

# Applications of Chiral Forces to Nuclear Matter and Neutron Stars

Berkeley  
UNIVERSITY OF CALIFORNIA

Christian Drischler

with K. Hebeler and A. Schwenk

CIPANP 2018

Palm Springs, June 1, 2018

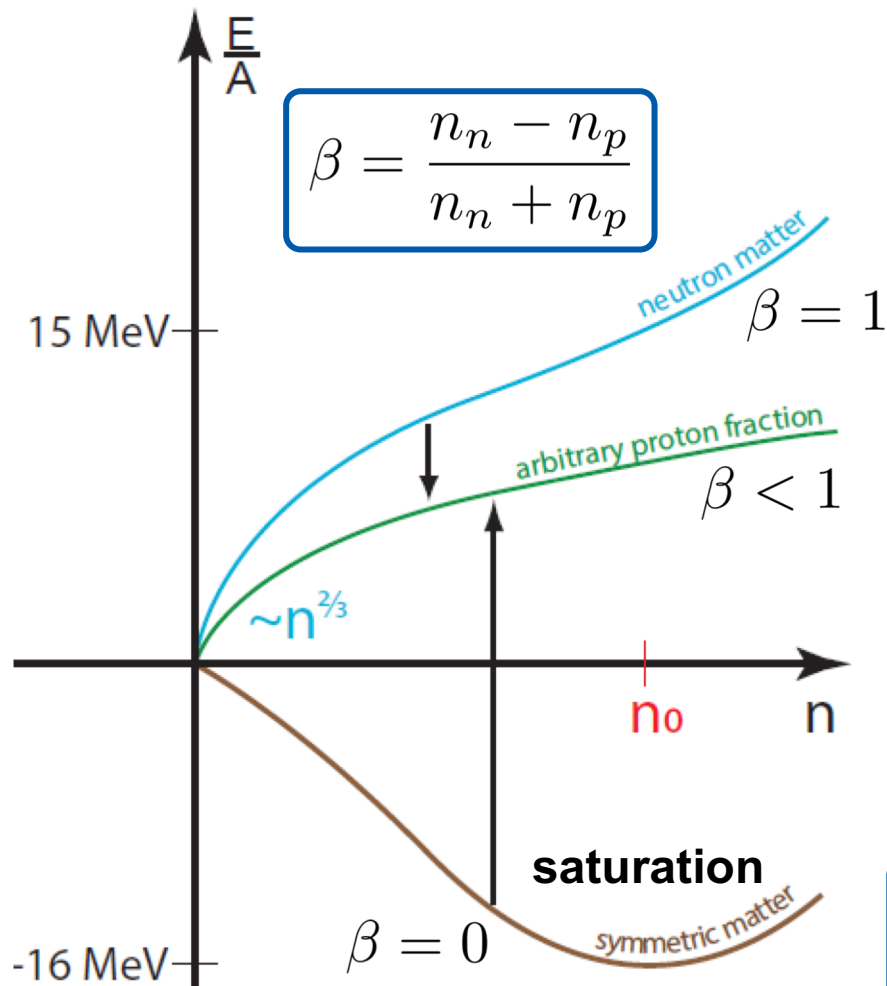


U.S. DEPARTMENT OF  
**ENERGY**

[Credit: ORNL]

# Applications of Chiral Forces to Nuclear Matter and Neutron Stars

Homogeneous nuclear matter



- theoretical **testbed** for benchmarking nuclear forces
  - saturation point ( $n_0, a_v$ )
  - incompressibility ( $K$ )
  - symmetry energy ( $S_v$ ) and its slope ( $L$ ) at saturation density
- **many-body perturbation theory**, but also in QMC, CC, SCGF, ...

for a recent review see:  
Hebeler *et al.*, *Annu. Rev. Nucl. Part. Sci.* **65**, 457

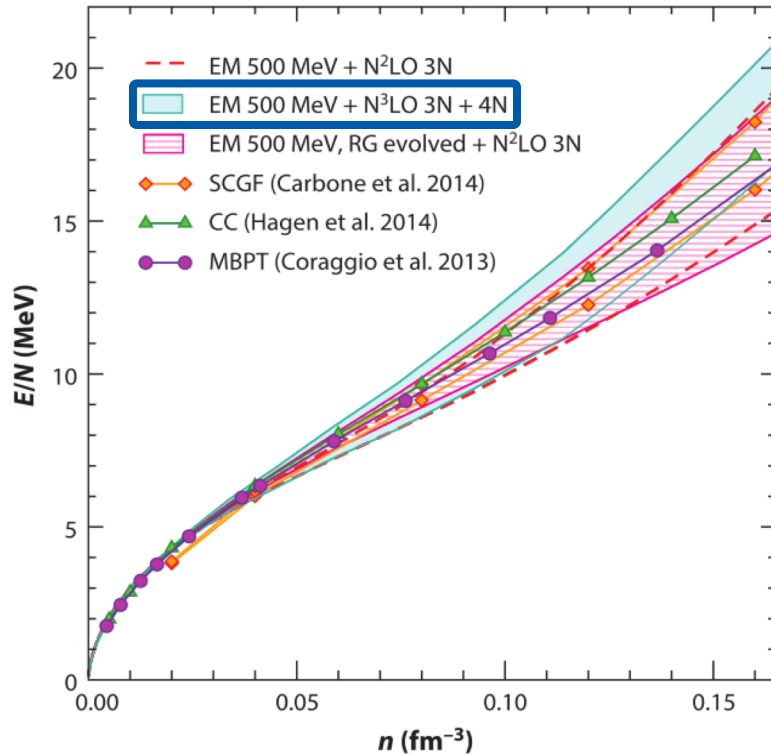
**Bethe–Weizsäcker formula**

$$\frac{E}{A}(\beta, n) = \frac{E}{A}(\beta = 0, n) + \beta^2 E_{\text{sym}}(n)$$

