Nucleon electromagnetic form factors in dispersively improved chiral EFT

C. Weiss (JLab), CIPANP 2018, Palm Springs, CA, 02-Jun-18

Summary: New method for calculating/analyzing nucleon FFs combining χEFT and dispersion theory

Implements analyticity in momentum transfer t

Includes $\pi\pi$ rescattering and ρ resonance through unitarity

Enables predictive calculations, controlled accuracy

Outline

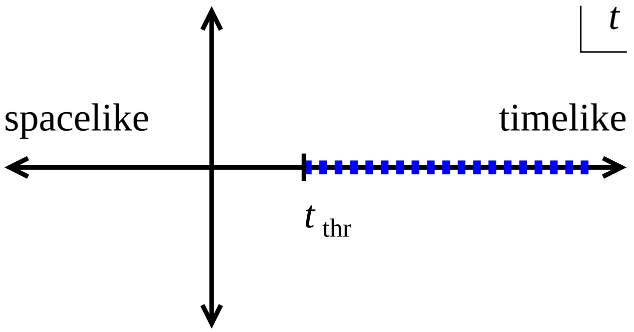
Method: Dispersive representation, elastic unitarity, χEFT

Results: Spectral functions, form factors

Applications: Proton radius extraction, other FFs

J. M. Alarcon, C. Weiss, PRC **97,** 055203 (2018); J. M. Alarcon, C. Weiss, arXiv:1803.09748;
J. M. Alarcon, A. N. Hiller Blin, M. Vicente Vacas, C. Weiss, NPA **964**, 18 (2017)

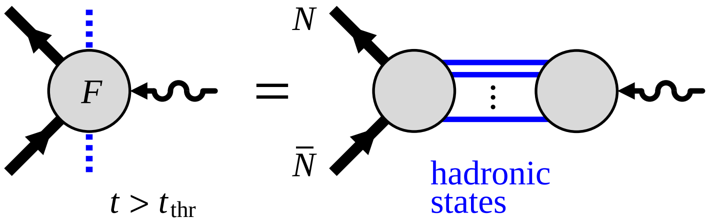
Method: Dispersive representation



- Dispersive representation

$$F_i(t) \;=\; \int\limits_{t_{\text{thr}}}^{\infty} \frac{dt'}{\pi}\, \frac{\text{Im}\, F_i(t')}{t' - t - i0}$$

Expresses analytic structure of $F_i(t)$



Isovector: $\pi\pi$ (incl. ρ), 4π, $K\bar{K}$, …
Isoscalar: 3π (incl. ω), $K\bar{K}$ (incl. ϕ), …

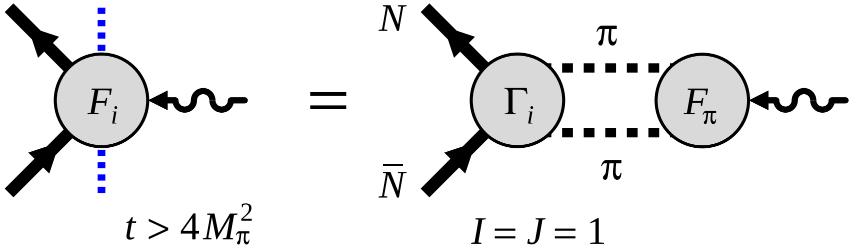
- Spectral functions $\text{Im}\, F_i(t)$

Current → hadronic states → $N\bar{N}$

Processes in unphysical region $t < 4M_N^2$

Spectral functions to be provided by theory
Frazer, Fulco 1960; Höhler et al 1975+

Method: Spectral functions on $\pi\pi$ cut



$$\mathrm{Im} F_i(t) = \frac{k_{\mathrm{cm}}^3}{\sqrt{t}} \, \Gamma_i(t) \;\; F_\pi^*(t)$$

$$= \frac{k_{\mathrm{cm}}^3}{\sqrt{t}} \, \underbrace{\frac{\Gamma_i(t)}{F_\pi(t)}}_{\chi\text{EFT}} \;\; \underbrace{|F_\pi(t)|^2}_{\text{Data}}$$

- Elastic unitarity relation

  $F_\pi(t)$ timelike pion FF, $\Gamma_i(t)$ partial-wave amplitude $\pi\pi \to N\bar{N}$

