Collective neutrino oscillations and nucleosynthesis

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Neutrino mixing, 3 active flavors

$$|\nu_{flavor}\rangle = T|\nu_{mass}\rangle$$

$$T = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{atmospheric\ neutrinos} \underbrace{\begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix}}_{reactor\ neutrinos} \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{solar\ neutrinos}$$

$$c_{ij} = \cos\theta_{ij} \quad s_{ij} = \sin\theta_{ij}$$

$$P(\nu_e \to \nu_e) = 1 - \sin^2 2\theta_{13}\left[\cos^2\theta_{12}\sin^2(\Delta_{31}L) + \sin^2\theta_{12}\sin^2(\Delta_{32}L)\right] - \cos^4\theta_{13}\sin^2 2\theta_{12}\sin^2(\Delta_{21}L)$$

$$\Delta_{ij} = \frac{\delta m_{ij}^2}{4E_\nu} = \frac{m_i^2 - m_j^2}{4E_\nu}, \qquad \Delta_{32} = \Delta_{31} - \Delta_{21}$$

Neutrinos in matter: the MSW Effect

In vacuum: $E^2 = \mathbf{p}^2 + m^2$

In matter:

$(E - V)^2 = (\mathbf{p} - \mathbf{A})^2 + m^2$

$\Rightarrow E^2 = \mathbf{p}^2 + m^2_{\text{eff}}$

V $\propto$ background density

$\mathbf{A} \propto \mathbf{J}_{\text{background}}$ (currents) or

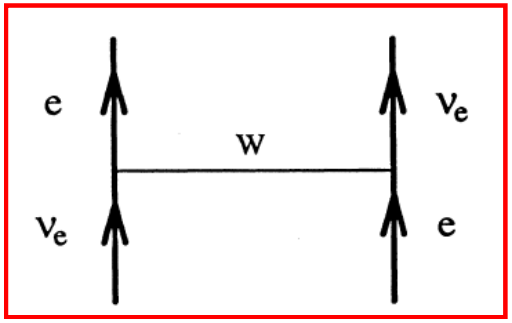
$\mathbf{A} \propto \mathbf{S}_{\text{background}}$ (spin)

In the limit of static, charge-neutral, and unpolarized background

V$\propto N_e$ and $\mathbf{A} = 0$

$\Rightarrow m^2_{\text{eff}} = m^2 + 2EV + \mathcal{O}(V^2)$

The potential is provided by the coherent forward scattering of ν_e's off the electrons in dense matter



There is a similar term with Z-exchange. But since it is the same for all neutrino flavors *at the tree level*, it does not contribute to phase differences *unless* we invoke sterile neutrinos.

Note the fine print!

Matter effects

$$i\frac{\partial}{\partial t}\begin{pmatrix}\psi_e\\ \psi_\mu\\ \psi_\tau\end{pmatrix}=\left[T\begin{pmatrix}E_1 & 0 & 0\\ 0 & E_2 & 0\\ 0 & 0 & E_3\end{pmatrix}T^\dagger+\begin{pmatrix}V_c+V_n & 0 & 0\\ 0 & V_n & 0\\ 0 & 0 & V_n\end{pmatrix}\right]\begin{pmatrix}\psi_e\\ \psi_\mu\\ \psi_\tau\end{pmatrix}$$

