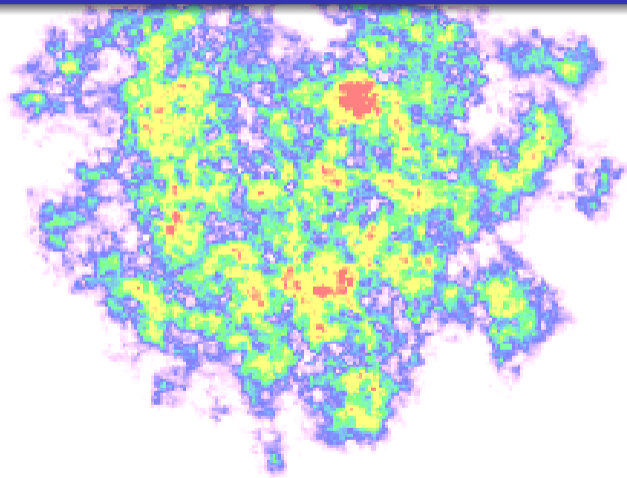


Towards 3D IP-Glasma



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*w/ Scott McDonald
and Charles Gale*

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McGill University
Montréal, QC, CANADA*



Based on NPA **1005**, 121771 (2021) (McDonald, Jeon and Gale), and Scott McDonald's thesis

- 壬新年 : King-New-Year
- 壬寅年 : 1962. Also 2022. Black-Tiger-Year
- What does 易經 (I-Jing) say about him? - Lots of Trees, Water and Earth
Independent. Brilliant. Sensitive. Strong leader. Soft outside, strong inside. Romantic. Could be stubborn (sometimes). Sunny disposition.

- 壬新年 : King-New-Year
- 壬寅年 : 1962. Also 2022. Black-Tiger-Year
- What does 核易經 (HIJING) say about him? - Lots of Tree(diagram)s.
*Independent. Brilliant. Sensitive. Strong leader. Soft outside, strong inside.
Romantic. Could be stubborn (sometimes). Sunny disposition.*

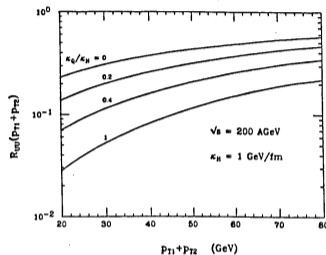


Fig. 7 **Dijet reduction factor** for central $U + U$ collisions at $\sqrt{s} = 200$ GeV/n as a function of the dijet energy $E = P_{T1} + P_{T2}$, for different values of κ_Q/κ_H assuming $\kappa_H = 1$ GeV/fm.

transverse coordinate, ϕ the azimuthal angle of the jet and $\tau_f(r, \phi)$ the escape time. Assuming only Bjorken[31] scaling longitudinal expansion and a Bag model equation of state[31], one can find the time dependence of $dE(\tau)/dx$ and get the reduction rate of jet production at fixed P_T by averaging over the initial coordinates (r, ϕ) [22],

$$R_{AA}(E) = \frac{\sigma^{jet}(E)_{quenching}}{\sigma^{jet}(E)_{no-quenching}}. \quad (11)$$

In the plasma phase, the temperature decreases as $T(\tau)/T_c = (\tau_Q/\tau)^{1/3}$. According

- First mention of R_{AA} I could find.
- Xin-Nian Wang and Miklos Gyulassy, *Jets in relativistic heavy ion collisions* in BNL RHIC Workshop 1990:0079-102 (QCD199:R2:1990)

Happy Birthday, Xin-Nian!

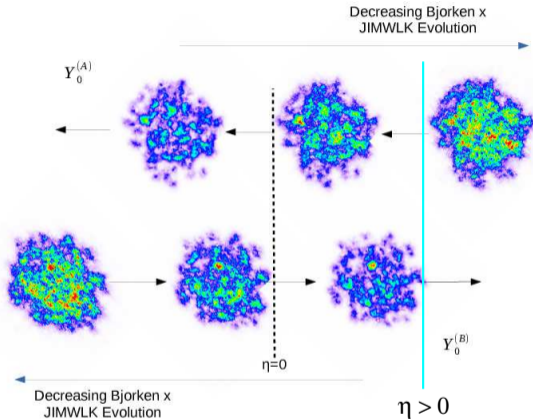
祝你生日快乐
新年大哥

Thank you.

You have been an inspiration and a big brother to many of us. You still are.