  Amplitudes have same phase from $\pi\pi$ rescattering — Watson's theorem

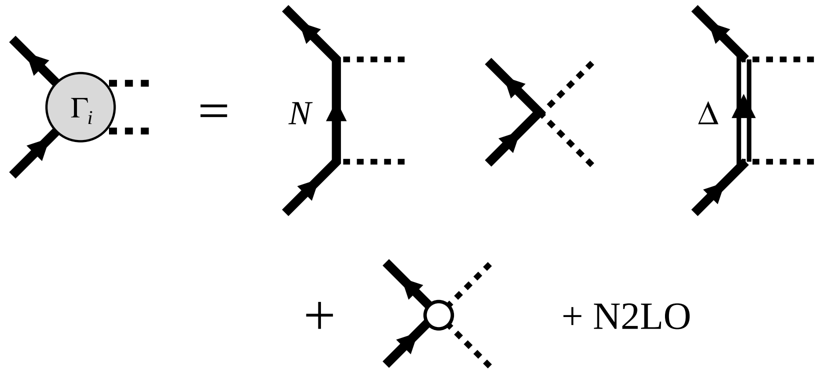
- Factorized representation (N/D method)

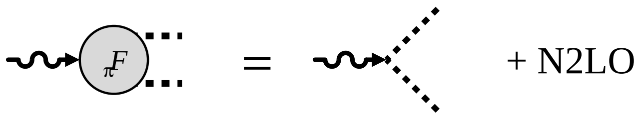
  Γ_i/F_π free of $\pi\pi$ rescattering → calculate in χEFT, well convergent

  $|F_\pi|^2$ includes $\pi\pi$ rescattering, ρ resonance → take from e^+e^- data, LQCD

- New χEFT-based approach

Method: Chiral EFT





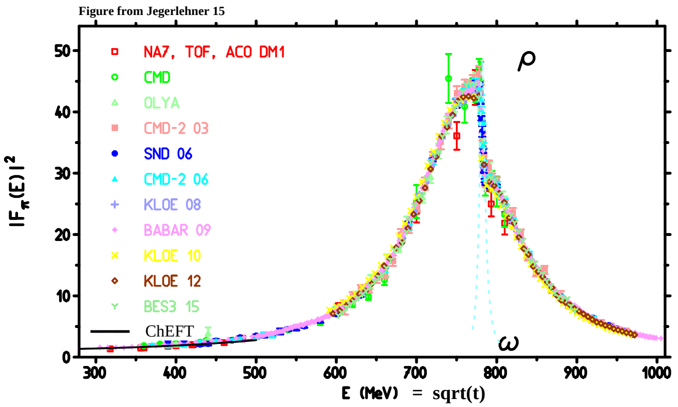
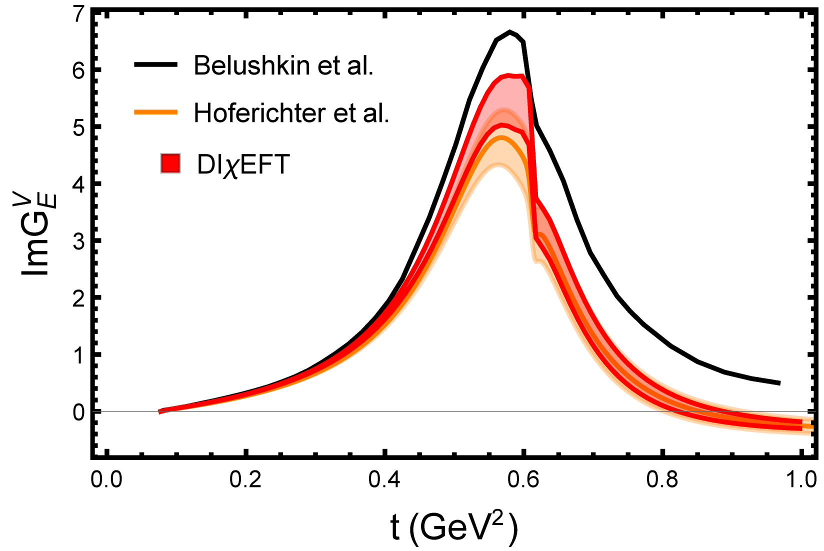
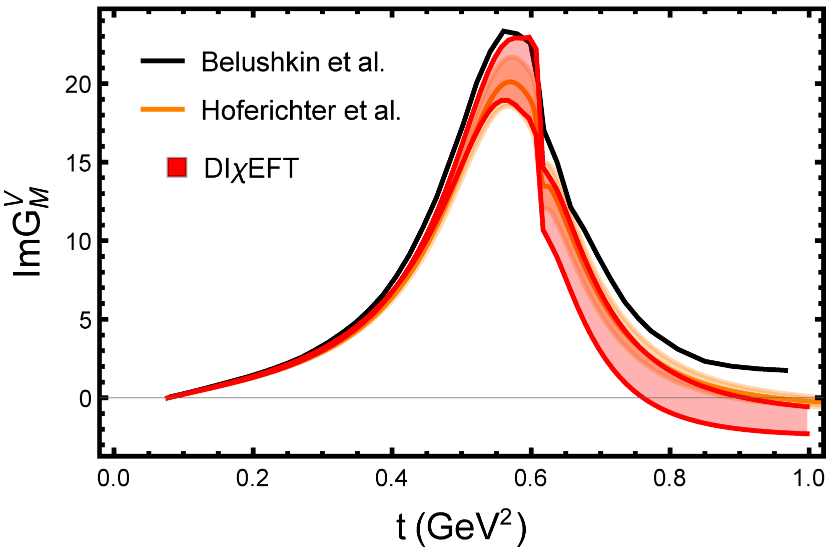


