$B \to \pi\ell\ell$ and $B \to K\ell\ell$ decay form factors from Lattice QCD

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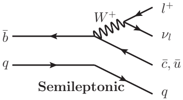
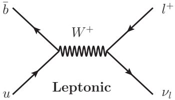
Outline

- Motivation and Introduction
- Lattice QCD form factor calculations
- FCNC Form factors
  - $B \to \pi$
  - $B \to K$
- Phenomenology
- Summary and outlook

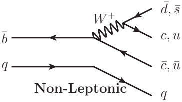
Examples of B decay Feynman Diagrams



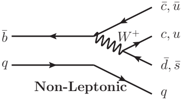
$B \to D^{(*)}\ell\nu$
$B \to \pi\ell\nu$
$B_s \to K\ell\nu$



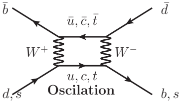
$B \to \tau\nu$



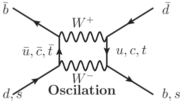
$B \to \pi\pi$
$B \to K\pi$



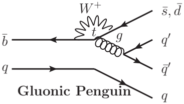
$B \to \pi\pi$
$B \to K\pi$



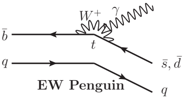
$B^0_{(s)} - \bar{B}^0_{(s)}$



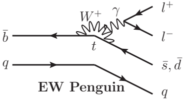
$B^0_{(s)} - \bar{B}^0_{(s)}$



$B \to \pi\pi$
$B \to K\pi$

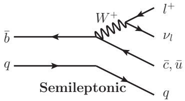


$B^0 \to K^{(*)0}\gamma$



$B \to \pi\ell\ell$
$B \to K^{(*)}\ell\ell$
$B_s \to \phi\ell\ell$

Will be covered in other talks ···



$B \to D^{(*)}\ell\nu$

$B \to \pi\ell\nu$

$B_s \to K\ell\nu$

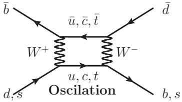
Semileptonic B_s Decays

inside Heavy Flavors and the CKM Matrix

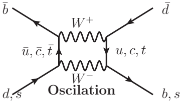
Friday, 15:00 - 15:20

Presenter(s): Oliver WITZEL (University of Colorado Boulder)

Will be covered in other talks ⋯



$B^0_{(s)} - \bar{B}^0_{(s)}$



$B^0_{(s)} - \bar{B}^0_{(s)}$

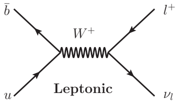
Short-Distance Matrix Elements for D^0-Meson Mixing from $N_f = 2 + 1$ Lattice QCD

inside Heavy Flavors and the CKM Matrix

Friday, 14:00 - 14:30

Presenter(s): Dr. Chia Cheng CHANG (LBL)

Will be covered in other talks ⋯



$B \to \tau\nu$

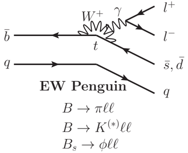
B and *D* Meson Leptonic Decay Constants and Quark Masses from Four-Flavor Lattice QCD

inside Heavy Flavors and the CKM Matrix

Friday, 14:30 - 15:00

Presenter(s): Carleton DETAR (University of Utah)

FCNC B decays



- Flavor-changing neutral current (FCNC) processes are forbidden at tree level in the standard model (SM).
- They only occur through loop (penguin, or box) diagrams in the SM.
- It is a promising probe of new physics with heavy particles: SUSY, non-SM Higgs et al.
- They can also be used to determine $|V_{ts}|$ and $|V_{td}|$.
- $b \to s$: $B \to K\ell\ell$
- $b \to d$: $B \to \pi\ell\ell$

Tensions with the Standard Model

- The ratio of branching fractions

$$R_K \equiv \frac{\mathcal{B}(B^+ \to K^+\mu^+\mu^-)}{\mathcal{B}(B^+ \to K^+e^+e^-)} \quad (1)$$

is 2.6σ lower than the SM(LHCb, arXiv:1406.6482, PRL 2014).