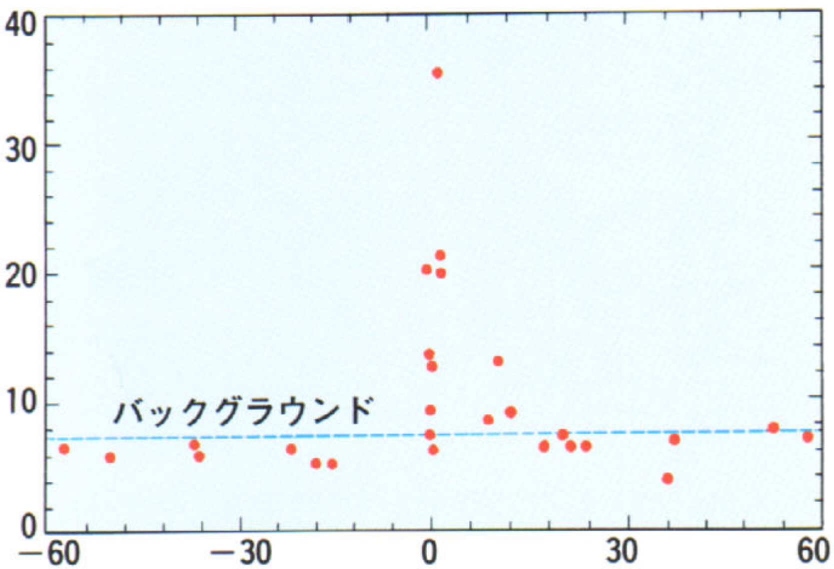
$$V_c=\sqrt{2}G_F N_e$$

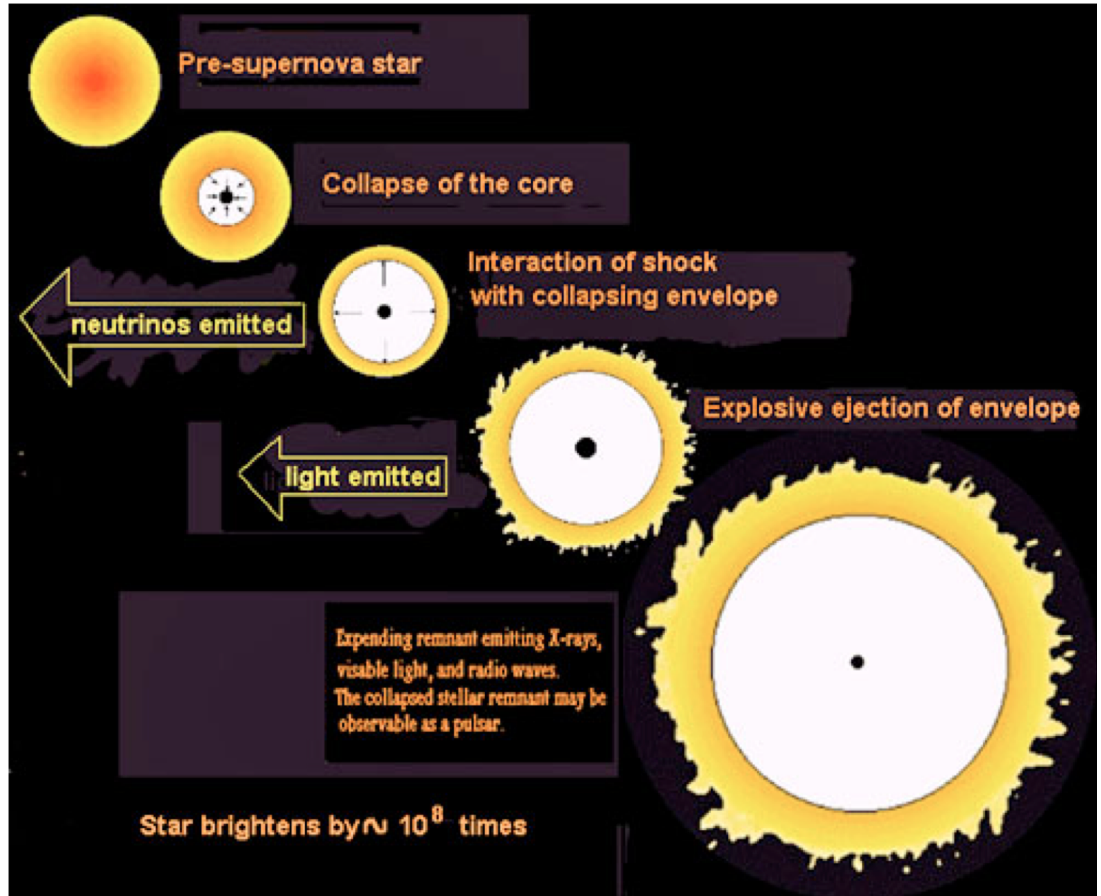
$$V_n=-\frac{1}{\sqrt{2}}G_F N_n$$

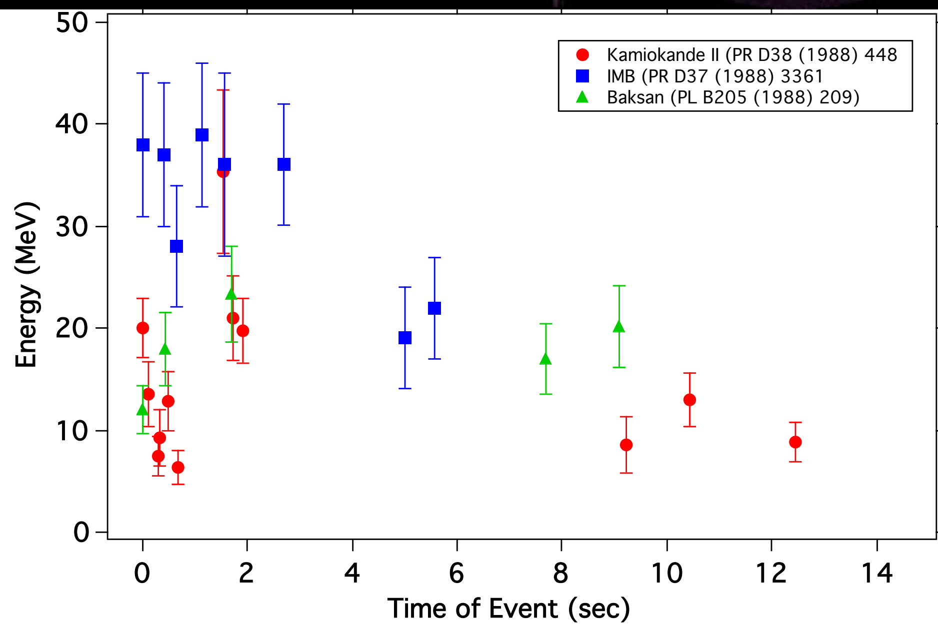
Two-flavor limit

$$H=\sum\left[\left(-\frac{\delta m^2}{4E_\nu}\cos 2\theta+\frac{1}{\sqrt{2}}G_F N_e\right)\left(a_e^\dagger a_e-a_\mu^\dagger a_\mu\right)+\frac{\delta m^2}{4E_\nu}\sin 2\theta\left(a_e^\dagger a_\mu+a_\mu^\dagger a_e\right)\right]$$

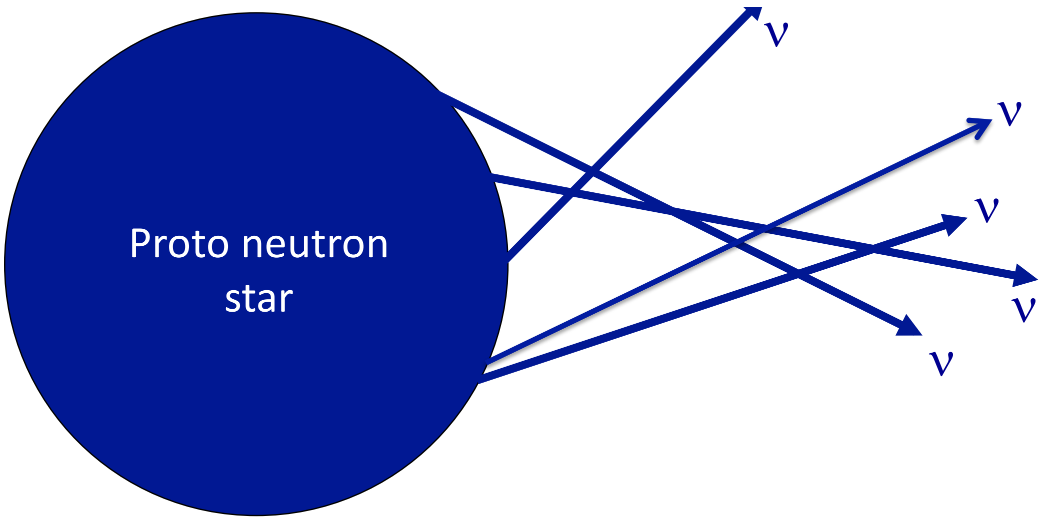
Neutrinos from core-collapse supernovae







- $M_{prog} \geq 8\ M_{sun} \Rightarrow \Delta E \approx 10^{53}$ ergs $\approx 10^{59}$ MeV
- 99% of the energy is carried away by neutrinos and antineutrinos with $10 \leq E_\nu \leq 30$ MeV $\Rightarrow 10^{58}$ neutrinos.
- We cannot ignore the interactions between these neutrinos!



Energy released in a core-collapse SN: $\Delta E \approx 10^{53}$ ergs $\approx 10^{59}$ MeV
99% of this energy is carried away by neutrinos and antineutrinos!
~ 10^{58} Neutrinos!
This necessitates including the effects of $\nu\nu$ interactions!

$$H = \underbrace{\sum a^{\dagger} a}_{\text{describes neutrino oscillations interaction with matter (MSW effect)}} + \underbrace{\sum (1-\cos\theta) a^{\dagger} a^{\dagger} a a}_{\text{describes neutrino-neutrino interactions}}$$

The second term makes the physics of a neutrino gas in a core-collapse supernova a very interesting many-body problem, driven by weak interactions.

Neutrino-neutrino interactions lead to novel collective and emergent effects, such as conserved quantities and interesting features in the neutrino energy spectra (spectral "swaps" or "splits").

Many neutrino system

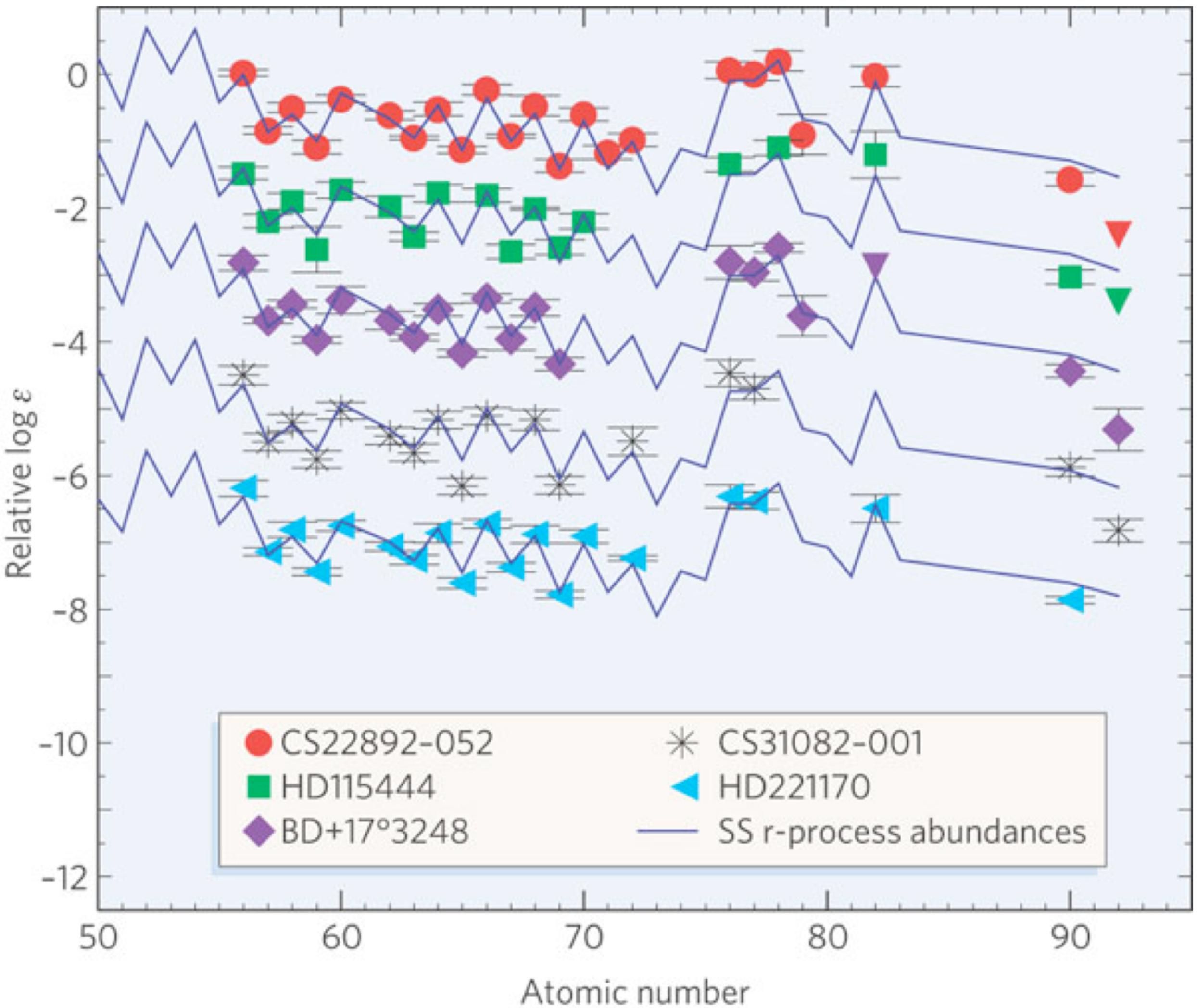
This is the only many-body system driven by the weak interactions:

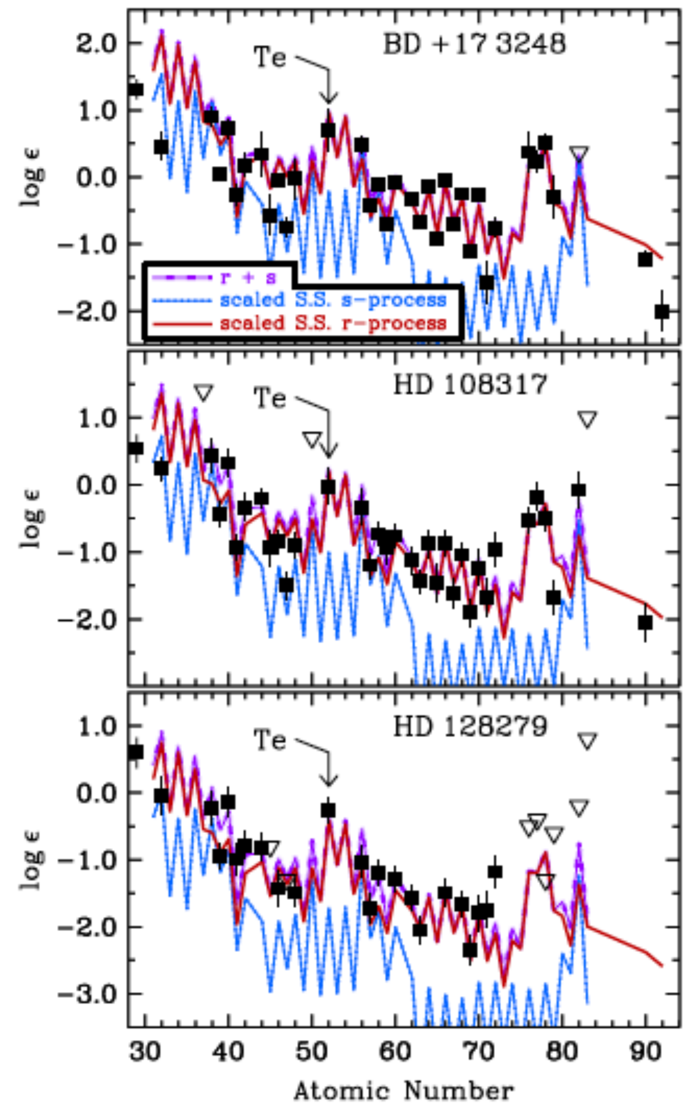
Table: Many-body systems

Nuclei	Strong	at most ~250 particles
Condensed matter	E&M	at most N_A particles
ν**'s in SN**	Weak	$\sim 10^{58}$particles

Astrophysical extremes allow us to test physics that cannot be tested elsewhere!

r-process nucleosynthesis

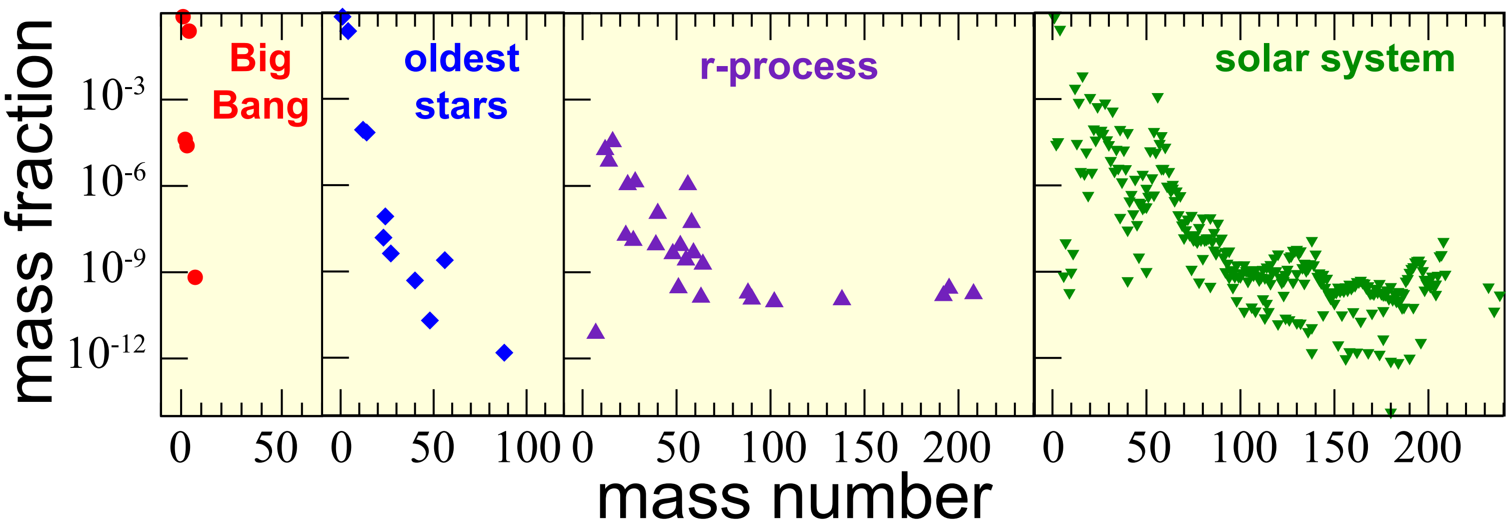


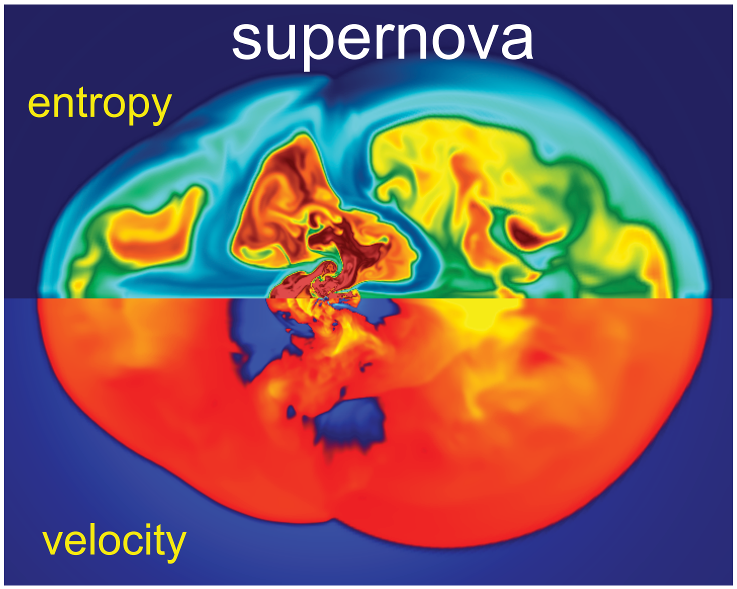


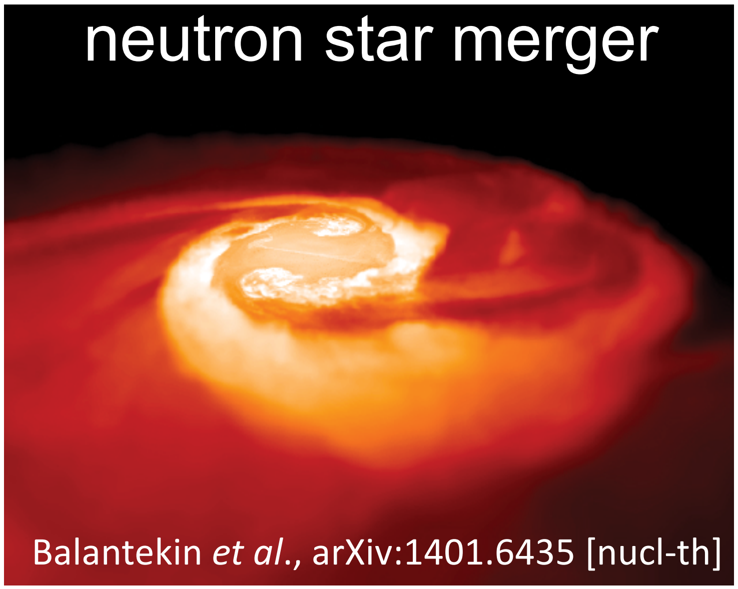
A > 100 abundance pattern fits the solar abundances well.

