Axial Vector Form Factors from Lattice QCD

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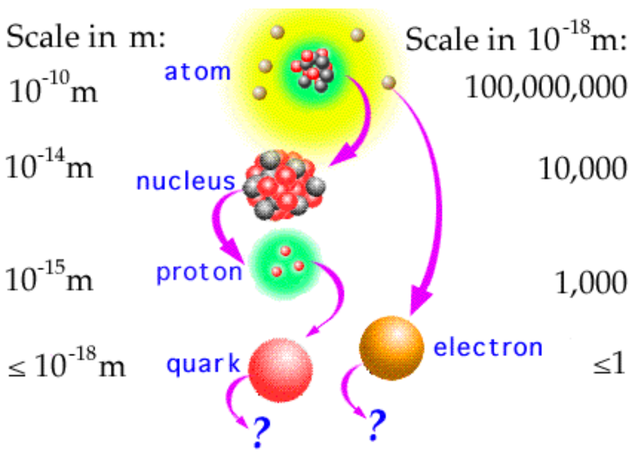
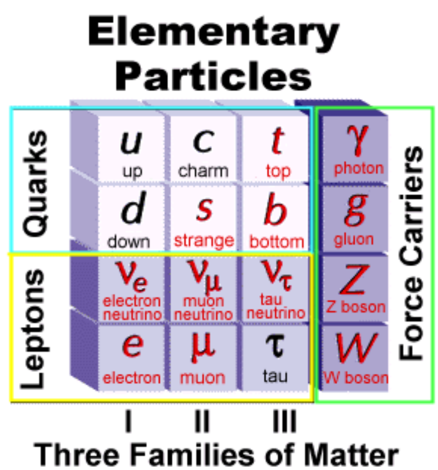
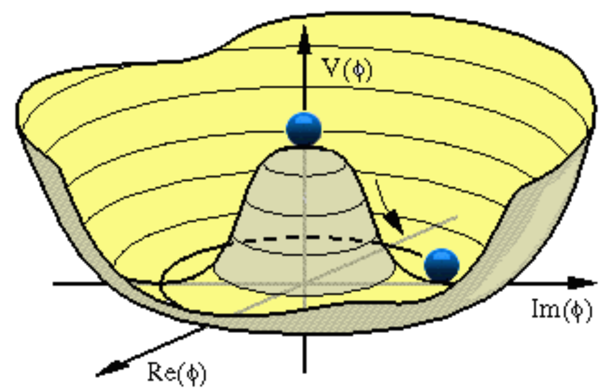






LA-UR 16-29009

CIPANP 2018, 31 May 2018, Palm Springs

The clover-on-HISQ calculations by the PNDME Collaboration

- Tanmoy Bhattacharya
- Vincenzo Cirigliano
- Yong-Chull Jang
- Huey-Wen Lin
- Boram Yoon

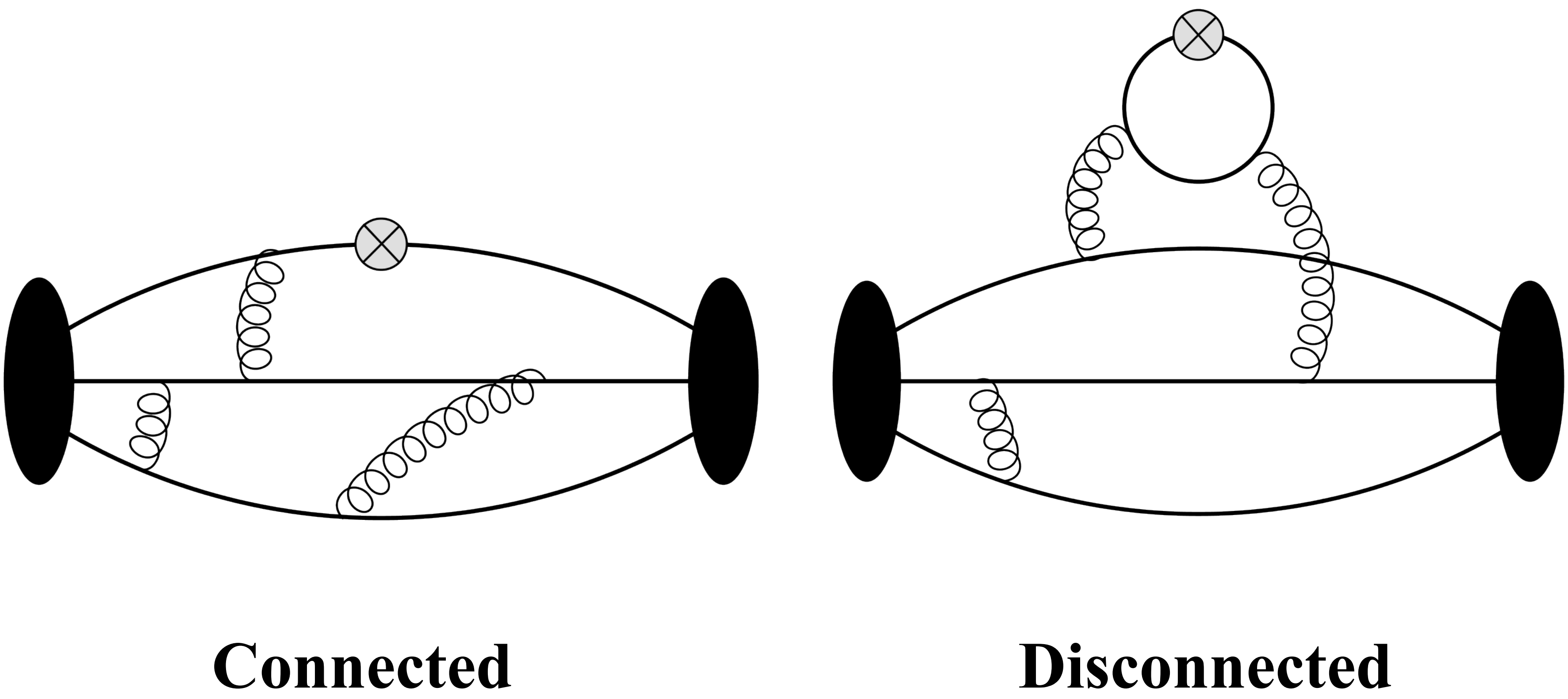
Bhattacharya et al, PRD85 (2012) 054512
Bhattacharya et al, PRD89 (2014) 094502
Bhattacharya et al, PRD92 (2015) 114026
Bhattacharya et al, PRL 115 (2015) 212002
Bhattacharya et al, PRD92 (2015) 094511
Bhattacharya et al, PRD94 (2016) 054508
Gupta et al, PRD96 (2017) 114503

Outline

- Physics Motivation
  - Axial vector form factors of nucleon needed for the analysis of neutrino-nucleus scattering:
    - Monitoring neutrino flux
    - Cross-section off various nuclear targets (LAr)

- Challenge: controlling systematic errors in the lattice QCD calculations of the matrix elements of composite operators within nucleon states

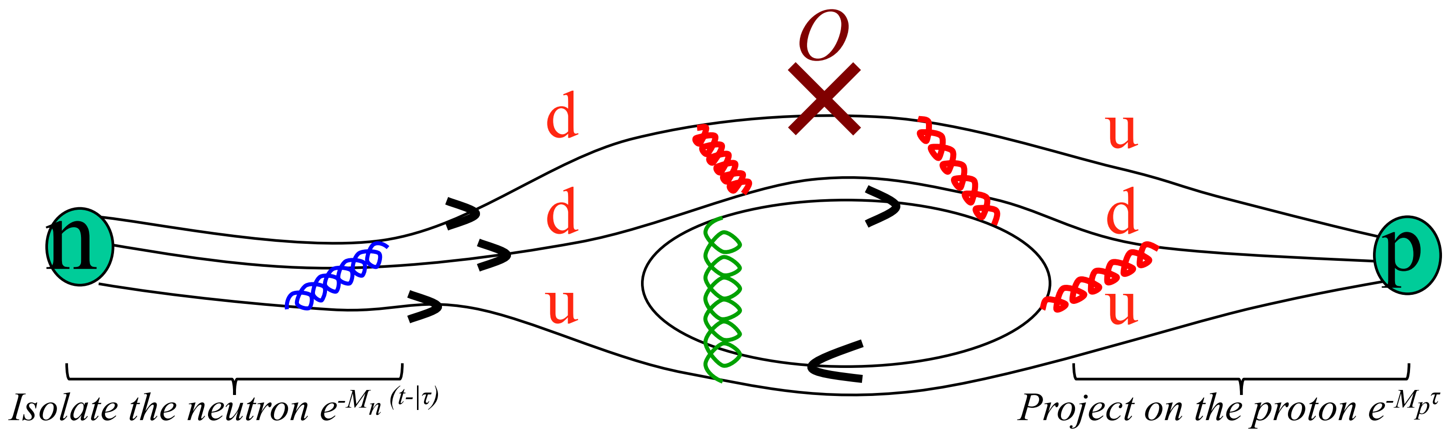
High precision estimates of the matrix elements of quark bilinear operators within the nucleon state, obtained from "connected" and "disconnected" 3-point correlation functions, needed to address a number of important physics questions



