Glue and Quark Spins from the Lattice

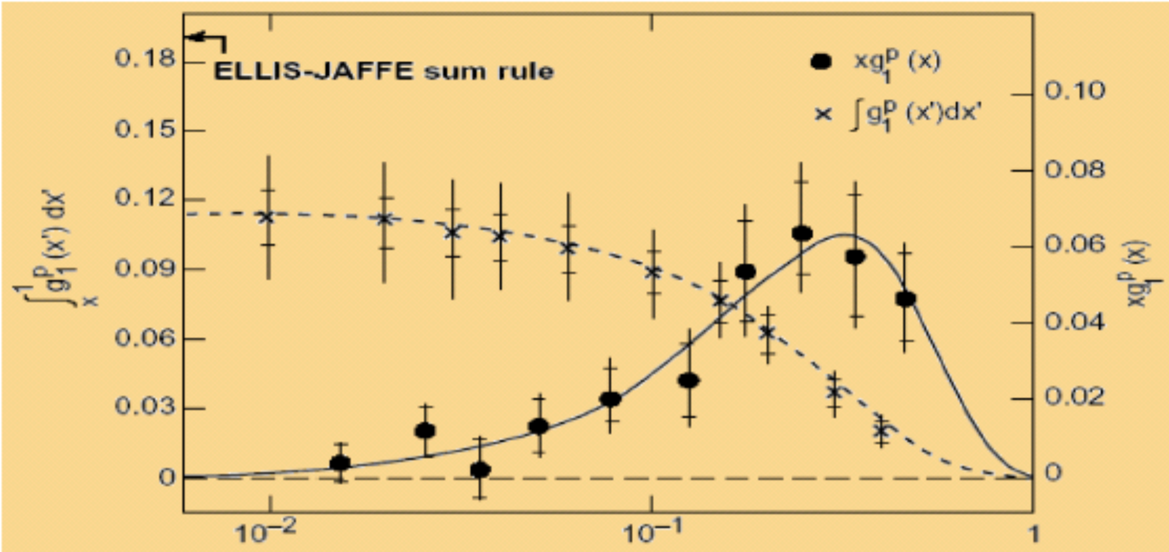
- Glue spin puzzle
- Proton spin sum rules
- Quark spin and anomalous Ward identity
- Proton spin decomposition

χ QCD Collaboration

CIPANP, May 31, 2018

Twenty $\wedge$9 years since the "spin crisis"

❑ **EMC experiment in 1988/1989 – "the plot":**



$$g_1(x) = \frac{1}{2}\sum_q e_q^2\,[\Delta q(x) + \Delta\bar{q}(x)] + \mathcal{O}(\alpha_s) + \mathcal{O}(1/Q)$$

$$\Delta q = \int_0^1 dx \Delta q(x) = \langle P, s_\| | \overline{\psi}_q(0)\gamma^+\gamma_5\psi_q(0)|P, s_\|\rangle$$

❑ **"Spin crisis" or puzzle:** $\Delta\Sigma = \sum_q \Delta q + \Delta\overline{q} \sim 0.3$

Proton Spin Crisis

- What's wrong with the quark model?
- Mixture from the glue spin?

Anomalous Ward Identity

$$\partial_\mu A_\mu^0 = i2 \sum_{i=u.d.s} m_i P_i - \frac{iN_f}{8\pi^2} G_{\mu\nu} \tilde{G}_{\mu\nu}$$

Take

$$q(x) = \frac{1}{16\pi^2} G_{\mu\nu} \tilde{G}_{\mu\nu} = \partial_\mu K_\mu, \quad K_\mu = \frac{1}{8\pi^2} \varepsilon_{\mu\nu\rho\sigma} tr[A_\nu (\partial_\rho A_\sigma + \frac{2}{3} A_\rho A_\sigma)]$$

$$\Longrightarrow \quad \partial_\mu (A_\mu^0 + 2iN_f K_\mu) = i2 \sum_{i=u.d.s} m_i P_i$$