- The ratio of branching fractions

$$R_{K^{*0}} \equiv \frac{\mathcal{B}(B^+ \to K^{*0}\mu^+\mu^-)}{\mathcal{B}(B^+ \to K^{*0}e^+e^-)} \quad (2)$$

is 2.1-2.3 and 2.4-2.5 σ lower than the SM(LHCb, arXiv:1705.05802, JHEP 2017).

- Independently, the branching ratio of the $B^+ \to K^+\mu^+\mu^-$ is about 45% (2σ) smaller than the SM prediction(LHCb, arXiv:1403.8044, JHEP 2014).

- Angular distribution of $B^0 \to K^{*0}\mu^+\mu^-$, P_5', differs from SM by 2.5σ in two bins(LHCb, arXiv:1308.1707, PRL 2013).

Effective action and Operator Product Expansion (OPE)

- The effective weak Hamiltonian of the $b \to s(d)\ell\ell$ transition under operator product expansion (OPE) with α_s and Λ/m_b corrections is

$$\mathcal{H}_{\rm eff} = -\frac{4G_F}{\sqrt{2}} V_{tb} V^*_{ts(d)} \sum_i C_i(\mu) Q_i(\mu) + \cdots . \tag{3}$$

  The standard model prediction has the following generic form

$$A(B \to P\ell\ell) = \langle P|\mathcal{H}_{\rm eff}|B\rangle = \sum_i (\text{prefactors}) \times (\text{CKM elements}) \times \langle P|Q_i(\mu)|B\rangle . \tag{4}$$

- $A(B \to P\ell\ell)$: quantities can be measured directly in experiments.
- Prefactors: Wilson coupling coefficients (short distance physics); sensitive to new physics.
- CKM elements: depend on the process.
- Hadronic matrix element operators: non-perturbative quantities, form factors (long distance physics). They can be calculated via Lattice QCD.
- Non-factorizable conributions need to be taken into account appropriately(D.Du et al. arXiv:1510.02349, PRD 2016).

Hadronic matrix elements and form factors

- The pseudoscalar-to-pseudoscalar transitions can be written in terms of three form factors

$$\langle P(p_P)|\bar{q}b|B(p_B)\rangle = \frac{M_B^2 - M_P^2}{m_b - m_q} f_0(q^2), \tag{5}$$

$$\langle P(p_P)|\bar{q}\gamma^\mu b|B(p_B)\rangle = f_+(q^2)\left[(p_B + p_P)^\mu - q^\mu \frac{M_B^2 - M_P^2}{q^2}\right] + f_0(q^2) q^\mu \frac{M_B^2 - M_P^2}{q^2},$$
$$= \sqrt{2M_B}\left[\frac{{P_B}^\mu}{M_B} f_\parallel(E_P) + \left(p_P^\mu - (p_P \cdot p_B) p_B^\mu \frac{E_P}{M_B}\right) f_\perp(E_P)\right], \tag{6}$$

$$\langle P(p_P)|i\bar{q}\sigma^{\mu\nu} b|B(p_B)\rangle = \frac{2}{M_B + M_P}(p_B^\mu p_P^\nu - p_B^\nu p_P^\mu) f_T(q^2). \tag{7}$$

- The form factors $f_0(q^2)$, $f_+(q^2)$, and $f_T(q^2)$ are functions of $q^2 = (p_B - p_P)^2$.
- For Lattice QCD, it is convenient to use $f_\parallel(E_P)$, $f_\perp(E_P)$, and $f_T(E_P)$.

$$f_\parallel(E_P) = \frac{\langle P|V^4|B\rangle}{\sqrt{2M_B}}, \tag{8}$$

$$f_\perp(E_P) = \frac{\langle P|V^i|B\rangle}{\sqrt{2M_B}} \frac{1}{p_P^i}. \tag{9}$$