In a nutshell

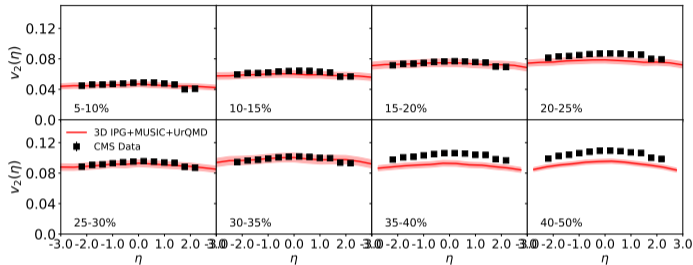
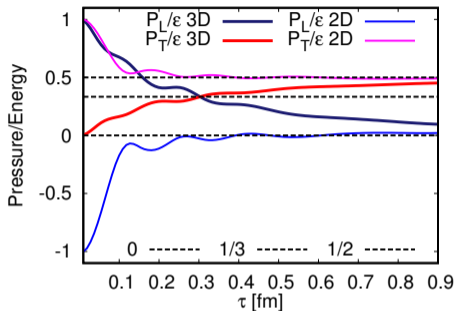


- Finite $\eta_s > 0 \implies$ Moving frame with $v^z = \tanh \eta_s$
- The target appears much denser than the projectile (JIMWLK) \implies Gives the initial condition at η_s and at $\tau = 0^+$.
- Longitudinal decorrelation is built in.

CGC & JIMWLK: Work by Venugopalan, McLerran, Jalilian-Marian, Iancu, Weigert, Leonidov, Kovner, Kovchegov, Dumitru, Gelis, Blaizot, Kharzeev, Nardi, Levin, Krasnitz, Nara, Lappi, Mäntysaari and many others.

The η slice initial condition: Phys. Rev. C94, 044907 (2016), Schenke and Schlichting

In a nutshell

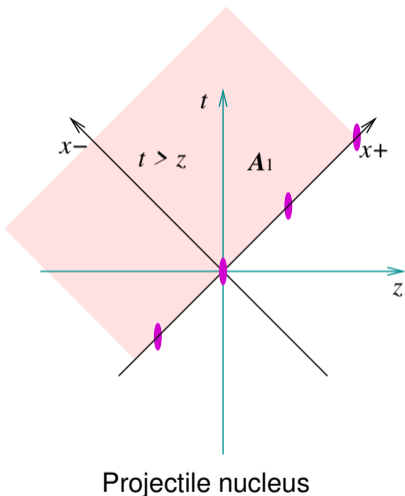


- $\mathbf{E}_\perp \neq 0, \mathbf{B}_\perp \neq 0$
- Pressure evolution is very different than 2D
- Transverse dynamics - Same quality
- Longitudinal dynamics: Global observables OK
- Differential observables in Longi.: Compute time hungry calculations

Why 3D?

- The world is 3D!
- Extended set of observables
- A lot of important physics in longitudinal dynamics (e.g. JIMWLK evolution, EoS)

Brief Review of the MV model



- Color sources on the x^+ axis or parallel to it

$$\mathcal{J}_P^\mu = \rho(x^-, \mathbf{x}_\perp) \delta^{\mu+}$$

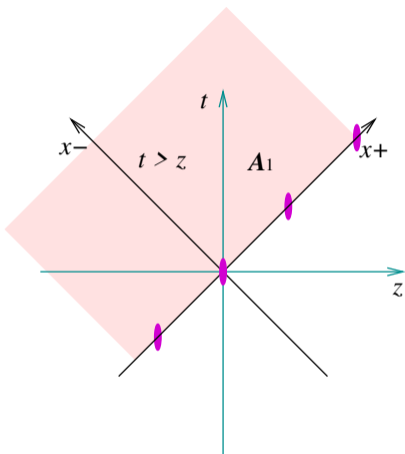
- Physics cannot depend on x^+ \implies Solvable
- Gluon field \mathcal{A}_1 present only for $x^- > 0$ or $t > z$
- Colour density distribution

$$\mathcal{P}[\rho] = \mathcal{N} \exp \left(- \int dx^- \int d^2x_\perp \frac{\rho_a(x^-, \mathbf{x}_\perp) \rho_a(x^-, \mathbf{x}_\perp)}{2\mu^2(x^-, \mathbf{x}_\perp)} \right)$$

- Saturation scale $Q_s \propto \mu$

Lightcone coordinates: $x^\pm = (t \pm z)/\sqrt{2}$

Brief Review of the MV model

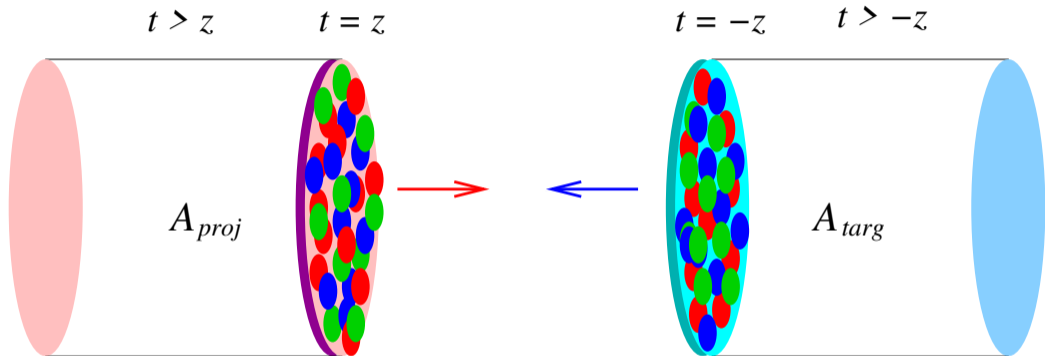


Projectile nucleus

- The MV model contains two separate concepts:
 - Infinite momentum (Boost invariant) EoM
 - Finite saturation scale $Q_s \propto \sqrt{s}^{\lambda/2}$

