Attractors

Ulrich Heinz

The Ohio State University

Berkeley Symposium on Hard Probes and Beyond
– in honor of Xin-Nian Wang's 60th birthday –
August 18-19, 2022

JETSCAPE

Unterstützt von / Supported by
Alexander von Humboldt
Stiftung / Foundation

BAND
Bayesian Analysis of Nuclear Dynamics

Overview

1 Prologue

2 Kinetic theory vs. hydrodynamics

3 Exact evolution in kinetic theory

4 Evolution in hydrodynamic approximation(s)
- Second-order Chapman Enskog hydrodynamics (CE hydro)
- Anisotropic hydrodynamics (aHydro)

5 Lessons learned

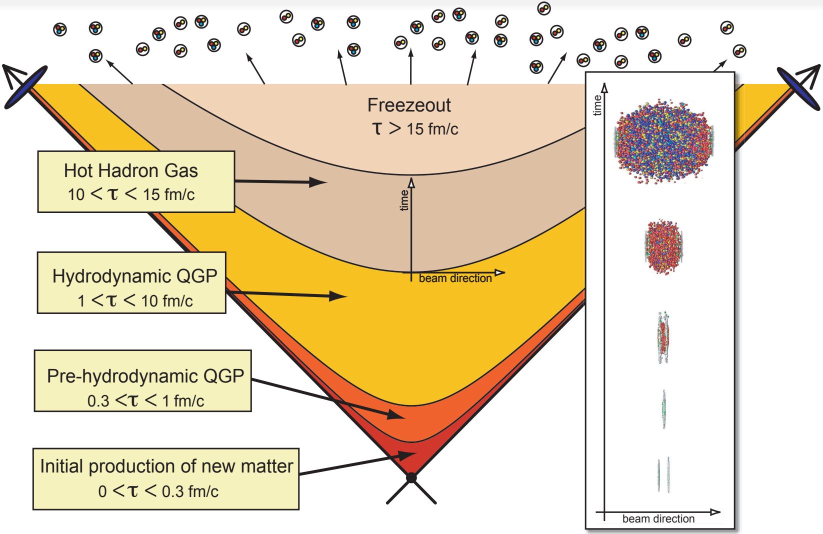
Prologue I

$\sim$ 15-20 years ago:

Discovery of the "unreasonable effectiveness of hydrodynamics" in describing ultrarelativistic heavy-ion collision dynamics

(hundreds of papers, too many to list here. . .)

Space-time diagram of a heavy-ion collision



(After M. Strickland, arXiv:1410.5786)

Causal dissipative relativistic fluid dynamics

Israel & Stewart '79; Muronga '02; Denicol, Niemi, Molnár, Rischke '12; Pang, Hatta, Wang, Xiao '15; & many others ...

Macroscopic evolution of densities and fluid velocity as functions of space and time:

$$\begin{aligned} T^{\mu\nu} &= e\,u^\mu u^\nu - (p+\Pi)\,\Delta^{\mu\nu} + \pi^{\mu\nu} \text{ where } \Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu = -(x^\mu x^\nu + y^\mu y^\nu + z^\mu z^\nu) \\ j^\mu &= n\,u^\mu + V^\mu \\ p &= p(e,n) \quad (EoS) \end{aligned} \tag{1}$$

- Conservation laws $\partial_\mu T^{\mu\nu} = 0 = \partial_\mu j^\mu \implies$ evolution of e, n, u^μ
- Relaxation equations for the dissipative flows $\pi^{\mu\nu}$, Π, V^μ, e.g.

$$D\pi^{\langle\mu\nu\rangle} = -\frac{1}{\tau_\pi}\left(\pi^{\mu\nu} - 2\eta\nabla^{\langle\mu}u^{\nu\rangle}\right) + \text{second order terms;}$$

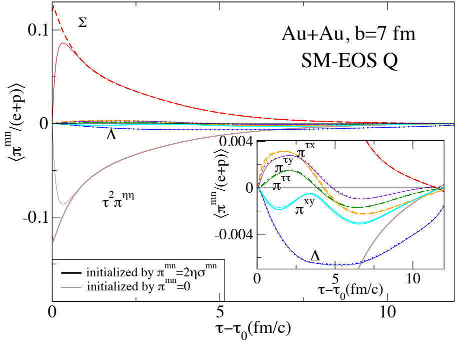
describe competition between collisions ($\to$ towards equilibrium) and expansion ($\to$ away from equilibrium)

- Large anisotropies in the expansion rate θ ($\theta_L \gg \theta_\perp$) keep the pressure anisotropy

$$P_L - P_\perp = \left(p + \pi^{zz}\right) - \left(p - \tfrac{1}{2}\pi^{zz}\right) = \tfrac{3}{2}\pi^{zz} < 0$$

large throughout the evolution history:

Large shear stress throughout the QGP phase! Song & UH '07



VISH2+1 (from H. Song's PhD thesis (arXiv:0908.3656))

$\Longrightarrow$ "hydrodynamization" $\neq$ "equilibration"

$\Longrightarrow$ hydrodynamics becomes valid well before local momentum isotropy and thermal equilibrium are reached

$\Longrightarrow$ "far-from-equilibrium hydrodynamics" (Romatschke 2018)

Anisotropic hydrodynamics

Martinez & Strickland '10; Florkowski & Ryblewski '11; Bazow et al. '14; Molnár et al. '16; McNelis et al. '18; and many others

...

