Nuclear-Matter Equation of State from Chiral Effective Field Theory

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with K. Hebeler and A. Schwenk

CIPANP 2018
Palm Springs, May 29, 2018

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[Credit: ORNL]

Nuclear-Matter Equation of State from Chiral Effective Field Theory

Homogeneous nuclear matter

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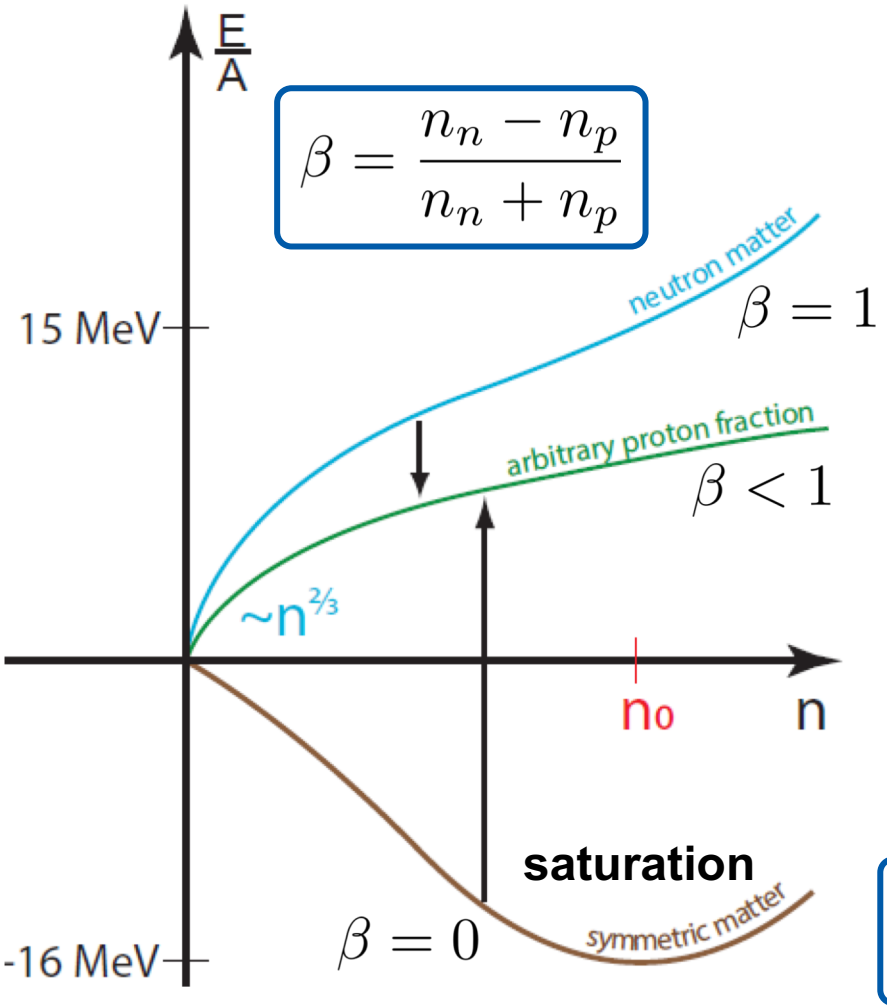


- theoretical **testbed** for benchmarking nuclear forces
  - saturation point (n_0, a_v)
  - incompressibility (K)
  - symmetry energy (S_v) and its slope (L) at saturation density
- **many-body perturbation theory,** but also in QMC, CC, SCGF, …

for a recent review see:
Hebeler *et al.*, Annu. Rev. Nucl. Part. Sci. **65**, 457

Bethe–Weizsäcker formula

$$\frac{E}{A}(\beta, n) = \frac{E}{A}(\beta = 0, n) + \beta^2 E_{\mathrm{sym}}(n)$$

