

Hadronic parity violation and the large- N_c expansion

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Hadronic parity violation

- Manifestation of PV quark interactions at hadronic level
- Interplay of weak and nonperturbative strong interactions
- Weak interactions short-ranged (~ 0.002 fm)
 - Sensitive to quark-quark correlations inside nucleon
 - No need to go to high energy
 - “Inside-out probe”
- Relative strength for NN case: $\sim G_F m_\pi^2 \approx 10^{-7}$
- Isolate through pseudoscalar observables ($\vec{p} \cdot \vec{\sigma}$)

Parity-violating NN interactions

- Meson-exchange model
 - Single-meson exchange (π, ρ, ω, \dots)
 - One PV, one PC meson-nucleon coupling
 - Most common framework: DDH approach
- Effective field theory
 - Pionless: 5 PV low-energy couplings at LO
 - Chiral:
 - 1 PV πN coupling at LO
 - 5 PV low-energy couplings at NLO
 - Couplings only very weakly constrained experimentally
 - Additional theoretical constraints?

Large- N_c QCD

QCD in limit $N_c \rightarrow \infty$

- Taken with $g^2 N_c$ fixed
- Simplifications
 - Color-singlet physical states
 - Mesons, glueballs: Weakly interacting $\sim 1/\sqrt{N_c}$
- Systematic expansion in $1/N_c$
- Seems to work well phenomenologically
- Baryons
 - Baryon mass $M \sim N_c$
 - SU(4) spin-flavor symmetry: $u \uparrow, u \downarrow, d \uparrow, d \downarrow$

NN potential in large- N_c expansion

$$V(\vec{p}_-, \vec{p}_+) = \langle (\vec{p}'_1, C), (\vec{p}'_2, D) | H | (\vec{p}_1, A), (\vec{p}_2, B) \rangle$$

with $\vec{p}_\pm = \vec{p}' \pm \vec{p}$

- Effective Hamiltonian

$$H = N_c \sum_{s,t,u} v_{stu} \left(\frac{S}{N_c} \right)^s \left(\frac{I}{N_c} \right)^t \left(\frac{G}{N_c} \right)^u$$

- Building blocks

$$S^i = q^\dagger \frac{\sigma^i}{2} q, \quad I^a = q^\dagger \frac{\tau^a}{2} q, \quad G^{ia} = q^\dagger \frac{\sigma^i \tau^a}{4} q$$

- Coefficients v_{stu}
 - Momentum dependent
 - Constrained by symmetries

Large- N_c scaling

- Nucleon matrix elements

$$\langle N' | S^i | N \rangle \sim \langle N' | I^a | N \rangle \sim 1,$$

$$\langle N' | G^{ia} | N \rangle \sim \langle N' | \mathbb{1} | N \rangle \sim N_c$$

- Momenta (in t-channel)

$$\vec{p}_- \sim 1$$

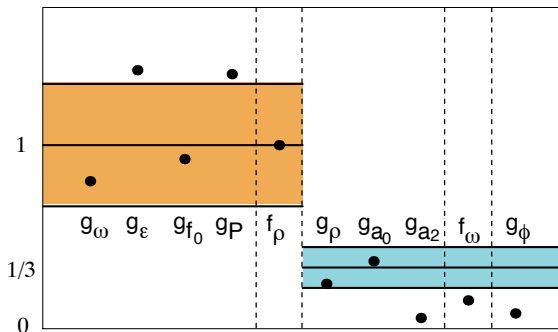
$$\vec{p}_+ \sim 1/M_N \sim 1/N_c$$

- Coefficients excluding momenta

$$\tilde{V}_{stu} \sim 1$$

$1/N_c$ expansion of NN potential

Comparison large- N_c scaling vs Nijmegen potential



PV operators in $1/N_c$ expansion

- Leading order [$\mathcal{O}(N_c)$]

$$\vec{p}_- \cdot (\vec{\sigma}_1 \times \vec{\sigma}_2) \vec{\tau}_1 \cdot \vec{\tau}_2$$

- Leading order' [$\mathcal{O}(N_c) \sin^2 \theta_W$]

$$\vec{p}_- \cdot (\vec{\sigma}_1 \times \vec{\sigma}_2) [\tau_1 \tau_2]_2^{ZZ}$$