# Applications of Chiral Forces to Nuclear Matter and Neutron Stars

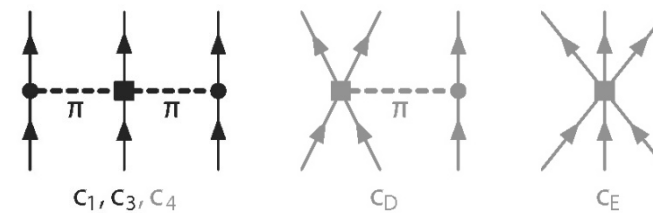
## Neutron-matter EOS

Hebeler *et al.*, Annu. Rev. Nucl. Part. Sci. 65, 457



**Remarkable agreement** between many-body frameworks and different Hamiltonians

### Leading 3N forces



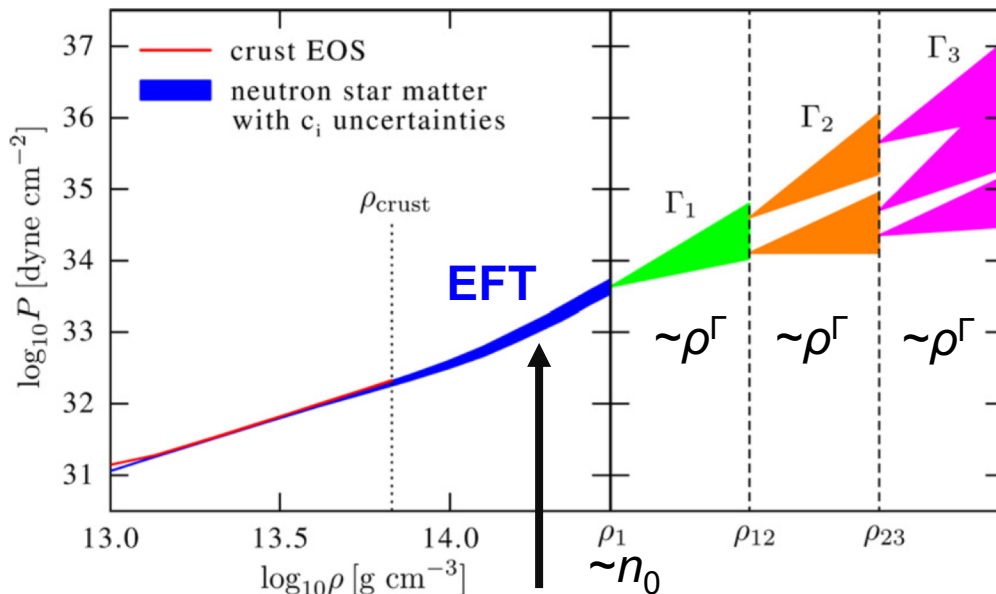
### Hierarchy of chiral forces

	2N force	3N force	4N force
LO		—	—
NLO		—	—
N <sup>2</sup> LO			—
N <sup>3</sup> LO			

# Applications of Chiral Forces to Nuclear Matter and Neutron Stars

## Mass–radius relation

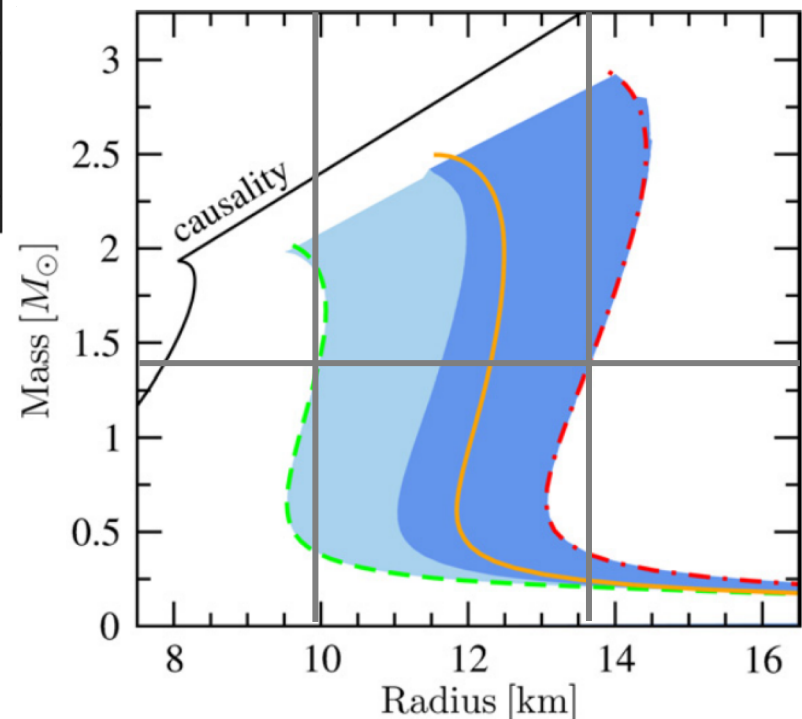
Hebeler *et al.*, *Astrophys. J.* **773**, 11



TOV eqs.

- +  $M_{\text{max}} \geq 1.97 M_{\text{sun}}$
- + causality
- + ...

mass–radius relation



Neutron matter extrapolated to  $\beta$  equilibrium

### Improvements needed:

- calculate asym. matter directly
- higher orders in the **chiral** and **perturbative expansion**