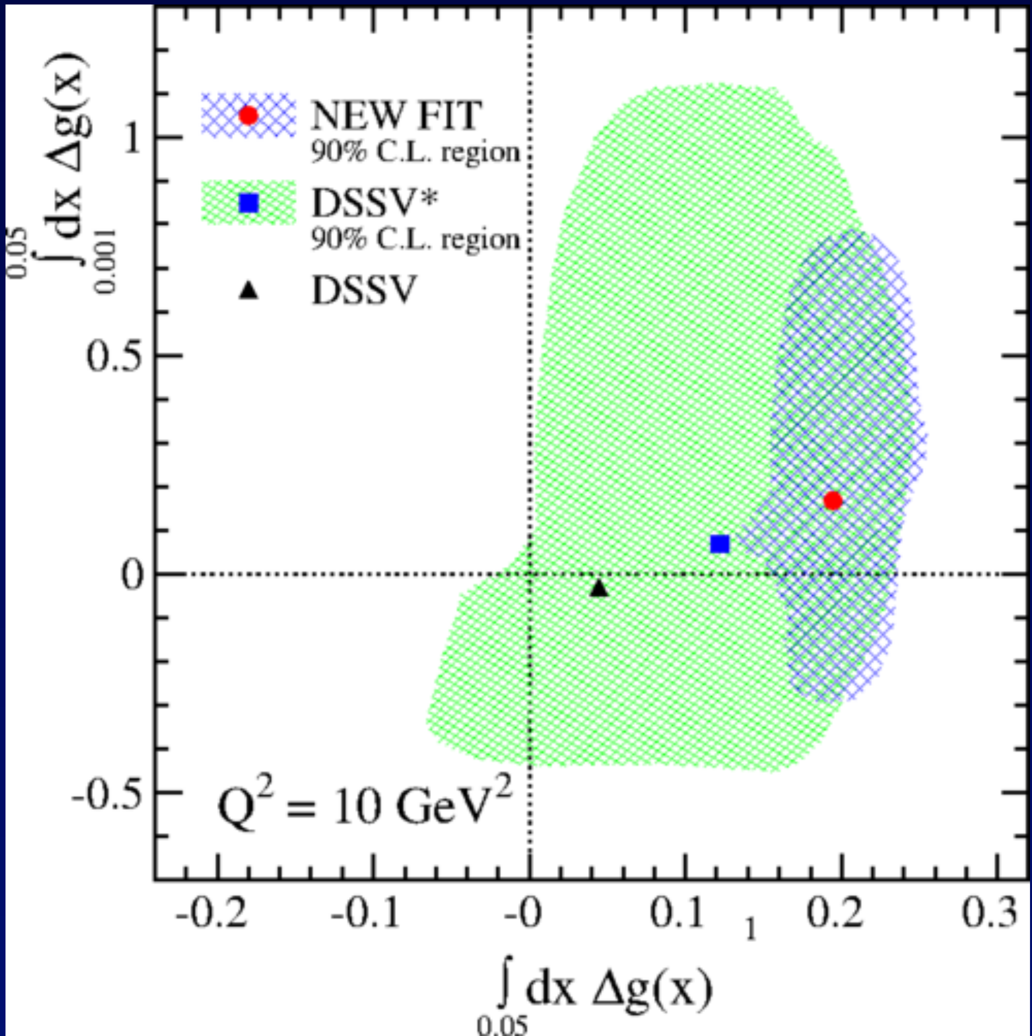
However, the Chern-Simons current is not gauge invariant.

- `Proton spin crisis is the graveyard of all hadronic models'

Where does the rest of the spin of the proton come from?

Glue spin
Quark orbital angular momentum
Glue orbital angular momentum

Glue Helicity ΔG



Experimental results from
STAR [1404.5134]
PHENIX [1402.6296]
COMPASS [1001.4654]

ΔG ~ 0.2 with large error

D. de Florian, R. Sassot, M. Stratmann, W. Vogelsang,
PRL 113, 012001 (2014)

Spin Sum Rules

- Jaffe and Manohar sum rule (1990)

$$J = \frac{\Sigma}{2} + L_q + S_G + L_G$$

$$\vec{J}_{Tot} = \int d^3x\ \psi^\dagger \frac{1}{2}\Sigma\psi + \int d^3x\ \vec{x}\times\psi^\dagger\vec{\nabla}\psi + \int d^3x\ \vec{E}^a\times\vec{A}^a$$
$$+ \int d^3x\ \vec{x}\times E^{aj}(\vec{x}\times\nabla)A^{aj}$$

  - Canonical EM tensor on light-cone with light-cone gauge
  - TMD formulation on the lattice to calculation (M. Engelhardt et al.)

- Ji sum rule (1997)

$$J = \frac{\Sigma}{2} + L_q + J_G$$

$$\vec{J}_{Tot} = \int d^3x\ \psi^\dagger \frac{1}{2}\Sigma\psi + \int d^3x\ \vec{x}\times\psi^\dagger\vec{D}\psi + \int d^3x\ \vec{x}\times\ (\vec{E}^a\times\vec{B}^a)$$

  - Symmetric EM tensor (Belinfante) → gauge invariant and frame independent.

Glue Spin and Helicity ΔG

- Jaffe and Manohar -- spin sum rule on light cone

$S_g = \int d^3x \, \vec{E} \times \vec{A}$ in light-cone gauge $(A^+ = 0)$ and IMF frame.

  - Not gauge invariant
  - Light cone not accessible on the Euclidean lattice.

- Manohar – gauge invariant light-cone distribution

$$\Delta g(x)\, S^+ = \frac{i}{2xP^+} \int \frac{d\xi^-}{2\pi} e^{-ixP^+\xi^-} \langle PS \,|\, F_a^{+\alpha}(\xi^-) L^{ab}(\xi^-,0) \tilde{F}^+_{\alpha,b}(0) \,|\, PS \rangle$$

  - After integration of x, the glue helicity operator is

$$\mathrm{H}_g(0) = \vec{E}^a(0) \times \left(\vec{A}^a(0) - \frac{1}{\nabla^+}\left(\vec{\nabla} A^{+,b}\right) L^{ba}(\xi^-,0) \right)$$

  - Non-local and on light cone

Glue Spin and Helicity ΔG

- X.S. Chen, T. Goldman, F. Wang (2008); Wakamatsu; Hatta, etc.

