Effect of collisions on collective neutrino oscillations

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Work in progress with Vincenzo Cirigliano and Mark Paris, arXiv:1806.xxxxx

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Outline

- Introduction and motivation
- Brief overview of collective neutrino oscillations
- Neutrino Bulb Model
- Single-angle approximation
- The Halo effect
- Results
- Conclusions

Introduction

- Core-collapse supernovae are one of the most intense sources neutrinos
- 10^{53} ergs (or 10^{58} neutrinos) are released in an interval of 10 seconds
- About 99% of energy of a core-collpase supernova is released in the form of neutrinos
- Core-collapse supernovae are a candidate site for R-process nucleosynthesis

Neutrinos and Supernovae

- Neutrino driven winds play a major role in supernova explosions
-
$$\nu_e + n \leftrightarrow p + e^-$$
$$\bar{\nu}_e + p \leftrightarrow n + e^+$$

  Neutrino oscillations can affect nucleosynthesis rate by changing the electron fraction
- It is vital to understand when and where neutrino flavor oscillations occur

Neutrino flavor evolution inside supernova

$$\rho = \begin{pmatrix} \langle \psi^*_{\nu_e} \psi_{\nu_e} \rangle & \langle \psi^*_{\nu_e} \psi_{\nu_\mu} \rangle \\ \langle \psi^*_{\nu_\mu} \psi_{\nu_e} \rangle & \langle \psi^*_{\nu_\mu} \psi_{\nu_\mu} \rangle \end{pmatrix} \quad \bar{\rho} = \begin{pmatrix} \langle \bar{\psi}^*_{\nu_e} \bar{\psi}_{\nu_e} \rangle & \langle \bar{\psi}^*_{\nu_e} \bar{\psi}_{\nu_\mu} \rangle \\ \langle \bar{\psi}^*_{\nu_\mu} \bar{\psi}_{\nu_e} \rangle & \langle \bar{\psi}^*_{\nu_\mu} \bar{\psi}_{\nu_\mu} \rangle \end{pmatrix}$$

$$\rho(t) = e^{-iHt} \rho(0) e^{iHt}$$

$$H_{vac} = \frac{1}{2} \begin{pmatrix} -\omega \cos(2\theta_{\mathrm{v}}) & \omega \sin(2\theta_{\mathrm{v}}) \\ \omega \sin(2\theta_{\mathrm{v}}) & \omega \cos(2\theta_{\mathrm{v}}) \end{pmatrix} \quad \omega = \frac{m_2^2 - m_1^2}{2E}$$

Hamiltonian

$$\begin{aligned} H &= H_{vac} + H_{mat} + H_{self} \quad \text{for neutrinos} \\ \bar{H} &= -H_{vac} + H_{mat} + H_{self} \quad \text{for anti-neutrinos} \end{aligned}$$

$$\begin{aligned} H_{vac} &= \frac{1}{2}\begin{pmatrix} -\omega\cos(2\theta_\text{v}) & \omega\sin(2\theta_\text{v}) \\ \omega\sin(2\theta_\text{v}) & \omega\cos(2\theta_\text{v}) \end{pmatrix} \\ H_{mat} &= \begin{pmatrix} \sqrt{2}G_F n_e & 0 \\ 0 & 0 \end{pmatrix} \\ H_{self} &= \sqrt{2}G_F \int d^3p'(1 - \vec{v}\cdot\vec{v}')(\rho_{p'} - \bar{\rho}_{p'}) \end{aligned}$$

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Collisions

$$\frac{d}{dt}\rho^{\vec{p}}(\vec{x},t) = -\frac{1}{2}\left\{\Pi^{\mathrm{loss}}, \rho^{\vec{p}}(\vec{x},t)\right\} + \frac{1}{2}\left\{\Pi^{\mathrm{gain}}, 1-\rho^{\vec{p}}(\vec{x},t)\right\}$$

- Neutrino traveling with momentum $\vec{p}$ can be scattered **out** to $\vec{p}'$. (Loss term)
- Neutrino traveling with $\vec{p}'$ can be scattered **in to** $\vec{p}$ momentum state. (Gain term)