$$
\begin{aligned}
T^{\mu\nu} &= e\, u^\mu u^\nu + P_L z^\mu z^\nu - P_\perp \Xi^{\mu\nu} + \pi_\perp^{\mu\nu} + 2 W_{\perp z}^{(\mu} z^{\nu)} \\
j^\mu &= n\, u^\mu + V_z^\mu + V_\perp^\mu \\
p &= p(e, n) \quad (EoS)
\end{aligned} \tag{2}
$$

where

$$
\begin{aligned}
\Xi^{\mu\nu} &= g^{\mu\nu} - u^\mu u^\nu + z^\mu z^\nu = -(x^\mu x^\nu + y^\mu y^\nu) \\
\Pi &= \tfrac{1}{3}(P_L + 2P_\perp) - p(e, n) \\
\pi^{\mu\nu} &= \pi_\perp^{\mu\nu} + 2 W_{\perp z}^{(\mu} z^{\nu)} + \tfrac{1}{3}(P_L - P_\perp)(z^\mu z^\nu - \Delta^{\mu\nu})
\end{aligned}
$$

- Conservation laws $\partial_\mu T^{\mu\nu} = 0 = \partial_\mu j^\mu \implies$ evolution of e, n, u^μ
- Relaxation equations for P_L, $P_\perp$, $\pi_\perp^{\mu\nu}$, $W_{\perp z}^\mu$, V^μ (Molnár et al. '16, McNelis et al. '18)

Public (3+1)-d code VAH available at https://github.com/mjmcnelis/cpu_vah.

Prologue II

$\sim$ 10 years ago:

Discovery of hydrodynamic attractors

A strong attractor:

PHYSICAL REVIEW D VOLUME 44, NUMBER 11 1 DECEMBER 1991

HIJING: A Monte Carlo model for multiple jet production in pp, pA, and AA collisions

Xin-Nian Wang* and Miklos Gyulassy

Nuclear Science Division, Mailstop 70A-3307, Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720

(Received 29 July 1991)

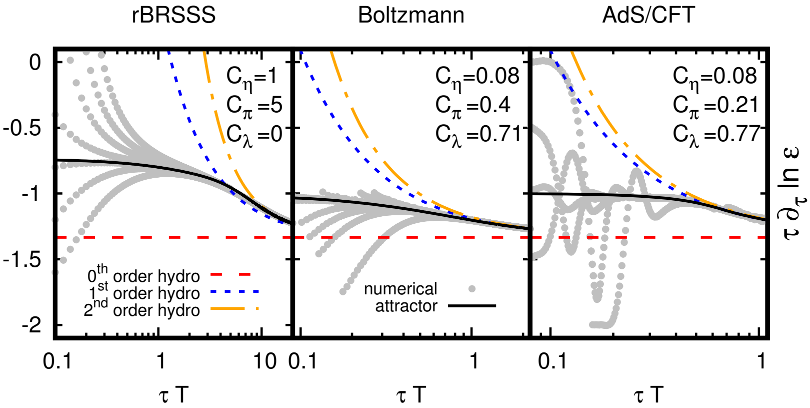
Combining perturbative-QCD inspired models for multiple jet production with low p_T multistring phenomenology, we develop a Monte Carlo event generator HIJING to study jet and multiparticle production in high energy pp, pA, and AA collisions. The model includes multiple minijet production, nuclear shadowing of parton distribution functions, and a schematic mechanism of jet interactions in dense matter. Glauber geometry for multiple collisions is used to calculate pA and AA collisions. The phenomenological parameters are adjusted to reproduce essential features of pp multiparticle production data for a wide energy range ($\sqrt{s}=5$–2000 GeV). Illustrative tests of the model on $p+A$ and light-ion $B+A$ data at $\sqrt{s}=20$ GeV/nucleon and predictions for Au+Au at energies of the BNL Relativistic Heavy Ion Collider ($\sqrt{s}=200$ GeV/nucleon) are given.

> 3,000 citations!

Hydrodynamic attractors

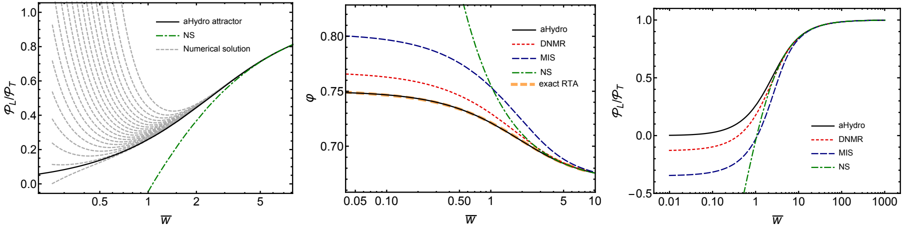
- Hydrodynamics is the effective theory of long-wavelength excitations which can be expressed as a hydrodynamic gradient series (Baier et al. (BRSSS) '08).
- While this gradient series was shown to be asymptotic (i.e. it diverges (Heller et al. '13 –'16, and others)), it can be Borel resummed, yielding an **attractor** (Heller et al. '13) to which the system converges on a microscopic relaxation time scale τ_R.
  Non-hydrodynamic moments of the underlying phase-space distribution decay on the same time scale τ_R (Strickland '18). The precise form of this decay depends on the microscopic collision dynamics (Romatschke '17).
- In the limit of small gradients, the attractor reduces to the low-order hydrodynamic gradient series solution. Navier-Stokes theory defines the unique attractor at first order in gradients.
- The existence and properties of the hydrodynamic attractor have been studied in greatest detail for conformal systems undergoing Bjorken flow (Heller et al. '13 –'18; Basar & Dunne '15; Romatschke '17; Denicol & Noronha '16; Strickland '18; ...). **Some pictures:**