Roederer *et al.*, Ap. J. Lett. **747**, L8 (2012)

The origin of elements



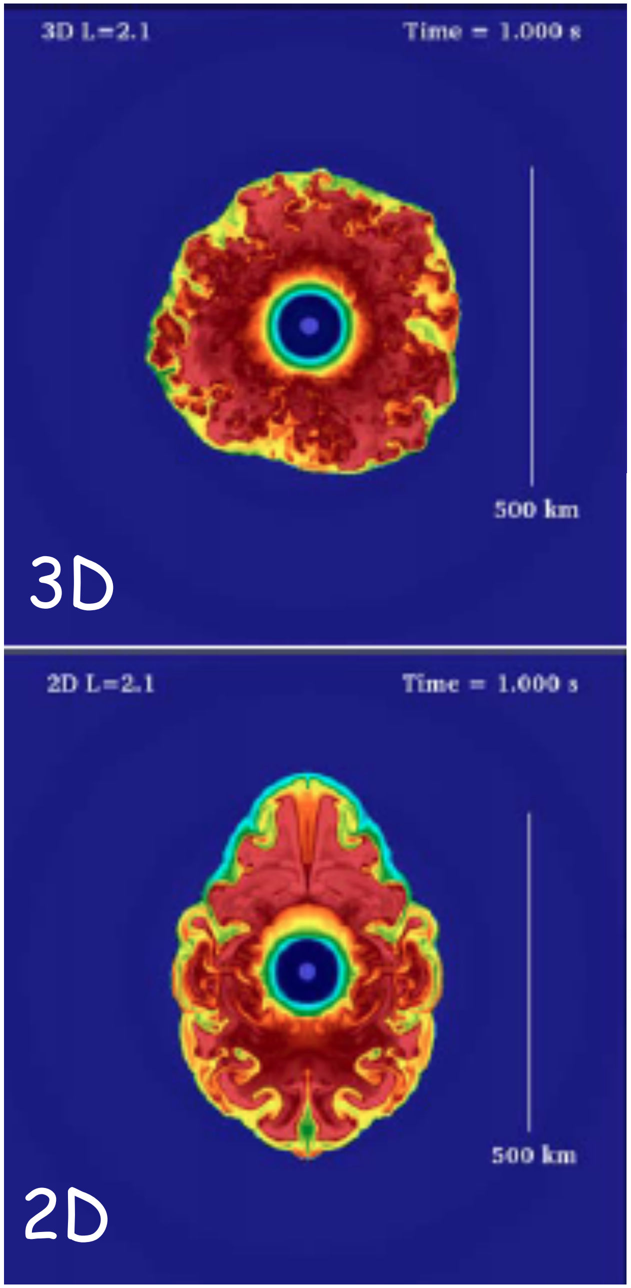




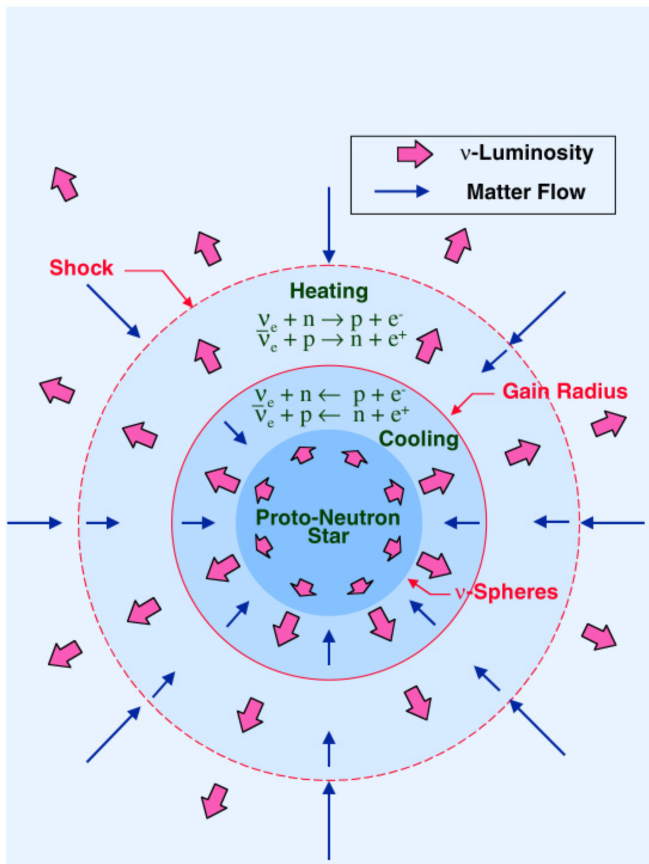
Possible sites for the r-process

Neutrinos not only play a crucial role in the dynamics of these sites, but they also control the value of the electron fraction, the parameter determining the yields of the r-process.

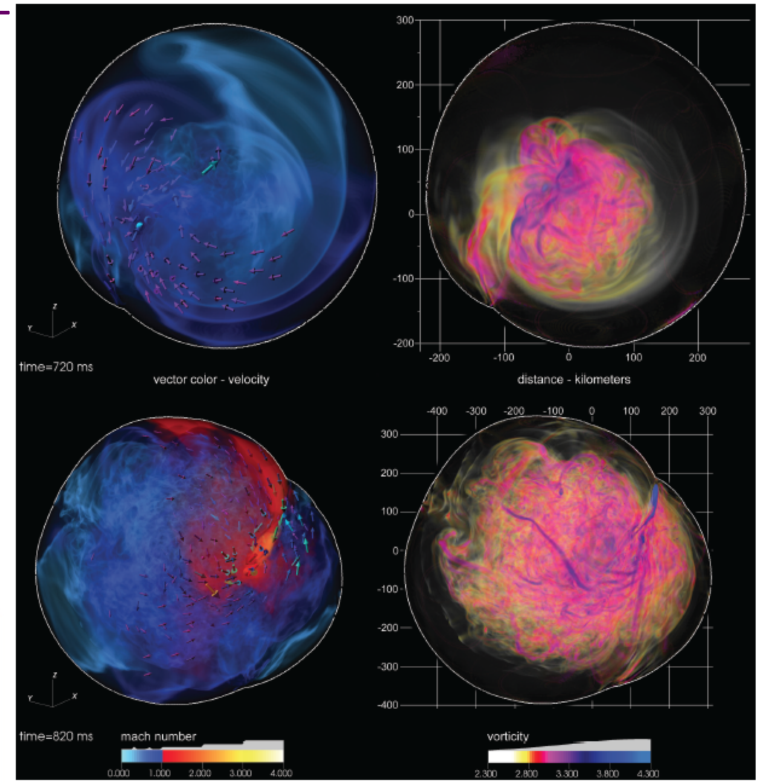
Development of 2D and 3D models for core-collapse supernovae: Complex interplay between turbulence, neutrino physics and thermonuclear reactions.