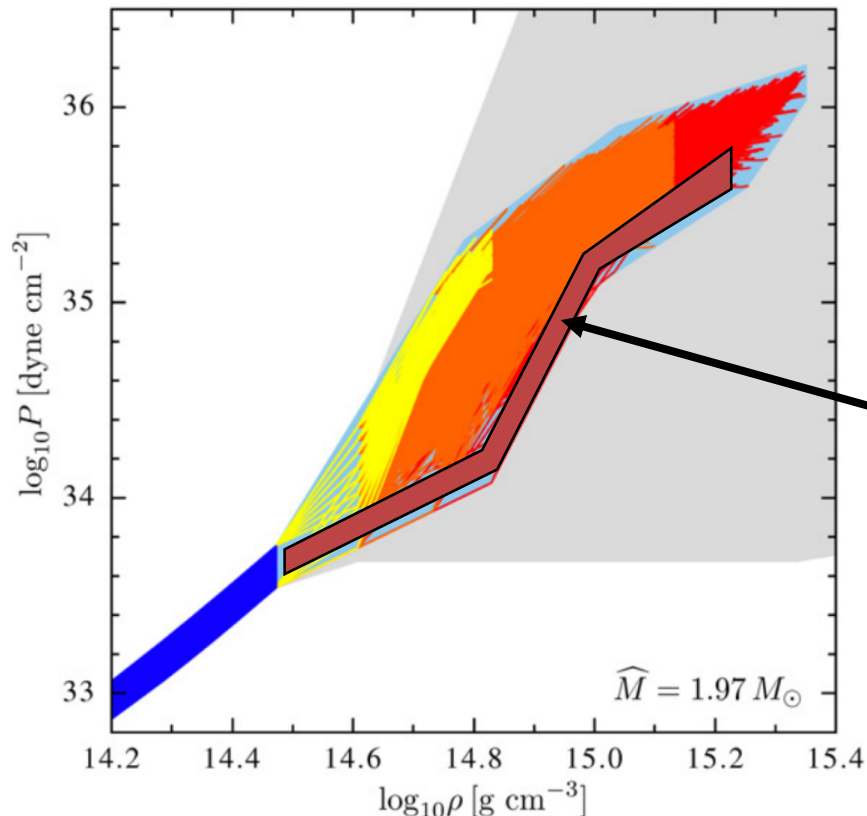


for QMC, see: Gandolfi *et al.*, *Phys. Rev. C* **85**, 032801

# Applications of Chiral Forces to Nuclear Matter and Neutron Stars

## Mass–radius relation

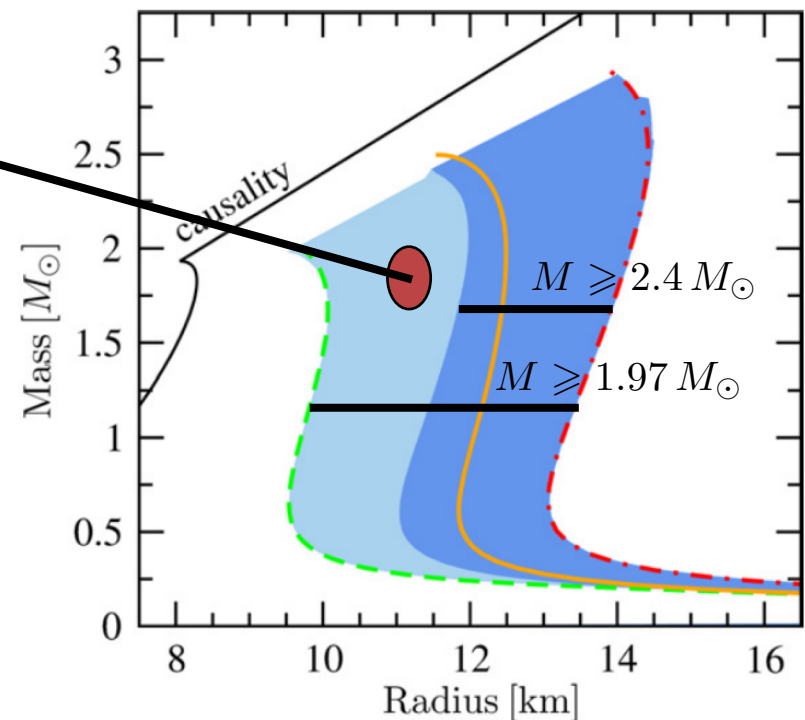
Hebeler *et al.*, *Astrophys. J.* **773**, 11



TOV eqs.

- +  $M_{\text{max}} \geq 1.97 M_{\text{sun}}$
- + causality
- + ...

