Spin hydrodynamics

Dirk H. Rischke

thanks to
David Wagner, Nora Weickgenannt, Enrico Speranza

based on
arxiv:2203.04766 [nucl-th], arXiv:2208.01955 [nucl-th]

Berkeley Symposium on Hard Probes and Beyond
LBNL, Aug. 18 – 19, 2022

ELEMENTS
GOETHE UNIVERSITÄT FRANKFURT AM MAIN
HFHF Helmholtz Forschungsakademie Hessen für FAIR
CRC-TR 211 Strong-interaction matter under extreme conditions

Xin-Nian is known for his seminal work on “Hard Probes”, but also on “Beyond”:

- hydrodynamics:

PHYSICAL REVIEW C **86**, 024911 (2012)

Effects of initial flow velocity fluctuation in event-by-event (3 + 1)D hydrodynamics

Longgang Pang,[1,2] Qun Wang,[2] and Xin-Nian Wang[1,3]
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(Received 24 May 2012; published 24 August 2012)

> 170 citations on hep-inspire

- chiral kinetic theory:

PRL **110,** 262301 (2013) PHYSICAL REVIEW LETTERS week ending 28 JUNE 2013

Berry Curvature and Four-Dimensional Monopoles in the Relativistic Chiral Kinetic Equation

Jiunn-Wei Chen,[1] Shi Pu,[1,2] Qun Wang,[2] and Xin-Nian Wang[3,4]
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(Received 16 March 2013; published 24 June 2013)

> 240 citations on hep-inspire

- hadron polarization:

PRL **94,** 102301 (2005) PHYSICAL REVIEW LETTERS week ending 18 MARCH 2005

Globally Polarized Quark-Gluon Plasma in Noncentral $A + A$ Collisions

Zuo-Tang Liang[1] and Xin-Nian Wang[2,1]
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(Received 25 October 2004; published 14 March 2005)

> 380 citations on hep-inspire

- Condensed matter: **Barnett effect**

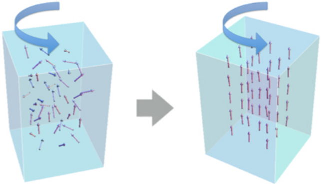


fig. by Mamoru Matsuo

$\Longrightarrow$ spins align under rotation (due to spin-orbit coupling)

$\Longrightarrow$ non-vanishing magnetization

- Non-central heavy-ion collisions: strong orbital angular momentum

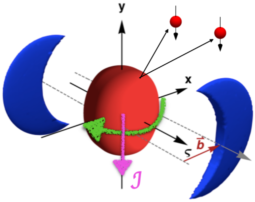
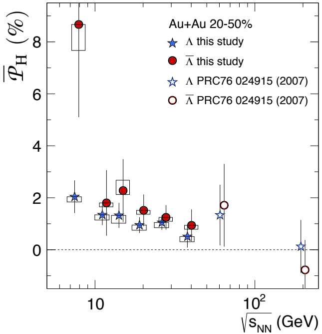


fig. by Radoslaw Ryblewski

Barnett effect in heavy-ion collisions?

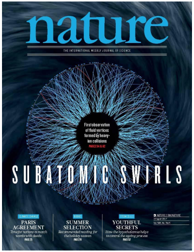
Λ polarization along angular-momentum direction ("global polarization")



L. Adamczyk et al. (STAR), Nature 548 (2017) 62

$\Longrightarrow$ QGP is "most vortical fluid ever observed"

$\omega \simeq (9 + 1) \times 10^{21} s^{-1}$



For comparison:

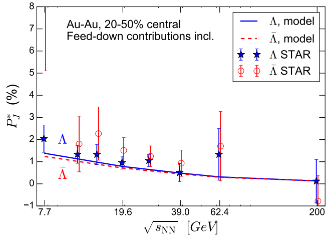
- Great Red Spot of Jupiter $\omega \simeq 10^{-4}\,s^{-1}$
- turbulent flow in superfluid He-II $\omega \sim 150\,s^{-1}$
- superfluid nanodroplets $\omega \sim 10^{7}\,s^{-1}$

Assuming local equilibrium on freeze-out hypersurface, hydrodynamics describes global polarization quite well . . .

$$\Pi^\mu \sim \epsilon^{\mu\nu\rho\sigma} k_\nu \int_{\Sigma_{\rm f.o.}} d\Sigma \cdot k \, \varpi_{\rho\sigma} f_{0\mathbf{k}}$$

