Introduction to the CKM Matrix

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$$\mathcal{L}_{\text{SM}} \sim \Lambda^4 + \Lambda^2 H^2 + \lambda H^4$$
$$+ \bar{\Psi} \not{D} \Psi + (D_\mu H)^2 + (F_{\mu\nu})^2 + F_{\mu\nu}\tilde{F}^{\mu\nu}$$
$$+ Y H\bar{\Psi}\Psi + \frac{1}{\Lambda}(LH)^2 + \frac{1}{\Lambda^2}\sum_i \mathcal{O}_i^{\text{dim6}}$$

Flavor in the SM and Beyond

Invariant under a SU(3)⁵ x U(1)⁵ flavor symmetry

$$\mathcal{L}_{\text{SM}} \sim \Lambda^4 + \Lambda^2 H^2 + \lambda H^4 + \bar{\Psi} \not{D} \Psi + (D_\mu H)^2 + (F_{\mu\nu})^2 + F_{\mu\nu} \tilde{F}^{\mu\nu}$$

$$+ Y H \bar{\Psi} \Psi + \frac{1}{\Lambda} (LH)^2 + \frac{1}{\Lambda^2} \sum_i \mathcal{O}_i^{\text{dim6}}$$

Invariant under a SU(3)^5 x U(1)^5 flavor symmetry

$$\mathcal{L}_{\rm SM} \sim \Lambda^4 + \Lambda^2 H^2 + \lambda H^4 + \bar{\Psi} \not{D} \Psi + (D_\mu H)^2 + (F_{\mu\nu})^2 + F_{\mu\nu}\tilde{F}^{\mu\nu}$$

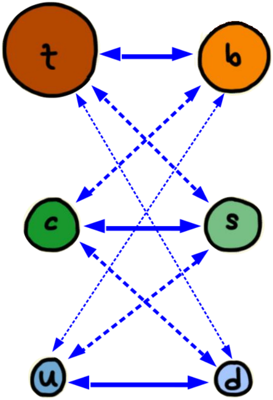
$$+ Y H \bar{\Psi} \Psi + \frac{1}{\Lambda}(LH)^2 + \frac{1}{\Lambda^2}\sum_i \mathcal{O}_i^{\rm dim6}$$

Only renormalizable source of flavor breaking

CKM matrix is the “remnant” of diagonalizing the Yukawa couplings

CKM in the SM

The CKM Matrix



Wolfenstein parametrization (defined to hold in all orders in λ and rephasing invariant)

$$\lambda^2 = \frac{|V_{us}|^2}{|V_{us}|^2 + |V_{ud}|^2}$$

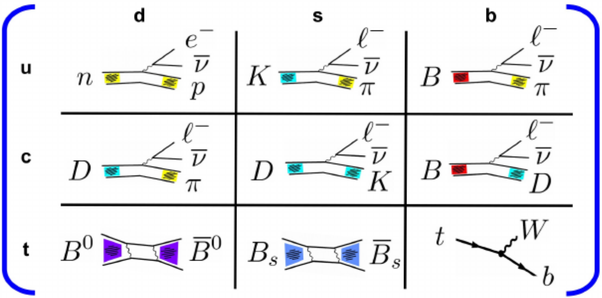
$$A^2\lambda^2 = \frac{|V_{cb}|^2}{|V_{us}|^2 + |V_{ud}|^2}$$

$$\bar\rho + i\bar\eta = -\frac{V_{ud} V_{ub}^*}{V_{cd} V_{cb}^*}$$

$$V_{\rm CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\bar\rho - i\bar\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\bar\rho-i\bar\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4)$$