Gauge invariant decomposition $\vec{J} = \vec{S}_q + \vec{L}_q + \vec{S}_G + \vec{L}_G$

$$S_g = \int d^3x \, \mathrm{Tr}\,(\vec{E} \times \vec{A}_{phys}), \quad A^\mu = A^\mu_{phys} + A^\mu_{pure}, \quad F^{\mu\nu}_{pure} = 0;$$

$$A^\mu_{phys} \;\to\; \mathrm{g}^\dagger A^\mu_{phys} g, \quad A^\mu_{pure} \;\to\; \mathrm{g}^\dagger A^\mu_{pure} g - \frac{i}{\mathrm{g}} g^\dagger \partial^\mu g$$

$$D^i A^i_{phys} = \partial^i A^i_{phys} - i\mathrm{g}\,[A^i, A^i_{phys}] = 0$$

  - Gauge invariant but frame dependent

- X. Ji, J.H. Zhang, Y. Zhao (2013); Y. Hatta, X. Ji, Y. Zhao

Infinite momentum frame

$$\vec{E}^a(0) \times \vec{A}^a_{phys} \xrightarrow[\text{light-cone}]{} \vec{E}^a(0) \times \left(\vec{A}^a(0) - \frac{1}{\nabla^+}\left(\vec{\nabla} A^{+,b}\right) L^{ba}(\xi^-, 0) \right)$$

Glue Spin and Helicity ΔG

- Large momentum limit

$$S_g = \frac{\langle PS | \int d^3x \; \mathrm{Tr}\, (\vec{E} \times \vec{A}_{phys})_z \, | PS \rangle}{2E_P} \xrightarrow[P_Z \to \infty]{} \Delta G$$

  – Calculate S_g at finite P_z
  – Match to MS-bar scheme at 10 GeV
  – Large momentum effective theory to match to IMF
  – Similar proof for the quark and glue orbital angular momenta which are related to form factors in generalized TMD (GTMD)

  (Y. Zhao, KFL, and Y. Yang, arXiv:1506.08832 (PRD))

- Solution of A_{phys} -- related to A in Coulomb gauge

$$U^{\mu}(x) = g_c(x) U_c^{\mu}(x) g_c^{-1}(x + a\hat{\mu}),$$

$$U_{pure}^{\mu}(x) \equiv g_c(x) g_c^{-1}(x + a\hat{\mu}),$$

$$A_{phys}^{\mu}(x) \equiv \frac{i}{a g_0}\left(U^{\mu}(x) - U_{pure}^{\mu}(x) \right) = g_c(x) A_c(x) g_c^{-1}(x) + O(a).$$

$$Tr(\vec{E} \times \vec{A}_{phys}) = Tr(\vec{E} \times g_c \vec{A}_c g_c^{-1}) = Tr(\vec{E}_c \times \vec{A}_c)$$

Lattice Details

- Overlap fermion on 2+1 flavor RBC/UKQCD Domain-wall fermion configurations

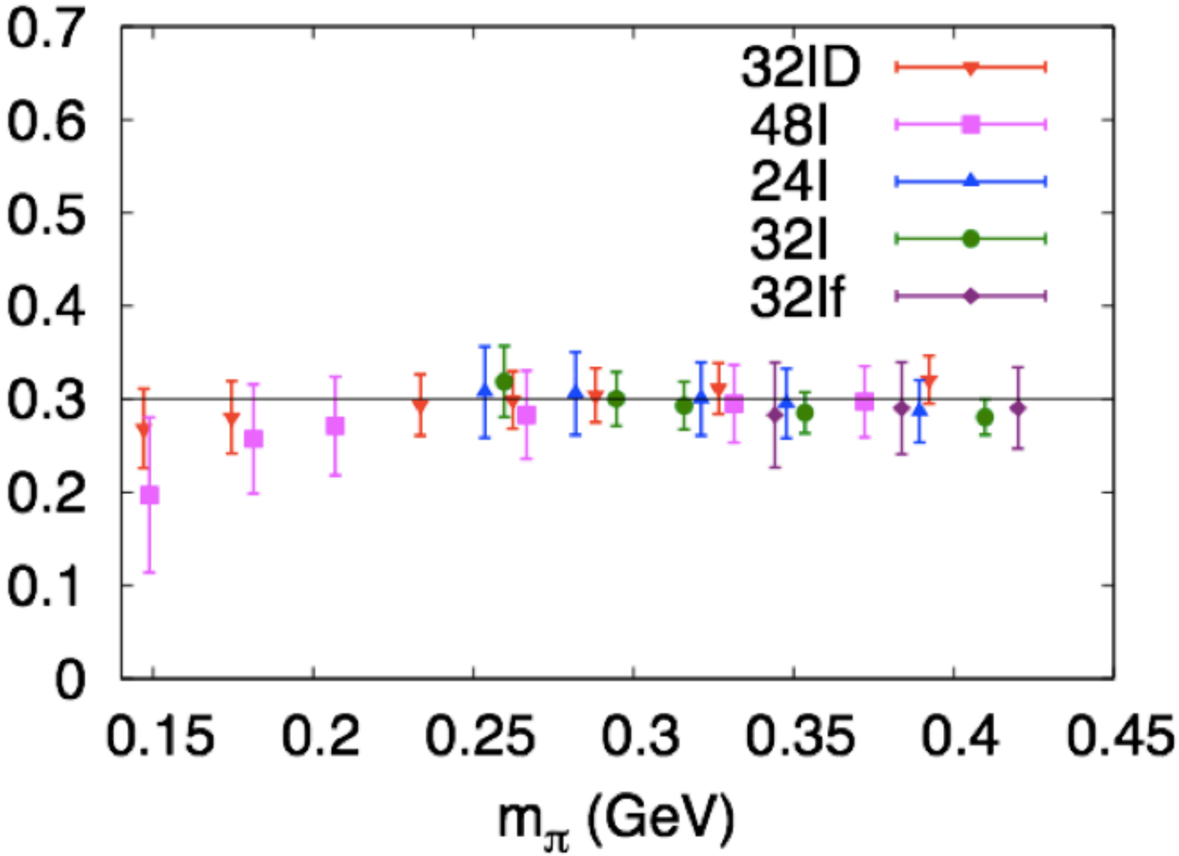
Symbol	$L^3 \times T$	a(fm)	$m_\pi^{(s)}$(MeV)	N_{cfg}
32ID	$32^3 \times 64$	0.1431(7)	170	200
48I	$48^3 \times 96$	0.1141(2)	140	81
24I	$24^3 \times 64$	0.1105(3)	330	203
32I	$32^3 \times 64$	0.0828(3)	300	309
32If	$32^3 \times 64$	0.0627(3)	370	238