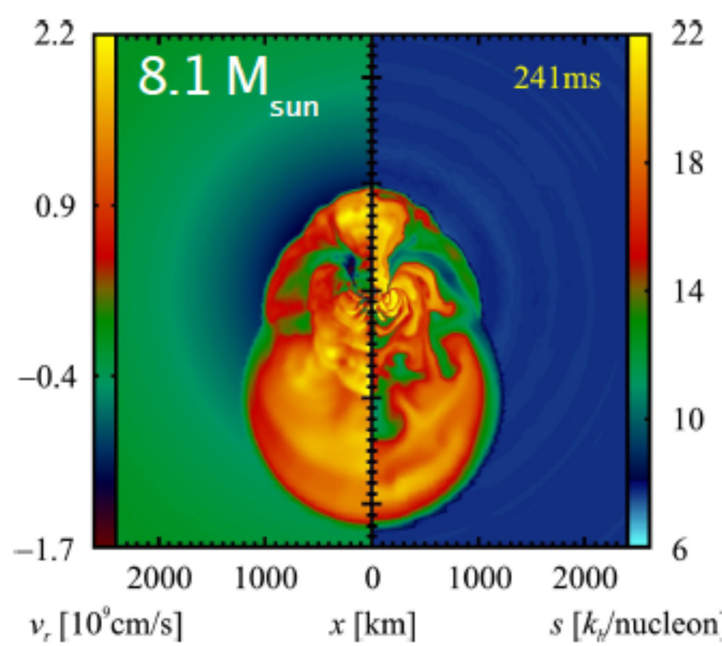


Princeton



ORNL/UT



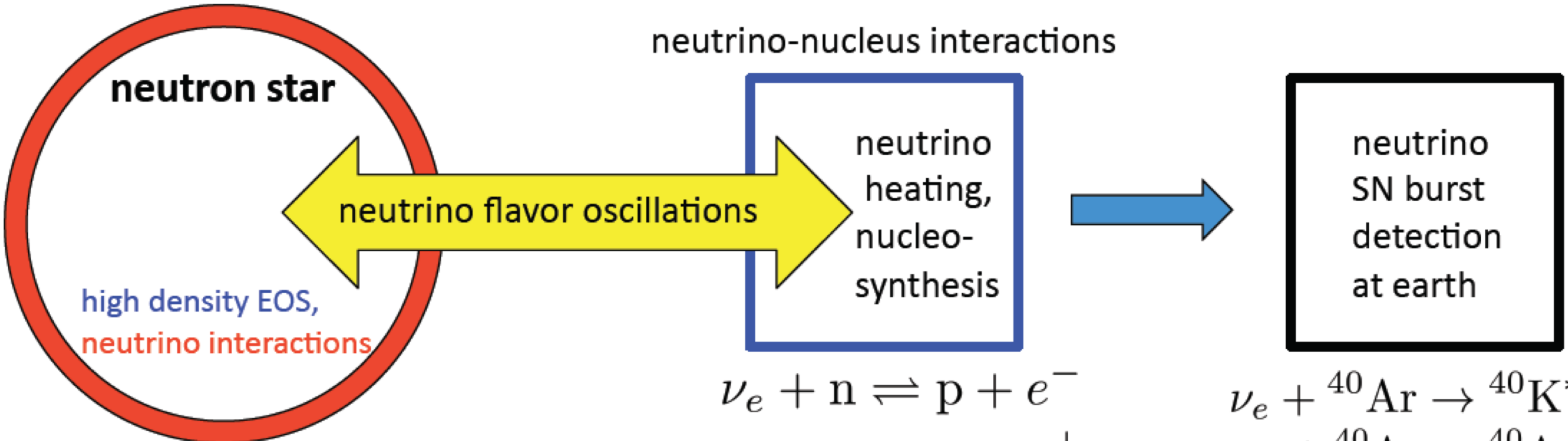


Munich

If we want to catch a supernova with neutrinos we'd better know what neutrinos do inside a supernova.

Understanding a core-collapse supernova and the nucleosynthesis it may host requires answers to a variety of questions!

Balantekin and Fuller, Prog. Part. Nucl. Phys. **71** 162 (2013)

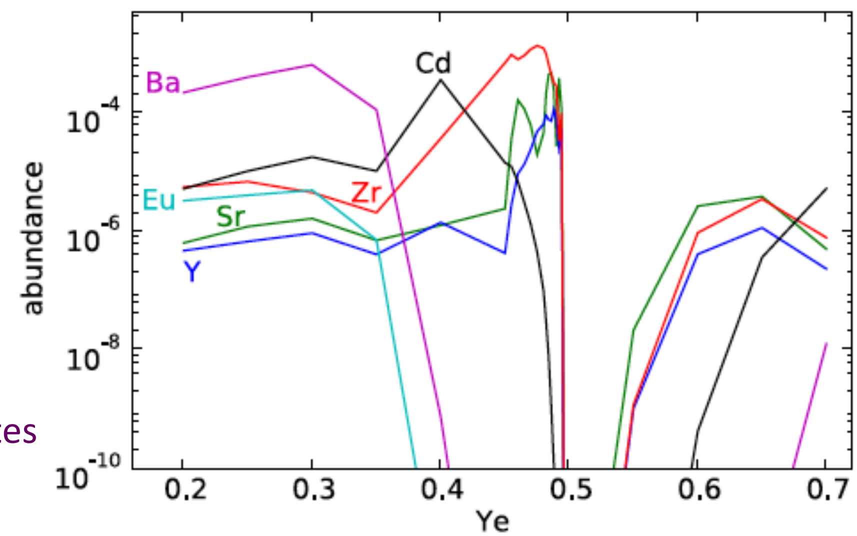


$$\nu_e + \mathrm{n} \rightleftharpoons \mathrm{p} + e^-$$
$$\bar{\nu}_e + \mathrm{p} \rightleftharpoons \mathrm{n} + e^+$$

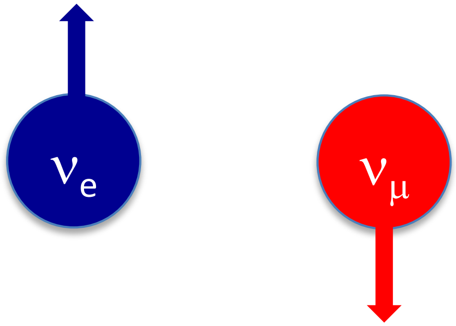
$$\nu_e + {}^{40}\mathrm{Ar} \rightarrow {}^{40}\mathrm{K}^* + e^-$$
$$\nu_x + {}^{40}\mathrm{Ar} \rightarrow {}^{40}\mathrm{Ar}^* + \nu_x$$

$$Y_e = \frac{N_p}{N_p + N_n} = \frac{1}{1 + \lambda_p / \lambda_n}$$

Arcones and Montes



Neutrino flavor isospin

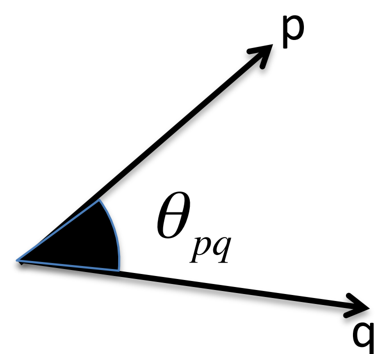


$$\hat{J}_+ = a_e^\dagger a_\mu \qquad \hat{J}_- = a_\mu^\dagger a_e$$

$$\hat{J}_0 = \frac{1}{2}\left(a_e^\dagger a_e - a_\mu^\dagger a_\mu\right)$$

These operators can be written in either mass or flavor basis

$$\hat{H}_{\nu\nu} = \frac{\sqrt{2}G_F}{V}\int dp\, dq \left(1-\cos\theta_{pq}\right)\vec{\mathbf{J}}_p \cdot \vec{\mathbf{J}}_q$$



$$\hat{H} = \int dp\left(\frac{\delta m^2}{2E}\vec{\mathbf{B}}\cdot\vec{\mathbf{J}}_p - \sqrt{2}G_F N_e \mathbf{J}_p^0\right) + \frac{\sqrt{2}G_F}{V}\int dp\, dq\left(1-\cos\theta_{pq}\right)\vec{\mathbf{J}}_p\cdot\vec{\mathbf{J}}_q$$

$$\vec{\mathbf{B}} = \left(\sin 2\theta,\ 0, -\cos 2\theta\right)$$

The duality between $H_{\nu\nu}$ and BCS Hamiltonians

The ν-ν Hamiltonian

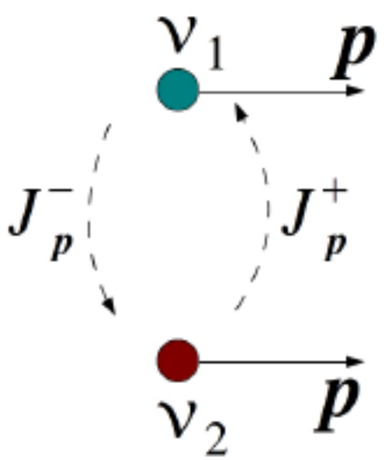
$$\hat{H} = \sum_p \frac{\delta m^2}{2p} \hat{B} \cdot \vec{J}_p + \frac{\sqrt{2} G_F}{V} \vec{J} \cdot \vec{J}$$