Measuring CKM Matrix Elements

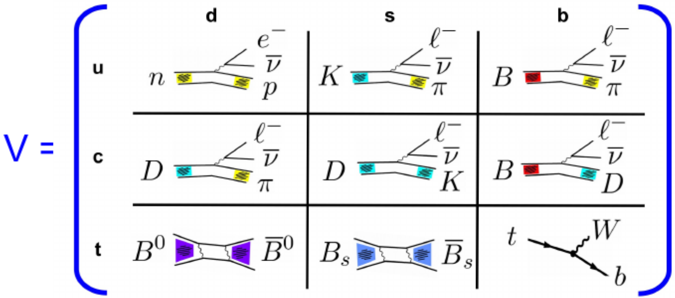
V =



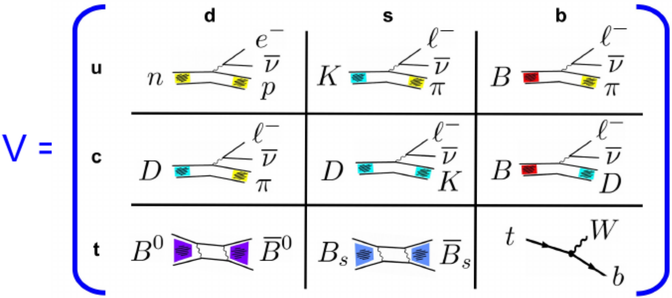
$|V_{ud}|$: from nuclear beta decay

see talks by J.C. Hardy and Kyle Leach (today 5:30pm, TSEI session)

Measuring CKM Matrix Elements



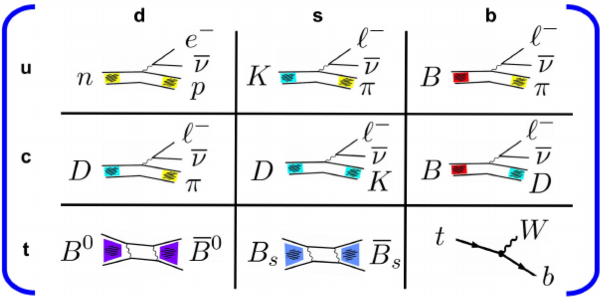
$|V_{us}|$: from from $K \to \pi \ell \nu$ branching ratios
+ $K \to \pi$ form factors from lattice



$|V_{us}/V_{ud}|$: from $K \to \mu\nu$ over $\pi \to \mu\nu$
+ ratio of Kaon and pion decay constants from lattice

Measuring CKM Matrix Elements

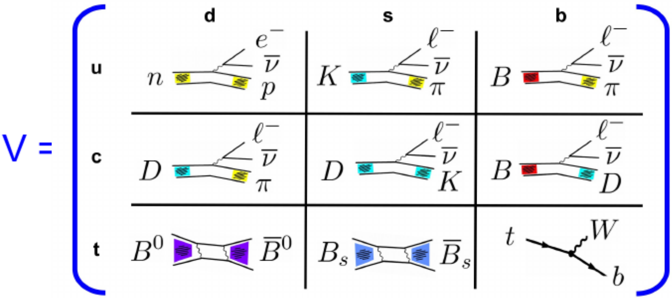
V =



$|V_{cd}|$: from $D \to \pi \ell \nu$ or $D \to \mu \nu$ decays
+ lattice input (form factors and decay constants)

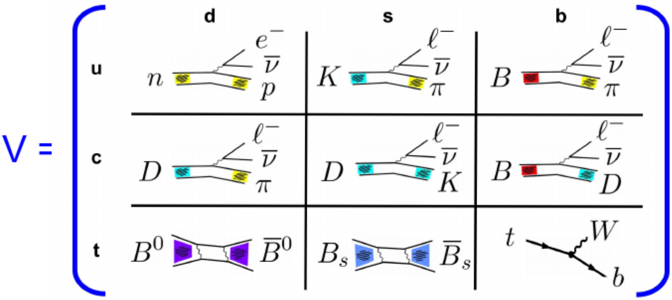
see talk by Carleton DeTar, (Friday 2:30pm, HFCKM session)

Measuring CKM Matrix Elements



$|V_{cs}|$: from $D \to K\ell\nu$ or $D_s \to \ell\nu$ decays
+ lattice input (form factors and decay constants)
see talk by Carleton DeTar, (Friday 2:30pm, HFCKM session)