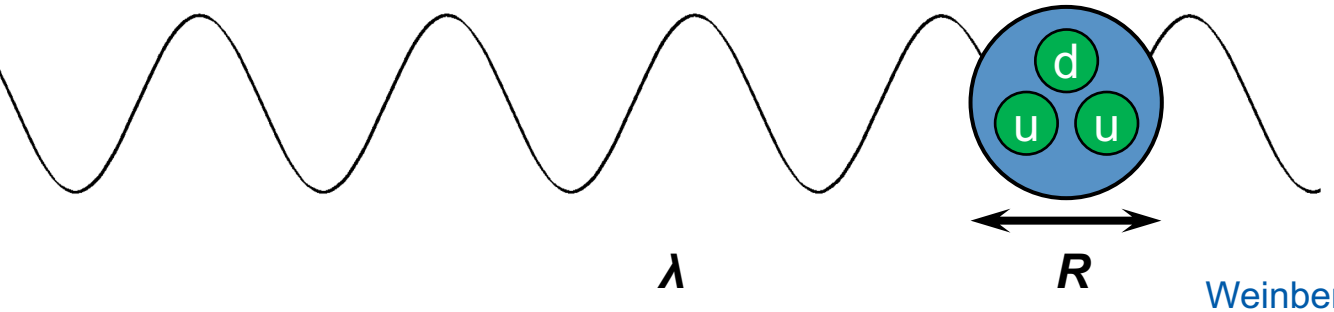
Nuclear-Matter Equation of State from Chiral Effective Field Theory

Chiral effective field theory

e.g., Epelbaum *et al.*, RMP **81**, 1773

Nuclear matter interacts via the strong interaction
(disregard Coulomb)

- QCD is non-perturbative at low energies of interest
- **modern approach**: **chiral EFT**
  - relevant degrees of freedom instead of quarks/gluons
  - use **nucleons** and **pions**



Steven Weinberg

Weinberg, Phys. Lett. B **251**, 288 (1990)
Weinberg, Nucl. Phys. B **363**, 3 (1991)
Weinberg, Phys. Lett. B **295**, 114 (1992)

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Chiral effective field theory

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Nuclear matter interacts via the strong interaction
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- QCD is non-perturbative at low energies of interest
- **modern approach**: **chiral EFT**
  - relevant degrees of freedom instead of quarks/gluons
  - use **nucleons** and **pions**
  - pion exchanges and short-range contact interactions
  - systematic expansion of nuclear forces:

$$Q = \max\left(\frac{p}{\Lambda_b}, \frac{m_\pi}{\Lambda_b}\right) \sim \frac{1}{3}$$

