so a complete approach to QCD is required. Presently, lattice QCD is the only available tool that provides the required nonperturbative evaluation of hadron observables. This article reviews progress in the study of few-hadron reactions in which resonances and bound states appear using lattice QCD techniques. The leading approach is described that takes advantage of the periodic finite spatial volume used in lattice QCD calculations to extract scattering amplitudes from the discrete spectrum of QCD eigenstates in a box. An explanation is given of how from explicit lattice QCD calculations one can rigorously garner information about a variety of resonance properties, including their masses, widths, decay couplings, and form factors. The challenges which currently limit the field are discussed along with the steps being taken to resolve them.

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