Lattice form factors

- For Lattice QCD, there is no difference between the tree level $B \to \pi\ell\nu$ and the FCNC $B \to \pi\ell\ell$ decays.
- In the SM, the tensor form factor, f_T, enters into the FCNC decays but not the tree level ones.
- The pseudoscalar to vector decays, such as $B \to K^*\ell\ell$,
  - have many more form factors;
  - not "gold-plated".
    - "Gold-plated": hadrons that have very small decay widths and are well below strong decay thresholds.
  - Theoretical framework now exists for semileptonic B decays to vector meson final states(Briceño et al. arXiv:1406.5965, PRD 2015; Agadjanov et al. arXiv:1605.03386, NPB 2016).
  - Lattice QCD calculations are underway.
- In the following, I will only focus on the $B \to \pi$ and $B \to K$ decay form factors.

Simplified procedure of getting the form factors

- Design (pick) a lattice action.
- Pick simulation parameters (a, m_q, L_x, L_t, g_0, $\cdots$) to generate the vacuum background fields (configurations), with "sea" quarks.
- Construct lattice interpolating operators for mesons (composed of "valence" quarks) and currents and then construct the correlation functions on the lattice.
- For each ensemble (with a set of fixed simulation parameters) :
  - Determine the lattice B meson masses, P meson masses and energies from the lattice two-point correlation functions.
  - Determine the lattice form factors $f_{\parallel}^{\mathrm{lat}}$ and $f_{\perp}^{\mathrm{lat}}$ at several discrete P meson momentum $\boldsymbol{p}_P$ from two- and three-point correlation functions.
- Obtain the continuum $f_{\parallel}$ and $f_{\perp}$ at a finite $\boldsymbol{p}_P$ by extrapolating the lattice form factors to physical quark masses and continuum (zero lattice spacing) limits, and matching the corresponding currents.
- Construct the continuum form factors f_+ and f_0 from $f_{\parallel}$ and $f_{\perp}$ and extrapolate to the whole kinematically allowed momentum transfer region, especially at $q^2 = 0$.

Comprehensive error analysis will be done in all the above steps.

Lattice actions

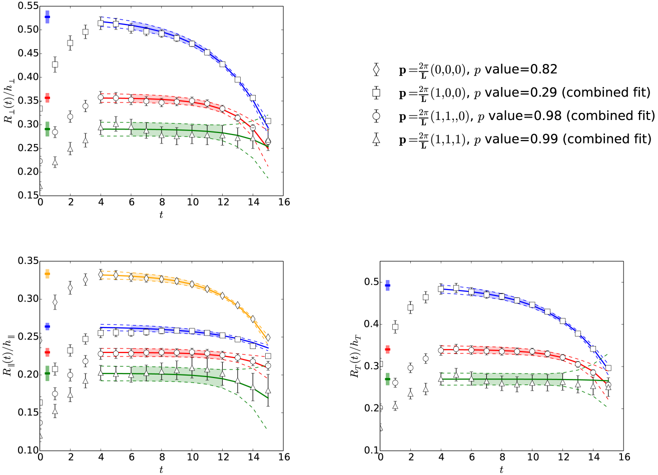
- Gauge actions for the gluon fields
  - Symanzik improved action: $\mathcal{O}(a^2)$-improved.
- Fermion actions for the "sea" and "valence" quarks
  - Light quarks ($m_\ell < \Lambda_{QCD}$):
    Staggered (asqtad, HISQ); Domain-Wall; Clover; Twisted-Mass Wilson, $\cdots$
  - Heavy quarks:
    - For c quarks: can use light quark methods, if action is sufficiently improved.
    - For b quarks:non-relativistic QCD (NRQCD); heavy quark effective theory (HQET); Relativistic heavy quark (HQ) actions (Fermilab, RHQ, $\cdots$); $\cdots$
- Different lattice actions have different discretization effects.
- The "sea" quarks usually include u, d, s and even c quarks:
  $n_f = 2+1, 1+1+1, 2+1+1,$ or $1+1+1+1$.
- The b quark appears as the "valence" quark for B decays.
- Partially quenched: the "sea" and "valence" quark masses are not equal.