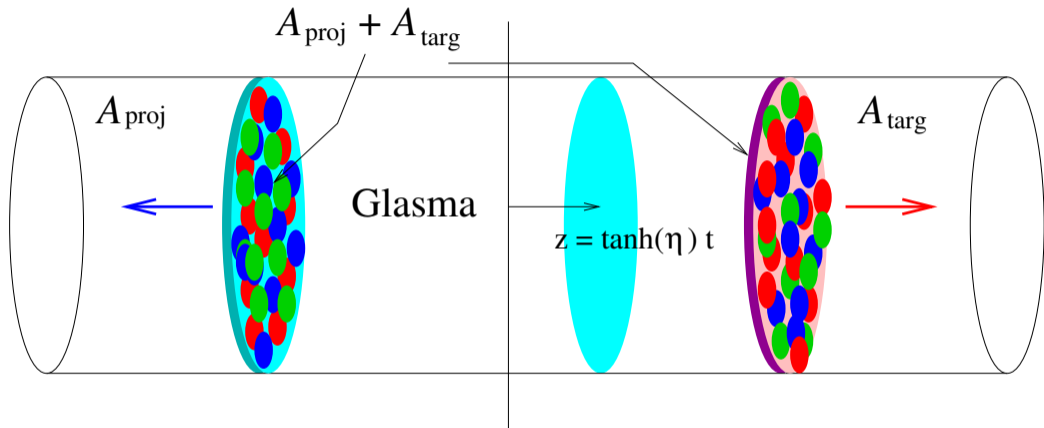
Lightcone coordinates: $x^\pm = (t \pm z)/\sqrt{2}$

Before the collision



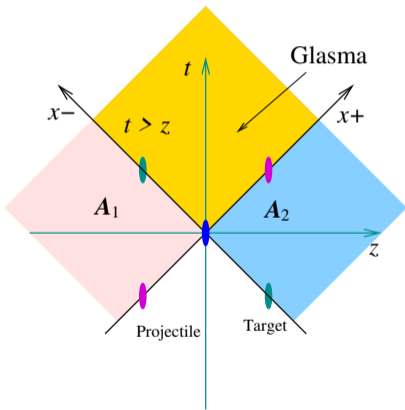
- Two nuclei approach accompanied by trailing gluon fields

After the collision



- Middle: Glasma - Result of interaction between A_{proj} and A_{targ}

Key Idea: Let the two gluon fields from the projectile and the target collide and evolve.



- In the forward light-cone region:

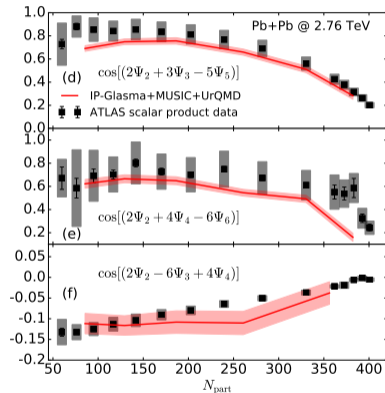
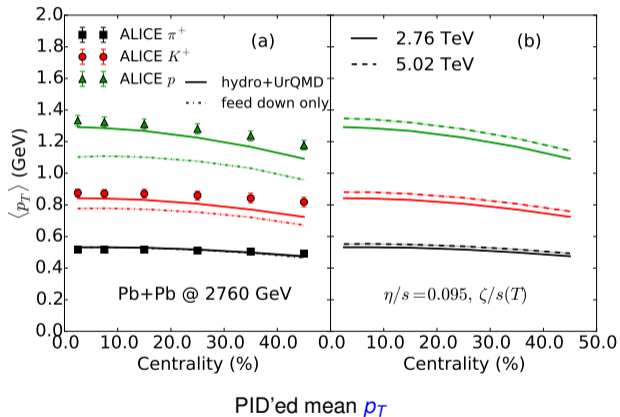
$$[D_\mu, G^{\mu\nu}] = 0$$

Initial conditions at $\tau = 0^+$

- $\mathcal{A}_i = \mathcal{A}_i^1 + \mathcal{A}_i^2$
- $\mathcal{E}^\eta = ig[\mathcal{A}_i^1, \mathcal{A}_i^2]$
- $\mathcal{E}^i = 0, \mathcal{B}^i = 0, \mathcal{A}_\eta = 0$
 \implies No initial transverse fields
- Boost-invariant \implies Relevant mostly for the mid-rapidity dynamics

2D IP-Glasma has been successful

[Phys. Rev. C 95, 064913 (2017), McDonald, Shen, Fillion-Gourdeau, Jeon, Gale]