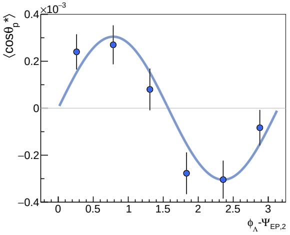
where $\varpi_{\rho\sigma} \equiv -\frac{1}{2}\left(\partial_\rho \beta_\sigma - \partial_\sigma \beta_\rho\right)$ thermal vorticity,
$\beta^\mu \equiv u^\mu / T$, $f_{0\mathbf{k}}$ local-equilibrium distribution function

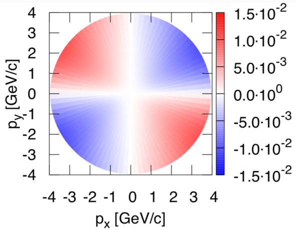
I. Karpenko, F. Becattini, NPA 967 (2017) 764



. . . but fails to describe azimuthal-angle dependence of polarization along the beam direction ("local longitudinal polarization")

F. Becattini, M.A. Lisa, Ann. Rev. Nucl. Part. Sci. 70 (2020) 395





Polarization and hydrodynamics

Recently, further (dissipative?) contributions to the polarization were found

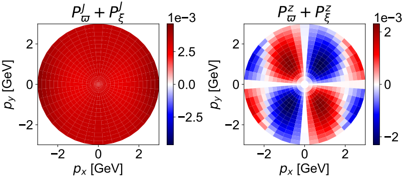
F. Becattini, M. Buzzegoli, G. Inghirami, I. Karpenko, A. Palermo, PRL 127 (2021) 272302

$$\Pi^\mu \sim \epsilon^{\mu\nu\rho\sigma} k_\nu \int_{\Sigma_{\text{f.o.}}} d\Sigma \cdot k \left(\varpi_{\rho\sigma} + 2\hat{t}_\rho \xi_{\sigma\lambda} \frac{k\lambda}{k_0} \right) f_{0\mathbf{k}}$$

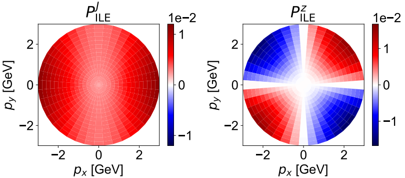
where $\xi_{\sigma\lambda} \equiv \frac{1}{2}\left(\partial_\sigma \beta_\lambda + \partial_\lambda \beta_\sigma\right)$ thermal shear tensor

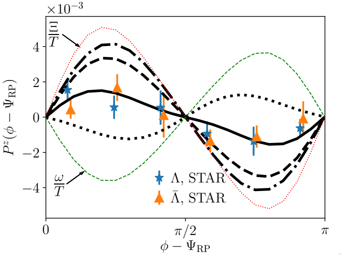
(see also S.Y.F. Liu, Y. Yin, JHEP 07 (2021) 188)

$\Longrightarrow$ this does not quite do the job ...



... but neglecting temperature gradients on $\Sigma_{\text{f.o.}}$ does!





Observations:

- derivation of formula for polarization ($\Pi^\mu \sim \varpi_{\rho\sigma}$) is strictly valid only for rotating global-equilibrium state

  F. Becattini, V. Chandra, L. Del Zanna, E. Grossi, Annals Phys. 338 (2013) 32

  $\Longrightarrow$ application of formula requires (infinitely) fast equilibration of spin degrees of freedom (relative to timescale of collision)

- dissipative effects influence (almost) all other observables in a heavy-ion collision, even appear explicitly in new term $\sim \xi_{\sigma\lambda}$ in formula for polarization

$\Longrightarrow$ Questions:

(I) How fast do spin degrees of freedom equilibrate?

(II) How is polarization influenced by dissipative effects?

$\Longrightarrow$ requires a theory of second-order dissipative spin hydrodynamics!

N. Weickgenannt, D. Wagner, E. Speranza, DHR, arxiv:2203.04766 [nucl-th], arXiv:2208.01955 [nucl-th]

Particle number conservation

$$\partial_\mu N^\mu = 0$$

where N^μ particle four-current

Energy-momentum conservation

$$\partial_\mu T^{\mu\nu} = 0$$

where $T^{\mu\nu}$ energy-momentum tensor

Angular-momentum conservation

$$\partial_\mu J^{\mu,\nu\lambda} = 0$$

where $J^{\mu,\nu\lambda}$ angular-momentum tensor

with $J^{\mu,\nu\lambda} \equiv x^\nu T^{\mu\lambda} - x^\lambda T^{\mu\nu} + \hbar S^{\mu,\nu\lambda}$ and energy-momentum conservation:

Equation of motion for spin tensor

$$\hbar \partial_\mu S^{\mu,\nu\lambda} = T^{[\lambda\nu]}$$

where $a^{[\lambda} b^{\nu]} \equiv a^\lambda b^\nu - a^\nu b^\lambda$

Conservation laws of spin hydrodynamics

Particle number conservation

$$\partial_\mu N^\mu = 0$$

1 equation, 4 unknowns

Energy-momentum conservation

$$\partial_\mu T^{\mu\nu} = 0$$

4 equations, 10 unknowns

Angular-momentum conservation

$$\partial_\mu J^{\mu,\nu\lambda} = 0$$

with $J^{\mu,\nu\lambda} \equiv x^\nu T^{\mu\lambda} - x^\lambda T^{\mu\nu} + \hbar S^{\mu,\nu\lambda}$ and energy-momentum conservation:

Equation of motion for spin tensor

$$\hbar \partial_\mu S^{\mu,\nu\lambda} = T^{[\lambda\nu]}$$

6 equations, 24 unknowns

where $a^{[\lambda} b^{\nu]} \equiv a^\lambda b^\nu - a^\nu b^\lambda$

Assume local equilibrium

$\Longrightarrow$ **Single-particle distribution function (to order $\mathcal{O}(\hbar)$)**

$$f_{\text{eq},\mathbf{k}\mathfrak{s}} = f_{0\mathbf{k}} \left(1 + \frac{\hbar}{4} \Omega_{\mu\nu} \Sigma^{\mu\nu}_{\mathbf{k}\mathfrak{s}} \right) \,, \qquad f_{0\mathbf{k}} = \exp\left(\alpha - \beta \cdot k\right)$$

where

- $\alpha \equiv \dfrac{\mu}{T}$ Lagrange multiplier for particle-number conservation
- $\beta^\mu \equiv \dfrac{u^\mu}{T}$ Lagrange multiplier for energy-momentum conservation, u^μ fluid 4-velocity
- $\Omega_{\mu\nu}$ spin potential, Lagrange multiplier for angular-momentum conservation
- $\Sigma^{\mu\nu}_{\mathbf{k}\mathfrak{s}} \equiv -\dfrac{1}{m} \epsilon^{\mu\nu\alpha\beta} k_\alpha \mathfrak{s}_\beta$ dipole-moment tensor of particle with mass m, (on-shell) 4-momentum k_α, and spin 4-vector $\mathfrak{s}_\beta$

Assume local equilibrium

$\Longrightarrow$ **Single-particle distribution function (to order $\mathcal{O}(\hbar)$)**

$$f_{\mathrm{eq},\mathbf{k}\mathfrak{s}} = f_{0\mathbf{k}} \left(1 + \frac{\hbar}{4}\Omega_{\mu\nu}\Sigma^{\mu\nu}_{\mathbf{k}\mathfrak{s}}\right) \,, \quad f_{0\mathbf{k}} = \exp\left(\alpha - \beta\cdot k\right)$$

where

- $\alpha \equiv \dfrac{\mu}{T}$ Lagrange multiplier for particle-number conservation
  1 parameter
- $\beta^\mu \equiv \dfrac{u^\mu}{T}$ Lagrange multiplier for energy-momentum conservation, u^μ fluid 4-velocity
  4 parameters
- $\Omega_{\mu\nu}$ spin potential, Lagrange multiplier for angular-momentum conservation
  6 parameters
- $\Sigma^{\mu\nu}_{\mathbf{k}\mathfrak{s}} \equiv -\dfrac{1}{m}\epsilon^{\mu\nu\alpha\beta}k_\alpha \mathfrak{s}_\beta$ dipole-moment tensor of particle with mass m, (on-shell) 4-momentum k_α, and spin 4-vector $\mathfrak{s}_\beta$

Extend phase space by spin degrees of freedom:

$$dK \equiv \frac{d^3\mathbf{k}}{(2\pi)^3} \longrightarrow d\Gamma \equiv dK\, dS(k)$$

with $$dS(k) \equiv \sqrt{\frac{k^2}{3\pi^2}}\, d^4\mathfrak{s}\, \delta(k\cdot\mathfrak{s})\, \delta(\mathfrak{s}\cdot\mathfrak{s}+3),$$

such that $\int dS(k) = 2\,,\ \int dS(k)\, \mathfrak{s}^\mu = 0\,,\ \int dS(k)\, \mathfrak{s}^\mu \mathfrak{s}^\nu = -2\left(g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}\right)$