- Gauge operators are from smeared plaquettes.

The dependence

Y. Yang, R. S. Sufian, et al, χQCD Collaboration, arXiv 1609.05937.

of m_π, a, and V

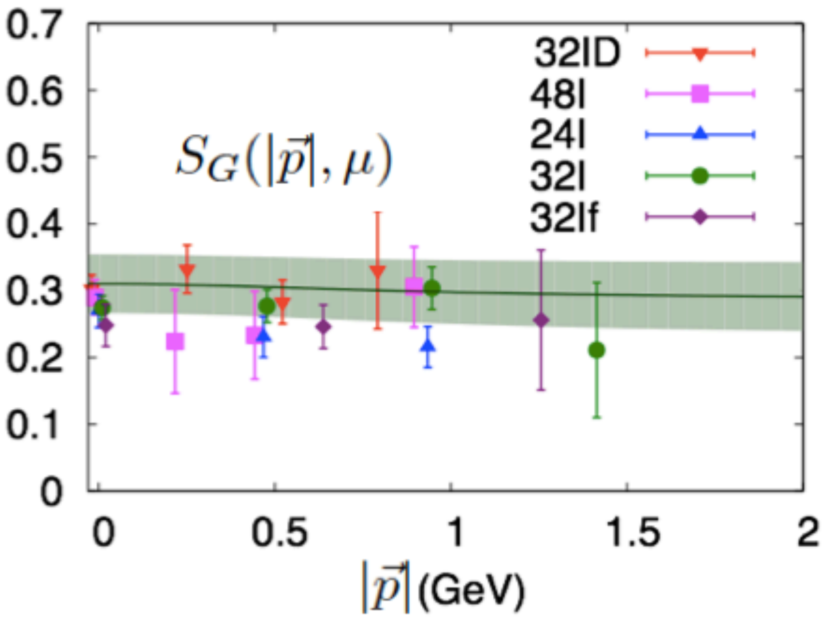


μ^2=10 GeV^2

In the rest frame, *the pion mass (both valence and sea), lattice spacing and volume dependences are mild.*

From glue spin to helicity

with *Large-momentum effective field theory*



X. Ji, J.-H. Zhang, and Y. Zhao, Phys. Lett. B743, 180 (2015)

$$S_G(|\vec{p}|,\mu) = \left[1 + \frac{g^2 C_A}{16\pi^2}\left(\frac{7}{3}\log\frac{(\vec{p})^2}{\mu^2} - 10.2098\right)\right]\Delta G(\mu) + \frac{g^2 C_F}{16\pi^2}\left(\frac{4}{3}\log\frac{(\vec{p})^2}{\mu^2} - 5.2627\right)\Delta\Sigma(\mu) + O(g^4) + O\left(\frac{1}{(\vec{p})^2}\right).$$

- **The large finite pieces indicates a convergence problem**
- **Large frame dependence need re-summation.**