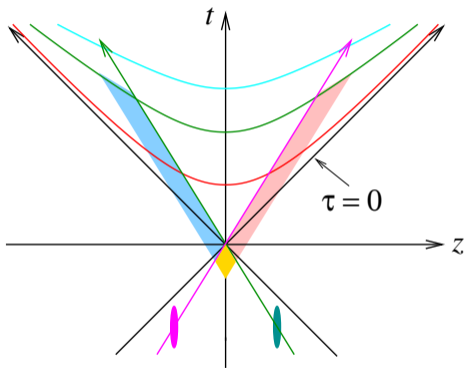


v_n correlation results: Prediction



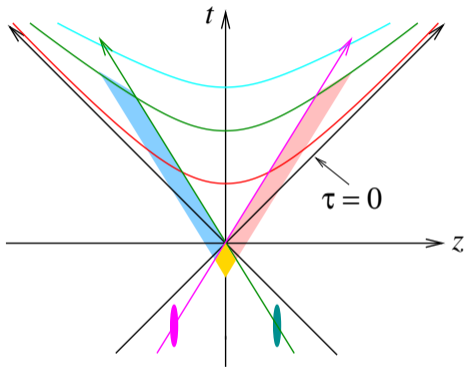
Going 3D

What we should be doing



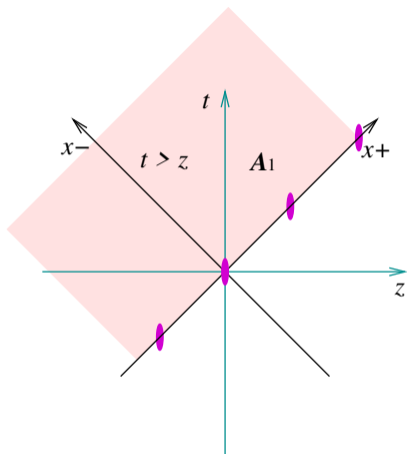
- In reality, the nuclei have subluminal velocities $v = z/t$
- Equivalently, finite (pseudo-)rapidities $\eta = \tanh^{-1} v$
- Boundary at constant $\pm\eta_{\text{beam}}$ lines – Not any fixed τ
- Sources are *not* infinitely thin
- Solve $[\mathcal{D}_\mu, \mathcal{F}^{\mu\nu}] = \mathcal{J}^\nu$ and $[\mathcal{D}_\mu, \mathcal{J}^\mu] = 0$ at the same time.

... But not that easy



- In general, no **MV-like** solutions exist
- What are the colour currents J^μ ?
- Where is the boundary and what is the boundary condition?

Recall

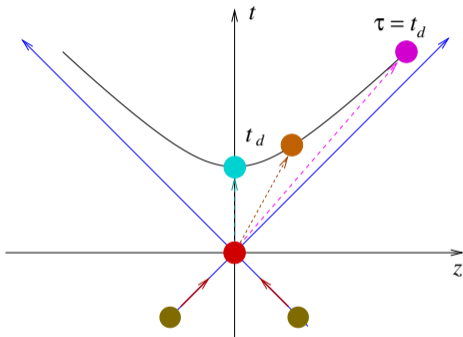


Projectile nucleus

- The MV model contains two separate concepts:
 - Infinite momentum (Boost invariant) EoM
 - Finite saturation scale $Q_s \propto \sqrt{s}^{\lambda/2}$
- It is a model for finite η_{beam} dynamics which uses the infinite momentum frame evolution as an approximation for the mid-rapidity dynamics

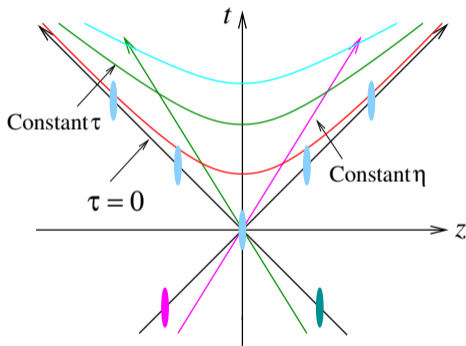
Lightcone coordinates: $x^\pm = (t \pm z)/\sqrt{2}$

Initial condition in τ



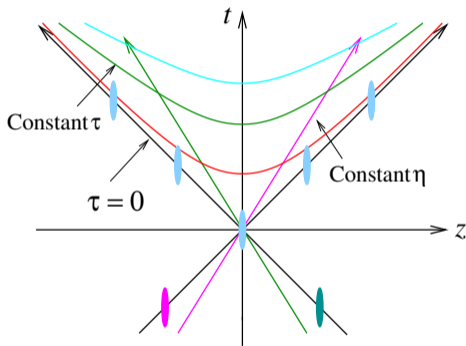
- If the three fireballs all start out at $t = 0, z = 0$ and evolve exactly the same way (e.g. thermalization), the state of the cyan at $t = t_d$ is the same as the state of the brown and magenta at $\tau = t_d$ due to *time dilation*
- If $\gamma = \infty$, then the initial state is infinitely thin
 \implies Longitudinal distribution must be uniform
- If $\gamma < \infty$, then the initial state has a finite width
 \implies Longitudinal distribution does not need to be uniform