Measuring CKM Matrix Elements

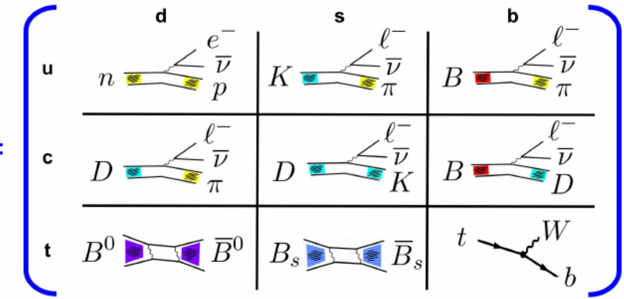


$|V_{cb}|$: from inclusive $B \to X_c \ell\nu$ or
exclusive $B \to D^{(*)}\ell\nu$, $B_s \to D_s \ell\nu$ + form factors

see talks by Matic Lubej (this session) + Oliver Witzel (Friday 3pm, HFCKM session)
+ plenary by Abi Soffer earlier today

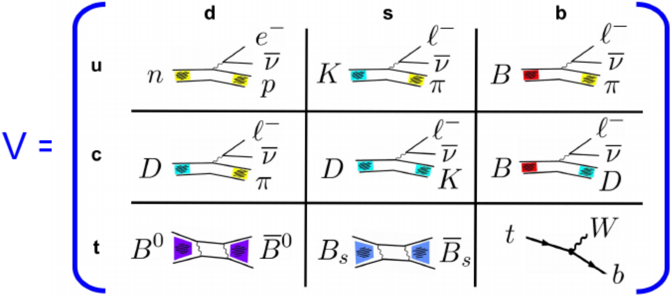
Measuring CKM Matrix Elements

V =



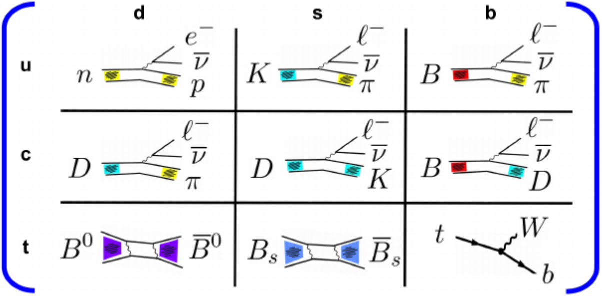
$|V_{ub}|$: from inclusive $B \to X_u \ell \nu$ or
exclusive $B \to \pi \ell \nu$, $B_s \to K \ell \nu$ + form factors

see talks by Matic Lubej (this session) + Oliver Witzel (Friday 3pm, HFCKM session)
+ plenary by Abi Soffer earlier today



$|V_{ub}/V_{cb}|$: from $\Lambda_b \to p\mu\nu$ over $\Lambda_b \to \Lambda_c\mu\nu$
+ form factor ratios from lattice

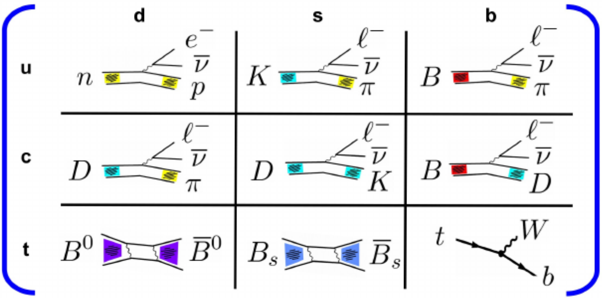
V =



$|V_{ts}|$ and $|V_{td}|$: from meson oscillations
+ hadronic matrix elements from lattice