Fluid-dynamical currents

$\Longrightarrow$

$$\begin{aligned} N^\mu &\equiv \langle k^\mu \rangle \\ T^{\mu\nu} &\equiv \langle k^\mu k^\nu \rangle + \mathcal{O}(\hbar^2) \\ S^{\mu,\nu\lambda} &\equiv \left\langle k^\mu \left(\frac{1}{2}\Sigma^{\nu\lambda}_{\mathbf{k}\mathfrak{s}} - \frac{\hbar}{4m^2} k^{[\nu}\partial^{\lambda]} \right) \right\rangle + \mathcal{O}(\hbar^2) \end{aligned}$$

where $\langle \cdots \rangle \equiv \int d\Gamma \cdots f(x,k,\mathfrak{s})$

$\Longrightarrow$ for $\langle \cdots \rangle_{\text{eq}} \equiv \int d\Gamma \cdots f_{\text{eq},\mathbf{k}\mathfrak{s}}$ $\Longrightarrow$ equations of motion are closed!

see, e.g., W. Florkowski, A. Kumar, R. Ryblewski, Prog. Part. Nucl. Phys. 108 (2019) 103709

Dissipative spin hydrodynamics

$\Longrightarrow$ provide additional equations of motion

$\Longrightarrow$ start from underlying microscopic theory, apply method of moments
see, e.g., G.S. Denicol, H. Niemi, E. Molnár, DHR, PRD 85 (2012) 114047

From equation of motion for the Wigner function, derive to first order in $\hbar$:
N. Weickgenannt, E. Speranza, X.-l. Sheng, Q. Wang, DHR,
PRL 127 (2021) 052301, PRD 104 (2021) 016022

Boltzmann equation for spin-1/2 particles with nonlocal collision term

$$
\begin{aligned}
k \cdot \partial f(x, k, \mathfrak{s}) &= \mathfrak{C}[f] \\
\mathfrak{C}[f] &= \int d\Gamma_1 d\Gamma_2 d\Gamma' \, \mathcal{W}_{\mathbf{kk}' \to \mathbf{k}_1 \mathbf{k}_2}^{\mathfrak{s}\mathfrak{s}' \to \mathfrak{s}_1 \mathfrak{s}_2} \, [f(x + \Delta_1, k_1, \mathfrak{s}_1) f(x + \Delta_2, k_2, \mathfrak{s}_2) \\
&\qquad - f(x + \Delta, k, \mathfrak{s}) f(x + \Delta', k', \mathfrak{s}')]
\end{aligned}
$$

where nonlocal position shift $\quad \Delta^\mu = -\dfrac{\hbar}{2m(k \cdot \hat{t} + m)} \epsilon^{\mu\nu\alpha\beta} k_\nu \hat{t}_\alpha \mathfrak{s}_\beta$

$\Longleftrightarrow$ Berry connection! see, e.g., M. Stone, V. Dwivedi, T. Zhou, PRD 91 (2015) 025004

Note: $\Delta \sim \dfrac{\hbar}{m}$ Compton wavelength!

Nonlocal collisions: allow mutual conversion of orbital angular momentum and spin

Usually, local-equilibrium distribution function f_{eq} is defined by condition

Local equilibrium

$$\mathfrak{C}\left[f_{\mathrm{eq}}\right] \equiv 0$$

However, as shown in N. Weickgenannt, E. Speranza, X.-l. Sheng, Q. Wang, DHR, PRL 127 (2021) 052301, PRD 104 (2021) 016022, nonlocal collision term vanishes only in

Global equilibrium

$$\begin{aligned} \alpha &= \text{const.} \\ \partial^\mu \beta^\nu + \partial^\nu \beta^\mu &= 0 \\ \Omega_{\mu\nu} &= \varpi_{\mu\nu} = \text{const.} \end{aligned}$$

$\Longrightarrow$ appears too restrictive: nonlocality $\Delta \lesssim r_{\mathrm{int}} \ll \lambda_{\mathrm{mfp}} \ll L_{\mathrm{hydro}}$

$\Longrightarrow$ nonlocality scale Δ much smaller than hydrodynamic scale L_{hydro}

$\Longrightarrow$

Generalized local equilibrium

$$\mathfrak{C}\left[f_{\mathrm{eq}}\right] \sim \mathcal{O}(\Delta / L_{\mathrm{hydro}})$$