What we are doing



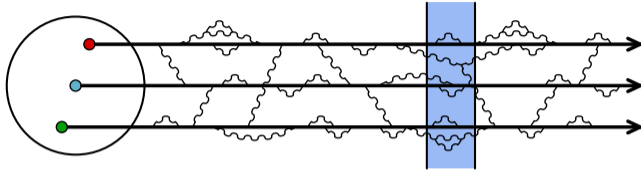
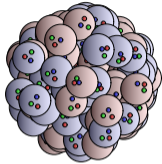
- The usual MV-model applies at mid-rapidity where two approaching nuclei have the same speed $v = \tanh(\eta_{\text{beam}})$.
 - Initial Glasma field is given by $\mathcal{A}_i = \mathcal{A}_i^P + \mathcal{A}_i^T$ where \mathcal{A}_i^P and \mathcal{A}_i^T are generated by the colour charge densities *observed in the CM frame* or $\eta_s = 0$. $Q_s^P = Q_s^T = Q_s^{CM}$
- Ask: How does the collision look like in a moving frame with the velocity $v = \tanh(\eta_s)$?
 - If $y_{\text{beam}} < \infty$, then the projectile is moving with $\gamma_P = \cosh(y_{\text{beam}} - \eta_s) < \cosh(y_{\text{beam}})$ and the target is moving with $\gamma_T = \cosh(y_{\text{beam}} + \eta_s)$
 - Can use the IP-Sat model to calculate $Q_s^P < Q_s^{CM}$ and $Q_s^T > Q_s^{CM}$

What we are doing

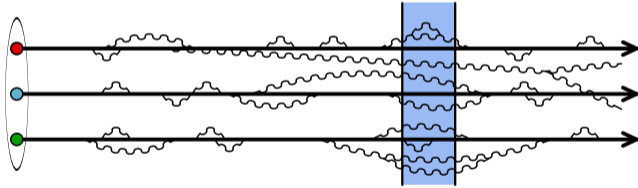
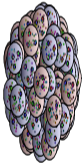


- The usual MV-model applies at mid-rapidity where two approaching nuclei have the same speed $v = \tanh(\eta_{\text{beam}})$.
 - Initial Glasma field is given by $\mathcal{A}_i = \mathcal{A}_i^P + \mathcal{A}_i^T$ where \mathcal{A}_i^P and \mathcal{A}_i^T are generated by the colour charge densities *observed in the CM frame* or $\eta_s = 0$.
 - Ask: How does the collision look like in a moving frame with the velocity $v = \tanh(\eta_s)$?
 - If $y_{\text{beam}} < \infty$, $\mathcal{A}_i = \mathcal{A}_i^P + \mathcal{A}_i^T$ where \mathcal{A}_i^P and \mathcal{A}_i^T are generated by the colour charge densities *observed in the moving frame with the rapidity η_s* . \implies JIMWLK
- [Phys. Rev. C94, 044907 (2016), Schenke and Schlichting]

JIMWLK Evolution



Small γ



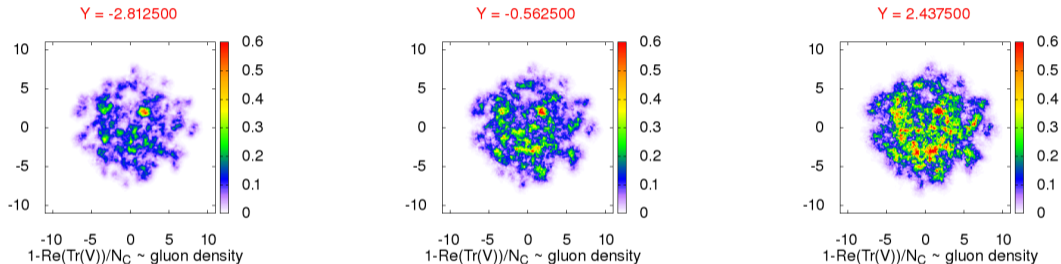
Large γ

- Time dilation: We see more denser “*real gluons*” as $\gamma = \cosh(\eta)$ increases

[Figures from Int. J. Mod. Phys. A, Vol. 28, No. 01, 1330001 (2013), F. Gelis]

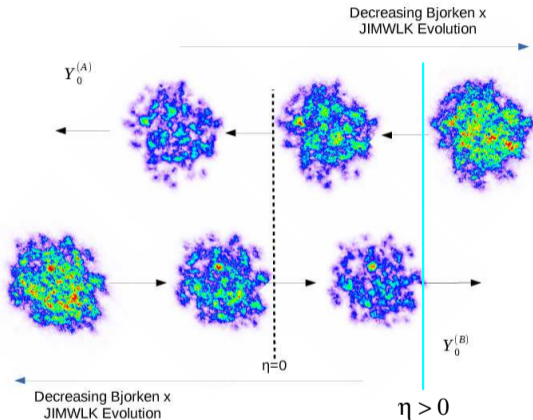
Target nucleus moving in the negative z direction with $-\eta_{\text{beam}}$

[Using the method by Lappi and Mäntysaari in Eur. Phys. J. C73 (2013) 2307]



- In the frame moving in the same direction as the target nucleus, the target nucleus looks sparser
- In the frame moving in the opposite direction to the target nucleus, the target nucleus looks denser

Conceptually



- JIMWLK: How the gluon density appears in a moving frame
- Finite $\eta_s > 0 \implies$ Moving frame with $v^z = \tanh \eta_s$
- The target appears much denser than the projectile \implies Gives the initial condition at η_s and at $\tau = 0^+$.
- Longitudinal decorrelation is built in.