Matrix elements within nucleon states required by many experiments

- Isovector charges g_A, g_S, g_T $\langle p | \bar{u} \Gamma d | n \rangle$
- Axial vector form factors $\langle p(q) | \bar{u} \gamma_\mu \gamma_5 d(q) | n(0) \rangle$
- Vector form factors $\langle p(q) | \bar{u} \gamma_\mu d(q) | n(0) \rangle$
- Flavor diagonal matrix elements $\langle p | \bar{q} q | p \rangle$
- nEDM: Θ-term, quark EDM, quark chromo EDM, Weinberg operator, 4-quark operators
- $0\nu\beta\beta$
- Generalized Parton Distribution Functions

Calculating matrix elements using Lattice QCD

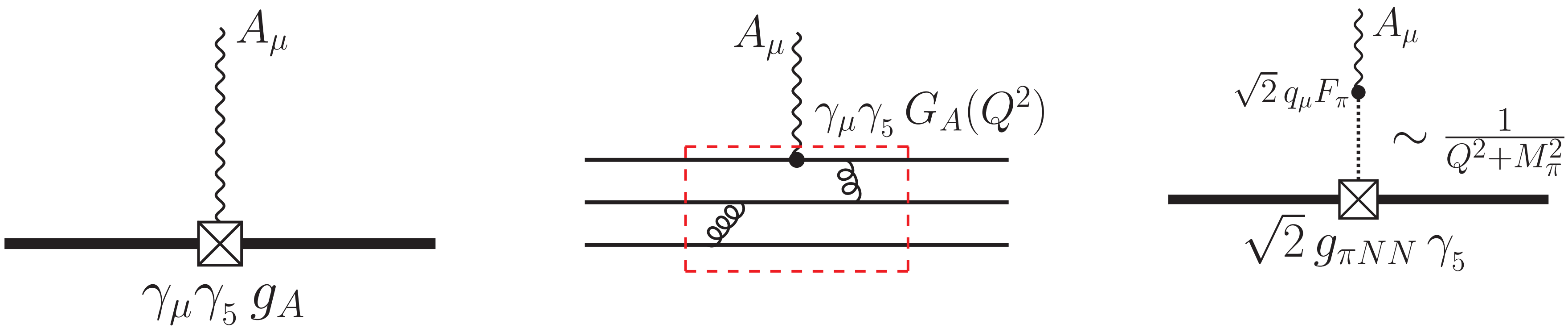


$$\langle \Omega | \hat{N}(t, p')\hat{O}(\tau, p'-p)\hat{N}(0,p) | \Omega \rangle =$$

$$\sum_{i,j} \langle \Omega | \hat{N}(p') | N_j \rangle e^{-\int dt\, H} \langle N_j | \hat{O}(\tau, p'-p) | N_i \rangle e^{-\int dt\, H} \langle N_i | \hat{N}(p) | \Omega \rangle =$$

$$\sum_{i,j} \langle \Omega | \hat{N}(p') | N_j \rangle e^{-E_j(t-\tau)} \langle N_j | \hat{O}(\tau, p'-p) | N_i \rangle e^{-E_i \tau} \langle N_i | \hat{N}(p) | \Omega \rangle$$

Axial-vector form factors



On the lattice we can calculate 3 form factors from ME of V_μ and A_μ:

- Axial: G_A
- Induced pseudoscalar: $\tilde{G}_P$
- Pseudoscalar: G_P

$$\langle N(p_f)|A^\mu(q)|N(p_i)\rangle = \bar{u}(p_f)\left[\gamma^\mu G_A(q^2) + q_\mu \frac{\tilde{G}_P(q^2)}{2M}\right]\gamma_5 u(p_i)$$

$$\langle N(p_f)|P(q)|N(p_i)\rangle = \bar{u}(p_f) G_P(q^2)\gamma_5\, u(p_i)$$

The 3 form factors are related by PCAC $\partial_\mu A_\mu = 2mP$

Challenges to obtaining high precision results for matrix elements within nucleon states

- **High Statistics: O(1,000,000) measurements**
- **Demonstrating control over all Systematic Errors:**
  - Contamination from excited states
  - Q^2 behavior of form factors
  - Non-perturbative renormalization of bilinear operators (RI_{smom} scheme)
  - Finite volume effects
  - Chiral extrapolation to physical m_u and m_d (simulate at physical point)
  - Extrapolation to the continuum limit (lattice spacing $a \to 0$)

Perform simulations on ensembles with multiple values of

- Lattice size: $M_\pi L \to \infty$
- Light quark masses: $\to$ physical m_u and m_d
- Lattice spacing: $a \to 0$

Toolkit

- Multigrid Dirac invertor → propagator $S_F = D^{-1}\eta$
- Truncated solver method with bias correction (AMA)
- Coherent source sequential propagator
- Deflation + hierarchical probing
- High Statistics
- 3-5 values of t_{sep} with smeared sources for S_F
- 2-, 3-, n-state fits to multiple values of t_{sep}
- Non-perturbative methods for renormalization constants
- Combined extrapolation in a, M_π, $M_\pi L$ (CCFV)
- Variation of results with CCFV extrapolation Ansatz

2+1+1 flavor HISQ lattices from MILC

m_s , m_c tuned to their physical values using M_{ss} and η_c

a (fm)	m_l/m_s	Volume	M_π L	M_π (MeV)	# Configs	HP	AMA
0.12 □	0.2	$24^3 \times 64$	4.55	310	1013	8,104	64,832
0.12 △	0.1	$24^3 \times 64$	3.29	225	1000/946	24,000	60,544
0.12 ◆	0.1	$32^3 \times 64$	4.38	228	958/744	7,664	47,616
0.12 ▽	0.1	40^3 x 64	5.49	228	1010	8,080	68,680
0.09 □	0.2	$32^3 \times 96$	4.51	313	881/2263	7,048	144,832
0.09 ◆	0.1	$48^3 \times 96$	4.79	226	890/964	7,120	123,392
0.09 ○	0.037	$64^3 \times 96$	3.90	138	883	7,064	84,768
0.06* □	0.2	$48^3 \times 144$	4.52	320	1000	8,000	64,000
0.06* ◆	0.1	$64^3 \times 144$	4.41	235	650	2,600	41,600
0.06 ○	0.037	$96^3 \times 192$	3.7	135	675	2700	43,220