Generalized local equilibrium

Define hydrodynamic scale $L_{\rm hydro}$ by $\frac{1}{m} k \cdot \partial f_{{\rm eq},\mathbf{k}\mathfrak{s}} \sim \frac{1}{L_{\rm hydro}} f_{{\rm eq},\mathbf{k}\mathfrak{s}}$

$\Longrightarrow$

$$\begin{aligned} \partial_\mu \alpha &\sim \mathcal{O}(L_{\rm hydro}^{-1}) \\ \partial^\mu \beta^\nu + \partial^\nu \beta^\mu &\sim \mathcal{O}((k L_{\rm hydro})^{-1}) \\ \partial_\lambda \Omega_{\mu\nu} &\sim \mathcal{O}(L_{\rm hydro}^{-1}) \end{aligned}$$

$\Longrightarrow$ using conservation of total angular momentum $J^{\mu\nu} \equiv \Delta^{[\mu} k^{\nu]} + \frac{\hbar}{2} \Sigma^{\mu\nu}_{\mathbf{k}\mathfrak{s}}$ in binary collisions, order $\mathcal{O}(\hbar)$ contribution to nonlocal collision term:

$$\sim \frac{\hbar}{4} \Omega_{\mu\nu} \Sigma^{\mu\nu}_{\mathbf{k}\mathfrak{s}} + \Delta^\mu \partial_\mu (\alpha - \beta_\nu k^\nu) = \frac{1}{2} \Delta^{[\mu} k^{\nu]} (\varpi_{\mu\nu} - \Omega_{\mu\nu}) + \mathcal{O}(\Delta / L_{\rm hydro})$$

$\Longrightarrow$ for generalized local equilibrium: $\Omega_{\mu\nu} \equiv \varpi_{\mu\nu} + \mathcal{O}((k L_{\rm hydro})^{-1})$

$\Longrightarrow$ consistent, as in global equilibrium $L_{\rm hydro} \to \infty$ and thus $\Omega_{\mu\nu} \to \varpi_{\mu\nu}$

Usually, all gradients of fluid-dynamical quantities are $\mathcal{O}(L_{\text{hydro}}^{-1})$

However, global-equilibrium conditions do not restrict value of thermal vorticity $\varpi_{\mu\nu}$
see, e.g., F. Becattini, L. Tinti, Annals Phys. 325 (2010) 1566

$\Longrightarrow$ vorticity does not follow usual power counting, $\varpi_{\mu\nu} \not\sim \mathcal{O}((kL_{\text{hydro}})^{-1})$

$\Longrightarrow$ define scale ℓ_{vort} set by vorticity: $\varpi_{\mu\nu} \sim \mathcal{O}((k\ell_{\text{vort}})^{-1})$

In principle, ℓ_{vort} can take any value from $\ell_{\text{vort}} \ll L_{\text{hydro}}$ to $\ell_{\text{vort}} \sim L_{\text{hydro}}$

However, in order for $\hbar-$expansion to apply: $\hbar\Omega_{\mu\nu}\Sigma^{\mu\nu}_{\mathbf{k}\mathfrak{s}} \sim \dfrac{\hbar}{m}\varpi_{\mu\nu}\epsilon^{\mu\nu\alpha\beta}k_\alpha \mathfrak{s}_\beta \sim \dfrac{\Delta}{\ell_{\text{vort}}} \ll 1$

$\Longrightarrow$ ℓ_{vort} cannot be arbitrarily small (as in global equilibrium)

$\Longrightarrow$ remember $\Delta \lesssim r_{\text{int}} \ll \lambda_{\text{mfp}} \ll L_{\text{hydro}}$ $\Longrightarrow$ ℓ_{vort} could be as small as λ_{mfp}!

$\Longrightarrow$ for the sake of simplicity assume $\dfrac{\Delta}{\ell_{\text{vort}}} \sim \dfrac{\lambda_{\text{mfp}}}{L_{\text{hydro}}} \equiv \text{Kn} \ll 1$

Extend method of moments developed in G.S. Denicol, H. Niemi, E. Molnár, DHR, PRD 85 (2012) 114047 by spin degrees of freedom