Measuring CKM Matrix Elements

V =

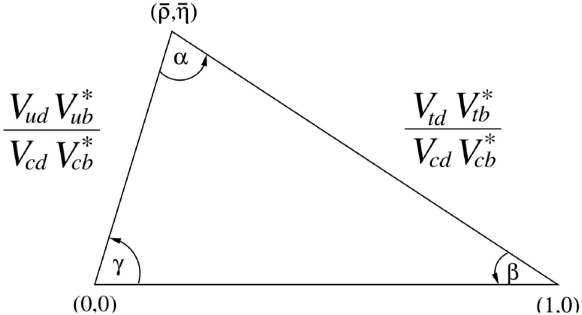


$|V_{tb}|$: from $t \to Wb$ or single top production

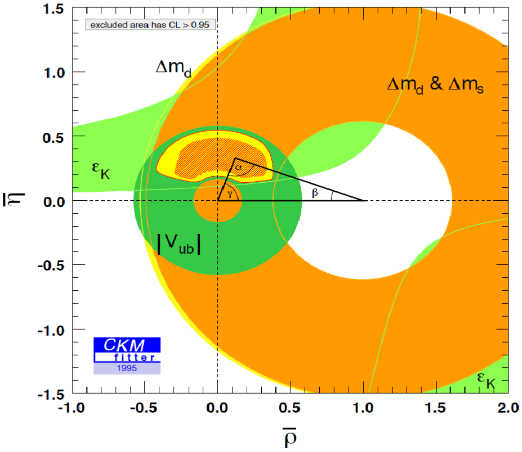
Graphical representation of the unitarity of the CKM matrix:

Unitary Triangles

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$

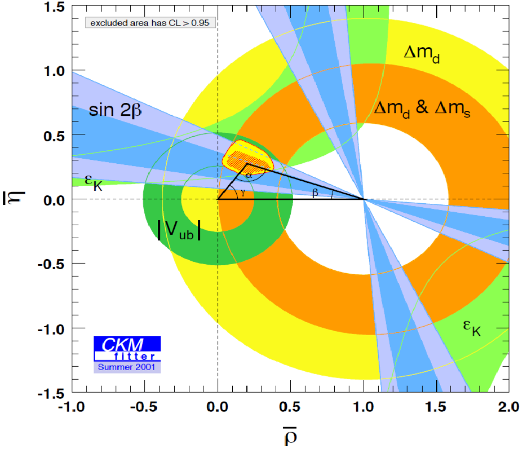


Two Decades of Unitarity Triangle Fits



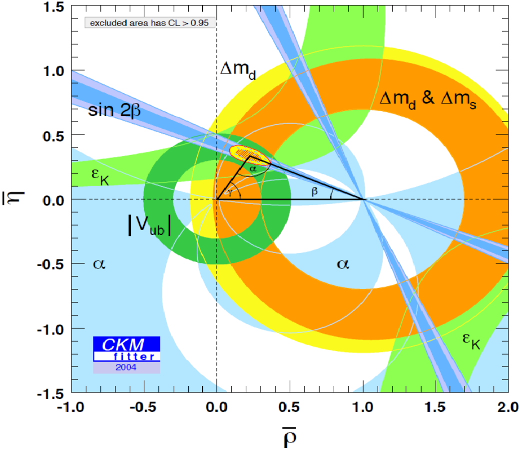
1995

Two Decades of Unitarity Triangle Fits



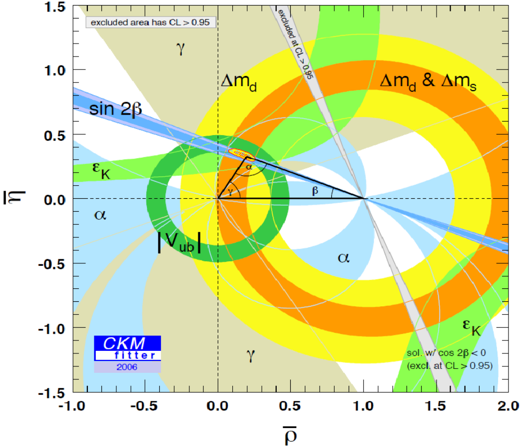
2001

Two Decades of Unitarity Triangle Fits

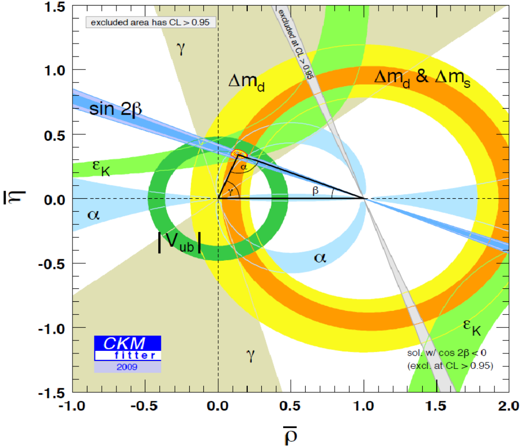


2004

Two Decades of Unitarity Triangle Fits

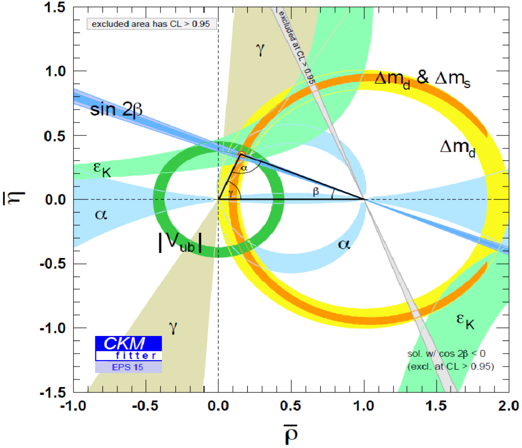


2006


excluded area has CL > 0.95
excluded at CL > 0.95
γ
Δm_d
Δm_d & Δm_s
sin 2β
ε_K
α
β
|V_ub|
sol. w/ cos 2β < 0
(excl. at CL > 0.95)
CKM fitter
2009
η̄
ρ̄
1.5
1.0
0.5
0.0
-0.5
-1.0
-1.5
2.0

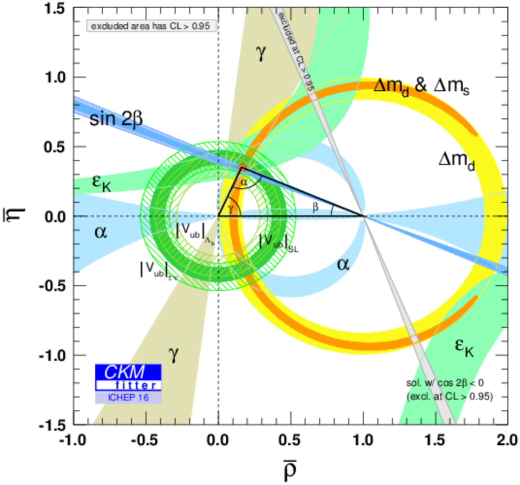
2009

Two Decades of Unitarity Triangle Fits



2015

Latest Results from CKMfitter



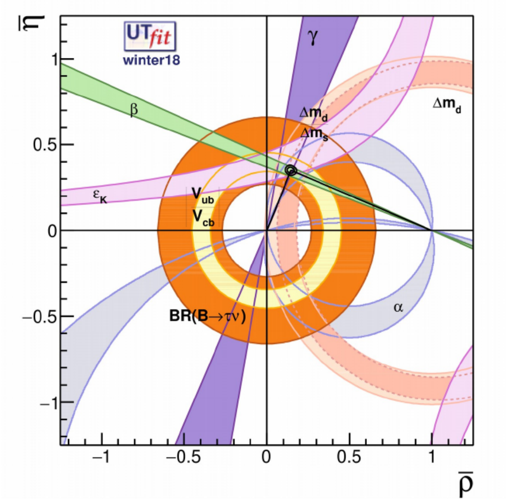
(http://ckmfitter.in2p3.fr/)

$$\lambda = 0.2251 \pm 0.0003$$

$$A = 0.825^{+0.007}_{-0.011}$$

$$\bar{\rho} = 0.160^{+0.008}_{-0.007}$$

$$\bar{\eta} = 0.350 \pm 0.006$$



(Luca Silvestrini @ LaThuile 2018 + http://www.utfit.org)