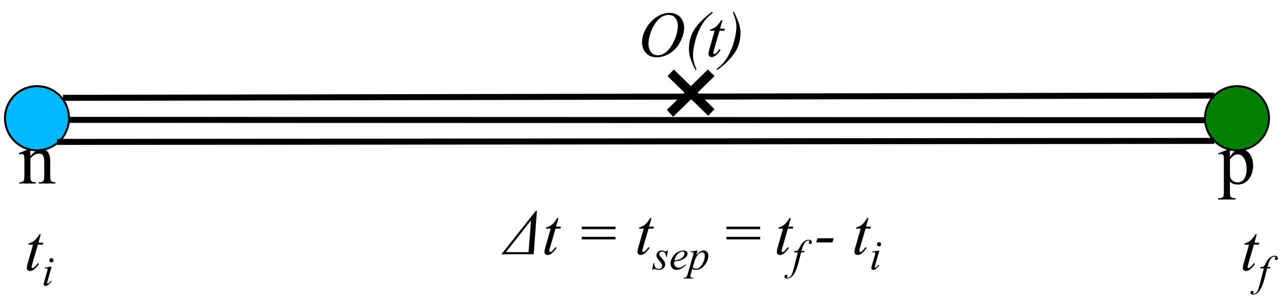
Controlling excited-state contamination: n-state fit

$$\Gamma^2(t) = |A_0|^2 e^{-M_0 t} + |A_1|^2 e^{-M_1 t} + |A_2|^2 e^{-M_2 t} + |A_3|^2 e^{-M_3 t} + \ldots.$$

$$\Gamma^3(t, \Delta t) = |A_0|^2 \langle 0|O|0\rangle e^{-M_0 \Delta t} + |A_1|^2 \langle 1|O|1\rangle e^{-M_1 \Delta t} + A_0 A_1^* \langle 0|O|1\rangle e^{-M_0 \Delta t} e^{-\Delta M (\Delta t - t)} + A_0^* A_1 \langle 1|O|0\rangle e^{-\Delta M t} e^{-M_0 \Delta t} + \ldots$$

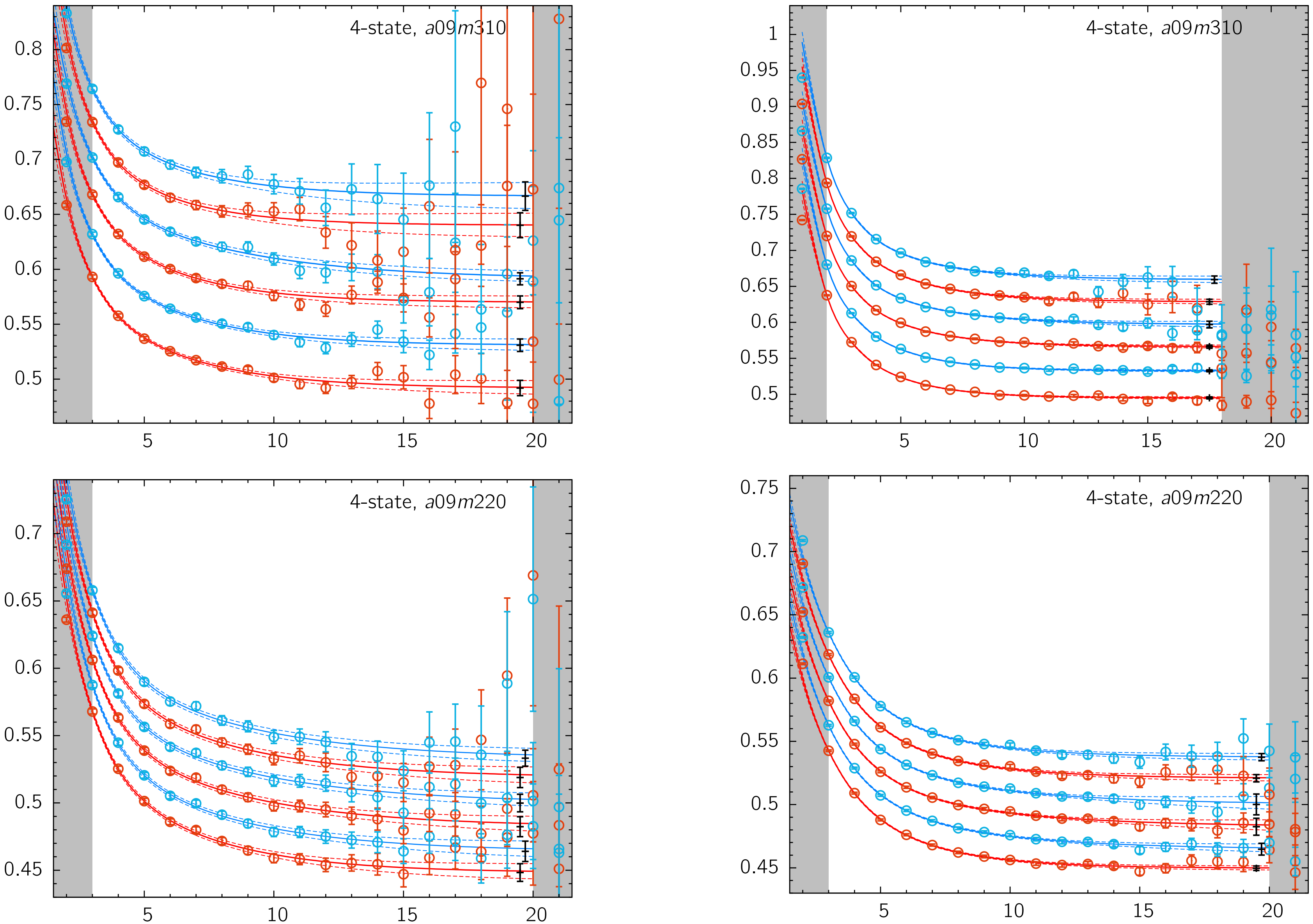
M_0, M_1, … masses of the ground & excited states