Hydrodynamic attractors for Bjorken flow



Romatschke, PRL 120 (2017) 012301

The anisotropic hydrodynamic attractor for Bjorken flow



Strickland & Noronha, PRD97 (2018) 036020

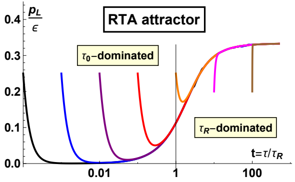
$$\varphi = \frac{1}{2}\left(\frac{(P_L/P_T)+3}{(P_L/P_T)+2}\right), \quad \bar{w} = \frac{\tau}{\tau_R} = \text{inverse Knudsen number}$$

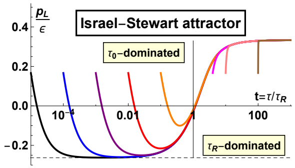
- Numerical solutions join attractor (i.e. lose memory of ICs) after $\tau \gtrsim (1{-}2)\tau_R$. At this point $P_L/P_T \lesssim 0.5$, i.e. shear stress effects are $\mathcal{O}(1)$.
- aHydro reproduces unterlying RTA Boltzmann transport almost perfectly, even for very large shear stress.
- Hydrodynamic attractors merge with Navier-Stokes after $\tau >$ few $\times\, \tau_R$ (not MIS).

Approach to attractor – I. Weak coupling

Jaiswal et al., PRC 100 (2019) 034901; Kurkela et al., PRL 124 (2020) 102301

- Consider an excursion δ about the attractor solution $\bar{\pi}(w) = \bar{\pi}_{\rm attr}(w) + \delta(w)$.
- At late times $\tau \gg \tau_R$ (small Knudsen numbers $\mathrm{Kn} \ll 1$) the excursion decays exponentially $\delta \propto \exp(-3\tau/2\tau_R)$
- At early times $\tau \ll \tau_R$ (large Knudsen numbers $\mathrm{Kn} \gg 1$) the excursion decays via a power law $\delta \propto (\tau/\tau_R)^{-2m}$ where m depends on transport coefficients.
- Late-time decay driven by interactions, decay period $\propto \tau_R$. Early-time decay driven by rapid medium expansion, decay period independent of τ_R:

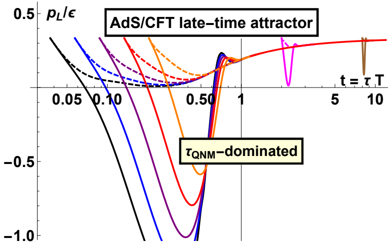




Approach to attractor – II. Strong coupling

Kurkela, van der Schee, Wiedemann, Wu, PRL 124 (2020) 102301

- No early-time attractor behavior in strongly coupled systems:



- Early-time dominated by oscillatory solutions: decay of quasinormal modes.
- Information of initial conditions lost only at $\tau T \gtrsim 1$.

Prologue brief summary:

- Attractor phenomenon has been extensively studied for conformal (massless) systems undergoing effectively 1-d expansion (Bjorken, Guber), in both RTA Boltzmann and hydrodynamics

- Attractors ubiquitous: (i) hydro flows; (ii) nonhydrodynamic kinetic moments; (iii) the phase-space distribution itself! (Strickland '18)

- **But:** different attractors for different hydrodynamic approximations of the same underlying kinetic theory; few studies for non-conformal systems or 3-d expansion, with inconclusive results, leaving many open questions

- $\Longrightarrow$ **Here:** focus on **non-conformal effects** in Bjorken flow, using a weakly coupled approach (RTA Boltzmann & various hydrodynamic approximations) that provides full control of all technical approximations.

Overview

1 Prologue

2 Kinetic theory vs. hydrodynamics

3 Exact evolution in kinetic theory

4 Evolution in hydrodynamic approximation(s)
- Second-order Chapman Enskog hydrodynamics (CE hydro)
- Anisotropic hydrodynamics (aHydro)

5 Lessons learned

Kinetic theory I

Relativistic Boltzmann eqn. in Relaxation Time Approximation (RTA BE):

Andersen & Witting '74

(gas of massive particles, Boltzmann statistics, no conserved charge ($\mu = 0$))

$$p^\mu \partial_\mu f(x,p) = -\frac{p\cdot u(x)}{\tau_R(x)}\Big(f(x,p) - f_{\rm eq}(x,p)\Big) \quad \text{with} \quad f_{\rm eq}(x,p) = \exp\Big(-\frac{p\cdot u(x)}{T(x)}\Big)$$

Nonlinear: $T(x)$, $u^\mu(x)$ fixed by Landau matching:

$$T^{\mu\nu}[f]u_\mu \equiv \langle p^\mu p^\nu \rangle_f u_\mu = e_{\rm eq}(T) u^\mu$$

Bjorken flow (longitudinal boost-invariance + transverse homogeneity):

ODE: $$\frac{df}{d\tau} = -\frac{f - f_{\rm eq}}{\tau_R(\tau)} \quad \text{with} \quad f_{\rm eq} = e^{-\sqrt{p_T^2 + (p_\eta/\tau)^2 + m^2}/T}$$