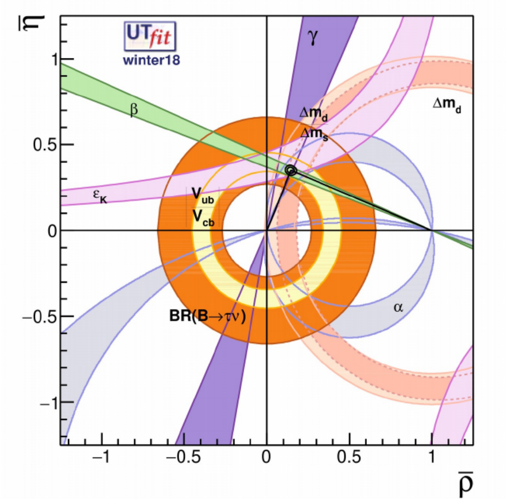
$$\lambda = 0.2250 \pm 0.0007$$

$$A = 0.833 \pm 0.012$$

$$\bar{\rho} = 0.145 \pm 0.014$$

$$\bar{\eta} = 0.349 \pm 0.010$$

Latest Results from UTfit



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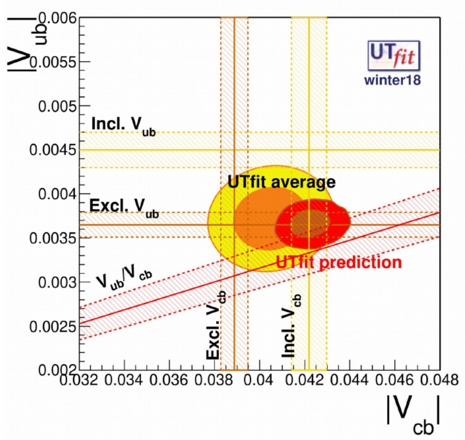
(Luca Silvestrini @ LaThuile 2018 + http://www.utfit.org)

for future prospects at Belle II see talk by Anselm Vossen (this session)

V_{ub} and V_{cb}: Inclusive vs. Exclusive



(Luca Silvestrini @ LaThuile 2018)

$$|V_{ub}|_{\text{incl}} = (4.50 \pm 0.20) \times 10^{-3}$$

$$|V_{ub}|_{\text{excl}} = (3.65 \pm 0.14) \times 10^{-3}$$

$$|V_{cb}|_{\text{incl}} = (42.19 \pm 0.78) \times 10^{-3}$$

$$|V_{cb}|_{\text{excl}} = (38.88 \pm 0.60) \times 10^{-3}$$

$$|V_{ub}/V_{cb}| = (7.90 \pm 0.57) \times 10^{-2}$$

"skeptical 2D combination"

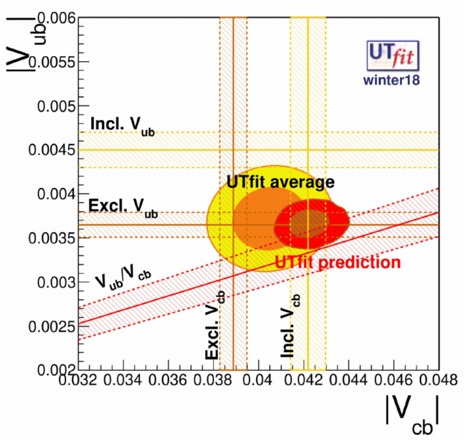
$$|V_{ub}| = (3.74 \pm 0.23) \times 10^{-3}$$

$$|V_{cb}| = (40.5 \pm 1.1) \times 10^{-3}$$

$$\rho = 0.09$$

see talks by Matic Lubej (this session) + Oliver Witzel (Friday 3pm, HFCKM session)
+ plenary by Abi Soffer earlier today

V_{ub} and V_{cb}: Inclusive vs. Exclusive



(Luca Silvestrini @ LaThuile 2018)

- Heavy Quark relations used in CLN parametrization of $B \to D^*$ form factors might be driving the discrepancy.

  BGL parametrization gives larger $|V_{cb}|$.
  Grinstein, Kobach 1703.08170
  Bigi, Gambino, Schacht arXiv:1703.06124

- Relaxing Heavy Quark relations appears to be in tension with lattice form factors at non-zero recoil. Bernlochner, Ligeti, Papucci, Robinson 1708.07134