mass–radius relation

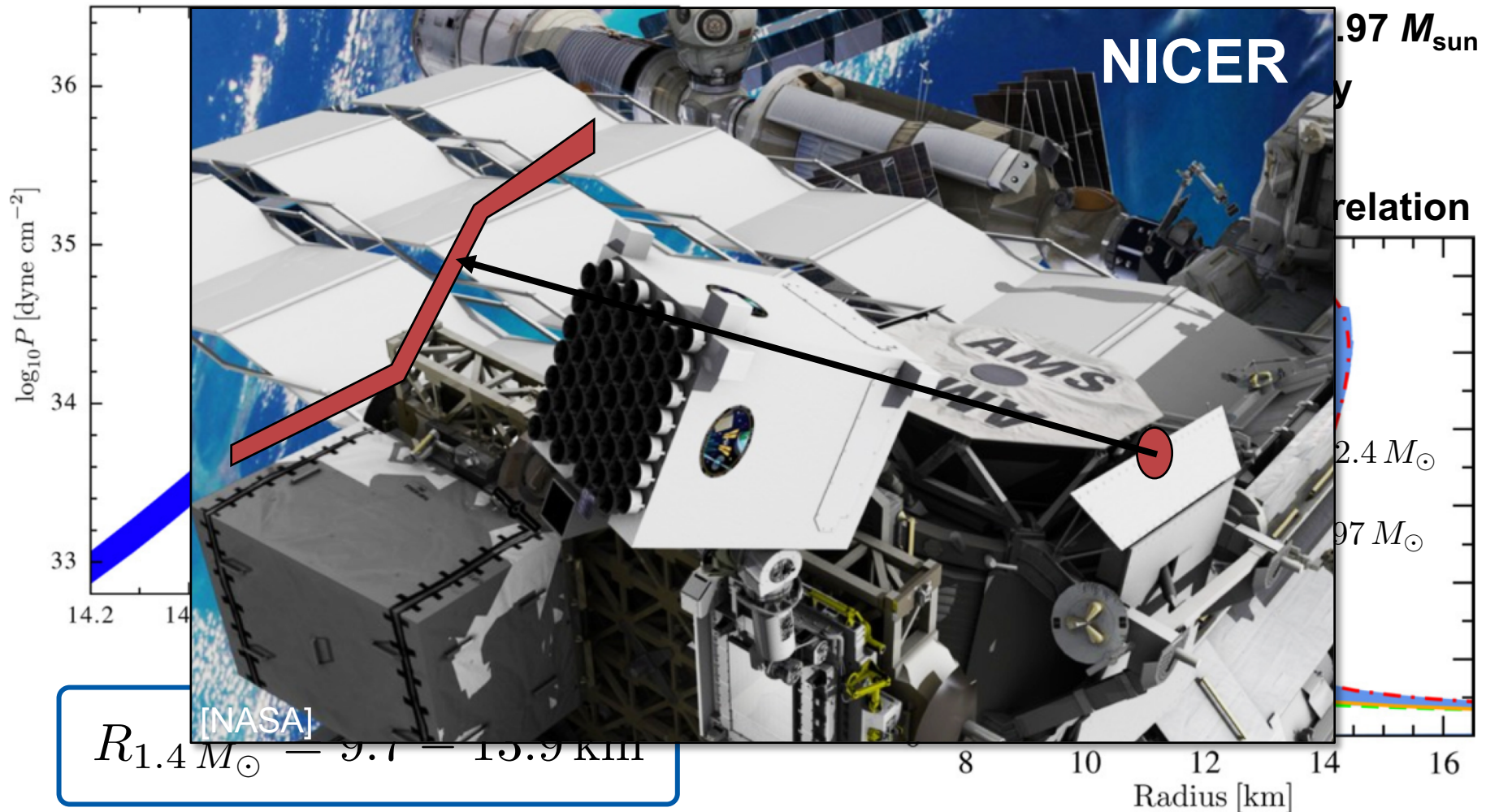


$$R_{1.4 M_{\odot}} = 9.7 - 13.9 \text{ km}$$

# Applications of Chiral Forces to Nuclear Matter and Neutron Stars

## Mass–radius relation

Hebeler *et al.*, *Astrophys. J.* 773, 11



# Applications of Chiral Forces to Nuclear Matter and Neutron Stars

3N forces beyond Hartree-Fock?

CD, Hebeler, Schwenk, Phys. Rev. C **93**, 054314

## Effective NN potentials

by summing *one* particle over the occupied states of the Fermi sea

» dominant 3N contributions

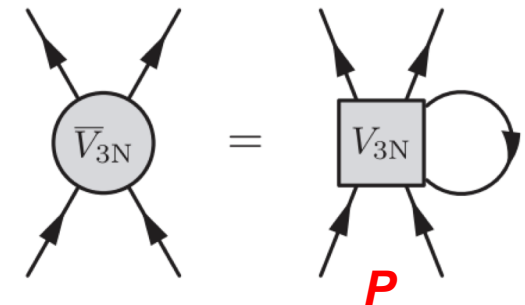
Holt *et al.*, PRC **81**, 024002  
Hebeler *et al.*, PRC **82**, 014314

so far: **only  $N^2LO$  3N** and  **$P = 0$**

## Improved method

- applicable to all nuclear forces
- $N^3LO$  3N forces due to recent partial-wave decomposition

Hebeler *et al.*, PRC **91**, 044001



some more applications:  
Wellenhofer *et al.*, PRC **92**, 015801  
Holt *et al.*, PPNP **73**, 35  
Hebeler *et al.*, ARNPS **65**, 457  
...



**towards *consistent*  
 $N^3LO$  calculations**

# Applications of Chiral Forces to Nuclear Matter and Neutron Stars

3N forces beyond Hartree-Fock?

CD, Hebeler, Schwenk, Phys. Rev. C **93**, 054314

PHYSICAL REVIEW C **95**, 024302 (2017)

## Pairing in neutron matter: New uncertainty estimates and three-body forces

C. Drischler,<sup>1,2,\*</sup> T. Krüger,<sup>1,2,†</sup> K. Hebeler,<sup>1,2,‡</sup> and A. Schwenk<sup>1,2,3,§</sup>

<sup>1</sup>*Institut für Kernphysik, Technische Universität Darmstadt, 64289 Darmstadt, Germany*

<sup>2</sup>*ExtreMe Matter Institute EMMI, GSI Helmholtzzentrum für Schwerionenforschung GmbH, 64291 Darmstadt, Germany*

<sup>3</sup>*Max-Planck-Institut für Kernphysik, Saupfercheckweg 1, 69117 Heidelberg, Germany*

## Improved method

- applicable to all nuclear forces
- N<sup>3</sup>LO 3N forces due to recent partial-wave decomposition



towards *consistent*  
N<sup>3</sup>LO calculations

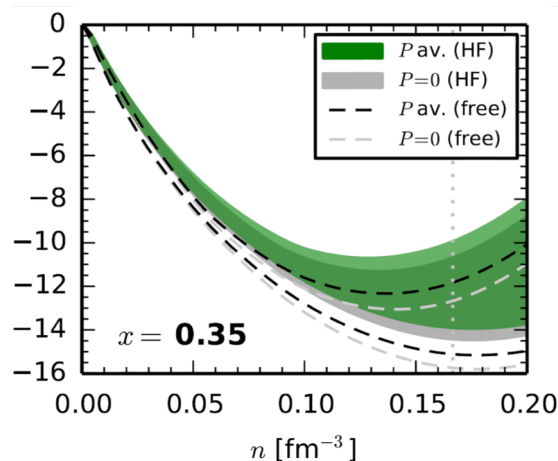
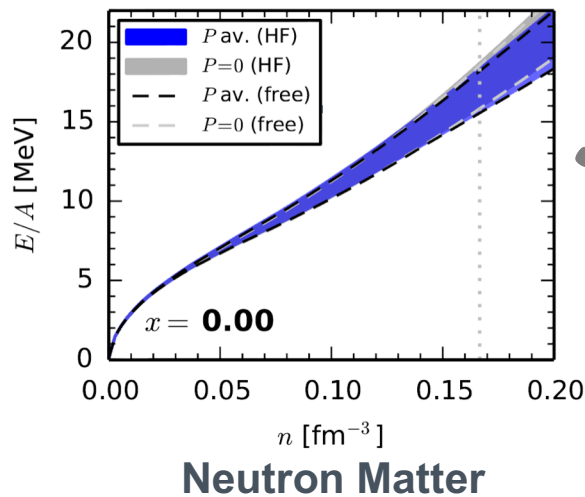
Hebeler *et al.*, PRC **91**, 044001