- Next-to-leading order [$\mathcal{O}(N_c^0) \sin^2 \theta_W$]

$$\vec{p}_+ \cdot (\vec{\sigma}_1 \tau_1^3 - \vec{\sigma}_2 \tau_2^3)$$

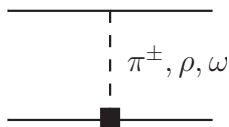
$$\vec{p}_- \cdot (\vec{\sigma}_1 + \vec{\sigma}_2) (\vec{\tau}_1 \times \vec{\tau}_2)^3$$

$$\vec{p}_- \cdot (\vec{\sigma}_1 \times \vec{\sigma}_2) (\vec{\tau}_1 + \vec{\tau}_2)^3$$

$$[(\vec{p}_+ \times \vec{p}_-) \cdot \vec{\sigma}_1 \vec{p}_- \cdot \vec{\sigma}_2 + (\vec{p}_+ \times \vec{p}_-) \cdot \vec{\sigma}_2 \vec{p}_- \cdot \vec{\sigma}_1] (\vec{\tau}_1 \times \vec{\tau}_2)^3$$

- Multiplied by independent functions $U_i(\vec{p}_-^2) \sim \mathcal{O}(1)$

Comparison with meson exchange potential



- Single-meson exchange with strong and weak vertex
- Structure not in DDH:

$$[(\vec{p}_+ \times \vec{p}_-) \cdot \vec{\sigma}_1 \vec{p}_- \cdot \vec{\sigma}_2 + (\vec{p}_+ \times \vec{p}_-) \cdot \vec{\sigma}_2 \vec{p}_- \cdot \vec{\sigma}_1] (\vec{\tau}_1 \times \vec{\tau}_2)^3$$

- Isovector/-tensor currents $\sim \sin^2 \theta_W \approx 0.23 \sim 1/3$ at electroweak scale \rightarrow running to hadronic scale?

Comparison with meson exchange potential

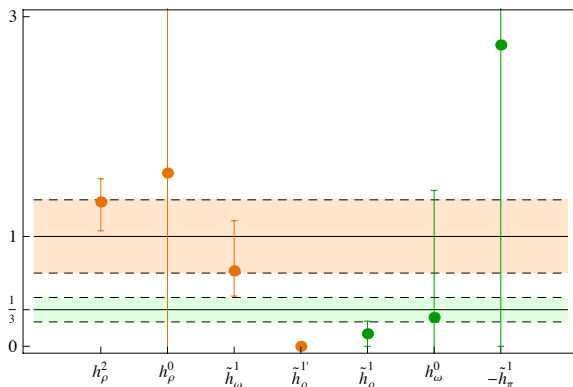
- Scaling of strong meson-nucleon couplings known
- Large- N_c scaling of weak meson-nucleon couplings

$$\begin{aligned} h_\rho^0 &\sim \sqrt{N_c} & h_\rho^2 &\sim \sqrt{N_c} [\sin^2 \theta_W] \\ h_\rho^{1'} &\lesssim \sqrt{N_c} \sin^2 \theta_W & h_\omega^1 &\sim \sqrt{N_c} \sin^2 \theta_W \\ h_\rho^1 &\lesssim \frac{1}{\sqrt{N_c}} \sin^2 \theta_W & h_\pi^1 &\lesssim \frac{1}{\sqrt{N_c}} \sin^2 \theta_W & h_\omega^0 &\sim \frac{1}{\sqrt{N_c}} \end{aligned}$$

Large- N_c suppression of PV πN coupling h_π^1

- $h_\rho^{1'}$ not necessarily small

Comparison with DDH best guesses



$$\tilde{h}_m^i = \frac{h_m^i}{\sin^2 \theta_W} \text{ [no } \sin^2 \theta_W \text{ for isotensor coupling]}$$

Large- N_c expansion and pionless EFT

- Leading-order parity-conserving EFT(π) interactions

$$\mathcal{L} = -\frac{1}{2}C_S(N^\dagger N)(N^\dagger N) - \frac{1}{2}C_T(N^\dagger \sigma^i N)(N^\dagger \sigma^i N)$$