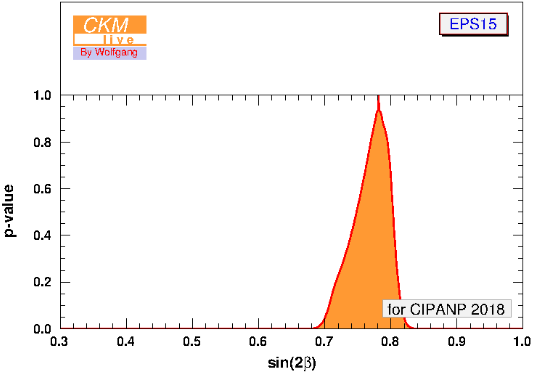
see talks by Matic Lubej (this session) + Oliver Witzel (Friday 3pm, HFCKM session)
+ plenary by Abi Soffer earlier today

perform your own CKM fit at http://ckmlive.in2p3.fr/

e.g. Standard Model prediction for $\sin 2\beta$

$$(\sin 2\beta)_{\rm SM} =$$

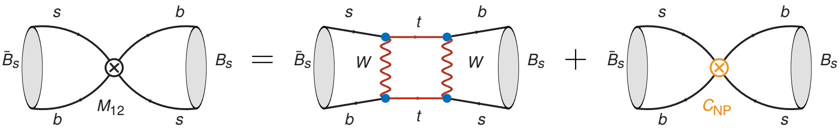
$$= 0.781^{+0.027}_{-0.050}$$



for joint BaBar/Belle measurement see talk by Tomonari Miyashita (this session)

New Physics

Basic Philosophy

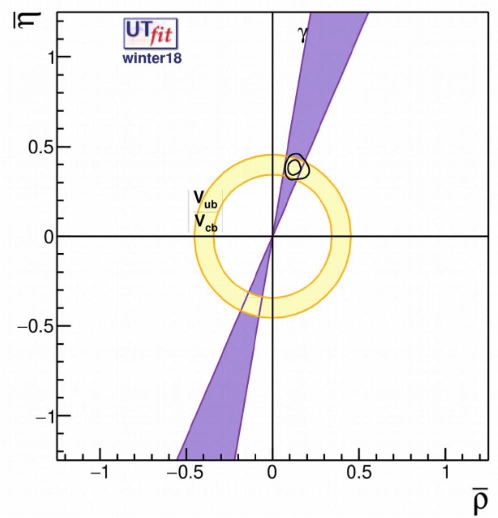


$$M_{12} \sim G_F \frac{g_2^2}{16\pi^2} \frac{m_t^2}{m_W^2} (V_{tb} V_{ts}^*)^2 + \frac{C_{NP}}{\Lambda_{NP}^2}$$

measure precisely

calculate precisely the SM contribution

get information on NP coupling and scale



(Luca Silvestrini @ LaThuile 2018)

New Physics Fit

$\bar{\rho} = 0.125 \pm 0.025$

$\bar{\eta} = 0.381 \pm 0.028$

Standard Model Fit

$\bar{\rho} = 0.145 \pm 0.014$

$\bar{\eta} = 0.349 \pm 0.010$

Phenomenological Parametrization of New Physics

$$\frac{\langle B_{d,s}|H_{\rm eff}|\bar{B}_{d,s}\rangle}{\langle B_{d,s}|H_{\rm eff}^{\rm SM}|\bar{B}_{d,s}\rangle} = C_{B_{d,s}} e^{2i\phi_{B_{d,s}}} = 1 + \frac{A_{d,s}^{\rm NP}}{A_{d,s}^{\rm SM}} e^{2i\phi_{d,s}^{\rm NP}}$$

$$\frac{{\rm Re}\langle K|H_{\rm eff}|\bar{K}\rangle}{{\rm Re}\langle K|H_{\rm eff}^{\rm SM}|\bar{K}\rangle} = C_{\Delta M_K} \quad , \quad \frac{{\rm Im}\langle K|H_{\rm eff}|\bar{K}\rangle}{{\rm Im}\langle K|H_{\rm eff}^{\rm SM}|\bar{K}\rangle} = C_{\epsilon_K}$$

Observables are modified: e.g.