Kinetic theory II

Analytic solution: Baym '84, Florkowski, Strickland et al. '13, '14

$$f(\tau; p_T, p_\eta) = D(\tau,\tau_0)\, f_{\rm in}(\tau_0; p_T, p_\eta) + \int_{\tau_0}^{\tau} \frac{d\tau'}{\tau_R(\tau')}\, D(\tau,\tau')\, f_{\rm eq}(\tau'; p_T, p_\eta)$$

with

$$D(\tau_2,\tau_1) = \int_{\tau_1}^{\tau_2} \frac{d\tau}{\tau_R(\tau)}$$

Here Romatschke & Strickland '03

$$f_{\rm in}(\tau_0; p_T, p_\eta) = \frac{1}{\alpha_0} \exp\left(-\frac{\sqrt{p_T^2 + (1+\xi_0)(p_\eta/\tau_0)^2 + m^2}}{\Lambda_0}\right)$$

with $\tau_0 = 0.1\,\mathrm{fm}/c$, $m = 200\,\mathrm{MeV}$, $T_0(\Lambda_0, \alpha_0, \xi_0, m) = 500\,\mathrm{MeV}$,

and various α_0, ξ_0 to control the initial bulk and shear stresses $\Pi_0, \pi_0^{\eta\eta}$.

Kinetic constraints on hydrodynamic moments I

Chattopadhyay et al., PLB 824 (2022) 136820

For Bjorken flow $T^{\mu\nu}(\tau) = \mathrm{diag}\big(e, P_T, P_T, P_L\big)$, with

$$e \equiv e_{\rm eq} = \big\langle (p{\cdot}u)^2 \big\rangle_{\rm eq}, \quad P \equiv P_{\rm eq} = -\frac{1}{3}\big\langle m^2 - (p{\cdot}u)^2 \big\rangle_{\rm eq} = \frac{e}{3} - \frac{m^2}{3}\langle 1 \rangle_{\rm eq},$$

$$P_L = P + \Pi - \pi, \quad P_T = P + \Pi + \pi/2, \quad \Pi = \frac{m^2}{3}\big\langle 1 \big\rangle_{\delta f}, \quad -\pi \equiv \tau^2 \pi^{\eta\eta} = \tau^2 \big\langle (p^\eta)^2 \big\rangle_{\delta f},$$

where $\delta f \equiv f - f_{\rm eq}$. Positivity of f implies the following kinetic theory constraints:

$$P_T = \frac{1}{2}\big\langle p_T^2 \big\rangle \geq 0, \quad P_L = \big\langle p_\eta^2/\tau^2 \big\rangle \geq 0,$$

$$P + \Pi = \frac{1}{3}\big\langle p_T^2 + p_\eta^2/\tau^2 \big\rangle \geq 0, \quad T^\mu_\mu = e - 3(P + \Pi) = m^2 \langle 1 \rangle \geq 0.$$

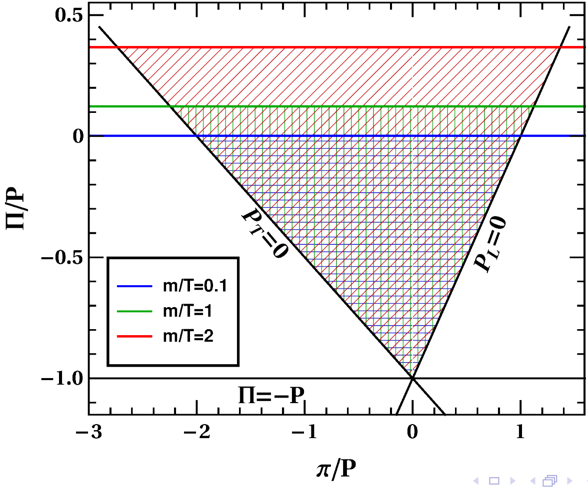
This implies for the normalized bulk and shear stresses $\bar{\Pi} = \Pi/P$, $\bar{\pi} = \pi/P$

$$\bar{\Pi} + \frac{1}{2}\bar{\pi} \geq -1, \quad \bar{\Pi} - \bar{\pi} \geq -1, \quad \bar{\Pi} \geq -1, \quad \bar{\Pi} \leq \frac{e}{3P} - 1$$ (depends on m)

Kinetic constraints on hydrodynamic moments II

Chattopadhyay et al., PLB 824 (2022) 136820

$$\bar{\Pi}+\frac{1}{2}\bar{\pi} \geq -1, \quad \bar{\Pi}-\bar{\pi} \geq -1, \quad \bar{\Pi} \geq -1, \quad \bar{\Pi} \leq \frac{e}{3P}-1$$



Overview

1 Prologue

2 Kinetic theory vs. hydrodynamics

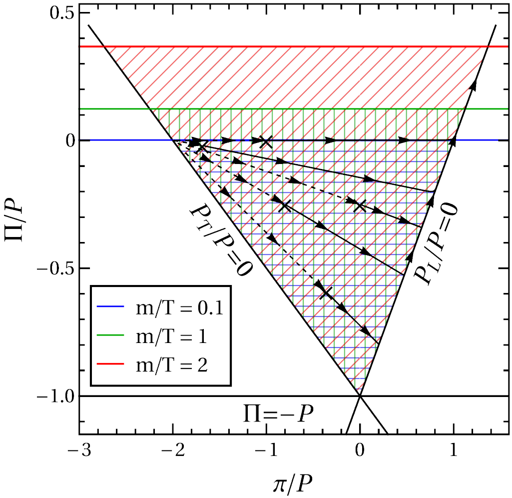
3 Exact evolution in kinetic theory

4 Evolution in hydrodynamic approximation(s)
- Second-order Chapman Enskog hydrodynamics (CE hydro)
- Anisotropic hydrodynamics (aHydro)