Lattice form factor calculations

- HPQCD:
  - $B \to K$ (f_+, f_0, f_T):
    MILC 2+1 asqtad ensembles; HISQ light valence; NRQCD b quarks.
    5 ensembles; $a \approx 0.12$ fm and 0.09 fm(arXiv:1306.2384, PRD 2013; arXiv:1306.0434, PRL 2013).
  - $B \to \pi$ (f_0 at zero recoil):
    MILC 2+1+1 HISQ ensembles; HISQ light valence; NRQCD b quarks.
    8 ensembles; $a \approx 0.15$ fm, 0.12 fm and 0.09 fm
    (arXiv:1510.07446, PRD 2016).
- RBC/UKQCD:
  - $B \to \pi$ (f_+, f_0):
    RBC/UKQCD 2+1 domain-wall Fermion(DWF) ensembles; DWF light valence; RHQ b quarks.
    5 ensembles; $a \approx 0.11$ fm and 0.09 fm(arXiv:1501.05373, PRD 2015).
- FNAL/MILC
  - $B \to \pi$ (f_+, f_0, f_T):
    MILC 2+1 asqtad ensembles; asqtad light valence; Fermilab b quarks.
    12 ensembles; $a \approx 0.12$ fm, 0.09 fm, 0.06 fm, and 0.045 fm
    (arXiv:1503.07839, PRD 2015; arXiv:1507.01618, PRL 2015).
  - $B \to K$ (f_+, f_0, f_T):
    MILC 2+1 asqtad ensembles; asqtad light valence; Fermilab b quarks.
    10 ensembles; $a \approx 0.12$ fm, 0.09 fm, 0.06 fm, and 0.045 fm
    (arXiv:1509.06235, PRD 2016).

$B \to \pi$ form factors: fit two- and three-point correlators

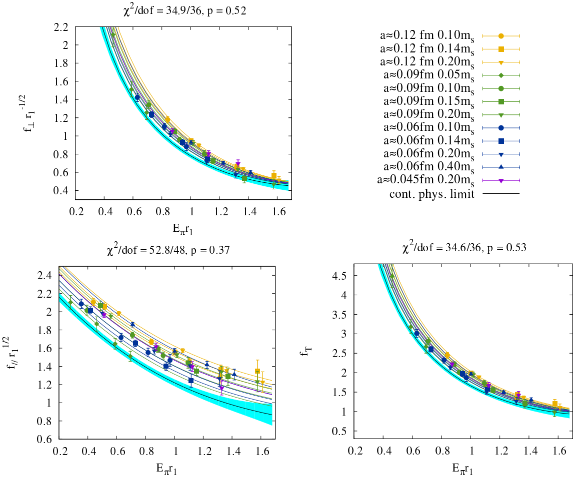
FNAL/MILC(arXiv:1503.07839, PRD 2015)



- Correlator fits. Determine the lattice form factors.
- Works in the B meson rest frame. The pions have finite discrete momenta.
- The quantities $R_{\parallel,\perp,T}$ are ratios of the two- and three-point correlators and related to the form factors.

$B \to \pi$ form factors: chiral-continuum extrapolation

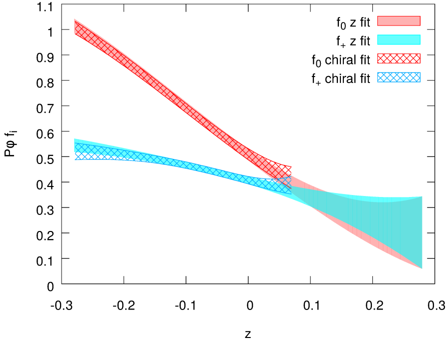
FNAL/MILC(arXiv:1503.07839, PRD 2015)



- Chiral-continuum extrapolation of lattice form factors.
- The extrapolated form factors are still in the large q^2 region ($17\ \mathrm{GeV}^2 \leq q^2 \leq 26\ \mathrm{GeV}^2$).