Figure from Jegerlehner 15

- Relativistic χEFT
  - Expansion in $\{M_\pi, k_\pi\}/\Lambda_\chi$
  - Include Δ isobar
- Calculation of $\Gamma_i(t)/F_\pi(t)$
  - LO: Born terms + Weinberg-Tomozawa
  - NLO: Contact term in $\Gamma_i(t)$
  - N2LO: Contact term and pion loops
  - Good convergence
- Pion timelike FF $|F_\pi(t)|^2$
  - Measured accurately in $e^+e^- \to \pi^+\pi^-$

Results: Spectral functions





- Spectral functions on $\pi\pi$ cut

  Include ρ resonance through $|F_\pi(t)|^2$

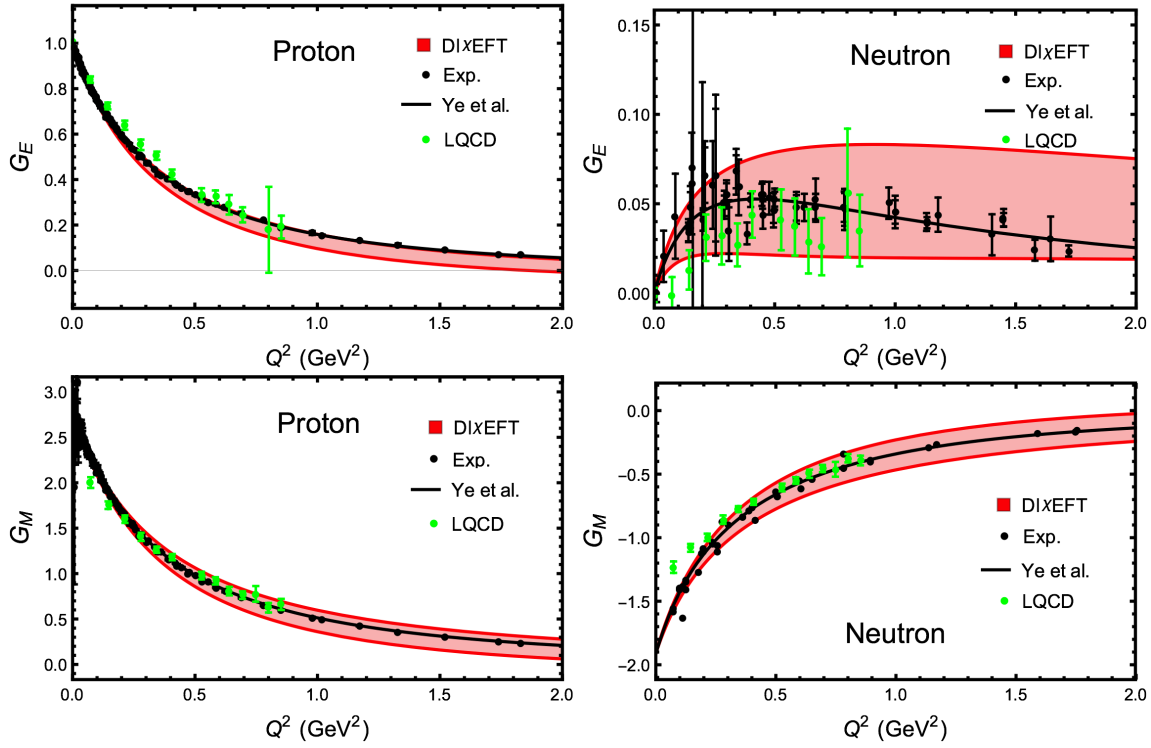
  Good agreement with Roy-Steiner analysis
  Hoferichter et al 2017