A_0, A_1, … corresponding amplitudes

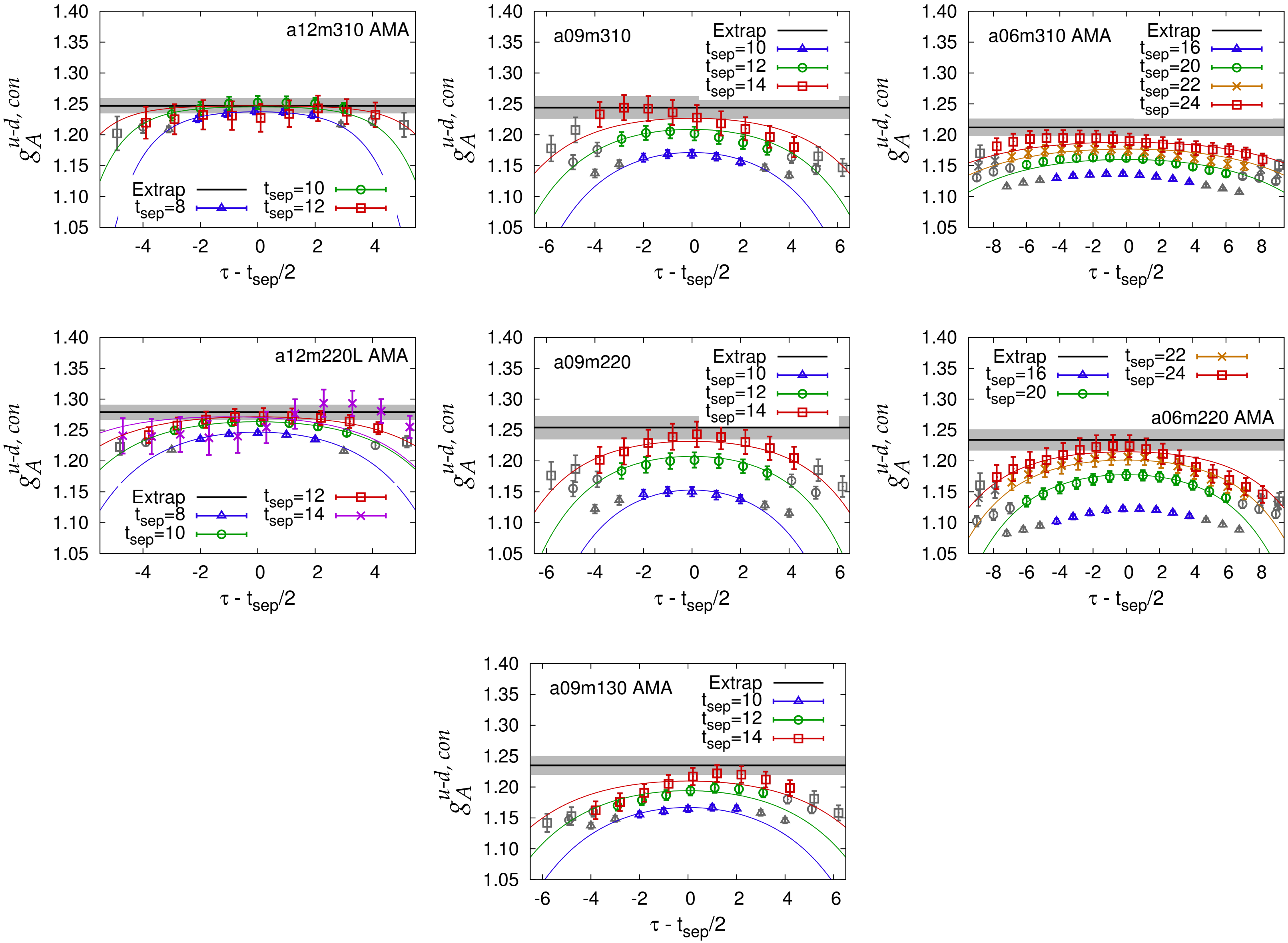


Make a simultaneous fit to data at multiple $\Delta t = t_{sep} = t_f - t_i$

4-state fits to 2-point correlation fn

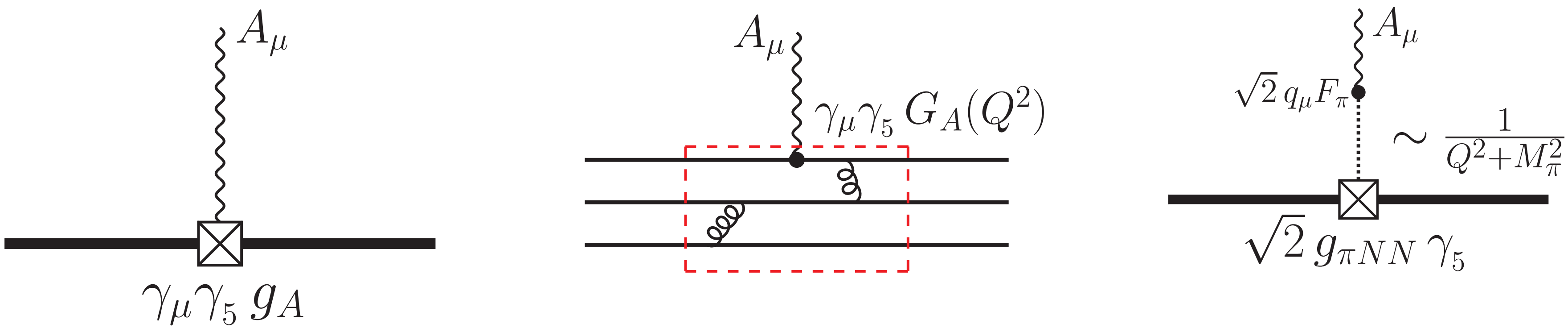


g_A: Excited State Contamination



Data and 2-state fits on 7 clover-on-HISQ ensembles: Bhattacharya et al, PRD94 (2016) 054508

Axial-vector form factors



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$$\langle N(p_f)|P(q)|N(p_i)\rangle = \bar{u}(p_f) G_P(q^2)\gamma_5\, u(p_i)$$

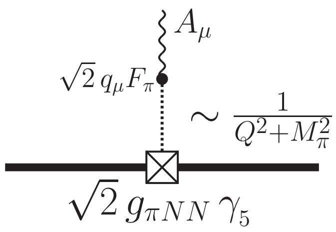
The 3 form factors are related by PCAC $\partial_\mu A_\mu = 2mP$

PCAC ($\partial_\mu A_\mu = 2\widehat{m}P$) requires

$$2\widehat{m}G_P(Q^2) = 2M_N G_A(Q^2) - \frac{Q^2}{2M_N}\tilde{G}_P(Q^2)$$

Pion pole-dominance hypothesis

$$\widetilde{G_P}(Q^2) = G_A(Q^2)\left[\frac{4M_N^2}{Q^2 + M_\pi^2}\right]$$



If pion pole-dominance holds

$\Rightarrow$ there is only one independent form factor

Goldberger-Treiman relation

$$F_\pi\ g_{\pi NN} = M_N\ g_A$$

Steps in the FF calculations

- Calculate matrix elements for different t_{sep}
- Control excited-state contamination: $p=0$, $p\neq 0$
- From different Lorentz components & the momentum dependence extract the form factors
- Fit Q^2 behavior of $G_i(q^2)$: (dipole, z-expansion, …)
- Calculate $r_i(a, M_\pi, M_\pi L)$: $\langle r_i^2 \rangle = -\frac{6}{dq^2}\left[\frac{\hat{G}_i(q^2)}{\hat{G}_i(0)}\right]_{q^2=0}$
- Extrapolate $r_i(\mathrm{a}\rightarrow 0,\ \mathrm{M}_\pi \mathrm{L}\rightarrow\infty,\ \mathrm{M}_\pi\rightarrow 135\mathrm{MeV})$

Dipole ansatz for q^2 behavior of G_E, G_M, G_A

$$G_i(q^2) = \frac{G_i(0)}{\left(1+\frac{q^2}{M_i^2}\right)^2}$$

M_i is the dipole mass

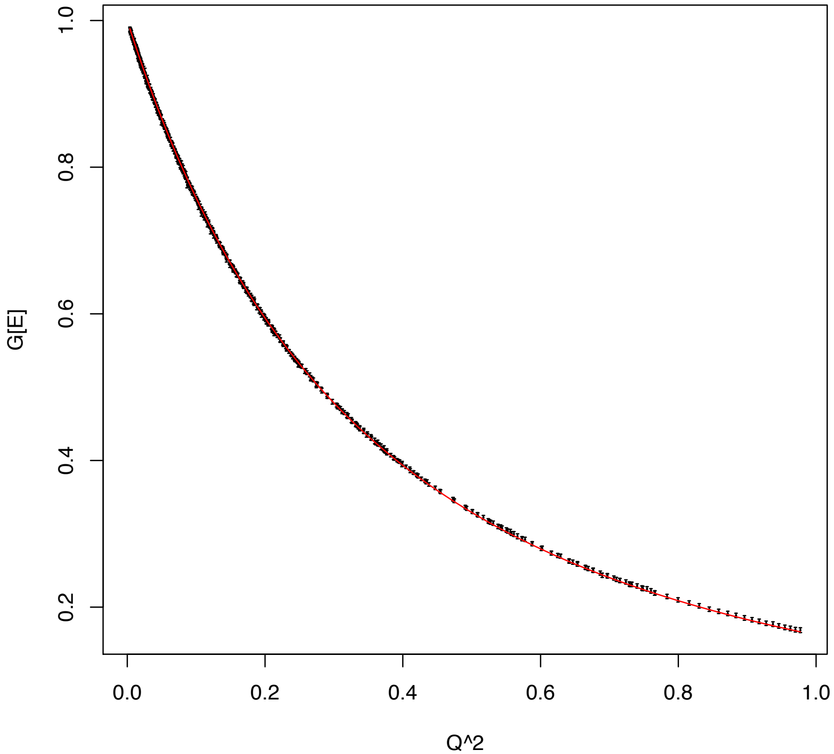
- Corresponds to exponential decaying distribution
- Has the desired $1/q^4$ behavior for $q^2 \rightarrow \infty$