# Applications of Chiral Forces to Nuclear Matter and Neutron Stars

## Equation of state

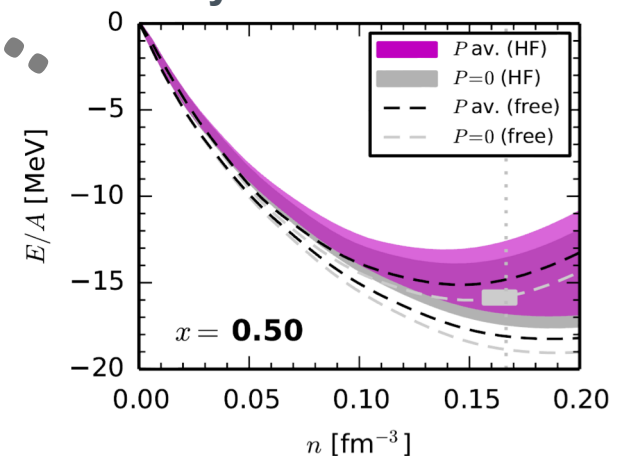
CD, Hebeler, Schwenk, Phys. Rev. C 93, 054314



**11 proton fractions**  
 $x = 0.0, 0.05, \dots, 0.5$   
up to second order

$$x = \frac{n_p}{n_n + n_p}$$

**Symmetric Matter**



**Uncertainty bands:** Hebeler *et al.*, PRC 83, 031301

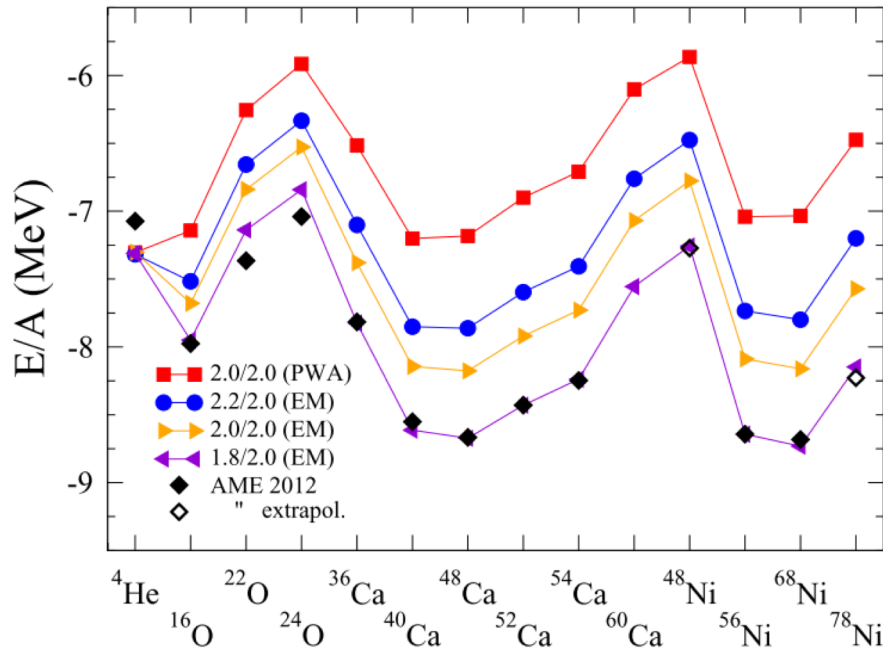
- 7 potentials: evolved  $N^3\text{LO NN}$  + bare  $N^2\text{LO 3N}$
- different combinations of  $\Lambda/\Lambda_{3N}$
- $c_D, c_E$  fit *only* to few-body data
- free and Hartree-Fock spectrum

# Applications of Chiral Forces to Nuclear Matter and Neutron Stars

Guiding finite nuclei

Simonis *et al.*, Phys. Rev. C 96, 014303

## ground-state energies

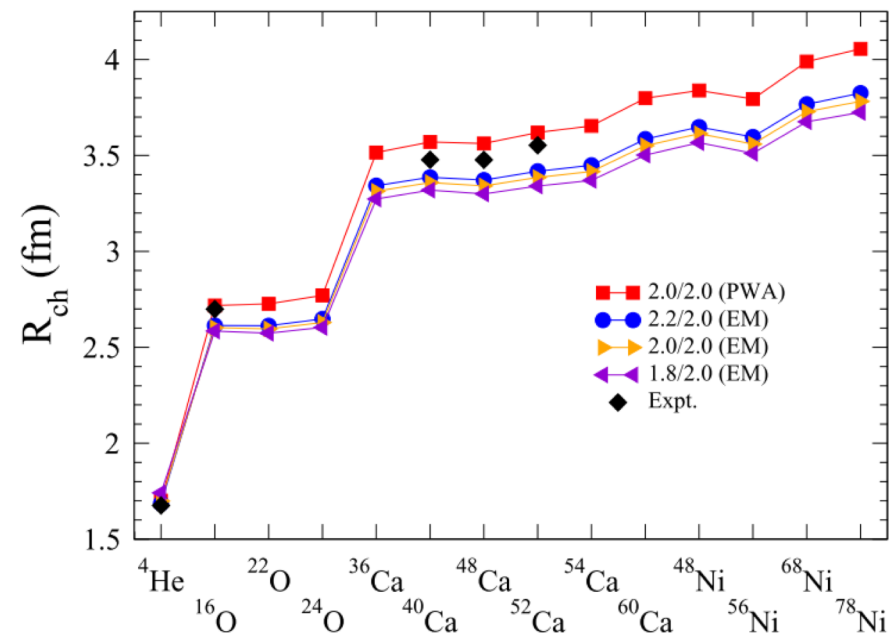


$\lambda / \Lambda_{3N} = 1.8 / 2.0$  (EM) exhibits good agreement with experiment