- Qualitative improvement compared to traditional χEFT

  $\pi\pi$ rescattering effects included

Alarcon, Weiss, arXiv:1803.09748

Results: Form factors



$$G_i(t) = \int_{4M_\pi^2}^{\infty} \frac{dt'}{\pi} \frac{\operatorname{Im} G_i(t')}{t' - t - i0}$$

Alarcon, Weiss, arXiv:1803.09748
Uncertainty bands: PDG range of nucleon radii

- Form factors evaluated using DR

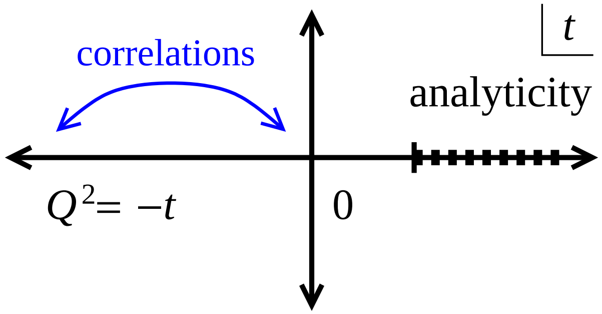
  $\pi\pi$ isovector spectral function calculated in DIχEFT

  High-mass states described by effective pole, strength fixed by sum rules (charges, radii)

- Excellent agreement with data

  Not fit, but dynamical prediction. Theoretical uncertainty estimates

Applications: Proton radius extraction



- Proton radius from electron scattering

  Data at $Q^2 > 0$ ↔ Slope at $Q^2 = 0$

  Several methods → review in this session

- Analyticity implies correlations

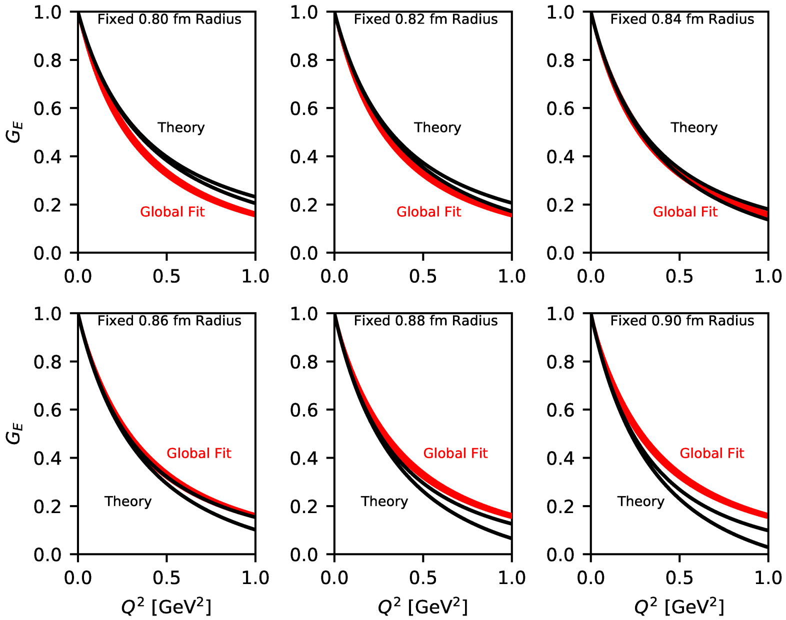
  Use data at "larger" $Q^2 \sim$ few 0.1 GeV^2 to constrain slope at $Q^2 = 0$

  Complement "extrapolation" methods

- DIχEFT-base extraction

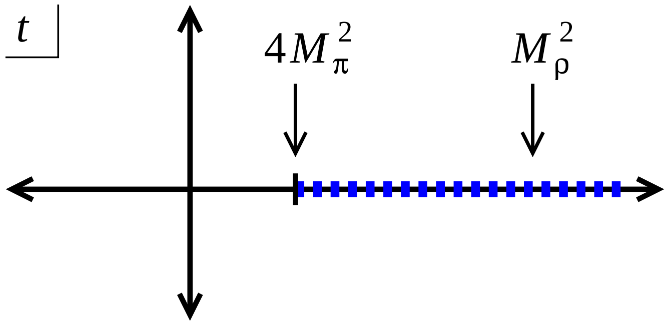
  Obtain $r_p = 0.85(1)$ fm (preliminary)