The charge radii are defined as

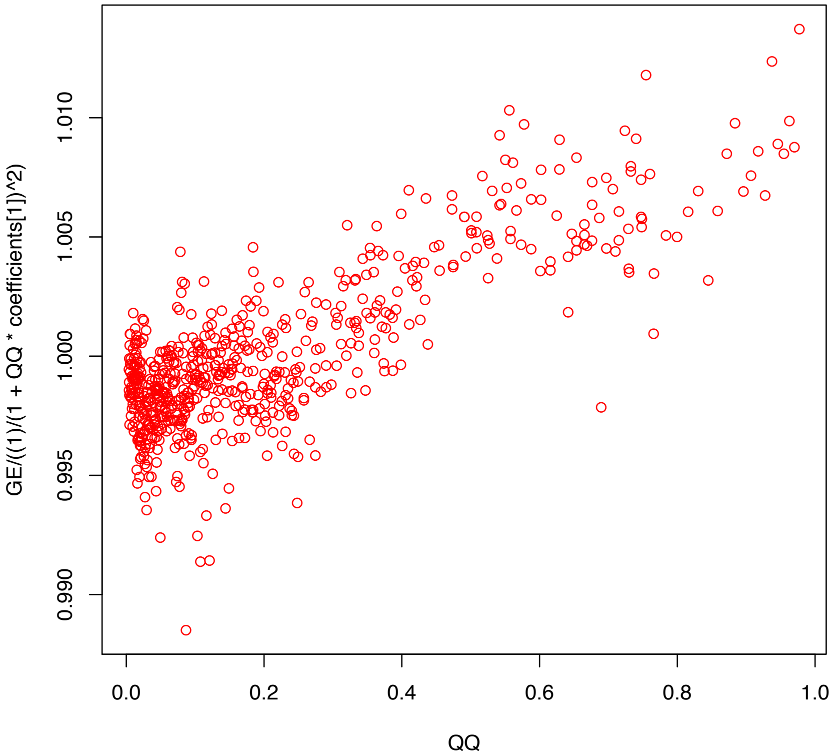
$$\left\langle r_i^2 \right\rangle = -\frac{6}{dq^2}\left[\frac{\hat{G}_i(q^2)}{\hat{G}_i(0)}\right]_{q^2=0}$$

$$\left\langle r_i^2 \right\rangle = \frac{12}{M_i^2}$$

Is dipole a good ansatz?



dipole fit to Mainz data



$$\frac{\text{Mainz data}}{\text{dipole fit}}$$

Thanks to D. Higinbotham for providing his binned version of the Mainz data

z-expansion

The form factors are analytic functions of Q^2 below a cut starting at n-particle threshold t_{cut}.

A model independent approach is the z-expansion:

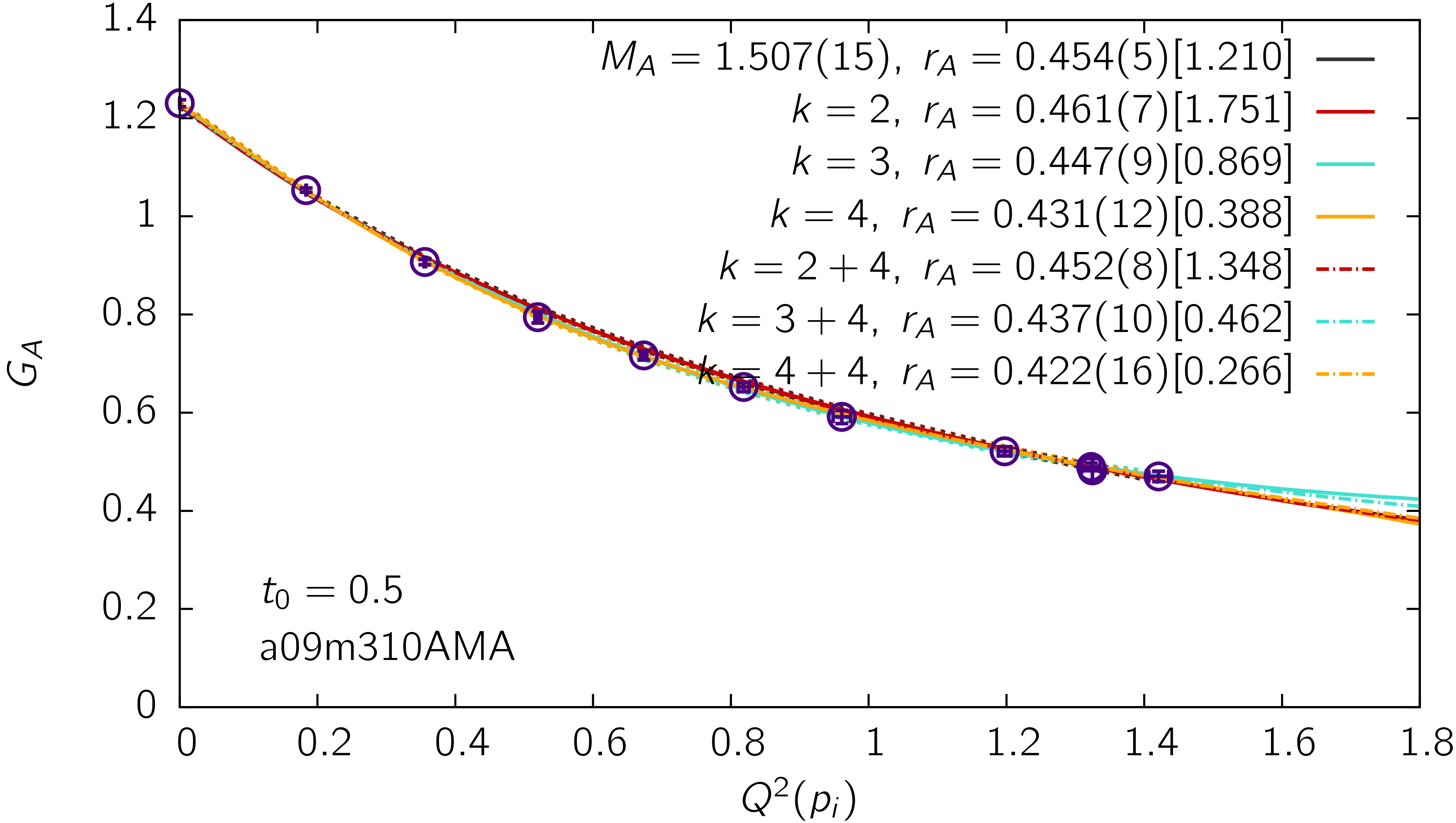
$$\hat{G}(Q^2) = \sum_{k=0}^{\infty} a_k z(Q^2)^k \qquad \text{with} \qquad z = \frac{\sqrt{t_{\text{cut}} + Q^2} - \sqrt{t_{\text{cut}} + Q_0^2}}{\sqrt{t_{\text{cut}} + Q^2} + \sqrt{t_{\text{cut}} + Q_0^2}}$$

with $t_{\rm cut} = 4m_\pi^2$ for $G_{E,M}$ and $t_{\rm cut} = 9m_\pi^2$ for G_A. We choose $Q_0 = 0$

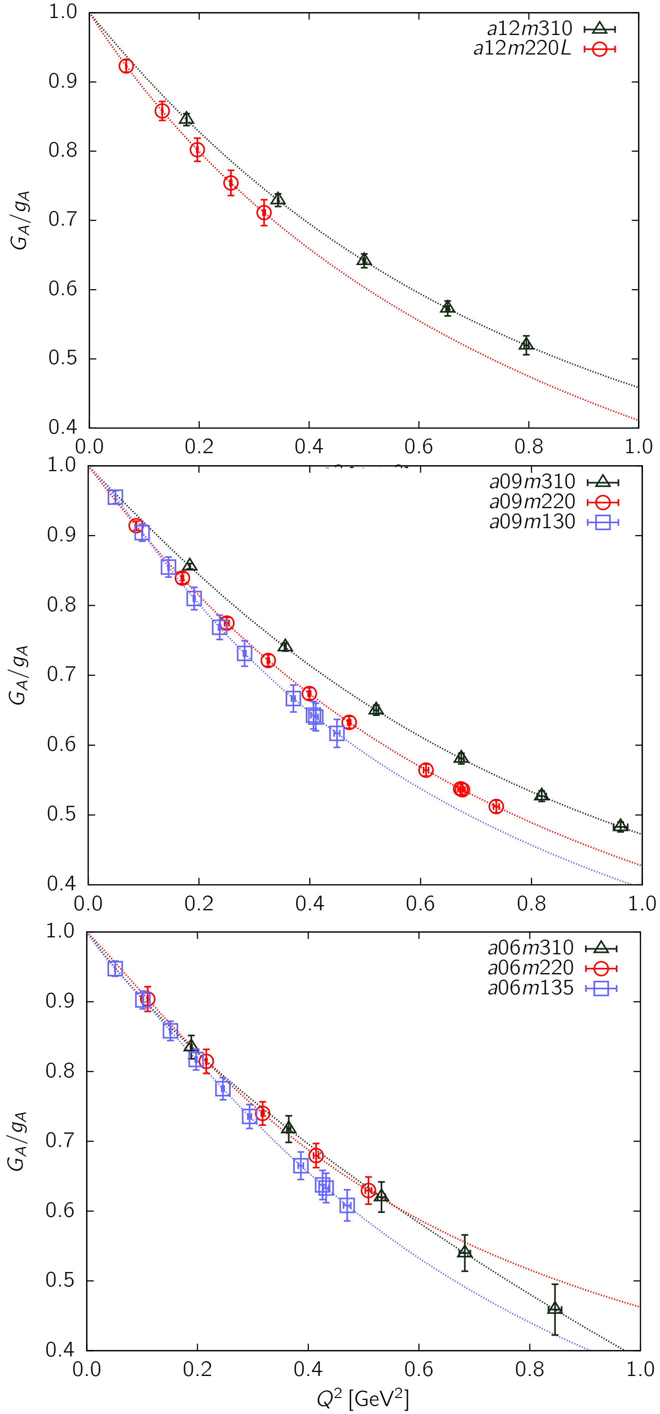
Incorporate $1/Q^4$ behavior as $Q^2 \to \infty$ via sum rules