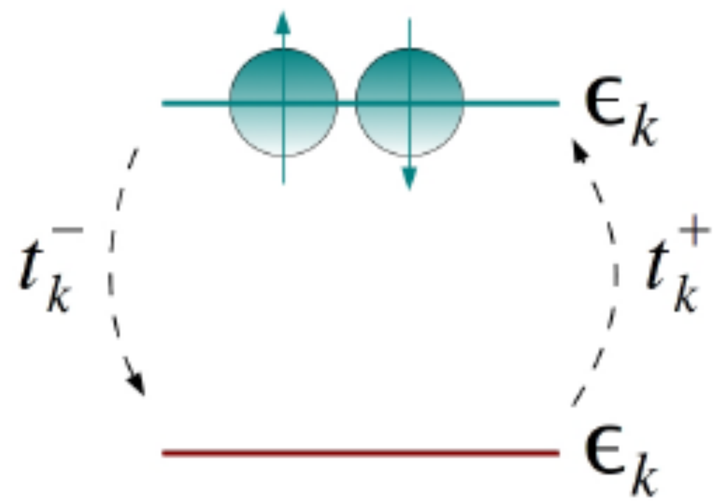
$\Longleftrightarrow$

The BCS Hamiltonian

$$\hat{H}_{\mathrm{BCS}} = \sum_k 2\epsilon_k \hat{t}_k^0 - |G| \hat{T}^+ \hat{T}$$

Same symmetries leading to Analogous (dual) dynamics!
Pehlivan, Balantekin, Kajino, and Yoshida, Phys.Rev. D **84**, 065008 (2011)





This symmetry naturally leads to splits in the neutrino energy spectra and was used to find conserved quantities in the single-angle case.

Neutrino Hamiltonian with $\nu - \nu$ interactions

$$\hat{H}_{\text{total}} = \sum_p \frac{\delta m^2}{2p} \hat{B} \cdot \vec{J}_p + \frac{\sqrt{2} G_F}{V} \sum_{\mathbf{p},\mathbf{q}} (1 - \cos \vartheta_{\mathbf{pq}}) \, \vec{J}_{\mathbf{p}} \cdot \vec{J}_{\mathbf{q}}$$

Single-angle approximation $\Rightarrow$

$$\hat{H}_{\text{total}} = \sum_p \frac{\delta m^2}{2p} \hat{B} \cdot \vec{J}_p + \frac{\sqrt{2} G_F}{V} \langle (1 - \cos \vartheta_{\mathbf{pq}}) \rangle \sum_{\mathbf{p} \neq \mathbf{q}} \vec{J}_{\mathbf{p}} \cdot \vec{J}_{\mathbf{q}}$$

Defining $\mu = \frac{\sqrt{2} G_F}{V} \langle (1 - \cos \vartheta_{\mathbf{pq}}) \rangle$, and $\omega_p = \frac{\delta m^2}{2p}$ one can write

$$\hat{H}_{\text{total}} = \sum_p \omega_p \hat{B} \cdot \vec{J}_p + \mu \sum_{\mathbf{p} \neq \mathbf{q}} \vec{J}_{\mathbf{p}} \cdot \vec{J}_{\mathbf{q}}$$

Single-angle approximation Hamiltonian:

$$H = \sum_p \frac{\delta m^2}{2p} J_p^0 + 2\mu \sum_{\substack{p,\, q \\ p \neq q}} \mathbf{J}_p \bullet \mathbf{J}_q$$

Eigenstates:

$$|x_i\rangle = \prod_{i=1}^N \sum_k \frac{J_k^\dagger}{\left(\delta m^2/2k\right) - x_i} |0\rangle$$

$$-\frac{1}{2\mu} - \sum_k \frac{j_k}{\left(\delta m^2/2k\right) - x_i} = \sum_{j \neq i} \frac{1}{x_i - x_j}$$

Bethe ansatz equations

$$\mu = \frac{G_F}{\sqrt{2}V} \langle 1 - \cos\Theta \rangle$$

Invariants:

$$h_p = J_p^0 + 2\mu \sum_{\substack{p,\, q \\ p \neq q}} \frac{\mathbf{J}_p \bullet \mathbf{J}_q}{\delta m^2 \left(\frac{1}{p} - \frac{1}{q} \right)}$$

Pehlivan, ABB, Kajino, & Yoshida
Phys. Rev. D 84, 065008 (2011)

What is the mean-field approximation?

$$\left[\hat{O}_1, \hat{O}_2\right] \cong 0$$

$$\hat{O}_1\hat{O}_2 \approx \hat{O}_1\left\langle \hat{O}_2 \right\rangle + \left\langle \hat{O}_1 \right\rangle \hat{O}_2 - \left\langle \hat{O}_1\hat{O}_2 \right\rangle$$

Expectation values should be calculated with a state $\left|\Psi\right\rangle$ chosen to satisfy:

$$\left\langle \hat{O}_1\hat{O}_2 \right\rangle = \left\langle \hat{O}_1 \right\rangle \left\langle \hat{O}_2 \right\rangle$$

This reduces the two-body problem to a one-body problem:

$$a^\dagger a^\dagger a a \Rightarrow \left\langle a^\dagger a \right\rangle a^\dagger a + \left\langle a^\dagger a^\dagger \right\rangle a a + \text{h.c.}$$

$$\hat{H}_{\nu\nu} = \frac{\sqrt{2}G_F}{V}\int dp\,dq\left(1-\cos\theta_{pq}\right)\vec{\mathbf{J}}_p \cdot \vec{\mathbf{J}}_q \cong \frac{\sqrt{2}G_F}{V}\int dp\,dq\left(1-\cos\theta_{pq}\right)\left\langle \vec{\mathbf{J}}_p \right\rangle \cdot \vec{\mathbf{J}}_q$$

Mean field

Neutrino-neutrino interaction

$$\bar{\psi}_{\nu L}\gamma^{\mu}\psi_{\nu L}\bar{\psi}_{\nu L}\gamma_{\mu}\psi_{\nu L} \Rightarrow \bar{\psi}_{\nu L}\gamma^{\mu}\psi_{\nu L}\left\langle \bar{\psi}_{\nu L}\gamma_{\mu}\psi_{\nu L} \right\rangle + \cdots$$

Antineutrino-antineutrino interaction

$$\bar{\psi}_{\bar{\nu} R}\gamma^{\mu}\psi_{\bar{\nu} R}\bar{\psi}_{\bar{\nu} R}\gamma_{\mu}\psi_{\bar{\nu} R} \Rightarrow \bar{\psi}_{\bar{\nu} R}\gamma^{\mu}\psi_{\bar{\nu} R}\left\langle \bar{\psi}_{\bar{\nu} R}\gamma_{\mu}\psi_{\bar{\nu} R} \right\rangle + \cdots$$

Neutrino-antineutrino interaction

$$\bar{\psi}_{\nu L}\gamma^{\mu}\psi_{\nu L}\bar{\psi}_{\bar{\nu} R}\gamma_{\mu}\psi_{\bar{\nu} R} \Rightarrow \bar{\psi}_{\nu L}\gamma^{\mu}\psi_{\nu L}\left\langle \bar{\psi}_{\bar{\nu} R}\gamma_{\mu}\psi_{\bar{\nu} R} \right\rangle + \cdots$$