- Large- N_c scaling

$$C_S \sim N_c, \quad C_T \sim 1/N_c$$

- In partial-wave basis

$$\mathcal{C}^{(1S_0)} = (C_S - 3C_T), \quad \mathcal{C}^{(3S_1)} = (C_S + C_T)$$

In large- N_c limit

$$\mathcal{C}^{(1S_0)} = \mathcal{C}^{(3S_1)}$$

Parity-conserving S -wave couplings

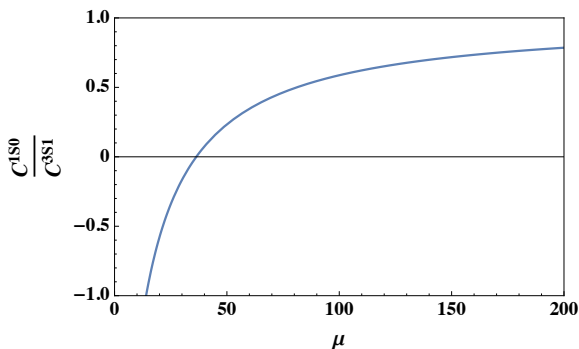
- In field theory LECs renormalization-scale dependent
- In PDS renormalization

$$\frac{C(^1S_0)}{C(^3S_1)} = \frac{\frac{1}{a(^3S_1)} - \mu}{\frac{1}{a(^1S_0)} - \mu}$$
$$\xrightarrow{\mu \rightarrow 0} \frac{a(^1S_0)}{a(^3S_1)} \approx -4.4$$

- Magnitude $\neq 1$
- Wrong sign

Parity-conserving S -wave couplings

- Large- N_c + EFT(π) requires suitable renormalization scale
- Agreement with large- N_c predicted errors for $\mu \gtrsim m_\pi$



Parity violation in pionless EFT

- In 'Girlanda basis'

$$\begin{aligned}\mathcal{L}_{PV}^{\min} = & \mathcal{G}_1(N^\dagger \vec{\sigma} N \cdot N^\dagger \overleftrightarrow{\nabla} N - N^\dagger N N^\dagger \overleftrightarrow{\nabla} \cdot \vec{\sigma} N) \\ & - \tilde{\mathcal{G}}_1 \epsilon_{ijk} N^\dagger \sigma_i N \nabla_j (N^\dagger \sigma_k N) \\ & - \mathcal{G}_2 \epsilon_{ijk} \left[N^\dagger \tau_3 \sigma_i N \nabla_j (N^\dagger \sigma_k N) + N^\dagger \sigma_i N \nabla_j (N^\dagger \tau_3 \sigma_k N) \right] \\ & - \tilde{\mathcal{G}}_5 \mathcal{I}_{ab} \epsilon_{ijk} N^\dagger \tau_a \sigma_i N \nabla_j (N^\dagger \tau_b \sigma_k N) \\ & + \mathcal{G}_6 \epsilon_{ab3} \vec{\nabla} (N^\dagger \tau_a N) \cdot N^\dagger \tau_b \vec{\sigma} N\end{aligned}$$

- 5 independent contact terms at LO

Fierz identities and large- N_c scaling

- Minimal form of Lagrangian derived using Fierz identities
- Fierz identities
 - Relate different (iso-)spin and momentum structures
 - Do not change EFT power counting
 - Change large- N_c counting
- Identify large- N_c scaling from non-minimal form of \mathcal{L}
- Example:

$$\begin{aligned}\mathcal{A}_1^+ \vec{p}_+ \cdot (\vec{\sigma}_1 - \vec{\sigma}_2) \\ \mathcal{A}_3^+ \vec{p}_+ \cdot (\vec{\sigma}_1 - \vec{\sigma}_2) \vec{\tau}_1 \cdot \vec{\tau}_2 \\ \mathcal{A}_3^- \vec{p}_- \cdot i(\vec{\sigma}_1 \times \vec{\sigma}_2) \vec{\tau}_1 \cdot \vec{\tau}_2\end{aligned}$$

- After Fierz transformation contribute to

$$-\mathcal{G}_1 \vec{p}_+ \cdot (\vec{\sigma}_1 - \vec{\sigma}_2) - i\tilde{\mathcal{G}}_1 \vec{p}_- \cdot (\vec{\sigma}_1 \times \vec{\sigma}_2)$$