Steven Weinberg

Weinberg, Phys. Lett. B **251**, 288 (1990)
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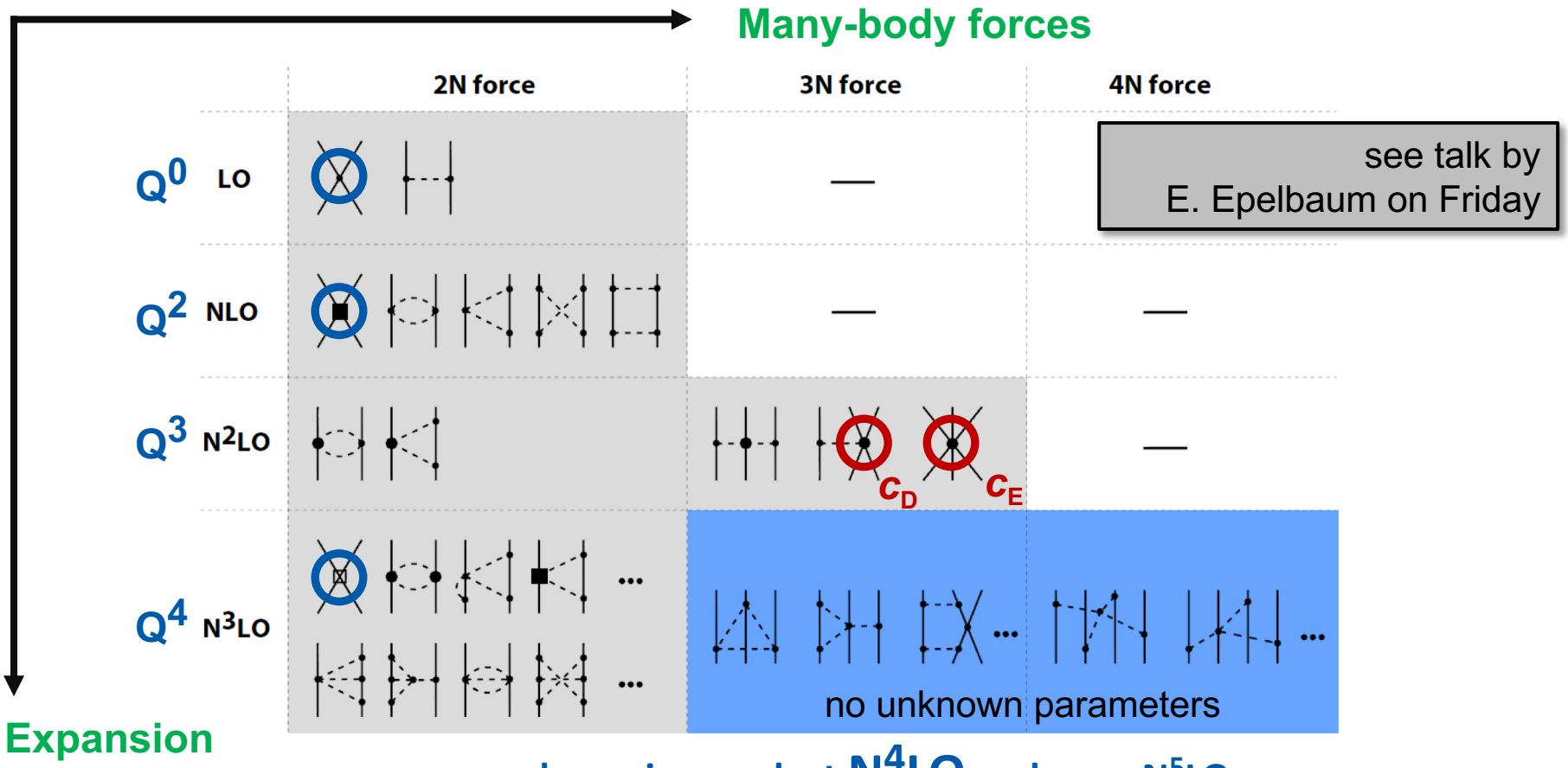
Hierarchy of nuclear forces in chiral EFT

e.g., Machleidt, Entem, Phys. Rep. **503**, 1



… and ongoing work at N^{4}LO and even N^{5}LO…

Weinberg, van Kolck, Kaplan, Savage, Wise, Epelbaum, Kaiser, Krebs, Machleidt, Meißner, ...

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Number of diagrams in MBPT

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P. D. Stevenson, Int. J. Mod. Phys. C **14**, 1135

The number of diagrams increases rapidly!

	1,	**3,**	**39,**	**840,**	**27 300,**	**1 232 280, …**
***n* =**	**2**	**3**	**4**	**5**	**6**	**7**

Integer sequence A064732:

Number of labeled Hugenholtz diagrams with *n* nodes.

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Significant challenges remain!

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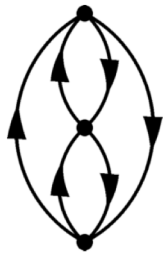
CD, Hebeler, Schwenk, arXiv:1710.08220

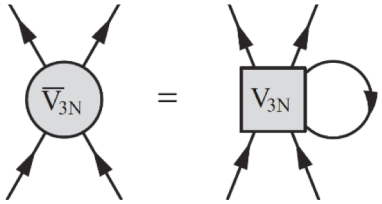
Higher orders: particle-hole contributions

Coraggio *et al.*, PRC **89**, 044321; Holt, Kaiser, PRC **95**, 034326



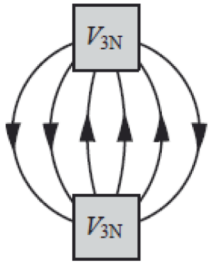
Approximated normal-ordering

Holt *et al.*, PRC **81**, 024002; Hebeler, Schwenk, PRC **82**, 014314



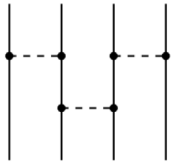
Neglected residual 3N diagrams

Hagen *et al.*, PRC **89**, 014319; Kaiser, EPJ A **48**, 58



Higher many-body forces

Hebeler *et al.*, PRC **91**, 044001



development of a novel Monte-Carlo framework

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Efficient Monte-Carlo framework