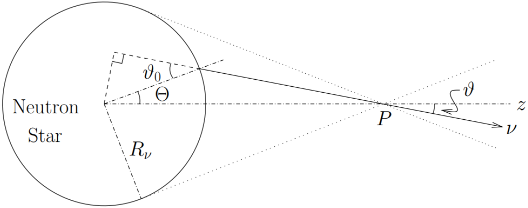
Full equations of motion

Attempt to find steady state solution

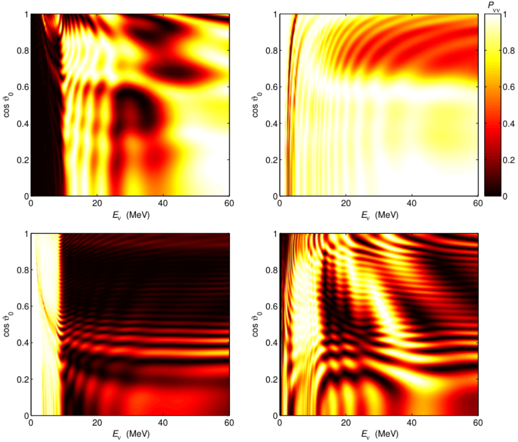
$$i\frac{d\rho(\vec{x},\vec{p})}{dl} = [H, \rho(\vec{x},\vec{p})] + i\mathcal{C}[\rho,\bar{\rho}]$$
$$i\frac{d\bar{\rho}(\vec{x},\vec{p})}{dl} = [\bar{H}, \bar{\rho}(\vec{x},\vec{p})] + i\bar{\mathcal{C}}[\rho,\bar{\rho}]$$

We are trying to solve 6-dimensional equations of motion in a non-trivial geometry. Collisions have never been satisfactorily included in studies.

Neutrino Bulb model



Neutrino bulb model (No collisions)



H. Duan, G. M. Fuller, J. Carlson and Y. Z. Qian, Phys. Rev. D **74**, 105014 (2006) doi:10.1103/Phy

[astro-ph/0606616].

Single angle approximation

- Single-angle approximation: all the emission angles are equivalent and a single angle can be used to represent all emission angles.
- Single angle approximation can reduce the number of equations and computational time required by a factor of 1000 or more

In the case of neutrino-bulb model single-angle approximation gives very similar results to multi-angle calculation

Flavor instability

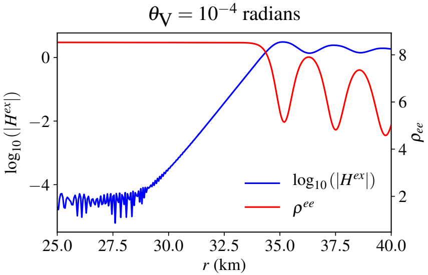
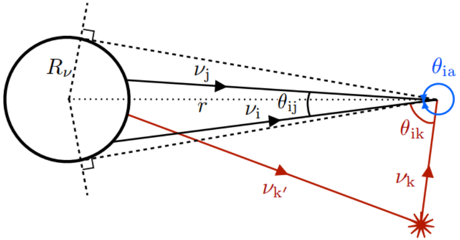


Figure: The blue line shows the evolution of the off-diagonal term of the Hamiltonian. The flavor instability is the exponential growth of the off-diagonal term of the Hamiltonian. The saturation of the off-diagonal term is accompanied by the onset of neutrino flavor oscillation (red line)

The Halo effect



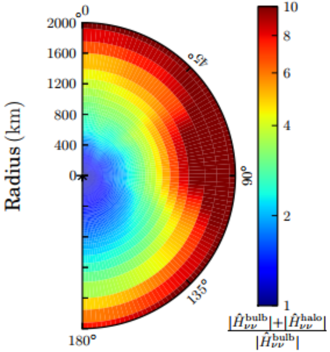
J. F. Cherry, J. Carlson, A. Friedland, G. M. Fuller and A. Vlasenko, Phys. Rev. Lett. **108**, 261104 (2012)

doi:10.1103/PhysRevLett.108.261104 [arXiv:1203.1607 [hep-ph]].

Importance and Challenges

- The back-scattered flux is proportional to the square of the Fermi constant ($\propto G_F^2$) while the self-interaction potential goes like Fermi constant ($\propto G_F$)
- The potential due to the back-scattered neutrino is $\mathcal{O}(1+\cos^2\theta)$ as opposed to self-interaction potential in neutrino-bulb model ($\mathcal{O}(1-\cos^2\theta)$)
- No longer an initial value problem. Usual numerical techniques; forward difference methods do not work.

Halo potential



J. F. Cherry, J. Carlson, A. Friedland, G. M. Fuller and A. Vlasenko, Phys. Rev. Lett. **108**, 261104 (2012)

doi:10.1103/PhysRevLett.108.261104 [arXiv:1203.1607 [hep-ph]].