With $|\vec{p}| = 1.5$ GeV and $\mu^2 = 10$ GeV^2,

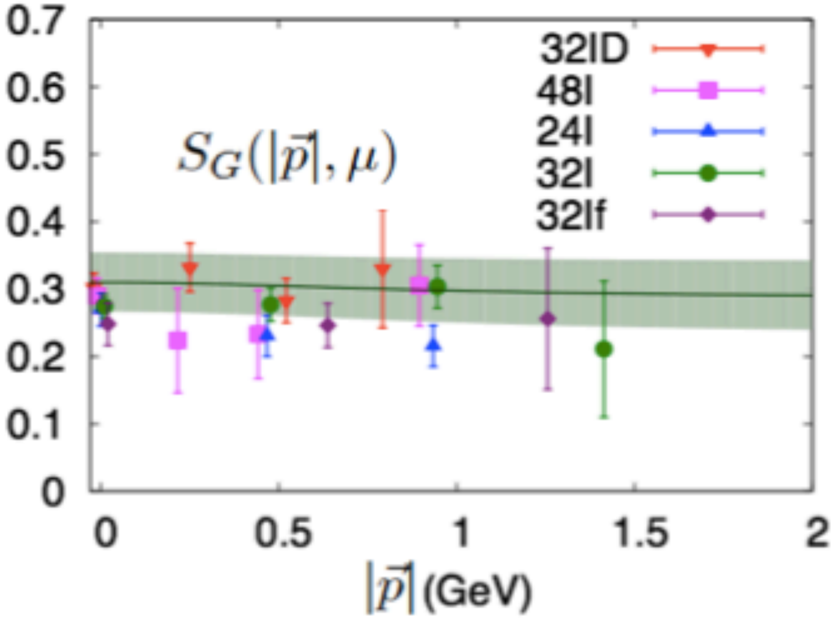
the factor before Δ_G is 0.22.

Glue spin

Y. Yang, R. S. Sufian, et al,
χQCD Collaboration,
arXiv 1609.05937.

The final result

PRL 118,102001 (2017) – Physic Viewpoint



We neglect the matching and use the following empirical form to fit our data,

$$S_G(|\vec{p}|) = S_G(\infty) + \frac{C_1}{M^2 + |\vec{p}|^2} + C_2(m^2_{\pi,vv} - m^2_{\pi,phys}) + C_3(m^2_{\pi,ss} - m^2_{\pi,phys}) + C_4 a^2$$

$m_{\pi,phys}$=0.139 GeV $\quad M = 0.939$ GeV

The glue spin at the large momentum limit for the renormalized value at μ^2=10GeV^2:

$S_G = 0.251(47)(16)$

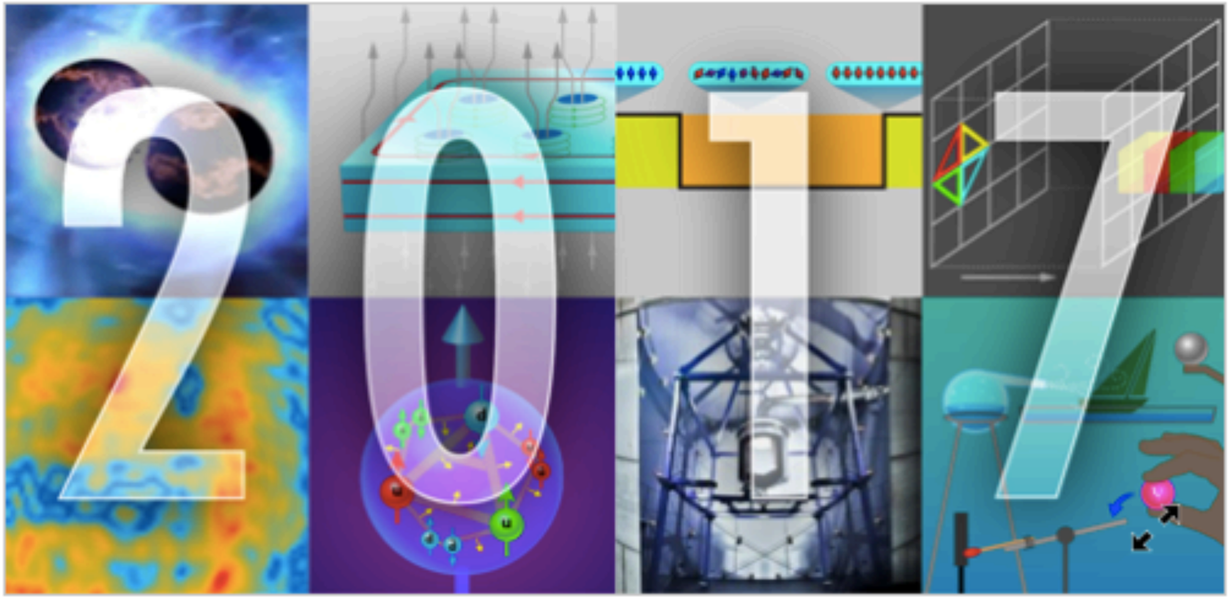
Present experiment

$\Delta G(Q^2=10\ \text{GeV}^2) \sim 0.2$,
de Florian et al., 2014

Highlights of the Year

December 18, 2017 • *Physics* 10, 137

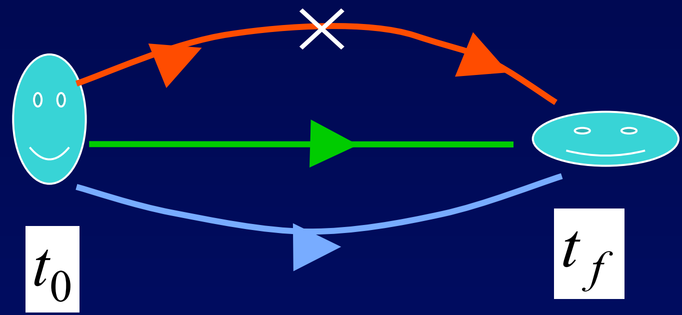
***Physics* looks back at its favorite stories from 2017.**