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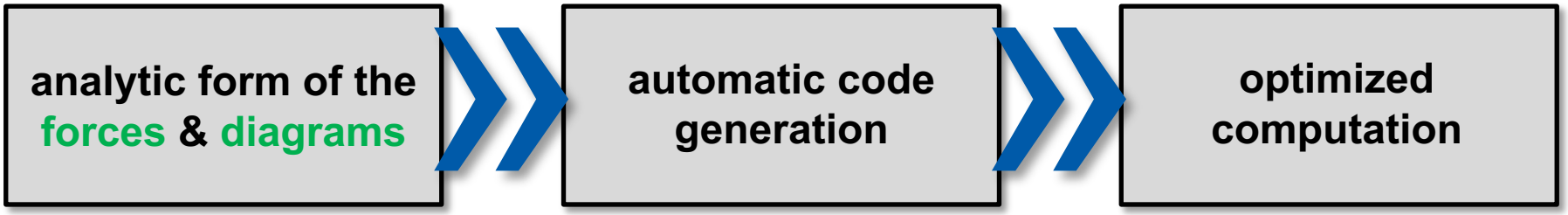
CD, Hebeler, Schwenk, arXiv:1710.08220

represent interactions as matrices **in spin-isospin space**

- based on analytic expressions, incl. **NN**, **3N**, and **4N** forces
- **no need for partial-wave decompositions**

efficient evaluation of **diagrams** in **MBPT** (single-particle basis)

- **implementing diagrams** has become **straightforward** (also ph)
- spin-isospin traces are fully automated; multidim. momentum integrals
- rapid increase of number of diagrams: 3 (3^{rd}), **39 (4^{th})**, 840 (5^{th})



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Nuclear saturation

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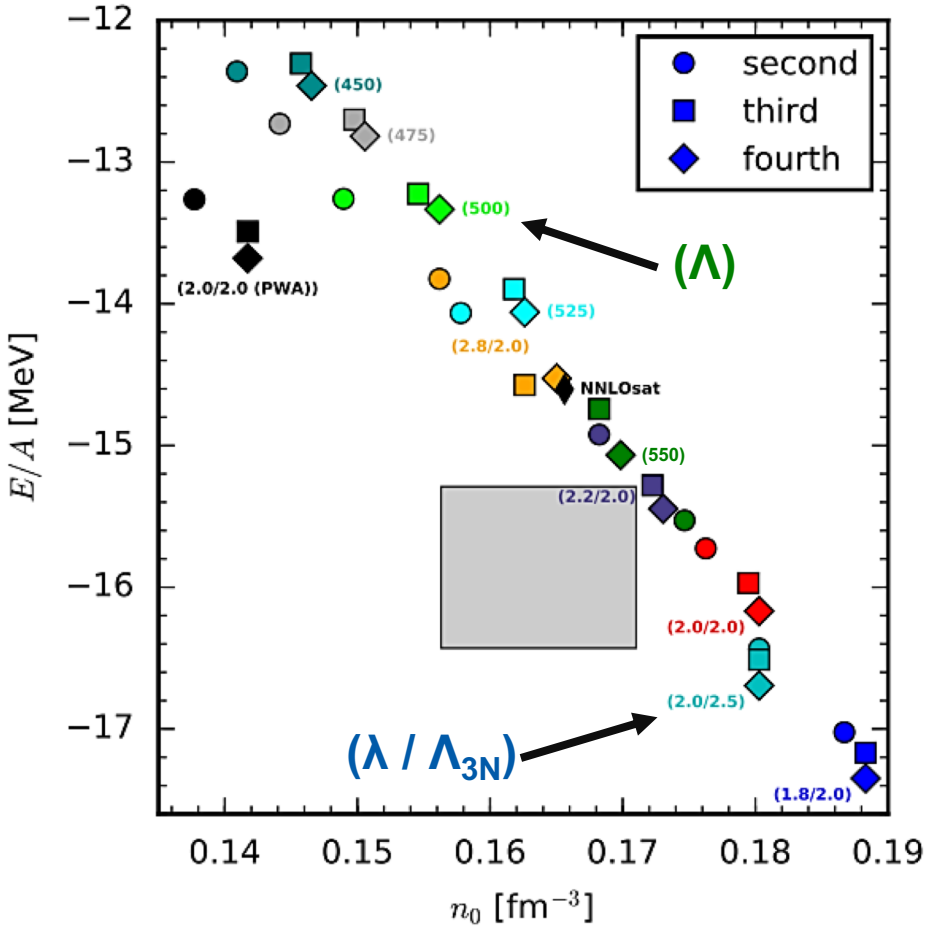
CD, Hebeler, Schwenk, arXiv:1710.08220