Truncation of the series in *z?*

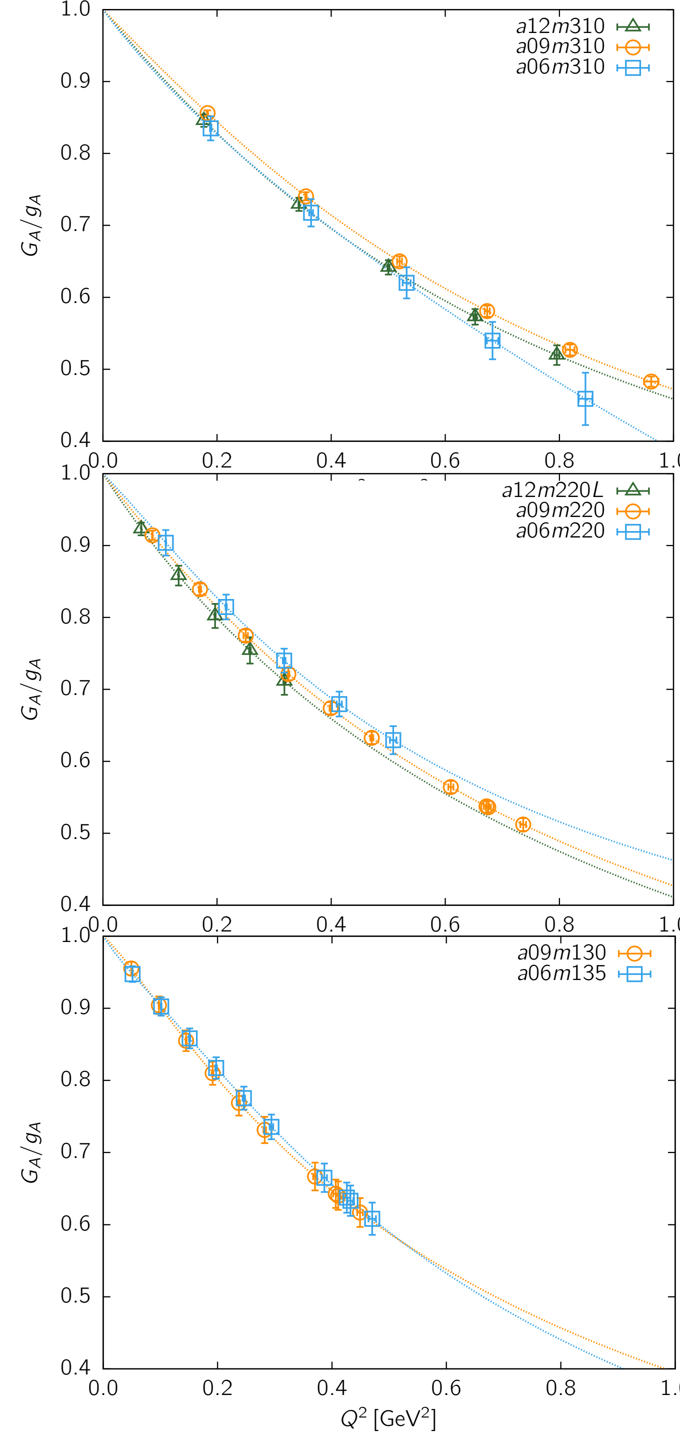
Q^2 behavior and fits



G_A



a12m310
a12m220L
a09m310
a09m220
a09m130
a06m310
a06m220
a06m135
G_A/g_A
Q^2 [GeV^2]



a12m310
a09m310
a06m310
a12m220L
a09m220
a06m220
a09m130
a06m135
G_A/g_A
Q^2 [GeV^2]

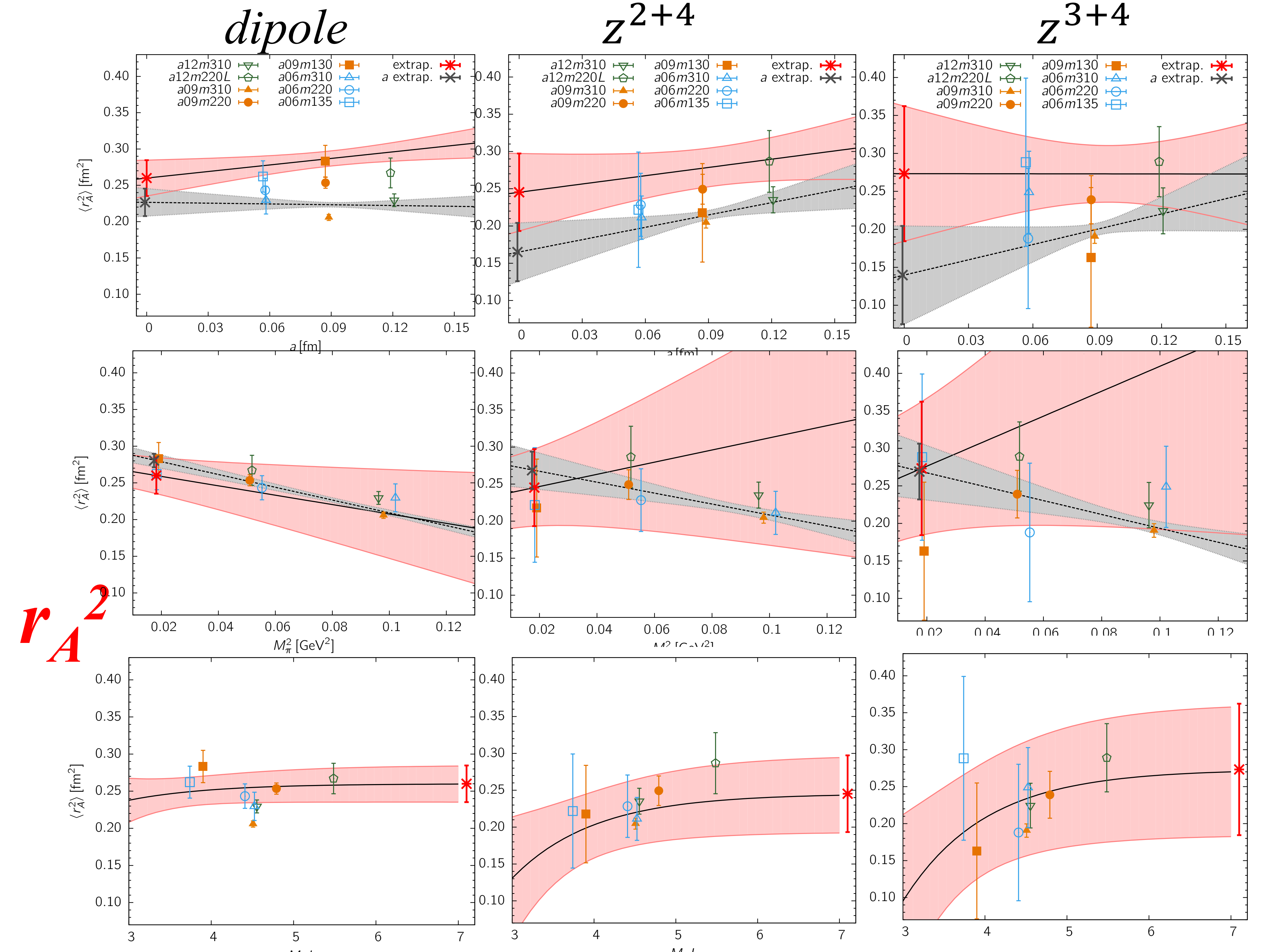
Analyzing lattice data $\Omega(a, M_\pi, M_\pi L)$:

Simultaneous CCFV fits versus $a, M_\pi^2, M_\pi L$

So far include lowest order corrections to fit lattice data w.r.t.