Large- N_c scaling of partial-wave LECs

$$C(^3S_1-^1P_1) \sim N_c$$

$$C_{(\Delta I=0)}(^1S_0-^3P_0) \sim N_c$$

$$C_{(\Delta I=2)}(^1S_0-^3P_0) \sim N_c \sin^2 \theta_W$$

$$C_{(\Delta I=1)}(^1S_0-^3P_0) \sim N_c^0 \sin^2 \theta_W$$

$$C(^3S_1-^3P_1) \sim N_c^0 \sin^2 \theta_W$$

$$C(^3S_1-^1P_1) = 3C_{(\Delta I=0)}(^1S_0-^3P_0) [1 + \mathcal{O}(1/N_c^2)]$$

PV LECs

- Renormalization-scale dependent
- Scale dependence driven by S-wave interactions
- “Wrong” choice of scale can hide large- N_c scaling

Application to measurements

Longitudinal asymmetry in $\vec{p}p$ scattering

- Experimental result

$$A_L^{\vec{p}p}(E = 13.6 \text{ MeV}) = (-0.93 \pm 0.21) \times 10^{-7}$$

- Constraint on LECs in large- N_c limit

$$\begin{aligned} (-1.5 \pm 0.3) \times 10^{-10} \text{ MeV}^{-1} &= \frac{4 \left[C_{(\Delta I=0)}^{(1S_0-3P_0)} + C_{(\Delta I=1)}^{(1S_0-3P_0)} + C_{(\Delta I=2)}^{(1S_0-3P_0)} \right]}{C^{(1S_0)}} \\ &\rightarrow \frac{4 \left[C^{(3S_1-1P_1)}/3 + C_{(\Delta I=2)}^{(1S_0-3P_0)} \right]}{C} \end{aligned}$$

- Induced circular polarization in $np \rightarrow d\vec{\gamma}$

$$P_\gamma = (1.8 \pm 1.8) \times 10^{-7}$$

$$\rightarrow -\frac{16M_N}{C} \frac{1}{\kappa_1(1 - \gamma a^{1S_0})} \left(C^{(3S_1-1P_1)} \left(1 - \frac{5}{9}\gamma a^{1S_0}\right) - \frac{2}{3}\gamma a^{1S_0} C_{(\Delta I=2)}^{(1S_0-3P_0)} \right)$$

- In large- N_c limit $C_{(\Delta I=2)}^{(1S_0-3P_0)} \sim C^{(3S_1-1P_1)}$
- If $C_{(\Delta I=2)}^{(1S_0-3P_0)} \ll C^{(3S_1-1P_1)}$ predict P_γ outside of current bound
- $\sin^2 \theta_W$ suppression for $\Delta I = 2$ coupling not significant?

Conclusion & Outlook

Large- N_c analysis

- Effects of embedding PV quark interactions in hadrons
- Establishes hierarchy of couplings
- Important constraints in absence of experimental data
- Gives trends, not exact predictions

Parity violation

- Two LECs at LO in combined EFT/large- N_c expansion
[$\sin^2 \theta_W$ for $\Delta I = 2$]
- Relation between two isoscalar LECs
- Isovector coupling suppressed by $\sin^2 \theta_W / N_c$

$1/N_c$ expansion of PV potential

Next-to-next-to-leading order [$\mathcal{O}(N_c^{-1})$]

$$\vec{p}_- \cdot (\vec{\sigma}_1 \times \vec{\sigma}_2)$$

$$\vec{p}_+^2 \vec{p}_- \cdot (\vec{\sigma}_1 \times \vec{\sigma}_2) \vec{\tau}_1 \cdot \vec{\tau}_2$$

$$\vec{p}_+ \cdot (\vec{\sigma}_1 - \vec{\sigma}_2)$$

$$\vec{p}_+ \cdot (\vec{\sigma}_1 - \vec{\sigma}_2) \vec{\tau}_1 \cdot \vec{\tau}_2$$

$$\vec{p}_+ \cdot (\vec{\sigma}_1 - \vec{\sigma}_2) [\tau_1 \tau_2]_2^{ZZ}$$

$$\vec{p}_+^2 \vec{p}_- \cdot (\vec{\sigma}_1 \times \vec{\sigma}_2) [\tau_1 \tau_2]_2^{ZZ}$$