include contributions from up to

- **NN (4th)**, NN plus 3N (3rd),
- **residual 3N–3N term (2nd)**

good many-body convergence

Hebeler *et al.*, PRC **83**, 031301
Carlsson *et al.*, PRX **6**, 011019

» **interactions are perturbative** for these densities

Coester-like linear correlation

Coester *et al.*, PRC **1**, 769

$$E_{\rm sym} = 31.1 - 32.5\,{\rm MeV}$$
$$L = 44.8 - 56.2\,{\rm MeV}$$

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Guiding finite nuclei

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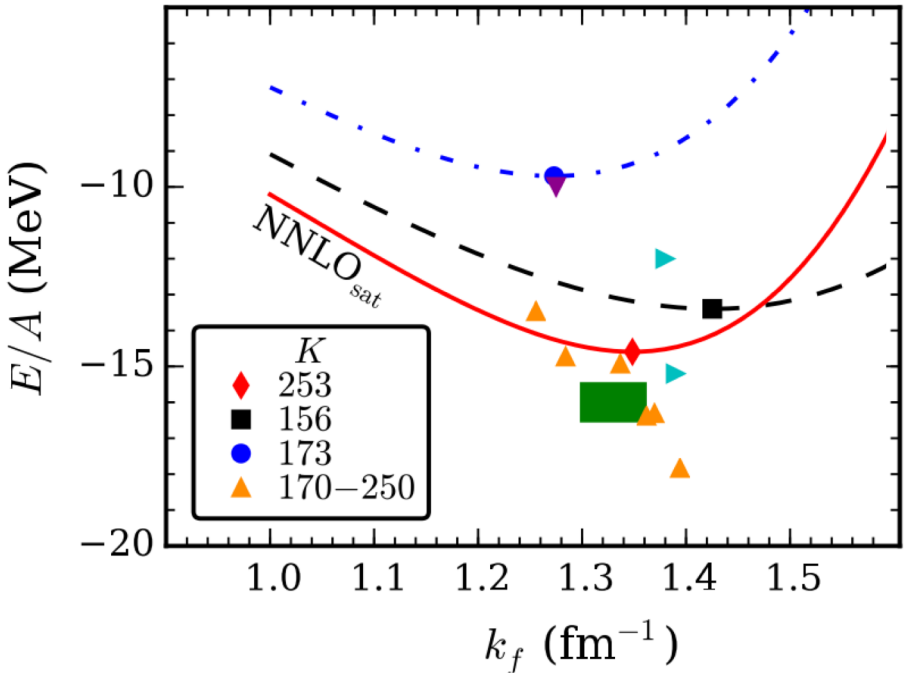
Ekström *et al.*, Phys. Rev. C **91**, 551301