5 Lessons learned

Free-streaming dynamics Chattopadhyay et al., PLB 824 (2022) 136820

- We first solve the RTA Boltzmann equation without interactions, $\tau_R \to \infty$:

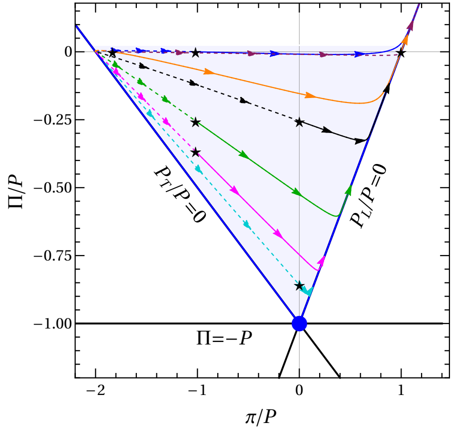


- **$P_L = 0$ is an attracting fixed line.**
- $P_T = 0$ is a repelling fixed line
- $(\Pi/P = 0, \pi/P = -2)$ is a repulsive fixed point
- At $(\Pi/P = -1, \pi/P = 0)$, all particles are at rest $f \sim \delta(|\vec{p}|)/\vec{p}^2$.

Turning on collisions

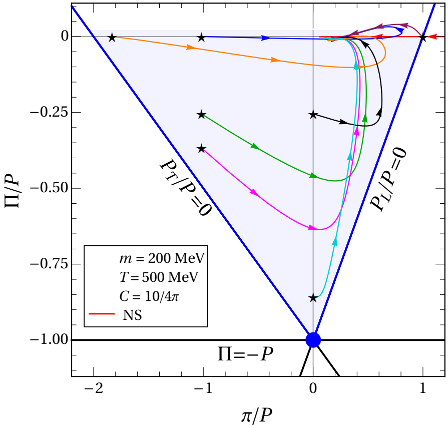
Jaiswal et al., PRC 105 (2022) 024911

$\tau_R \to \infty$:



$T\tau_R = 5C$, $C = 10/4\pi$:

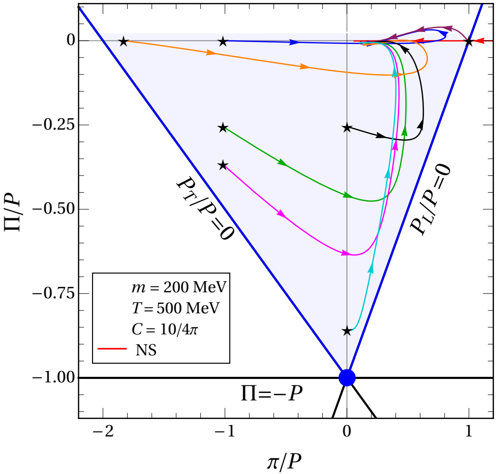
$(C = \eta/s$ when $m = 0)$



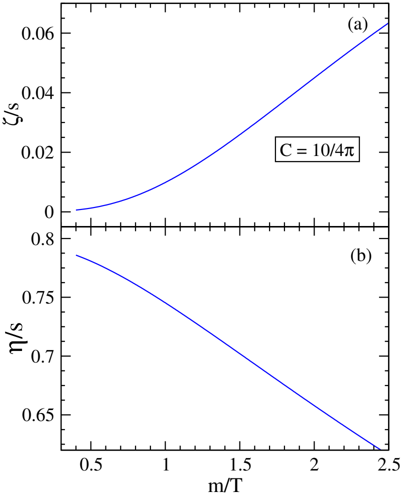
Turning on collisions

Jaiswal et al., PRC 105 (2022) 024911

$T\tau_R = 5C, \; C = 10/4\pi$:

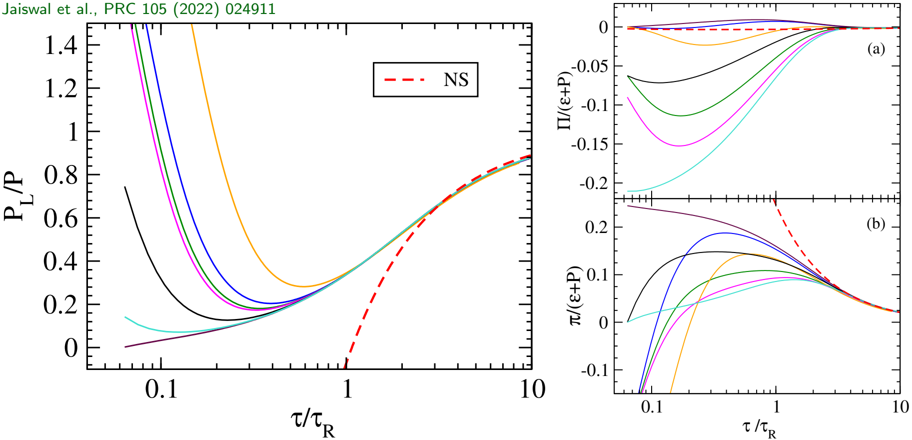


$\zeta = \frac{5}{3}\eta - \tau_R c_s^2 (e+P)$



Early-time attractor for $P_L = P+\Pi-\pi$, but not for π, Π!

Jaiswal et al., PRC 105 (2022) 024911



Strong bulk-shear coupling effects lead to strong sensitivity of early-time ($\tau < \tau_R$) trajectories to initial conditions

At late times ($\tau \gtrsim (2{-}3)\tau_R$), when π, Π are small, convergence to NS attractor.