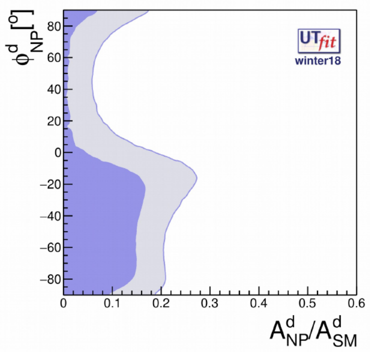
$$(\Delta M_{d,s})_{\rm exp} = C_{B_{d,s}} (\Delta M_{d,s})_{\rm SM} \quad , \quad (\Delta M_K)_{\rm exp} = C_{\Delta M_K} (\Delta M_K)_{\rm SM}$$

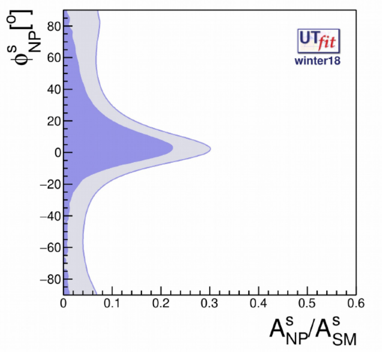
$$(\sin 2\beta)_{\rm exp} = \sin(2\beta + 2\phi_{B_d}) \quad , \quad (\phi_s)_{\rm exp} = \beta_s - \phi_{B_s}$$

$$(\epsilon_K)_{\rm exp} = C_{\epsilon_K} (\epsilon_K)_{\rm SM}$$

...

Constraints on the New Physics Amplitude





(Luca Silvestrini @ LaThuile 2018)

few % - 30% NP contribution allowed in meson mixing
(depending on the NP phase)

Parametrization in Terms of Contact Interactions

4 fermion contact interactions contributing to meson mixing

$$\frac{C_1}{\Lambda^2}(\bar{q}\gamma_\mu P_L q')(\bar{q}\gamma^\mu P_L q')\,, \quad \frac{\tilde{C}_1}{\Lambda^2}(\bar{q}\gamma_\mu P_R q')(\bar{q}\gamma^\mu P_R q')$$

$$\frac{C_2}{\Lambda^2}(\bar{q}P_L q')(\bar{q}P_L q')\,, \quad \frac{\tilde{C}_2}{\Lambda^2}(\bar{q}P_R q')(\bar{q}P_R q')$$

$$\frac{C_3}{\Lambda^2}(\bar{q}_\alpha P_L q'_\beta)(\bar{q}_\beta P_L q'_\alpha)\,, \quad \frac{\tilde{C}_3}{\Lambda^2}(\bar{q}_\alpha P_R q'_\beta)(\bar{q}_\beta P_R q'_\alpha)$$

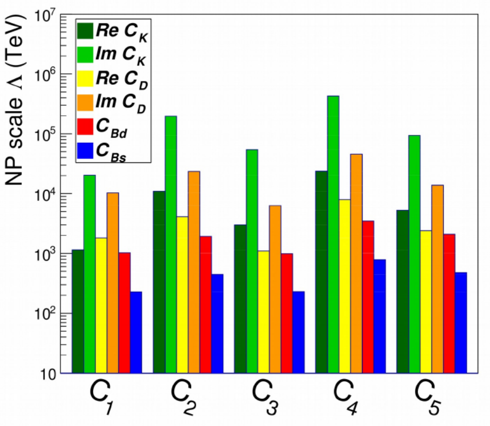
$$\frac{C_4}{\Lambda^2}(\bar{q}P_L q')(\bar{q}P_R q')$$

$$\frac{C_5}{\Lambda^2}(\bar{q}_\alpha P_L q'_\beta)(\bar{q}_\beta P_R q'_\alpha)$$

need hadronic matrix element from lattice to determine NP parameters

see talk by Chia Cheng Chang (Friday 2:00pm, HFCKM session)

Constraints on the New Physics Scale



(Luca Silvestrini @ LaThuile 2018)

- Best bound from ϵ_K (limited by CKM uncertainty)
- CPV in charm follows (limited by exp. uncertainty)
- Then B_d and B_s mixing (CKM + hadronic uncertainties)
- For NP couplings $C_i \sim 1$ bounds on the NP scale up to 100,000 TeV!

Summary

- The SM CKM picture of flavor and CP violation is consistent at the O(10%) level
- NP in meson mixing can in some cases be probed up to scales of 100,000 TeV
- Strong constraints on many “flavorful” NP models at the TeV scale