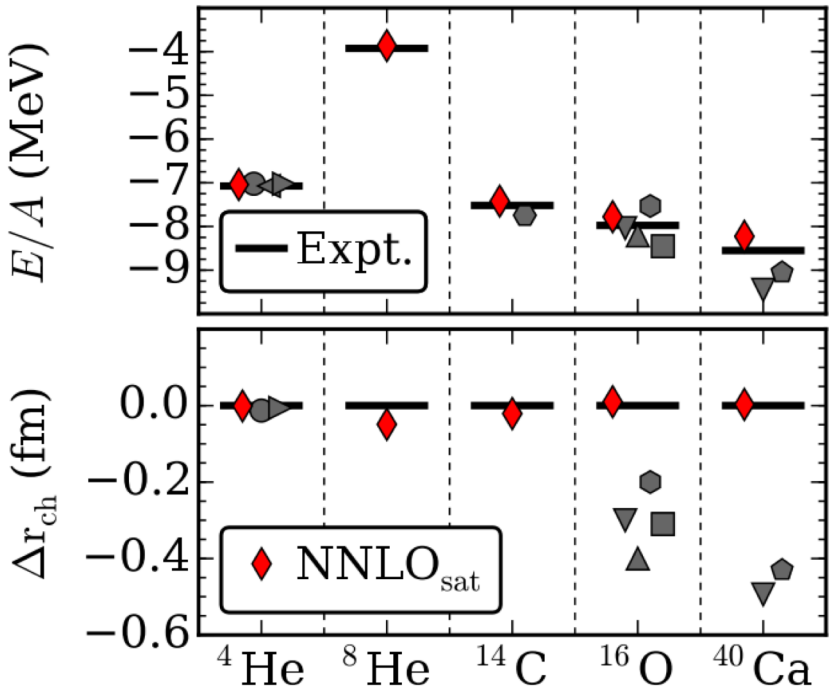
Infinite Matter



***Ab initio* calculations overbind medium-mass and heavy nuclei, underestimate charge radii**

$$\Delta r_{\rm ch} = r_{\rm ch}^{\rm theo} - r_{\rm ch}^{\rm exp}$$

Finite Nuclei



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Fits to saturation region

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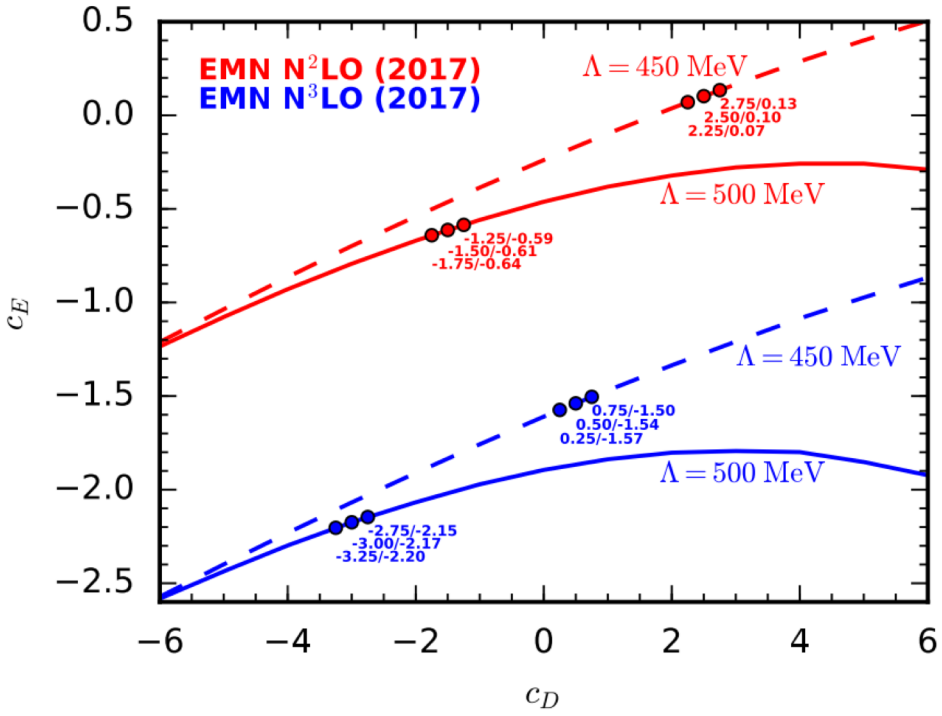
CD, Hebeler, Schwenk, arXiv:1710.08220



use the Monte-Carlo framework to constrain 3N LECs

- **N^2LO / N^3LO EMN potentials** with Λ = 450 MeV & Λ = 500 MeV

  Entem, Machleidt, Nosyk, PRC **96**, 024004

- **fit to ^3H binding energy:** $c_E(c_D)$ consistently at N^2LO / N^3LO

- **study saturation properties:** 3rd order contribution important !