CGC & JIMWLK: Work by Venugopalan, McLerran, Jalilian-Marian, Iancu, Weigert, Leonidov, Kovner, Kovchegov, Dumitru, Gelis, Blaizot, Kharzeev, Nardi, Levin, Krasnitz, Nara, Lappi, Mäntysaari and many others

Finding the 3D Initial conditions

Goals

- Stay as close to the 2D initial conditions as possible
- Energy deposition only when there is overlap

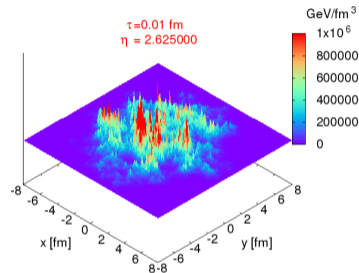
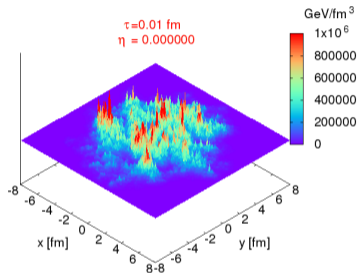
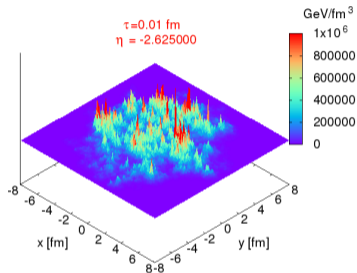
2D Initial conditions

- $\mathcal{A}_i^{P,T} = (i/g) V_{P,T} \partial_i V_{P,T}^\dagger$
- $\mathcal{A}_i = \mathcal{A}_i^P + \mathcal{A}_i^T$
- $\mathcal{E}^\eta = ig[\mathcal{A}_i^P, \mathcal{A}_i^T]$
- $\mathcal{E}^i = 0, \mathcal{A}_\eta = 0$

3D Initial conditions

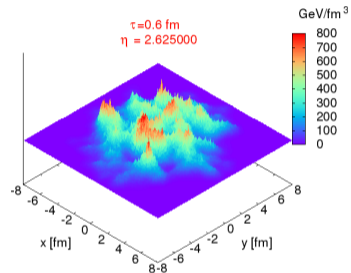
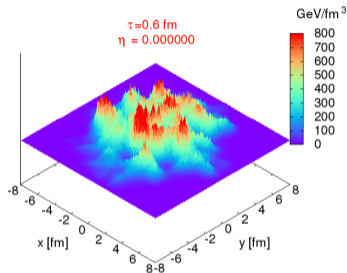
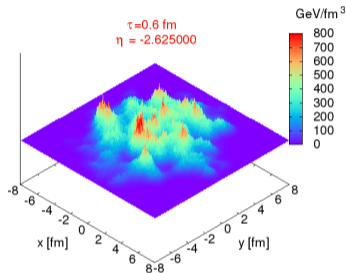
- $\mathcal{A}_i^{P,T} = (i/g) V_{P,T} \partial_i V_{P,T}^\dagger$
- $\mathcal{A}_\eta^{P,T} = (i/g) V_{P,T} \partial_\eta V_{P,T}^\dagger$
- $\mathcal{A}_i(\eta_s) = \mathcal{A}_i^P(\eta_s) + \mathcal{A}_i^T(\eta_s)$
- $\mathcal{A}_\eta(\eta_s) = \mathcal{A}_\eta^P(\eta_s) + \mathcal{A}_\eta^T(\eta_s)$
- $\mathcal{E}^\eta(\eta_s) = ig[\mathcal{A}_i^P(\eta_s), \mathcal{A}_i^T(\eta_s)]$
- $[\mathcal{D}_\eta, \mathcal{E}^\eta] + [\mathcal{D}_i, \mathcal{E}^i] = 0$

Initial energy distribution



- $\sqrt{s_{NN}} = 2.76$ TeV
- This is within the “plateau”

Energy distribution after YM evolution



- $\sqrt{s_{NN}} = 2.76$ TeV
- This is within the “plateau”