$B \to \pi$ form factors: kinematic range extrapolation

FNAL/MILC(arXiv:1503.07839, PRD 2015)



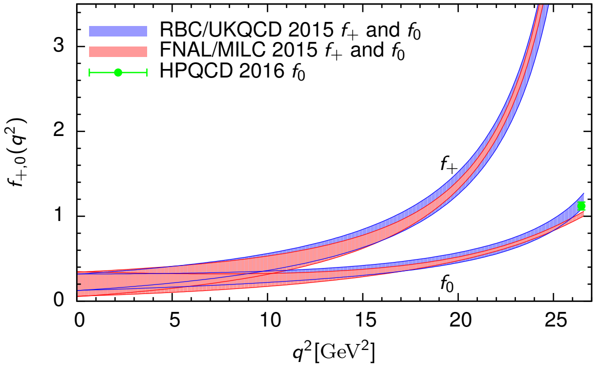
- Extrapolate the continuum form factors to the whole kinematically allowed region, especially at $q^2 = 0$ (right most region in the above z-plane).
- Model independent z-expansion is used for the extrapolation. Based on unitarity and analyticity of the form factors.
- Central values, errors and correlation matrix of the coefficients of the form factors are provided. The form factors can be reconstructed easily.

$B \to \pi$ form factors: f_+ and f_0

RBC/UKQCD(arXiv:1501.05373, PRD 2015)

FNAL/MILC(arXiv:1503.07839, PRD 2015)

HPQCD(arXiv:1510.07446, PRD 2016)

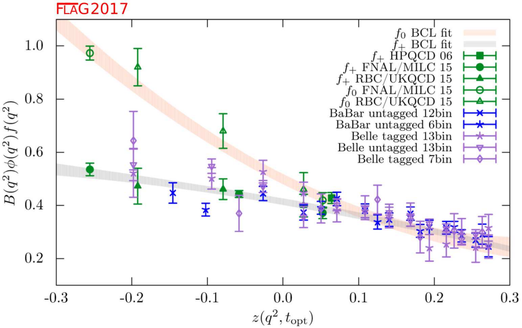


- Comparison among RBC/UKQCD, FNAL/MILC, and HPQCD form factors.
- RBC/UKQCD and FNAL/MILC form factors are in good agreement.
- At q^2_{max}, HPQCD agrees too.

$B \to \pi$ form factors: f_+ and f_0

Flavor Lattice Averaging Group (FLAG)

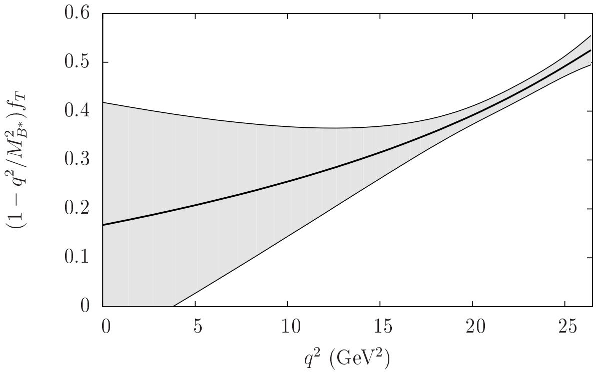
(arXiv:1607.00299, EPJC 2017; Web update)



- Experimental data are rescaled by $|V_{ub}|^2$.
- Shape of f_+(gray band) agrees with experimental data.