Lattice Calculations of Quark and Glue Spins

- Quark and Glue Momentum and Angular Momentum in the Nucleon

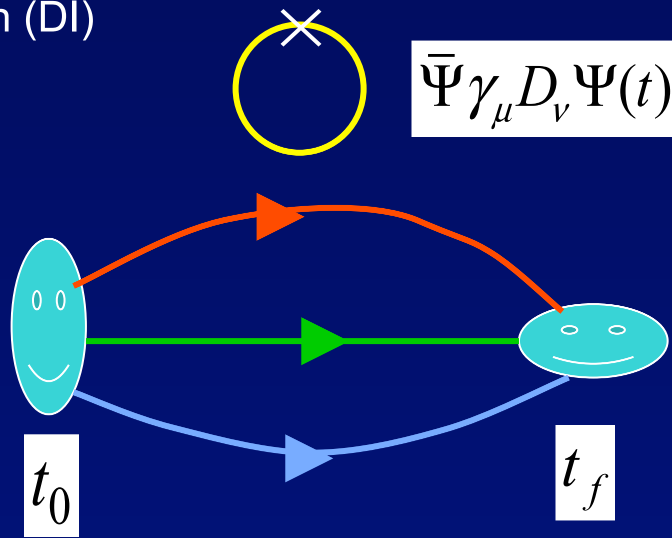
$(\bar{u}\gamma_\mu D_\nu u + \bar{d}\gamma_\mu D_\nu d)(t)$



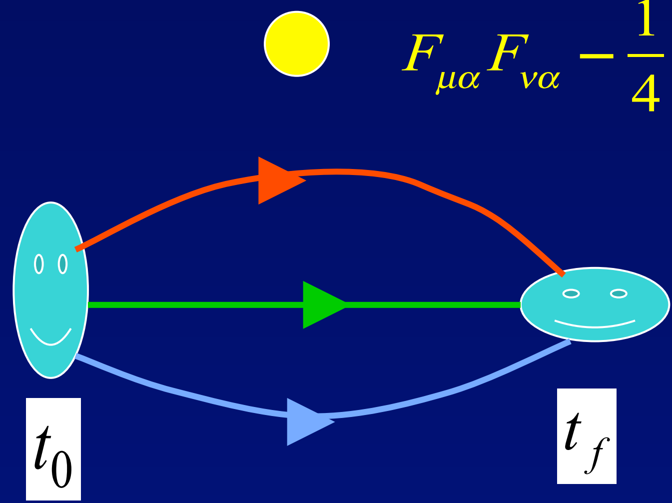
Connected insertion (CI)

Disconnected insertion (DI)

$\bar{\Psi}\gamma_\mu D_\nu \Psi(t)(u,d,s)$



$F_{\mu\alpha}F_{\nu\alpha} - \frac{1}{4}\delta_{\mu\nu}F^2$



Quark Spin and Anomalous Ward Identify

- Calculation of the point axial-vector in the DI is not sufficient.
- AWI needs to be satisfied. $\partial_\mu A_\mu^0 = i2mP - \frac{iN_f}{8\pi^2} G_{\mu\nu}\tilde{G}_{\mu\nu}$
- Unrenormalized AWI for overlap fermion for point current

$$\kappa_A \partial_\mu A_\mu^0 = i2mP - iN_f 2q(x)$$

Renormaliztion and mixing:

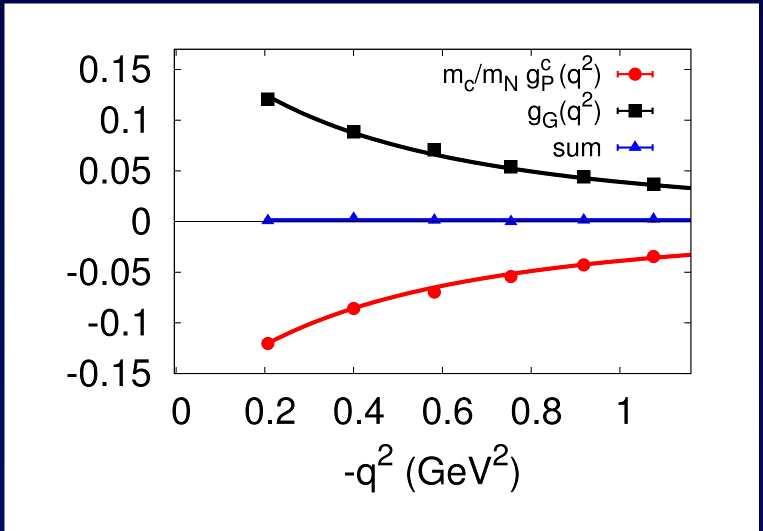
$$Z_A^0 \kappa_A \partial_\mu A_\mu^0 = i2Z_m m Z_P P - iN_f 2(Z_q q(x) + \lambda \partial_\mu A_\mu^0)$$

- Overlap fermion --> mP is RGI ($Z_m Z_P$=1)
- Overlap operator for $q(x) = -1/2\, \mathrm{Tr}\gamma_5 D_{ov}(x,x)$ has no multiplicative renormalization.
- Espriu and Tarrach (1982) $Z_A^0(2-\mathrm{loop}) = 1 - \left(\frac{\alpha_s}{\pi}\right)^2 \frac{3}{8} C_2(R) N_f \frac{1}{\varepsilon},$