Evolution of the distribution function

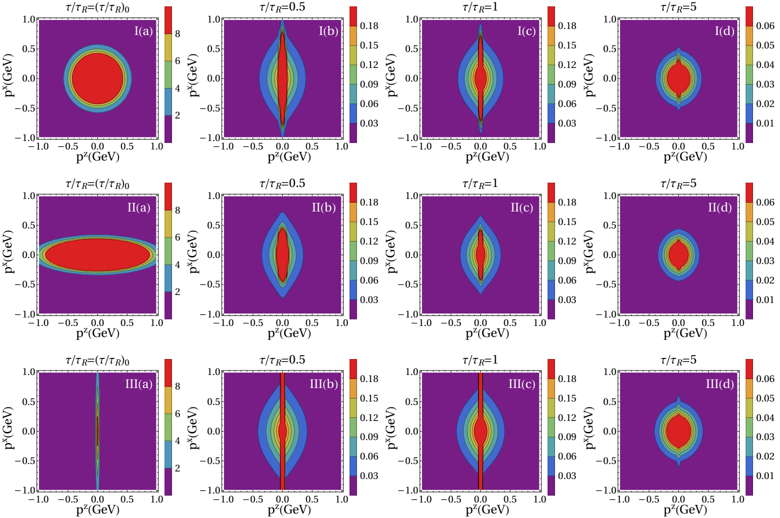
Jaiswal et al., PRC 105 (2022) 024911

$(\bar{\pi}_0, \bar{\Pi}_0)$:

$(0, -0.25)$

$(-1, -0.25)$

$(0, 99, 0)$



Free-streaming shrinks the distribution function in p_z direction $\Longrightarrow P_L \to 0$.

Key insights

- Rapid longitudinal expansion of Bjorken flow rapidly shrinks the width of the p_z-distribution at early times.
- This results in P_L/P quickly decreasing at early times, approaching, with power-law decay, a universal far-off-equilibrium attractor at $\tau \lesssim \tau_R$.
- This is a feature of the expansion profile, and it is independent of the particles being massless (conformal) or massive (non-conformal).
- The dissipative hydrodynamic moments $\bar{\pi}$, $\bar{\Pi}$ do not individually exhibit universality; shear-bulk coupling causes strong initial-state sensitivity of the early-time, far-off-equilibrium dynamics.
- In conformal systems, P_L and π are interchangeable, and the attractor in P_L/P maps onto the well-studied attractor for the normalized shear stress $\bar{\pi}$: $(P_L/P)_{\mathrm{conf}} = 1 - 4\bar{\pi}$.

Overview

1 Prologue

2 Kinetic theory vs. hydrodynamics

3 Exact evolution in kinetic theory

4 Evolution in hydrodynamic approximation(s)
- Second-order Chapman Enskog hydrodynamics (CE hydro)
- Anisotropic hydrodynamics (aHydro)

5 Lessons learned

Non-conformal Bjorken hydrodynamics S. Jaiswal et al., PRC 105 (2022) 024911

Second-order CE hydro: A. Jaiswal et. al., PRC 90 (2014) 044908

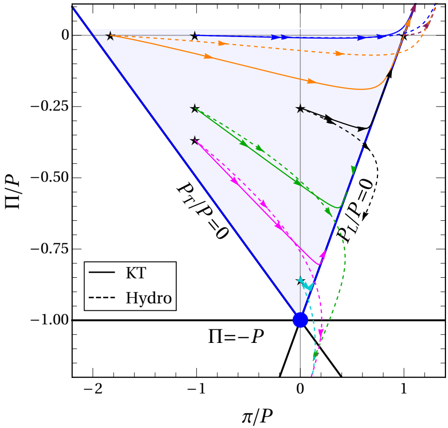
$$\frac{de}{d\tau} = -\frac{1}{\tau}(e+P+\Pi-\pi), \quad \frac{d\Pi}{d\tau} + \frac{\Pi}{\tau_R} = -\frac{\beta_\Pi}{\tau} - \delta_{\Pi\Pi}\frac{\Pi}{\tau} + \lambda_{\Pi\pi}\frac{\pi}{\tau},$$
$$\frac{d\pi}{d\tau} + \frac{\pi}{\tau_R} = \frac{4}{3}\frac{\beta_\pi}{\tau} - \left(\frac{1}{3}\tau_{\pi\pi}+\delta_{\pi\pi}\right)\frac{\pi}{\tau} + \frac{2}{3}\lambda_{\pi\Pi}\frac{\Pi}{\tau}.$$

Transport coefficients from kinetic theory for a massive Boltzmann gas.