Single-particle distribution function

$$f(x,k,\mathfrak{s}) = f_{\text{eq},\mathbf{k}\mathfrak{s}} + \delta f_{\mathbf{k}\mathfrak{s}}$$

$$\delta f_{\mathbf{k}\mathfrak{s}} = f_{0\mathbf{k}} \sum_{\ell=0}^{\infty} \sum_{n\in\mathbb{S}_\ell} \mathcal{H}_{\mathbf{k}n}^{(\ell)} \left[\rho_n^{\mu_1\cdots\mu_\ell} - \tau_n^{\langle\mu\rangle,\mu_1\cdots\mu_\ell}\left(g_{\mu\nu} - \frac{k_{\langle\mu\rangle}u_\nu}{E_\mathbf{k}}\right)\mathfrak{s}^\nu\right] k_{\langle\mu_1}\cdots k_{\mu_\ell\rangle}$$

where

- $k_{\langle\mu_1}\cdots k_{\mu_\ell\rangle}$ irreducible tensors
- $\rho_n^{\mu_1\cdots\mu_\ell} \equiv \left\langle E_\mathbf{k}^n k^{\langle\mu_1}\cdots k^{\mu_\ell\rangle}\right\rangle_\delta$ irreducible moments, $\langle\cdots\rangle_\delta \equiv \langle\cdots\rangle - \langle\cdots\rangle_{\text{eq}}$
- $\tau_n^{\mu,\mu_1\cdots\mu_\ell} \equiv \left\langle \mathfrak{s}^\mu E_\mathbf{k}^n k^{\langle\mu_1}\cdots k^{\mu_\ell\rangle}\right\rangle_\delta$ spin moments
- $E_\mathbf{k} \equiv k\cdot u$ energy of particle in fluid rest frame
- $A^{\langle\mu\rangle} \equiv \Delta^{\mu\nu}A_\nu$, with $\Delta^{\mu\nu} \equiv g^{\mu\nu} - u^\mu u^\nu$ projector onto 3-space orthogonal to u^μ
- $A^{\langle\mu_1\cdots\mu_\ell\rangle} \equiv \Delta^{\mu_1\cdots\mu_\ell}_{\nu_1\cdots\nu_\ell} A^{\nu_1\cdots\nu_\ell}$,
  $\Delta^{\mu_1\cdots\mu_\ell}_{\nu_1\cdots\nu_\ell}$ symmetric, traceless rank-2ℓ projection operators built from $\Delta^{\mu\nu}$
- $\mathcal{H}_{\mathbf{k}n}^{(\ell)}$ polynomials in $E_\mathbf{k}$ of rank N_ℓ

Inserting $f(x,k,\mathfrak{s}) = f_{\text{eq},\mathbf{k}\mathfrak{s}} + \delta f_{\mathbf{k}\mathfrak{s}}$ into definition of spin tensor

Spin tensor

$\Longrightarrow$
$$S^{\mu,\nu\lambda} = u^\mu \tilde{\mathfrak{N}}^{\nu\lambda} + \tilde{\mathfrak{P}}^{\langle\mu\rangle\nu\lambda} + u_\alpha \tilde{\mathfrak{H}}^{\mu\nu\lambda\alpha} + u^\mu \tilde{\mathfrak{H}}_\alpha{}^{\nu\lambda\alpha} + \tilde{\mathfrak{Q}}^{\mu\nu\lambda} - \frac{\hbar}{4m^2}\partial^{[\lambda} T^{\nu]\mu}$$

where

- $\tilde{\mathfrak{N}}^{\nu\lambda} \equiv \epsilon^{\nu\lambda\alpha\beta}\mathfrak{N}_{\alpha\beta}$,
  with $\mathfrak{N}^{\alpha\beta} \equiv -\dfrac{1}{2m} u^\alpha \left[\left\langle E_{\mathbf{k}}^2 \mathfrak{s}^\beta \right\rangle_{\text{eq}} + \tau_2^\beta\right]$ spin energy tensor
- $\tilde{\mathfrak{P}}^{\mu\nu\lambda} \equiv \epsilon^{\mu\nu\lambda\alpha}\mathfrak{P}_\alpha$,
  with $\mathfrak{P}^\alpha \equiv -\dfrac{1}{6m}\left[\left\langle \left(m^2 - E_{\mathbf{k}}^2\right)\mathfrak{s}^\alpha \right\rangle_{\text{eq}} + m^2\tau_0^\alpha - \tau_2^\alpha\right]$ spin pressure vector
- $\tilde{\mathfrak{H}}^{\mu\nu\lambda\alpha} \equiv \epsilon^{\nu\lambda\alpha\beta}\mathfrak{H}^\mu{}_\beta$,
  with $\mathfrak{H}^{\mu\beta} \equiv -\dfrac{1}{2m}\left[\left\langle E_{\mathbf{k}} k^{\langle\mu\rangle}\mathfrak{s}^\beta \right\rangle_{\text{eq}} + \tau_1^{\beta,\mu}\right]$ spin diffusion tensor
- $\tilde{\mathfrak{Q}}^{\mu\nu\lambda} \equiv \epsilon^{\nu\lambda\alpha\beta}\mathfrak{Q}^\mu{}_{\alpha\beta}$,
  with $\mathfrak{Q}^{\mu\alpha\beta} \equiv -\dfrac{1}{2m}\left[\left\langle k^{\langle\mu}k^{\alpha\rangle}\mathfrak{s}^\beta \right\rangle_{\text{eq}} + \tau_0^{\beta,\mu\alpha}\right]$ spin stress tensor