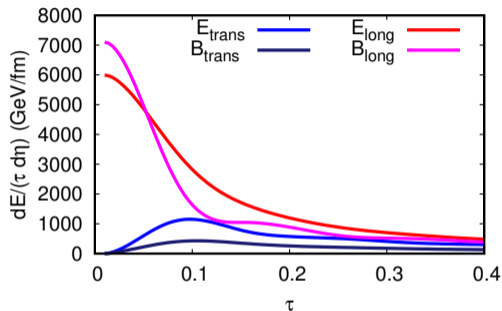
3D-Glasma Results

A bit of technical detail

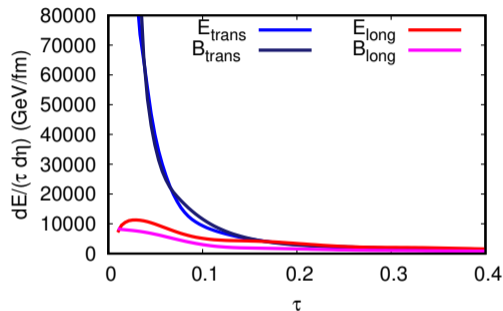
- New implementation of 3D SU(3) real-time CYM in $\tau, \eta, \mathbf{x}_\perp$
- Fully in-house code
- Time-evolution method: Leap-frog
- Gauss law solver: non-Abelian Jacobi method
- Running coupling JIMWLK following Lappi and Mäntysaari
- Initial y for JIMWLK: ± 4.25
- Hydro: MUSIC in 3+1D mode
- Hadronic afterburner: UrQMD
- Going 3D also means two orders of magnitude more compute time...
- More statistics and more centralities coming soon

Field Evolution

Energy in Fields in (2+1)-D



Energy in Fields in (3+1)-D

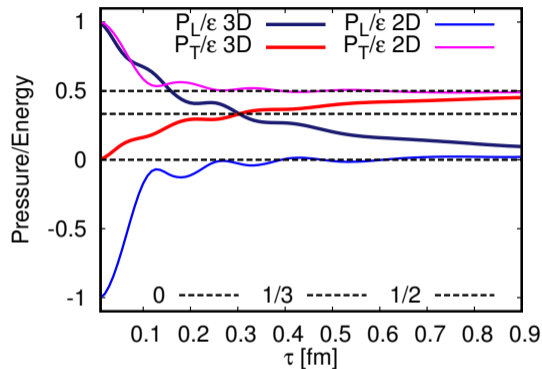
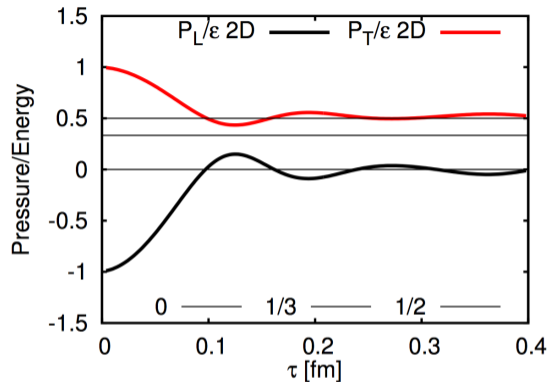


- Note the scale – 3D initial energy is *much* higher

- This is because
$$E = \int d\eta d^2x_{\perp} \tau \left(\frac{1}{2} \left((\mathcal{E}^{\eta})^2 + (\mathcal{B}^{\eta})^2 \right) + \frac{1}{2\tau^2} \left(\mathbf{E}_{\perp}^2 + \mathbf{B}_{\perp}^2 \right) \right)$$

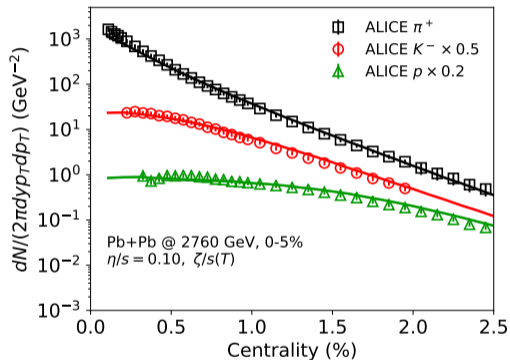
- In 3D, one *cannot* set $\mathbf{E}_{\perp} = 0$ and $\mathbf{B}_{\perp} = 0$

Pressure Evolution

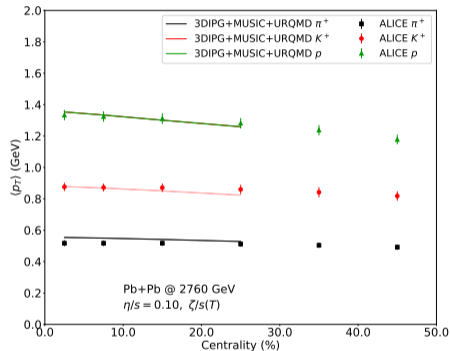


- In 2D, $P_L = \epsilon_\eta$ and $P_L = -\epsilon_\eta$ at τ_0
- In 3D, $P_L \approx \epsilon_x + \epsilon_y$ and $P_L \approx \epsilon_x - \epsilon_y$ at τ_0
- Note the crossing at the isotropic point $P_T = P_L = 1/3$
- Large τ behaviours are similar