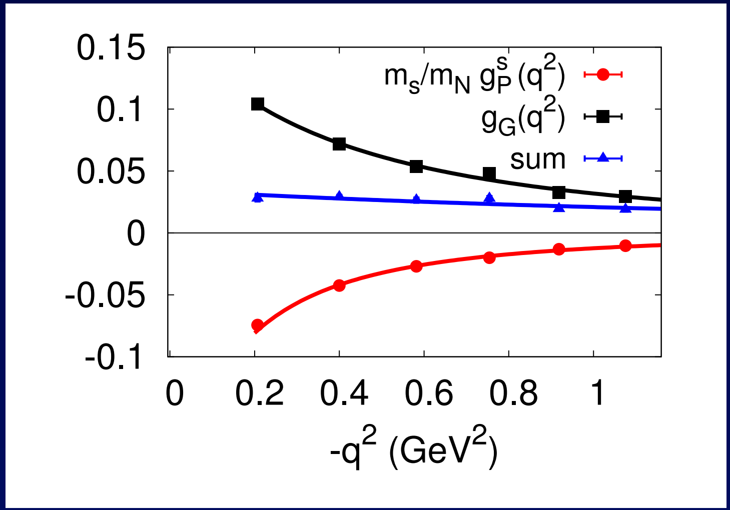
$$\lambda = -\left(\frac{\alpha_s}{\pi}\right)^2 \frac{3}{16} C_2(R) \frac{1}{\varepsilon}$$

Anomaly and Pseudoscalar Form Factors

Charm



Strange



Check Anomalous Ward Identity

$$\left\langle N(p')|\kappa_A \partial_\mu A_\mu|N(p)\right\rangle_{CI} = \left\langle N(p')|2mP|N(p)\right\rangle_{CI}$$

$$\left\langle N(p')|\kappa'_A \partial_\mu A_\mu|N(p)\right\rangle_{DI} = \left\langle N(p')|2mP-2iq|N(p)\right\rangle_{DI}$$

$$\kappa'_A = \kappa_A$$

Quark Spin Components $\overline{MS}$ (2 GeV)

g_A	Δ(u+d) CI	Δ(u/d) DI	Δs	Δu	Δd	$g_A = \Delta u - \Delta d$	ΔΣ
J. Green			-0.0240 (45)	0.863 (7)(14)	-0.345 (6)(9)	1.206 (20)	0.494 (11)(15)
C. Alexandrou	0.598 (24)(6)	-0.077 (15)(5)	-0.042 (10)(2)	0.830 (26)(4)	-0.386 (16)(6)	1.216 (31)(7)	0.402 (34)(10)
χ QCD	0.582 (13)(28)	-0.073 (13)(15)	-0.035 (8)(7)	0.846 (18)(32)	-0.410 (16)(18)	1.256 (16)(30)	0.401 (25)(37)
NNPDFpol1.1 (Q^2=10 GeV^2)			-0.10 (8)	0.76 (4)	-0.41 (4)	1.2723 (23)	0.25 (10)
DSSV			-0.012 +(56)-(62)	0.793 +(28)-(34)	-0.416 +(35)-(25)	1.2723 (23)	0.366 +(62)-(42)

J. Green et al., N_F=2+1, Clover fermion, m_π = 317 MeV, one lattice

C. Alexandrou et al., N_F=2, twisted mass fermion, , m_π = 131 MeV, one lattice

χ QCD , N_F=2+1, Overlap fermion, , m_π = 170, 290, 330 MeV, 5 - 6 valence quarks for each of the three lattices, non-perturbative renormalization

Quark Spin

- Lattice calculation with chiral fermion which satisfies the anomalous Ward identity is able to reveal the origin of the smallness of the quark spin – the disconnected insertion is large and negative.
- The interplay between the pseudoscalar and topological charge couplings in the anomalous Ward identity is the origin for the negative DI contribution – another example of U(1) anomaly at work.

Momenta and Angular Momenta of Quarks and Glue

- Energy momentum tensor operators decomposed in quark and glue parts gauge invariantly --- Xiangdong Ji (1997)

$$T^q_{\mu\nu} = \frac{i}{4}\left[\bar{\psi}\gamma_\mu \overleftrightarrow{D}_\nu \psi + (\mu \leftrightarrow \nu)\right] \rightarrow \vec{\mathrm{J}}_q = \int d^3x \left[\frac{1}{2}\bar{\psi}\vec{\gamma}\gamma_5\psi + \vec{x}\times\bar{\psi}\gamma_4(-i\vec{D})\psi\right]$$

$$T^g_{\mu\nu} = F_{\mu\lambda}F_{\lambda\nu} - \frac{1}{4}\delta_{\mu\nu}F^2 \qquad \rightarrow \vec{\mathrm{J}}_g = \int d^3x \left[\vec{x}\times(\vec{E}\times\vec{B})\right]$$