Define local-equilibrium state via

Landau matching conditions

$$\begin{aligned} N^{\mu} u_{\mu} &= N^{\mu}_{\text{eq}} u_{\mu} \\ T^{\mu\nu} u_{\nu} &= T^{\mu\nu}_{\text{eq}} u_{\nu} \\ J^{\mu,\nu\lambda} u_{\mu} &= J^{\mu,\nu\lambda}_{\text{eq}} u_{\mu} \end{aligned}$$

$\Longrightarrow$ relates α, β^{μ}, and $\Omega_{\mu\nu}$ in $f_{\text{eq},\mathbf{k}\mathfrak{s}}$ to fluid-dynamical variables

$\Longrightarrow$ determine α, β^{μ}, and $\Omega_{\mu\nu}$ via conservation laws!

$\Longrightarrow$ still need to derive equations of motion for dissipative currents Π, n^{μ}, and $\pi^{\mu\nu}$, as well as spin moments τ_0^{α}, τ_2^{α}, $\tau_1^{\beta,\mu}$, and $\tau_0^{\beta,\mu\alpha}$ appearing in $S^{\mu,\lambda\nu}$

Equations of motion for standard dissipative currents Π, n^{μ}, and $\pi^{\mu\nu}$

$\Longrightarrow$ see G.S. Denicol, H. Niemi, E. Molnár, DHR, PRD 85 (2012) 114047

$\Longrightarrow$ relaxation-type equations, e.g., $\dot{\Pi} + \frac{1}{\tau_{\Pi}}\Pi = \ldots$, with $\dot{\Pi} \equiv u^{\mu}\partial_{\mu}\Pi$

Take spin moments of Boltzmann equation:

Equations of motion for spin moments

$$\Longrightarrow \quad \dot{\tau}_n^{\langle\mu\rangle,\langle\mu_1\cdots\mu_\ell\rangle} - \mathfrak{C}_{n-1}^{\langle\mu\rangle,\mu_1\cdots\mu_\ell} = \ldots$$

$$\mathfrak{C}_{n-1}^{\mu,\mu_1\cdots\mu_\ell} = \int d\Gamma\, E_{\mathbf{k}}^{n-1} k^{\langle\mu_1} \cdots k^{\mu_\ell\rangle} \mathfrak{s}^\mu \, \mathfrak{C}[f]$$

Linearized collision integral

$$\mathfrak{C}_{n-1}^{\mu,\mu_1\cdots\mu_\ell} = \mathfrak{C}_{n-1,\mathrm{local}}^{\mu,\mu_1\cdots\mu_\ell} + \mathfrak{C}_{n-1,\mathrm{nonlocal}}^{\mu,\mu_1\cdots\mu_\ell}$$

$$\Longrightarrow \quad \mathfrak{C}_{n-1,\mathrm{local}}^{\mu,\mu_1\cdots\mu_\ell} = -\sum_{r\in\mathbb{S}_\ell} B_{nr}^{(\ell)} \tau_r^{\mu,\mu_1\cdots\mu_\ell}$$

$$\mathfrak{C}_{n-1,\mathrm{nonlocal}}^{\mu,\mu_1\cdots\mu_\ell} = \int d\Gamma_1 d\Gamma_2 d\Gamma d\Gamma' \, \mathcal{W}_{\mathbf{k}\mathbf{k}'\to\mathbf{k}_1\mathbf{k}_2}^{\mathfrak{s}\mathfrak{s}'\to\mathfrak{s}_1\mathfrak{s}_2} E_{\mathbf{k}}^{n-1} f_{0\mathbf{k}} f_{0\mathbf{k}'} \times k^{\langle\mu_1} \cdots k^{\mu_\ell\rangle} \mathfrak{s}^\mu \left[\frac{\hbar}{4}(\varpi_{\alpha\beta} - \Omega_{\alpha\beta})\Sigma_{\mathbf{k}\mathfrak{s}}^{\alpha\beta} + \xi_{\alpha\beta}\Delta^\alpha k^\beta\right]$$