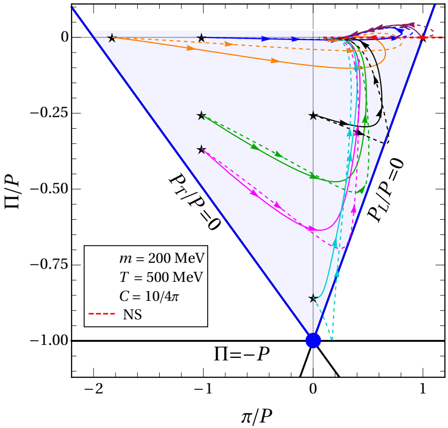
Kinetic vs. hydrodynamic evolution

Jaiswal et al., PRC 105 (2022) 024911

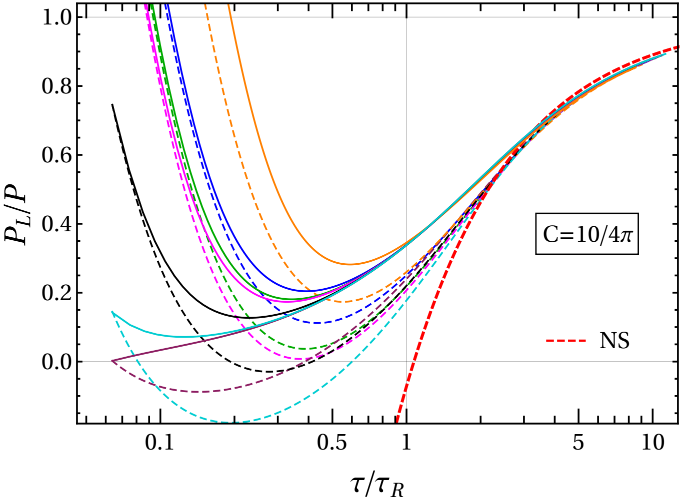
$\tau_R \to \infty$:



$T\tau_R = 5C,\ C = 10/4\pi$:



No early-time attractor for P_L in hydro!? Jaiswal et al., PRC 105 (2022) 024911



This is for CE hydro; similar results for MIS and DNMR hydro.

Anisotropic hydrodynamics

Jaiswal et al., PRC 105 (2022) 024911

- Starting from BE $\partial_\tau f = -(f - f_{\rm eq})/\tau_R$, obtain the evolution of $e,\ P_L,\ P_T$:

$$\frac{d\epsilon}{d\tau} = -\frac{e+P_L}{\tau}, \quad \text{where } e = \int_p (p^\tau)^2 f,$$

$$\frac{dP_L}{d\tau} = -\frac{P_L - P}{\tau_R} + \frac{\bar\zeta_z^L}{\tau}, \quad \frac{dP_T}{d\tau} = -\frac{P_T - P}{\tau_R} + \frac{\bar\zeta_z^\perp}{\tau}$$

- The couplings $\bar\zeta_z^L$ and $\bar\zeta_z^\perp$ involve higher-order moments:

$$\bar\zeta_z^L = -3P_L + \int dP\, E_p^{-2}\, p_z^4\, f,$$

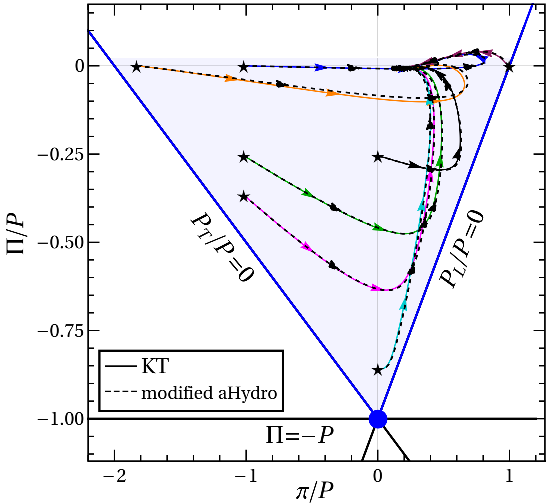
$$\bar\zeta_z^\perp = -P_T + \frac{1}{2}\int dP E_p^{-2}\, p_z^2\, p_T^2\, f.$$

- Close the system of equations by using the leading order ansatz

$$\tilde f_a(\tau, p_T, p_\eta) = \frac{1}{\alpha(\tau)} \exp\left(-\frac{\sqrt{p_T^2 + (1+\xi(\tau))\,(p_\eta/\tau)^2 + m^2}}{\Lambda(\tau)}\right),$$

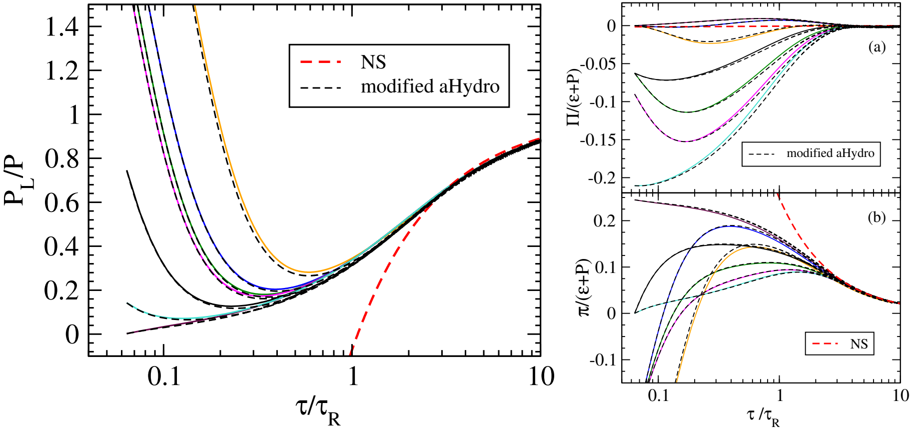
using the method of moments. Molnár, Niemi, Rischke '16

Anisotropic hydrodynamics vs. kinetic theory I



Near-perfect agreement with kinetic theory! Bounds maintained throughout.

Anisotropic hydrodynamics vs. kinetic theory II



Near-perfect agreement with KT! Far-off-equilibrium attractor for P_L but not for π, Π.