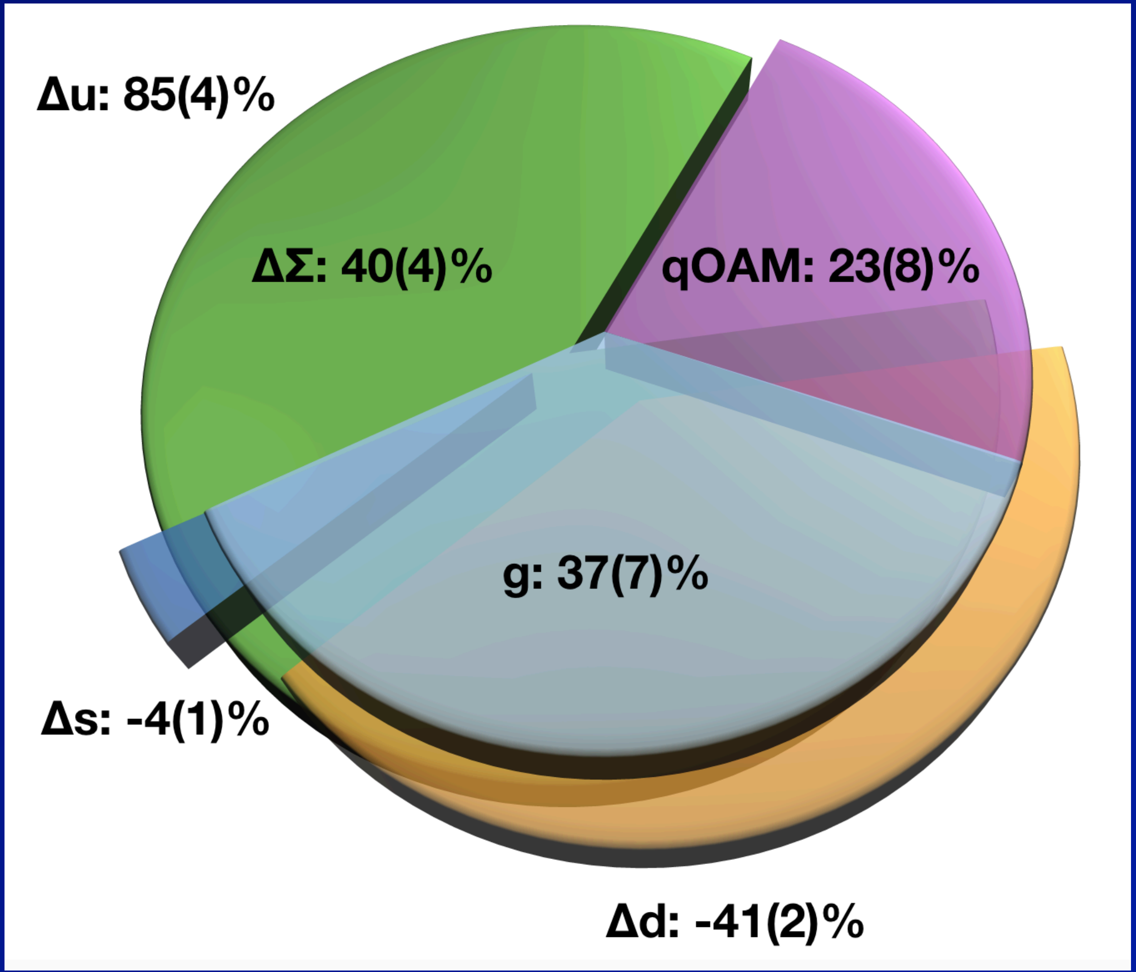
- Nucleon form factors

$$\left\langle p,s \,|\, T_{\mu\nu} \,|\, p's'\right\rangle = \bar{u}(p,s)[T_1(q^2)\gamma_\mu \bar{p}_\nu - T_2(q^2)\bar{p}_\mu \sigma_{\nu\alpha}q_\alpha / 2m$$
$$-\mathrm{i}\mathrm{T}_3(q^2)(q_\mu q_\nu - \delta_{\mu\nu}q^2)/m + T_4(q^2)\delta_{\mu\nu} m/2]u(p's')$$

- Momentum and Angular Momentum

$$Z_{q,g} T_1(0)_{q,g} \;\left[\mathrm{OPE}\right] \rightarrow \left\langle x\right\rangle_{q/g}(\mu, \overline{\mathrm{MS}}),\quad Z_{q,g}\left[\frac{T_1(0)+T_2(0)}{2}\right]_{q,g} \rightarrow J_{q/g}(\mu, \overline{\mathrm{MS}})$$

Quark Spin, Orbital Angular Momentum, and Gule Angular Momentum in X. Ji's Sum Rules



Summary and Challenges

- Lattice calculations of the physical 2+1 flavor dynamical fermions at the physical pion point and with extrapolations to continuum and infinite volume limits are becoming available even with chiral fermions.
- Decomposition of proton spin and hadron masses into quark and glue components on the lattice is feasible, pending reasonalbe statistics of non-perturbative renormalization. Large momentum frame for the proton to calculate glue helicity remains a challenge.
- Together with evolution, factorization, perturbative QCD for the global fitting of PDFs, lattice QCD results with small enough statistical and systematic errors can compare directly with experiments and have an impact in advancing our understanding of the underline physics of the hadron structure (form factors, PDF, neutron electric dipole moment, muon g-2, etc).