$B \to \pi$ form factors: f_T

FNAL/MILC(arXiv:1507.01618, PRL 2015)

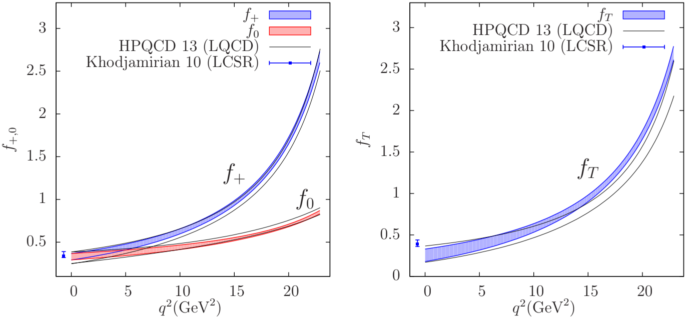


- FNAL/MILC also calculated the $B \to \pi$ tensor form factor f_T.

$B \to K$ form factors: f_+, f_0, f_T

HPQCD(arXiv:1306.2384, PRD 2013; arXiv:1306.0434, PRL 2013)

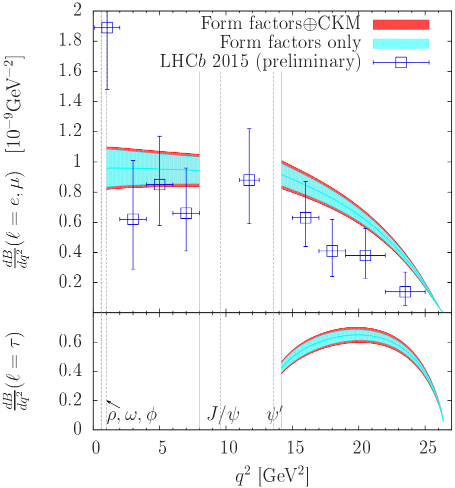
FNAL/MILC(arXiv:1509.06235, PRD 2016)



- Comparison between HPQCD and FNAL/MILC form factors.
- All three form factors are consistent with each other.
- Consistent with LCSR(Khodjamirian et al. arXiv:1006.4945, JHEP 2010).

Phenomenology for $B \to \pi\ell\ell$

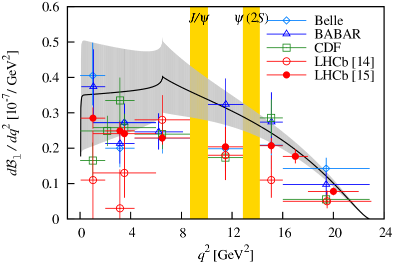
FNAL/MILC(arXiv:1507.01618, PRL 2015)



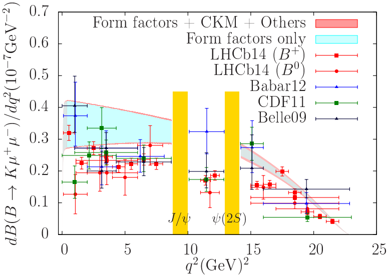
- FNAL/MILC: SM partial branching fraction for $B^+ \to \pi^+\mu^+\mu^-$ and $B^+ \to \pi^+\tau^+\tau^-$.
- They agree with LHCb(arXiv:1509.00414, JHEP 2015).

Phenomenology for $B \to K\ell\ell$

HPQCD(arXiv:1306.0434, PRL 2013)



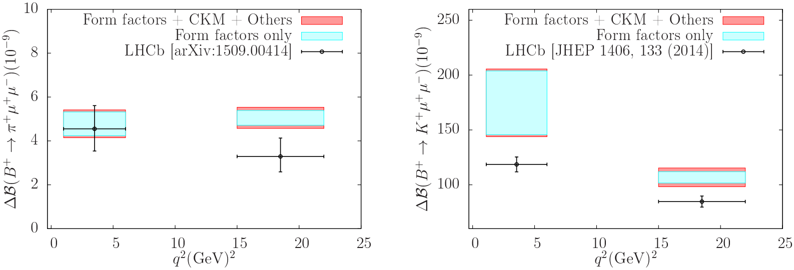
FNAL/MILC(arXiv:1507.01618, PRL 2015)



- SM differential branching fraction for $B \to K\mu^+\mu^-$.
- The μ mode experimental results are smaller than the SM prediction.