Balantekin and Pehlivan, JPG **34**,1783 (2007)

Neutrino-antineutrino can also have an additional mean field

$$\bar{\psi}_{\nu L}\gamma^{\mu}\psi_{\nu L}\bar{\psi}_{\bar{\nu}R}\gamma_{\mu}\psi_{\bar{\nu}R} \Rightarrow \bar{\psi}_{\nu L}\gamma^{\mu}\left\langle \psi_{\nu L}\bar{\psi}_{\bar{\nu}R}\gamma_{\mu}\right\rangle \psi_{\bar{\nu}R} + \cdots$$

However note that

$\left\langle \psi_{\nu L}\bar{\psi}_{\bar{\nu}R}\gamma_{\mu}\right\rangle \propto m_{\nu}$ (negligible if the medium is isotropic)

Fuller *et al.*
Volpe

Single-angle approximation Hamiltonian:

$$H = \sum_p \frac{\delta m^2}{2p} J_p^0 + 2\mu \sum_{\substack{p,\,q \\ p\neq q}} \mathbf{J}_p \bullet \mathbf{J}_q$$

Eigenstates:

$$|x_i\rangle = \prod_{i=1}^{N} \sum_k \frac{J_k^\dagger}{\left(\delta m^2/2k\right) - x_i} |0\rangle$$

$$-\frac{1}{2\mu} - \sum_k \frac{j_k}{\left(\delta m^2/2k\right) - x_i} = \sum_{j\neq i} \frac{1}{x_i - x_j}$$

Bethe ansatz equations

$$\mu = \frac{G_F}{\sqrt{2}V} \langle 1 - \cos\Theta \rangle$$

Invariants:

$$h_p = J_p^0 + 2\mu \sum_{\substack{p,\,q \\ p\neq q}} \frac{\mathbf{J}_p \bullet \mathbf{J}_q}{\delta m^2 \left(\frac{1}{p} - \frac{1}{q}\right)}$$

Pehlivan, ABB, Kajino, & Yoshida
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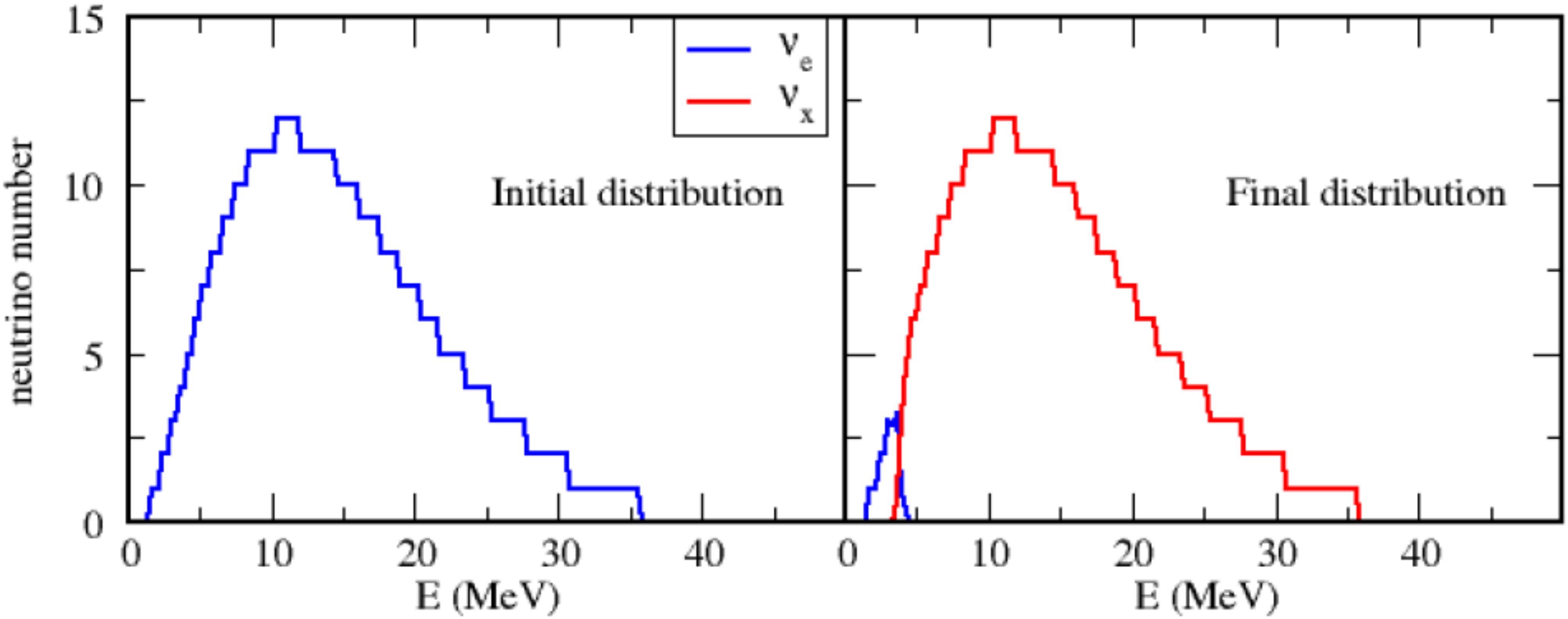
Away from the mean-field: First adiabatic solution of the *exact* many-body Hamiltonian