The Halo effect in single angle approximation

- The calculation of neutrino flavor oscillations in neutrino-bulb model with the Halo effect is extremely difficult
- In order to gain insight in to the physics due to the Halo effect we study the Halo effect in single-angle approximation
- A single emission angle is used to represent all emission angles and the backscattered neutrinos travel along the same path but in the opposite direction

Numerical method

- The neutrino flux is evolved from $r_{\min}$ to $r_{\max}$ and the density matrices are stored as spline curves
- At each point $\mathcal{C} = \sqrt{2} G_F n_e 10^{-6} \rho$
- The spline curves are used to calculate the flavor evolution is the backward direction
- The density matrices are evolved in the outward direction again with backscattered neutrinos taken in to account
- Iterate till necessary

Numerical results

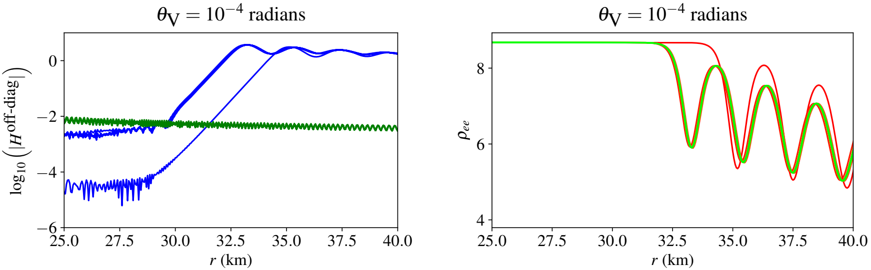


Figure: Blue: off-diagonal term of the Hamiltonian for various iterations. Green: Same for the Halo Hamiltonian only. Right: Evolution of the flux for 8.5 MeV bin. Red to Green: 0 to 15^{th} iteration

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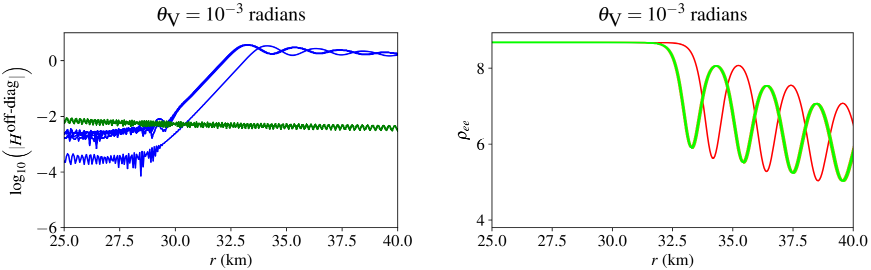

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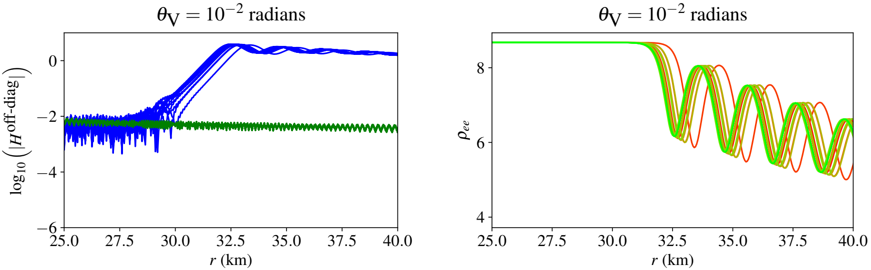

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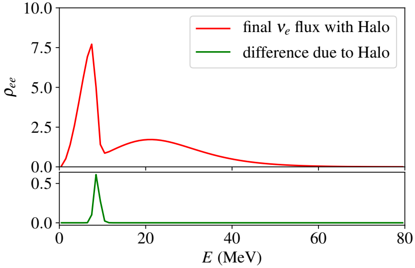
Final flux



10.0
7.5
5.0
2.5
0.0
0.5
0.0
ρ_{ee}
final ν_e flux with Halo
difference due to Halo
0
20
40
60
80
E (MeV)

Discussion

- Flavor instability is determined by the diagonal elements of the density matrices.
- There is a small relative change is the magnitude of the diagonal elements of the Hamiltonian in the region of flavor instability.
- Back-scattered neutrinos do not significantly modify the final neutrino spectrum because they have a large effect in a region where there are no neutrino flavor oscsillations.

Conclusion

- We have calculated the neutrino flavor evolution in single-angle approximation
- The Halo effect causes increases the effective mixing angle and as a result pushes the onset of neutrino flavor oscillations to a slightly smaller radius.
- We do not find new flavor instability due to the Halo effect
- Multi-angle calculations have to be performed to be sure that Halo effect modifies the neutrino flavor in a limited fashion