## “Hebeler *et al.*” interactions

- $N^3\text{LO}$  NN (SRG) +  $N^2\text{LO}$  3N forces
- $c_D$ ,  $c_E$  *only* fit to few-body data

## charge radii



# Applications of Chiral Forces to Nuclear Matter and Neutron Stars

Symmetry energy and slope parameter

quadratic expansion  $\beta = 1 - 2x$

$$\frac{E}{A}(n, \beta) = \frac{E_{\text{SNM}}(n)}{A} + S_2(n)\beta^2 + \dots$$

$$S_2(n) = S_v + \frac{L}{3} \left( \frac{n - n_0}{n_0} \right) + \dots$$

tight constraints

$$S_v = (30.9 \pm 1.4) \text{ MeV}$$

$$L = (45.0 \pm 7.1) \text{ MeV}$$

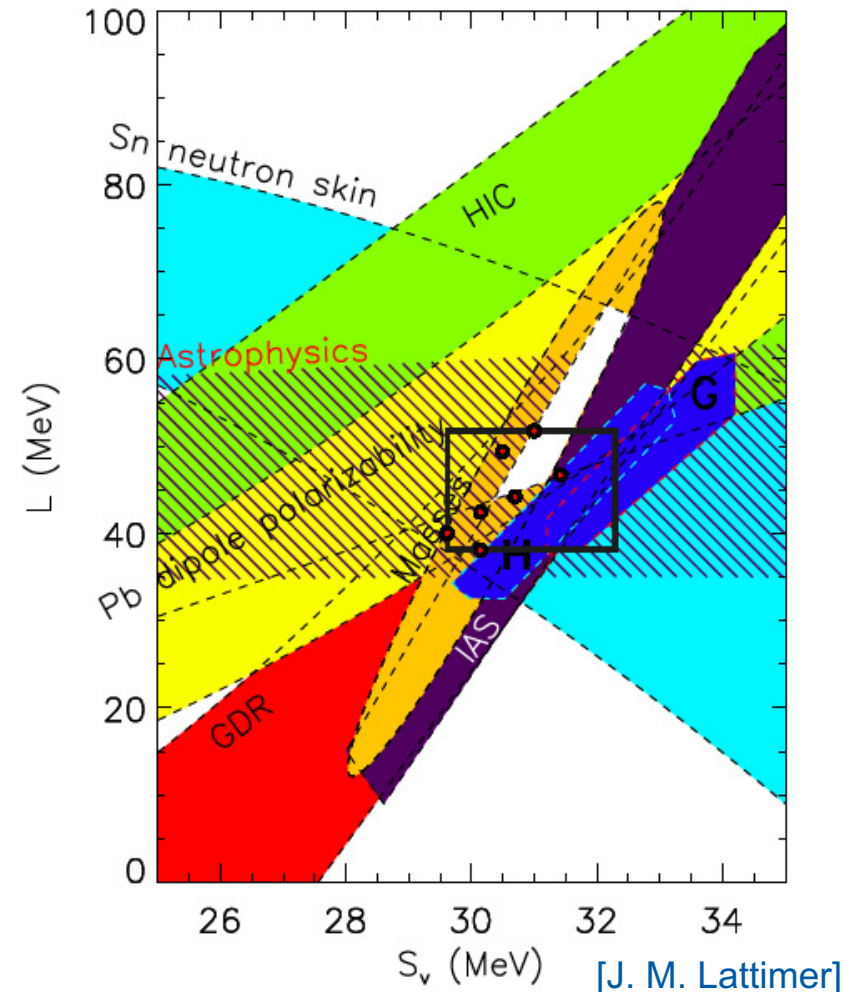
in agreement with quadratic expansion

Lattimer, Lim, *Astrophys. J.* **771**, 51

quadratic expansion is reliable; but  
nonanalytical quartic term:  $\beta^4 \ln |\beta|$

Kaiser, *PRC* **91**, 065201

Wellenhofer *et al.*, *PRC* **93**, 055802



# Applications of Chiral Forces to Nuclear Matter and Neutron Stars

## Radius estimates for neutron stars

Hagen *et al.*, Nat. Phys. 12, 186

### empirical relation by Lattimer & Prakash

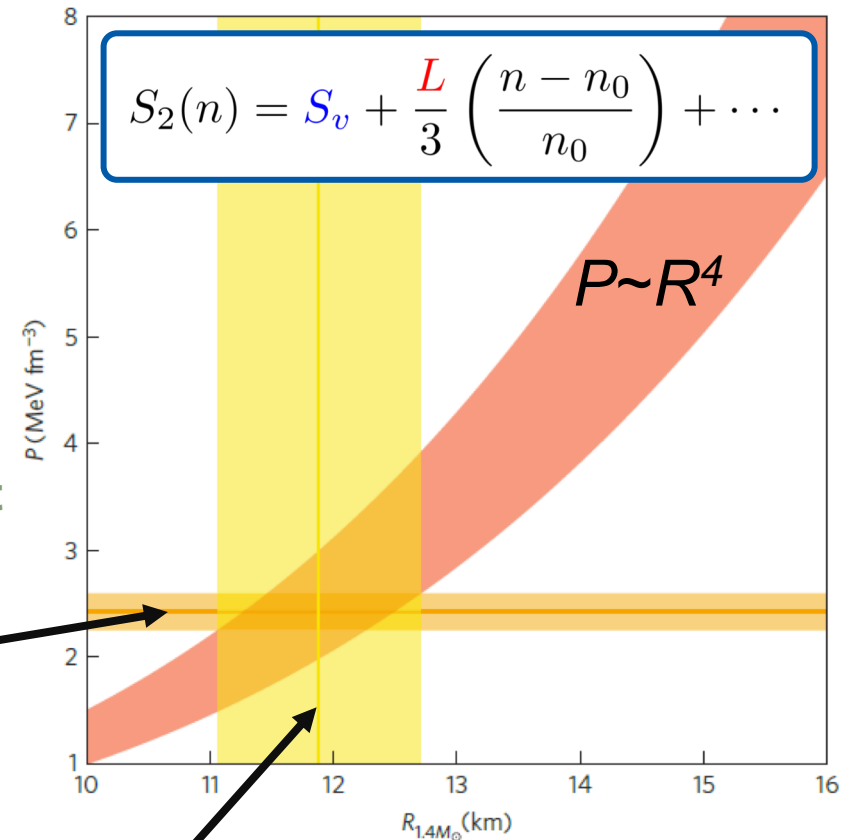
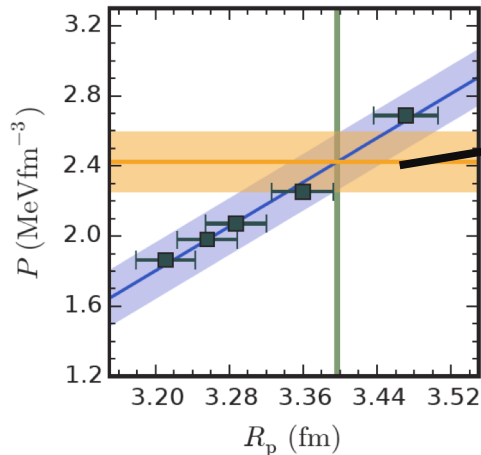
$$P(\beta_{\text{eq}}, n_0) \simeq \frac{L}{3} n_0 \propto R_{1.4 M_\odot}^4$$