Phenomenology for $B \to \pi \ell\ell$ and $B \to K \ell\ell$

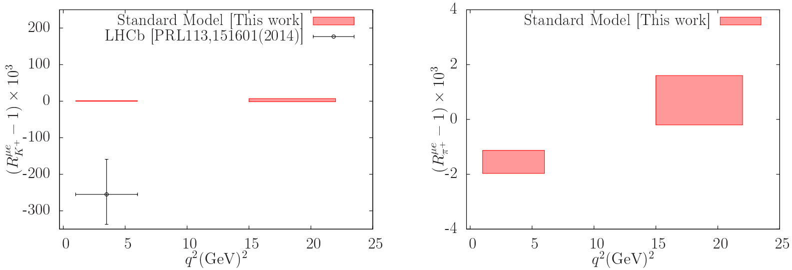
D.Du et al.(arXiv:1510.02349, PRD 2016)



- SM partially integrated branching ratios for $B^+ \to \pi^+\mu^+\mu^-$ and $B^+ \to K^+\mu^+\mu^-$.
- FNAL/MILC form factors are used.
- 1-2σ tension between SM theory and LHCb experimental measurement (arXiv:1509.00414, JHEP 2015; arXiv:1403.8044, JHEP 2014).

Phenomenology for $B \to \pi\ell\ell$ and $B \to K\ell\ell$

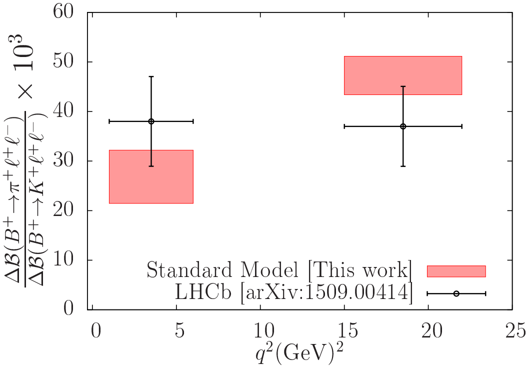
D.Du et al.(arXiv:1510.02349, PRD 2016)



- SM lepton-flavor-violating ratios.
- The SM prediction of R_K is unity up to corrections of order $(m_\ell^2/M_B^2, m_\ell^4/q^4)$.
- FNAL/MILC form factors are used.
- 2.6 σ tension between SM theory and LHCb experimental measurement(LHCb arXiv:1406.6482, PRL 2014).

Phenomenology for $B \to \pi\ell\ell$ and $B \to K\ell\ell$

D.Du et al.(arXiv:1510.02349, PRD 2016)



- Ratio of partially integrated branching ratios.
- FNAL/MILC form factors are used.
- Some tension between SM theory and LHCb experimental measurement.

Summary

- Lattice QCD results for $B \to \pi$ and $B \to K$ scalar, vector, and tensor form factors are available.
- The form factors can be used to calculate SM observables for the $B \to K(\pi)\ell\ell$ process and compared with experimental measurements.
- There is still tension between experimental measurements and SM calculations for several physical quantities.
- New methods are being developed.
- New Lattice QCD calculations are underway.