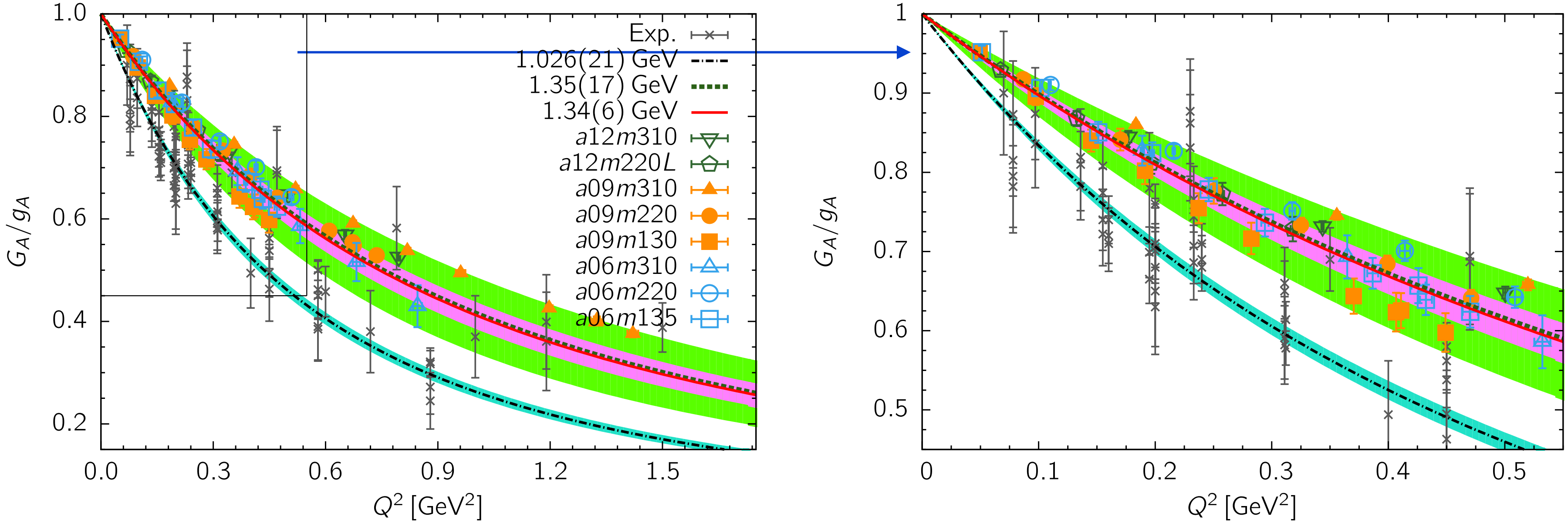
- Lattice spacing: a
- Dependence on light quark mass: $m_q \sim M_\pi^2$
- Finite volume: $M_\pi L$

$$r^2{}_A(a, M_\pi, M_\pi L) = c_0 + c_1 \mathrm{a} + c_2\, M_\pi^2 + c_3 M_\pi^2\, e^{-M_\pi L} + \ldots$$



dipole
z^{2+4}
z^{3+4}
a12m310
a12m220L
a09m310
a09m220
a09m130
a06m310
a06m220
a06m135
extrap.
a extrap.
⟨r_A^2⟩ [fm^2]
a [fm]
M_π^2 [GeV^2]
r_A^2

Axial Charge Radius $\langle r_A \rangle$ (clover-on-HISQ)



Experimental data from:
Bernard, Elouadrhiri, and Meissner, J. Phys. G28, R1 (2002), arXiv:hep-ph/0107088

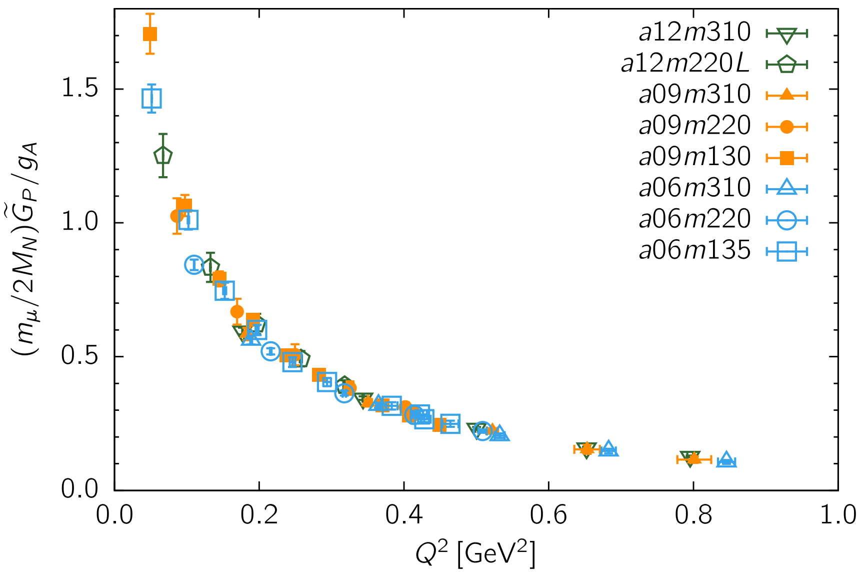
	$\langle r_A \rangle$ fm	M_A GeV
dipole	0.51(2)	1.34(6)
$\langle z^{2+4}, z^{3+4} \rangle$	0.51(3)	1.35(9)
Expt+Pheno	0.68(3)	1.026(21)
MiniBooNE		1.35(17)