pressure of neutron-star matter

» pin down  $L$  is important

### Pressure constrained by *ab initio*

### CC results for $^{48}\text{Ca}$ radii and measurement

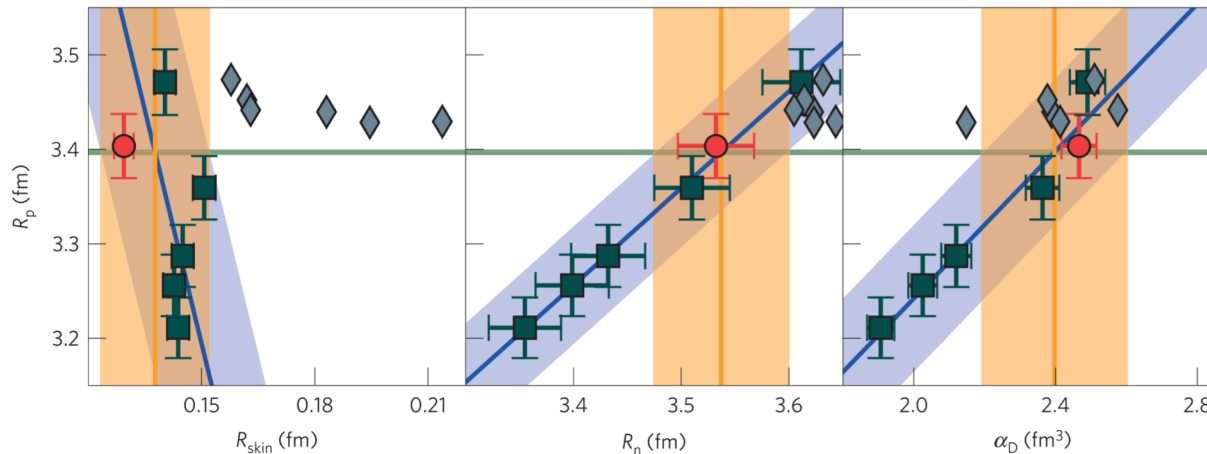


$$11.1 \text{ km} \leq R_{1.4 M_\odot} \leq 12.7 \text{ km}$$

# Applications of Chiral Forces to Nuclear Matter and Neutron Stars

Constraints from  $^{48}\text{Ca}$  calculations

Hagen *et al.*, Nat. Phys. 12, 186



see also Birkhan *et al.*, PRL 118, 252501

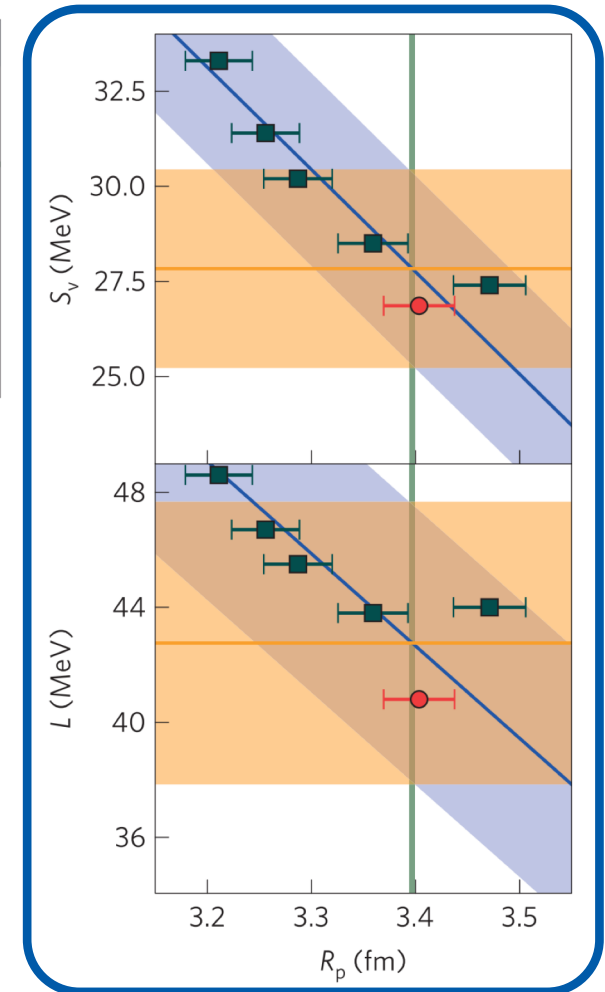
nature  
physics

ARTICLES

PUBLISHED ONLINE: 2 NOVEMBER 2015 | DOI: 10.1038/NPHYS3529

## Neutron and weak-charge distributions of the $^{48}\text{Ca}$ nucleus

G. Hagen<sup>1,2\*</sup>, A. Ekström<sup>1,2</sup>, C. Forssén<sup>1,2,3</sup>, G. R. Jansen<sup>1,2</sup>, W. Nazarewicz<sup>1,4,5</sup>, T. Papenbrock<sup>1,2</sup>, K. A. Wendt<sup>1,2</sup>, S. Bacca<sup>6,7</sup>, N. Barnea<sup>8</sup>, B. Carlsson<sup>3</sup>, C. Drischler<sup>9,10</sup>, K. Hebeler<sup>9,10</sup>, M. Hjorth-Jensen<sup>4,11</sup>, M. Miorelli<sup>6,12</sup>, G. Orlandini<sup>13,14</sup>, A. Schwenk<sup>9,10</sup> and J. Simonis<sup>9,10</sup>

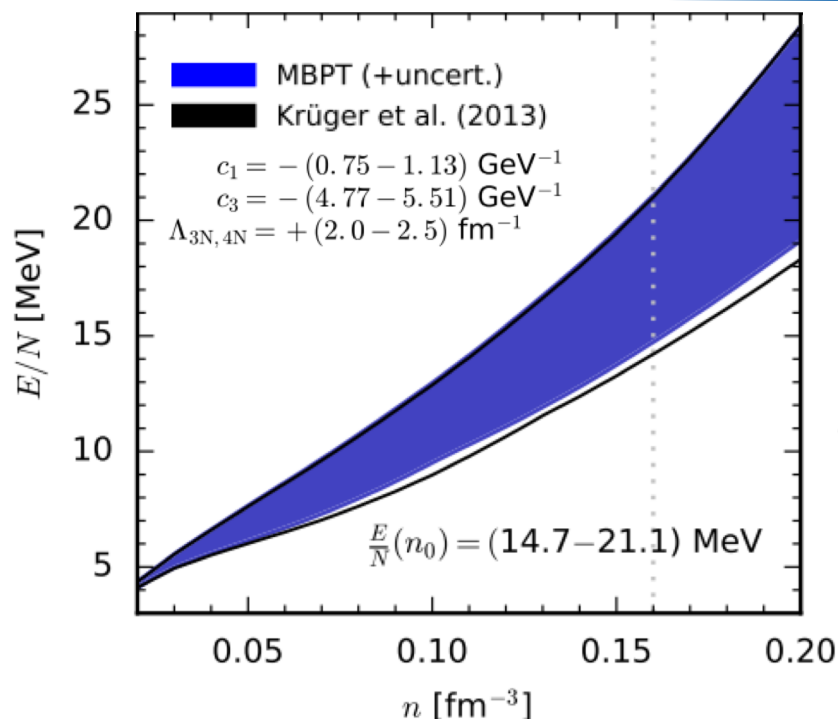