reasonable fits to saturation at N^2LO & N^3LO identified

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Fits to saturation region

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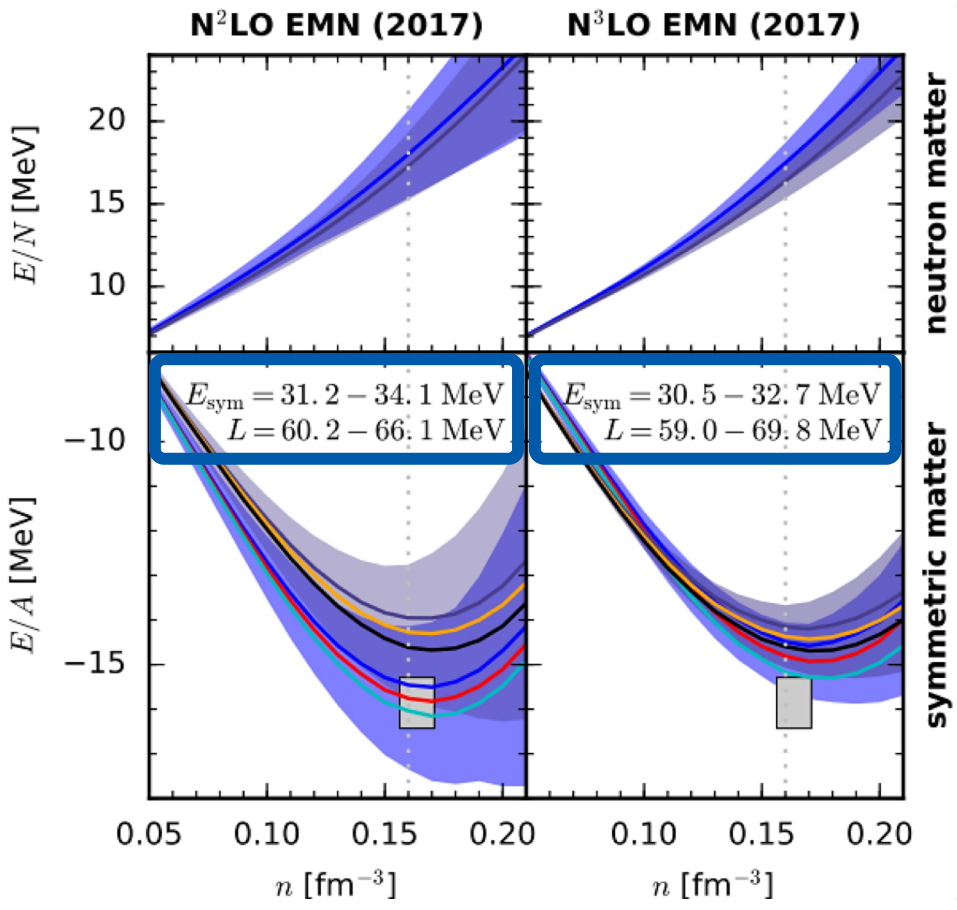

Neutron and symmetric matter with consistent NN + 3N forces

- 4N HF energy ~150 keV @ n_0
- **narrow ranges** for $E_{\rm sym}$ and L
- uncertainties from chiral EFT

Epelbaum *et al.*, EPJ A **51**, 53

Symmetric matter @ N³LO:

- **reduced cutoff dependence**
- **reduced theo. uncertainties**

left column:

Λ/c_D [MeV]/[1]	
450/2.25	500/−1.75
450/2.50	500/−1.50
450/2.75	500/−1.25

right column:

Λ/c_D [MeV]/[1]	
450/0.25	500/−3.25
450/0.50	500/−3.00
450/0.75	500/−2.75

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Outlook

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1. **Apply the new Hamiltonians to finite nuclei**
   calculations will provide additional insights …
2. **Perform calculations in asymmetric matter**
   extract astrophysical quantities, mass-radius relations, …
3. **Extend framework to finite temperatures**
   study thermal properties, tabulate equation of state, …
4. **Study dilute Fermi gas at fourth order**
   with Wellenhofer, Hebeler, Schwenk, in preparation

Thank you for your attention!

Collaborators:

K. Hebeler
K. McElvain
A. Schwenk
C. Wellenhofer

erc European Research Council Established by the European Commission

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