On-going and relevant projects

- FNAL/MILC HISQ: $B \to \pi$, $B \to K$, $B_s \to K$ (arXiv:1710.09442, EPJC 2018; arXiv:1711.08085, EPJC 2018).
- HPQCD: $B_{(s)} \to D^*_{(s)}$(arXiv:1711.11013, PRD 2018).
- RBC/UKQCD: $B_s \to \phi$, $B_{(s)} \to D^{(*)}_{(s)}$(arXiv:1612.05112).
- ALPHA: $B_s \to K$ (arXiv:1701.03923; arXiv:1601.04277, PLB 2016).
- Horgan et al.: $B \to K^*$, $B_s \to \phi$(arXiv:1310.3887, PRL 2013; arXiv:1310.3722, PRD 2014).
- Detmold, Meinel et al.: $\Lambda_b \to \Lambda$(arXiv:1212.4827, PRD 2013; arXiv:1602.01399, PRD 2016; arXiv:1608.08110).

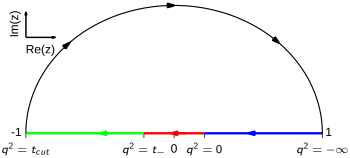
Thank You!

BACKUP

z-parametrization

- Map the whole complex q^2 plane onto the unit disk in the z plane.

$$
\begin{aligned}
z(q^2, t_0) &= \frac{\sqrt{t_{\rm cut} - q^2} - \sqrt{t_{\rm cut} - t_0}}{\sqrt{t_{\rm cut} - q^2} + \sqrt{t_{\rm cut} - t_0}}, \\
q^2 &= t_{\rm cut} - (\frac{1+z}{1-z})^2 (t_{\rm cut} - t0), \\
t_{\rm cut} &= (M_B + M_\pi)^2, \\
t_- &= (M_B - M_\pi)^2, \\
t_0 &= t_{\rm cut}(1 - \sqrt{1 - t_-/t_{\rm cut}}).
\end{aligned}
$$



- $t_{\rm cut}$ is the $B\pi$ pair-production threshold.

- t_- is the maximum momentum-transfer squared allowed in the $B \to \pi$ decay.

- t_0 is chosen such that the full kinematic range for $B \to \pi$ decay is centered around the origin $z = 0$, i.e., by solving $z(q^2 = 0, t_0) = -z(q^2 = t_-, t_0)$.

- Kinematically allowed range:
  $z(q^2 = t_-, t_0) \leq z \leq z(q^2 = 0, t_0)$.

By analyticity and positivity properties of vacuum polarization functions, the form factors can be expanded as (BGL)

$$f_+(q^2) = \frac{1}{B(q^2)\phi(q^2, t_0)} \sum_{n=0}^{\infty} a_n(t_0) z^n , \tag{10}$$

where $B(q^2) = z(q^2, M_{B*}^2)$ is the Blaschke factor,which takes the pole(s) into account; $\phi(q^2, t_0)$ is a complicated outer function, computable via perturbative QCD and the operator product expansion.
From unitarity and crossing symmetry, one gets (unitarity condition):

$$\sum_{n=0}^{\infty} a_n^2(t_0) \leq 1. \tag{11}$$

An alternative simpler parametrization is

$$f_+(q^2) = \frac{1}{1 - q^2/m_{B*}^2} \sum_{k=0}^{K} b_k(t_0)\, z^k . \tag{12}$$

From angular momentum conservation and analycity, one can get $\frac{\partial f_+}{\partial z}|_{z=-1} = 0$, which means $b_K = \sum_{k=0}^{K-1} (-1)^{k-K-1} \frac{k}{K} b_k$.

Therefore, Eq. (12) can be written as (BCL)

$$f_+(q^2) = \frac{1}{1 - q^2/m_{B*}^2} \sum_{k=0}^{K-1} b_k \left[z^k - (-1)^{k-K} \frac{k}{K} z^K \right] , \tag{13}$$

f_0 can be expanded as $\sum_{k=0}^{K} b_k z^k$ or as in Eq. (12) depending on the importance of the scalar pole.
The unitarity condition in BGL Eq. (11) becomes

$$\sum_{j,k=0}^{K} B_{jk}(t_0) b_j(t_0) b_k(t_0) \leq 1, \tag{14}$$

where the B_{jk} is calculable via the outer function $\phi(q^2, t_0)$.