Table updated from that in Gupta et al, PRD96 (2017) 114503

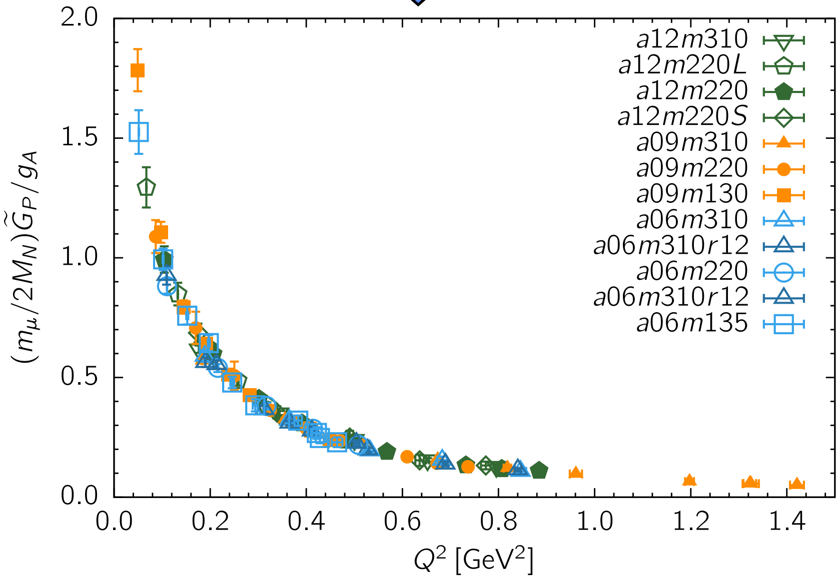
Experimental Results

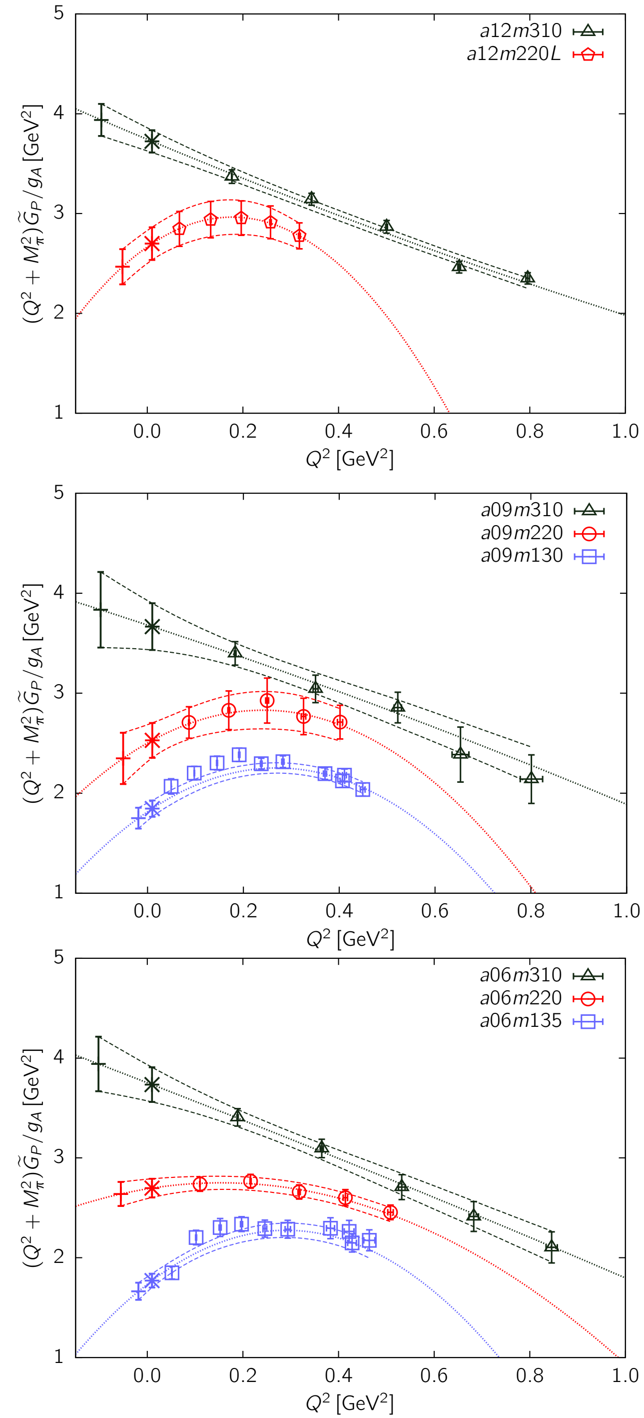
$r_A = 0.80(17)$ fm	ν scattering
$r_A = 0.74(12)$ fm	Electroproduction
$r_A = 0.68(16)$ fm	Deuterium target

$\widetilde{G}_P \to g_P^*, g_{\pi NN}$

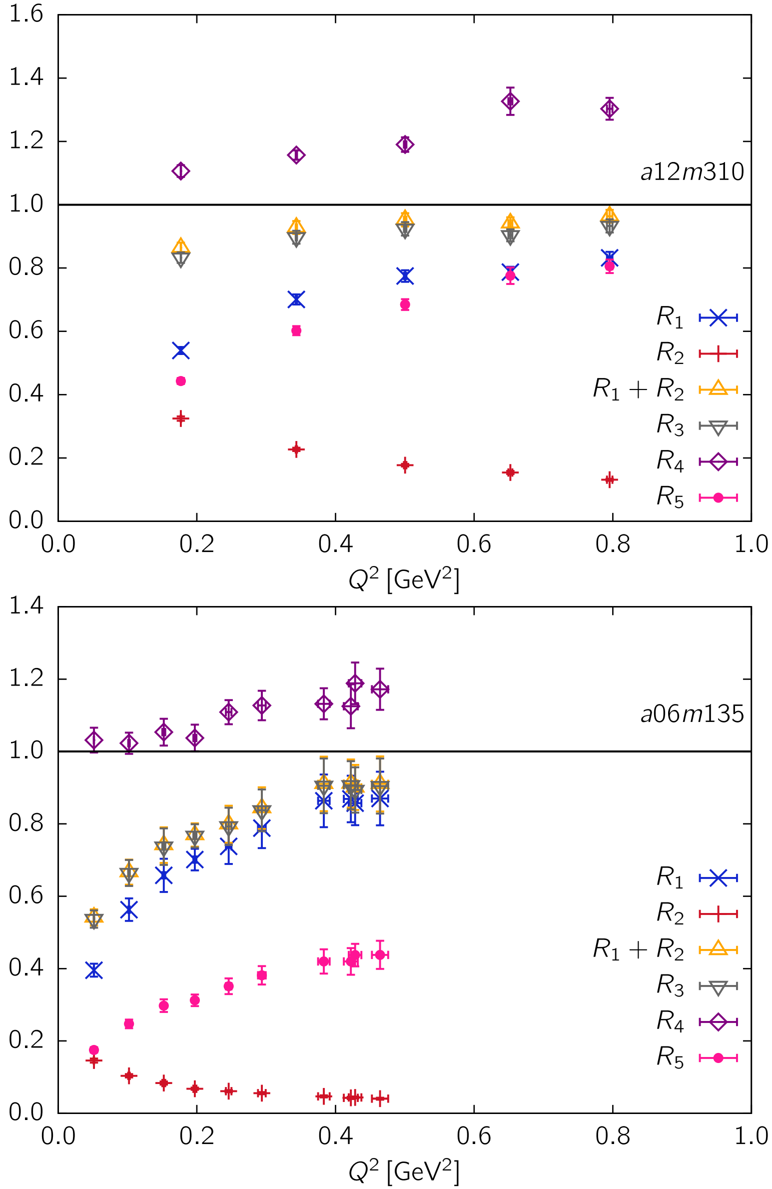


2018





Do G_A, G_P and $\widetilde{G_P}$ satisfy PCAC?



$$2\widehat{m}G_P(Q^2) = 2M_N G_A(Q^2) - \frac{Q^2}{2M_N}\tilde{G}_P(Q^2)$$

$$\frac{Q^2}{4M_N^2}\frac{\tilde{G}_P(Q^2)}{G_A(Q^2)} + \frac{2\widehat{m}}{2M_N}\frac{G_P(Q^2)}{G_A(Q^2)} = 1$$

Pion pole-dominance?

$$2\widehat{m}\, G_P(Q^2) = (M_\pi^2/2M_N)\,\tilde{G}_P^I(Q^2)$$

$$R_1 = \frac{Q^2}{4M_N^2}\frac{\tilde{G}_P(Q^2)}{G_A(Q^2)},$$

$$R_2 = \frac{2\widehat{m}}{2M_N}\frac{G_P(Q^2)}{G_A(Q^2)},$$

$$R_3 = \frac{Q^2 + M_\pi^2}{4M_N^2}\frac{\tilde{G}_P(Q^2)}{G_A(Q^2)},$$

$$R_4 = \frac{4\widehat{m}M_N}{M_\pi^2}\frac{G_P(Q^2)}{\tilde{G}_P(Q^2)},$$

$$R_5 = \frac{aQ^2}{4M_N}\frac{G_P(Q^2)}{G_A(Q^2)},$$

Gupta et al, arXiv:1705:06834

Summary

- Data for the isovector charges and form factors are becoming precise at the few percent level
- Need to understand why the 3 form factors G_A, $\widetilde{G}_P$, G_P do not satisfy PCAC
- Lattice values of the charge radii r_A are smaller than phenomenological estimates. Are all the systematics under control?
- Increase statistics + simulate on larger lattices to get data at smaller Q^2 to improve calculation of $< r_i^2 >$
- Disconnected contributions reaching similar maturity