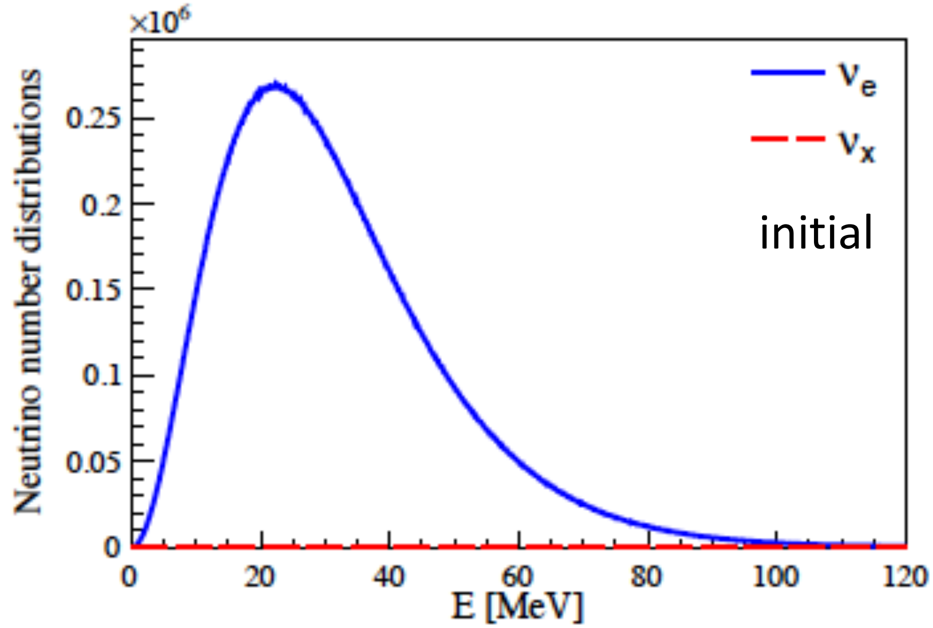


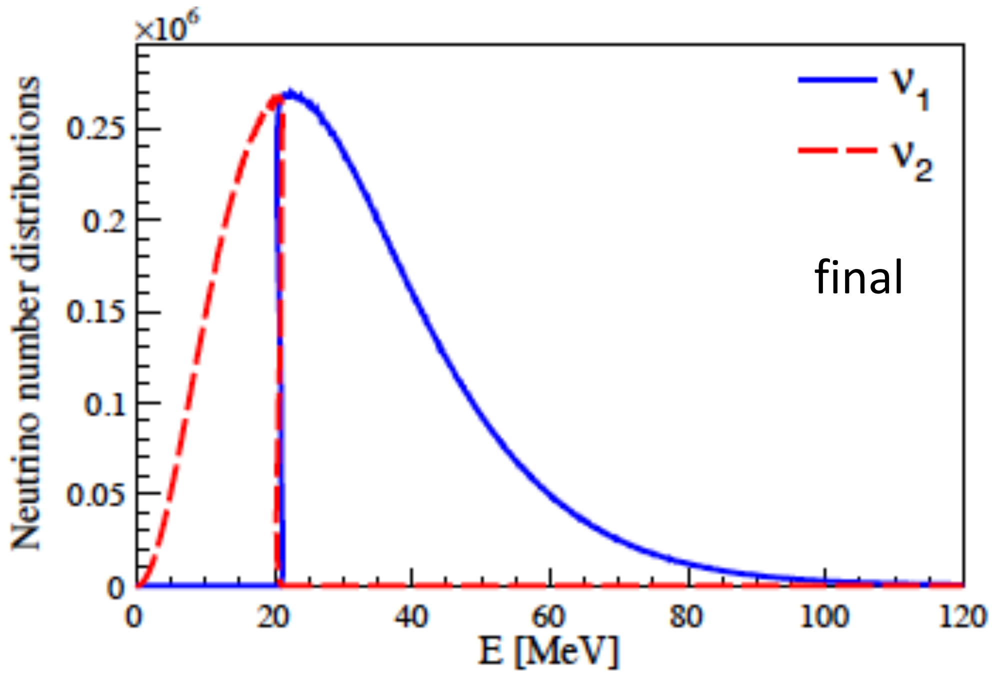
- Solutions of the Bethe ansatz equations for 250 neutrinos. Same behavior as the mean-field.
- Two flavors only
- Inverted hierarchy, no matter effect

2015

Adiabatic evolution of an initial thermal distribution (T = 10 MeV) of electron neutrinos. 10^8 neutrinos distributed over 1200 energy bins with solar neutrino parameters and normal hierarchy.

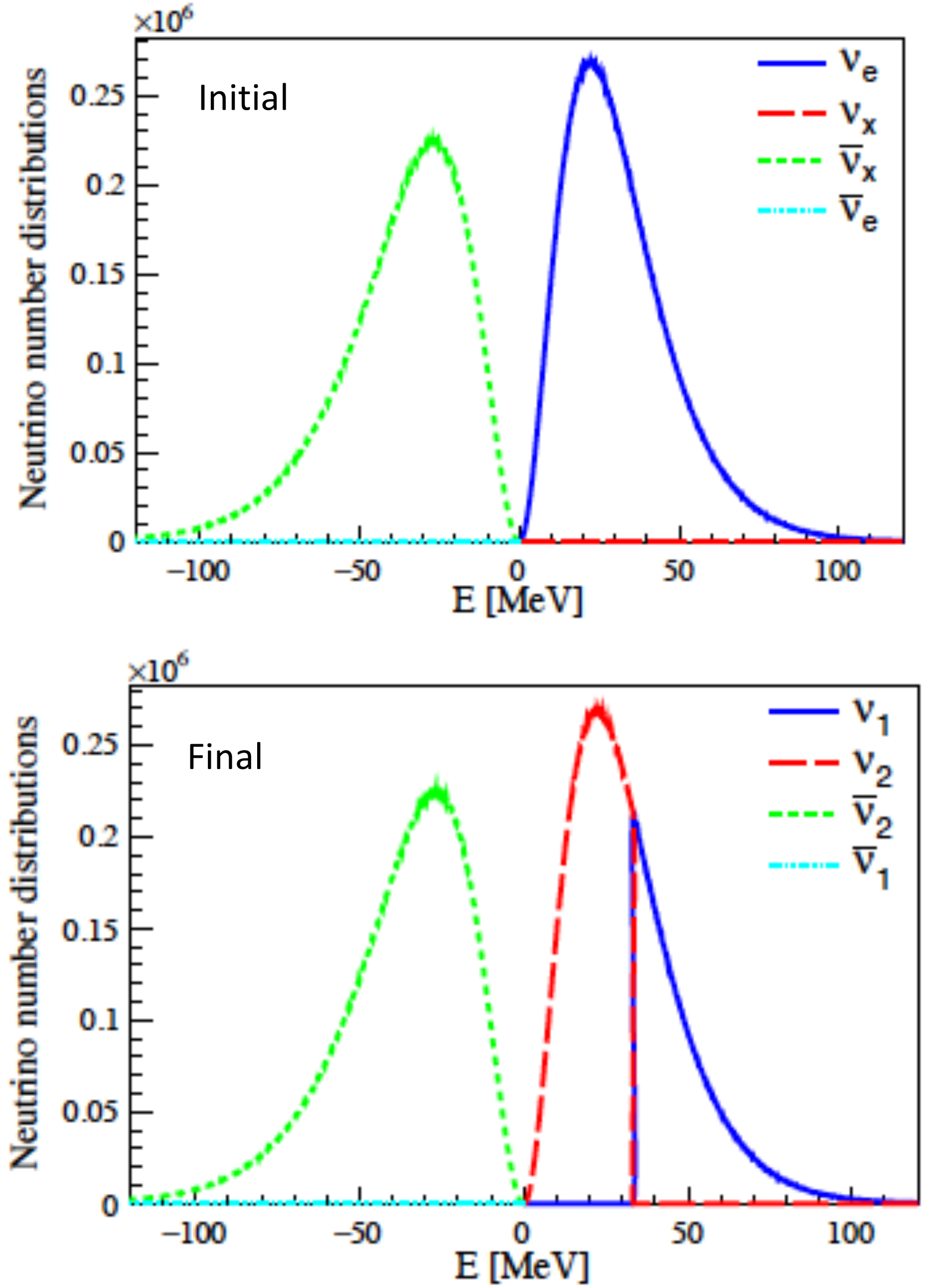
Birol, Pehlivan, Balantekin, Kajino
arXiv:1805.11767





Adiabatic evolution of an initial thermal distribution of electron neutrinos (T=10 MeV) and antineutrinos of another flavor (T=12MeV). 10^8 neutrinos distributed over 1200 energy bins both for neutrinos and antineutrinos with solar neutrino parameters and normal hierarchy.

Birol, Pehlivan, Balantekin, Kajino
arXiv:1805.11767



An alternative approach

Patwardhan, Cervia, Balantekin 2018

$$-\frac{1}{2\mu} - \sum_{p}^{M} \frac{j_p}{\omega_p - \lambda_i} = \sum_{j \neq i}^{N} \frac{1}{\lambda_i - \lambda_j}$$

$$P(\lambda) = \prod_i (\lambda - x_i) = \exp\left(\sum_i \log(\lambda - x_i)\right)$$

$$\Lambda(\lambda) = \frac{dP/d\lambda}{P(\lambda)} = \sum_{i=1}^{N} \frac{1}{\lambda - x_i}$$

$$\Lambda^2(\lambda) + \Lambda'(\lambda) + \frac{1}{\mu}\Lambda(\lambda) = 2\sum_p \frac{j_p}{\lambda - \omega_p}\left[\Lambda(\lambda) - \Lambda(\omega_p)\right]$$

Key idea: Calculate Λ at $\lambda=\omega_p$

$$\Lambda^2(\omega_q) + (1-2j_q)\Lambda'(\omega_q) + \frac{1}{\mu}\Lambda(\omega_q) = 2\sum_{p\neq q} j_p \frac{\Lambda(\omega_q)-\Lambda(\omega_p)}{\omega_q-\omega_p}.$$

Note that for $j_q = 1/2$ the derivative term vanishes and one gets an algebraic equation!

$$\begin{aligned} 2\Lambda(\omega_q)\Lambda'(\omega_q) &+& (1-j_q)\Lambda''(\omega_q) + \frac{\Lambda'(\omega_q)}{\mu} \\ &=& 2\sum_{p\neq q} j_p \left[\frac{\Lambda'(\omega_q)}{\omega_q-\omega_p} + \frac{\Lambda(\omega_q)-\Lambda(\omega_p)}{(\omega_q-\omega_p)^2}\right] \end{aligned}$$

The second derivative vanishes for $j_q = 1$. For higher j_q values keep taking derivatives.

How can we make further progress?

- We examined the many-neutrino gas both from the exact many-body perspective and an effective one-body description following introduction of a mean field. In the limit of the single angle approximation, both pictures possess constants of motion.
- At least in the single angle approximation, we can solve the full many-body problem in the adiabatic limit for a few simple cases. To go beyond those special cases we need to solve the Bethe ansatz equations.


Thank you very much!