# Applications of Chiral Forces to Nuclear Matter and Neutron Stars

New mass–radius constraints

Most, Weih, Rezzolla, Schaffner-Bielich, arXiv:1803.00549

## Neutron matter at full N<sup>3</sup>LO



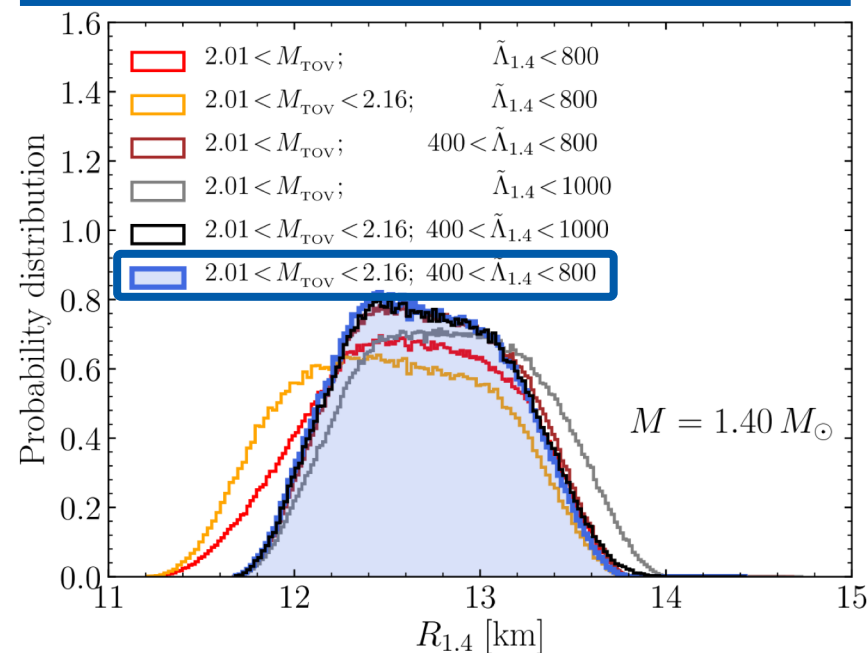
NN+3N contributions at 3rd order and 4N Hartree-Fock energies

CD, Carbone, Hebeler, Schwenk, PRC **94**, 054307

Agreement with our constraints

$$R_{1.4 M_{\odot}} = 12.00 - 13.45 \text{ km}$$

## Neutron-star radii



see also Annala *et al.*, PRL **120**, 172703

# Applications of Chiral Forces to Nuclear Matter and Neutron Stars

## Outlook

- 1 Study asymmetric matter at full N<sup>3</sup>LO**  
extract astrophysical quantities, mass-radius relations, ...
- 2 Explore finite temperatures**  
study thermal properties, single-particle energies, ...
- 3 Improve EFT uncertainty quantification**  
Bayesian inference, uncertainty propagation, ...
- 4 Tabulate the equation of state**  
make EOS accessible in astrophysical simulations, ...

**Thank you  
for your attention!**



European Research Council  
Established by the European Commission



### Collaborators:

K. Hebeler

K. McElvain

A. Schwenk

C. Wellenhofer