  Quantified thy and exp uncertainties



Zhihong Ye, Higinbotham, Alarcon, CW; in progress
Global fit with fixed radius adapted from Ye et al 2017

Applications: Higher derivatives



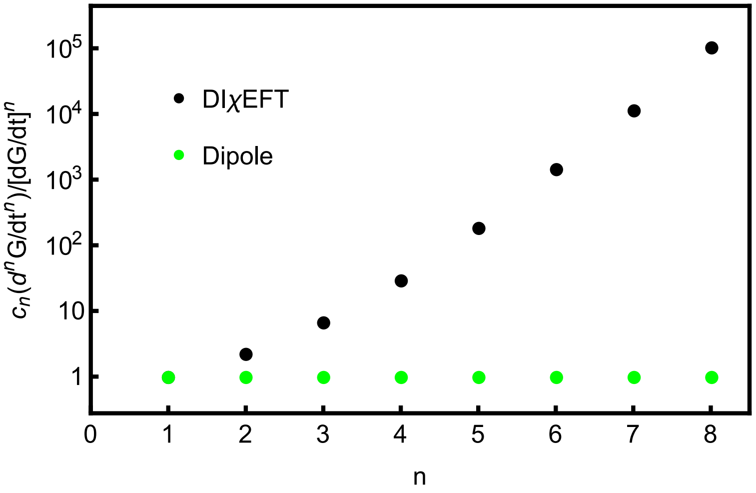
- Form factor derivatives from DR

$$\left.\frac{d^n G_i^V(t)}{dt^n}\right|_{t=0} = \int_{4M_\pi^2}^{\infty} \frac{dt'}{\pi} \frac{\mathrm{Im}\, G_i^V(t')}{t'^{n+1}}$$

- Two dynamical scales

$4M_\pi^2$	two-pion threshold
M_ρ^2	maximum of spectral function

Relative weight depends on n



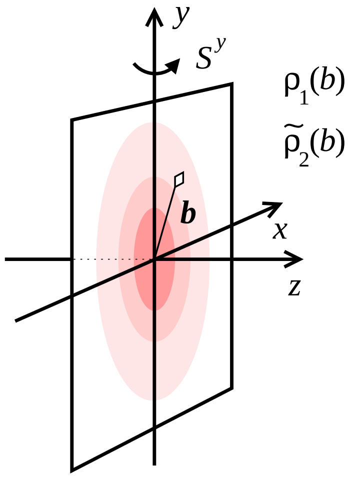
Alarcon, Weiss, PRC **97,** 055203 (2018)

- Unnatural size of higher derivatives

Model-independent prediction

Could be tested in polynomial fits

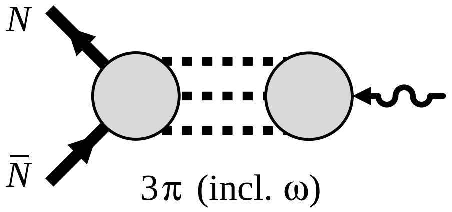
Applications: Densities, scalar FF



- Nucleon transverse charge/magnetization densities
  Alarcon, Hiller Blin, Vicente Vacas, Weiss, NPA **96,** 18 (2017); Alarcon, Weiss, in progress
- Nucleon scalar FF
  Alarcon, Weiss, PRC **96,** 055206 (2017)

Extensions

- FFs with 3π cut, e.g. isoscalar vector FF
  Use methods of 3-body unitarity, presently being developed



- Resonance transition FFs, e.g. $N \to \Delta$

- Methodological extension: Timelike pion FF $|F_\pi(t)|^2$ from Lattice QCD
  H. Meyer 2011

Summary

- DIχEFT new method for calculating $\pi\pi$ spectral functions of nucleon FFs
  - Uses elastic unitarity and N/D method
  - Includes $\pi\pi$ rescattering in t-channel through timelike pion FF
  - Overcomes main limitation of traditional χEFT
- Excellent description of spacelike nucleon EM FF data up to $Q^2 \sim 1$ GeV^2
  - Implements analyticity
  - Provides theoretical uncertainty estimates
- Proton radius extraction from moderate–Q^2 data
- Many applications