Overview

1 Prologue

2 Kinetic theory vs. hydrodynamics

3 Exact evolution in kinetic theory

4 Evolution in hydrodynamic approximation(s)
- Second-order Chapman Enskog hydrodynamics (CE hydro)
- Anisotropic hydrodynamics (aHydro)

5 Lessons learned

Lessons learned

- In weakly coupled systems that can be described by kinetic theory, Bjorken flow induces a far-from-equilibrium attractor for P_L/P.
- No early-time, far-from-equilibrium attractor in AdS/CFT for strongly coupled systems with Bjorken flow.
- At $\tau \ll \tau_R$ this attractor is controlled by approximate free-streaming dynamics (expansion-driven rapid cooling of p_z-distribution); P_L/P-excursions decay by power law.
- It smoothly connects the free-streaming attractor at $\tau \ll \tau_R$ with the late-time Navier-Stokes attractor at $\tau \gg \tau_R$.
- Only $P_L = P + \Pi - \pi$ exhibits attractive behavior; π, Π don't (bulk-shear coupling).
- Standard dissipative hydrodynamics (based on expansion around isotropic local equilibrium) does not reproduce the early-time far-off-equilibrium attractor, only shows the late-time Navier-Stokes attractor.
- Anisotropic hydrodynamics (based on expansion around an anisotropic momentum distribution, optimized to capture the effects of early-time free-streaming in Bjorken expansion) accurately reproduces the early-time far-from-equilibrium attractor for P_L/P as well as the non-attractive dynamics of π and Π.

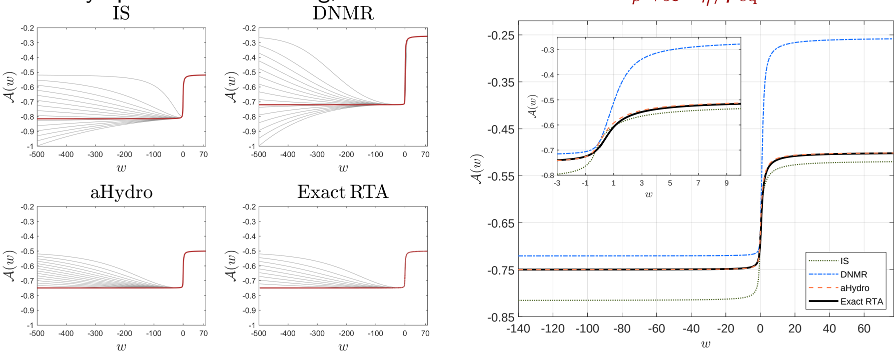
Happy 60th birthday, Xin-Nian, and many returns!

Thank you!

Extras

Hydrodynamic attractors for Gubser flow

- Gubser flow (Gubser '10) = long. boost-invariant + azimuthally symmetric, strong transverse flow
- Opposite to Bjorken flow, Knudsen number increases with time (exponentially) $\Longrightarrow$ asymptotic free-streaming, shear stress saturates at $\lim_{\rho\to\infty} \pi^\eta_\eta / p_{\rm eq} = 2$.



Behtash, Cruz-Camacho, Martinez PRD97 (2018) 044041

$$w = \tanh\rho/\hat{T} \sim \text{Knudsen number}, \qquad \mathcal{A}(w) = d\ln\hat{T}/d\ln\cosh\rho$$

- aHydro attractor and time evolution agree almost perfectly with exact RTA Boltzmann equation even in the free-streaming limit!

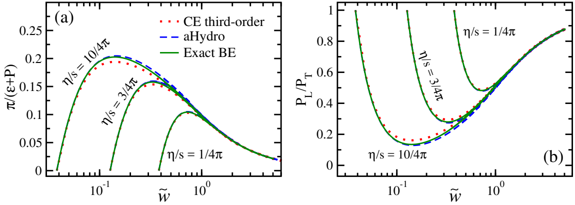
Hydrodynamic attractors: Bjorken vs. Gubser

Chattopadhyay, UH, Pal, Vujanovic, PRC97 (2018) 064909

Bjorken:

$$\tilde{w} = \frac{\tau\, T(\tau)}{4\pi\eta/s}$$

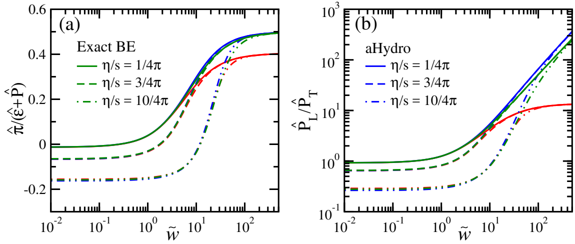
(inverse Knudsen number)



Gubser:

$$\tilde{w} = \frac{2\tanh\rho}{\hat{T}(\rho)}\,\frac{4\pi\eta}{s}$$

(Knudsen number)



Anisotropic hydrodynamics describes the underlying kinetic theory accurately even well before the evolution trajectory joins the attractor!

Some speculative comments:

- Similar far-from-equilibrium attractors may exist, for different pressure components, in other effectively one-dimensional, weakly coupled systems when the Knudsen number is large (for example, 3d Hubble flow).
- IMHO unlikely that any of this generalizes to less symmetric expansion geometries (such as generic (3+1)-d expansion).
- By quickly decaying to the far-off-equilibrium attractor, memory of initial P_L/P value is rapidly lost before $\tau \lesssim \tau_R$. Memories of the initial shear and bulk stresses, π and Π, linger until after $\tau \sim (2{-}3)\tau_R$. Phenomenological implications for the apparent "unreasonable effectiveness of hydrodynamics in heavy-ion collisions" deserve additional study.