- $\mathfrak{C}_{n-1,\mathrm{local}}^{\mu,\mu_1\cdots\mu_\ell}$ $\Longrightarrow$ inverting $B_{nr}^{(\ell)}$ yields relaxation times
- $\mathfrak{C}_{n-1,\mathrm{nonlocal}}^{\mu,\mu_1\cdots\mu_\ell}$ $\Longrightarrow$ gives rise to Navier-Stokes terms

Infinite set of moment equations needs to be truncated

$\Longrightarrow$ lowest-order truncation:

14 standard fluid-dynamical moments + 24 moments for components of spin tensor

$\Longrightarrow$ (14+24)–moment approximation

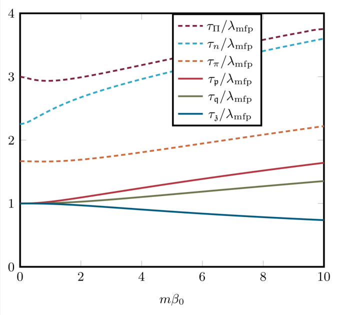
Independent spin moments: $\mathfrak{p}^\mu \equiv \tau_0^{\langle\mu\rangle}$, $\mathfrak{z}^{\mu\nu} \equiv \tau_1^{\langle\mu\rangle,\nu} + \tau_1^{\langle\nu\rangle,\mu}$, $\mathfrak{q}^{\mu\nu\lambda} \equiv \tau_0^{\langle\mu\rangle,\nu\lambda}$

3 + 6 + 15 components

Equations of motion for independent spin moments

$$\Longrightarrow \begin{aligned} \tau_{\mathfrak{p}}\, \dot{\mathfrak{p}}^{\langle\mu\rangle} + \mathfrak{p}^\mu &\sim \epsilon^{\mu\nu\alpha\beta} \left(\varpi_{\alpha\beta} - \Omega_{\alpha\beta}\right) u_\nu + \ldots \\ \tau_{\mathfrak{z}}\, \dot{\mathfrak{z}}^{\langle\mu\rangle\langle\nu\rangle} + \mathfrak{z}^{\mu\nu} &\sim \ldots \\ \tau_{\mathfrak{q}}\, \dot{\mathfrak{q}}^{\langle\mu\rangle\langle\nu\lambda\rangle} + \mathfrak{q}^{\mu\nu\lambda} &\sim \xi_\alpha{}^{\langle\nu} \epsilon^{\lambda\rangle\mu\alpha\beta} u_\beta + \ldots \end{aligned}$$

Spin relaxation times



$\Longrightarrow$ spin relaxation times of the same order (but somewhat smaller) than relaxation times for Π, n^μ, $\pi^{\mu\nu}$

$\Longrightarrow$ spin degrees of freedom equilibrate (i.e., approach their Navier-Stokes values) as fast (or even faster) than Π, n^μ, $\pi^{\mu\nu}$

$\Longrightarrow$ answers Question (I)

Pauli-Lubanski vector (spin polarization vector!) in Navier-Stokes limit

$$\Pi^\mu_{\rm NS} \sim \int_{\Sigma_{\rm f.o.}} d\Sigma \cdot k\, f_{0\mathbf{k}} \left\{ \epsilon^{\mu\nu\rho\sigma} k_\nu \Omega_{\rho\sigma} + \left(\delta^\mu_\nu - \frac{u^\mu k_{\langle\nu\rangle}}{E_\mathbf{k}} \right) \times \left[\kappa_{\mathfrak{p}} \epsilon^{\nu\rho\alpha\beta} \left(\Omega_{\alpha\beta} - \varpi_{\alpha\beta} \right) u_\rho + \kappa_{\mathfrak{q}} \xi_\alpha^{\ \langle\rho} \epsilon^{\sigma\rangle\nu\alpha\beta} u_\beta k_{\langle\rho} k_{\sigma\rangle} \right] \right\}$$

$\Longrightarrow$ novel dissipative corrections $\sim \Omega_{\alpha\beta} - \varpi_{\alpha\beta}$ and $\xi_{\alpha\beta}$ $\Longrightarrow$ answers Question (II)

- Starting from Boltzmann equation with nonlocal collision term, and using method of moments, derived equations of motion of relativistic second-order dissipative spin hydrodynamics in (14+24)–moment approximation
- Spin degrees of freedom relax as fast as usual dissipative quantities
- Polarization vector is influenced by dissipative corrections
- Need to quantify influence of dissipative corrections on polarization observables!
- Causality and stability of equations of motion of spin hydrodynamics?