Transverse dynamics

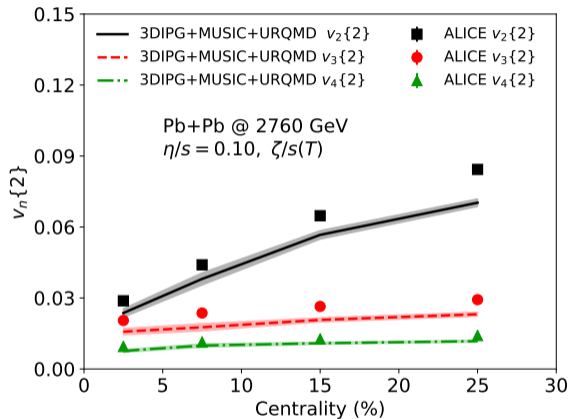


Spectra OK

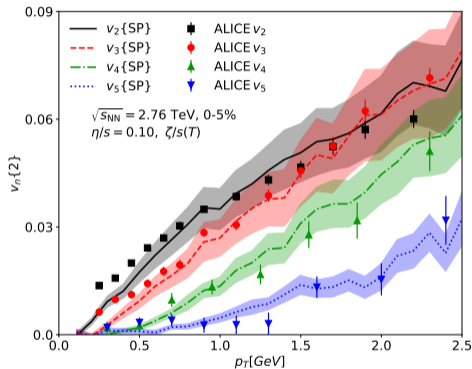


Mean p_T OK

Transverse dynamics



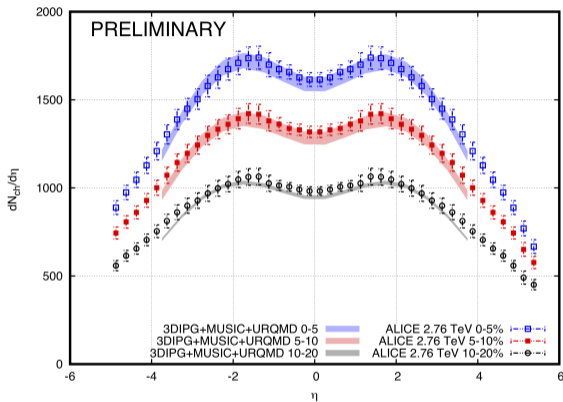
Integrated v_n



Differential v_n

- Needs a small bit of tweaking. For instance the value of η/s – Getting there

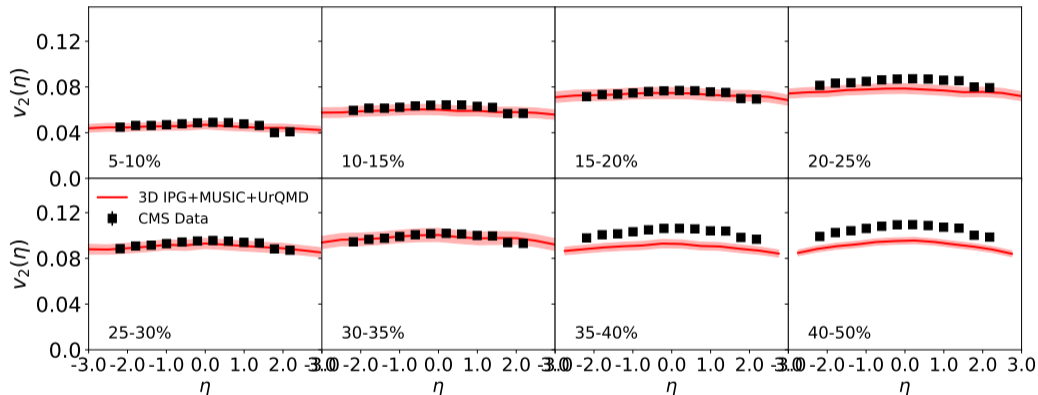
Longitudinal dynamics



- Rapidity distribution
- Global longitudinal dynamics is being captured

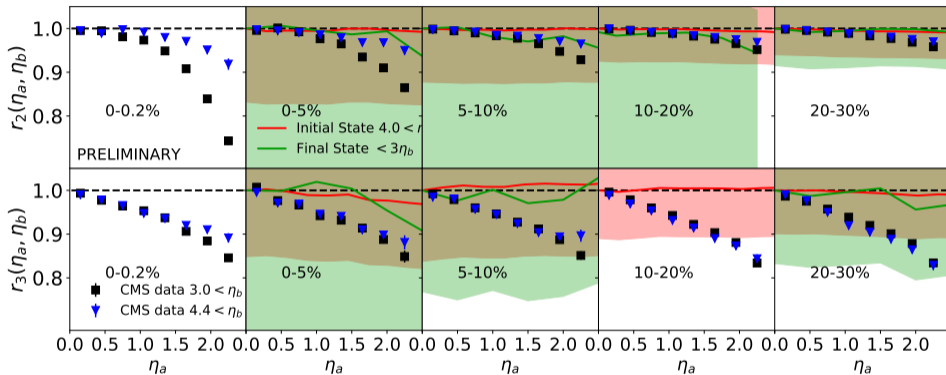
Initial hydro condition beyond $y = \pm 4.25$: Smooth fall-off

Longitudinal dynamics



● $v_2(\eta)$ OK

New Results – Longitudinal dynamics



- Lower centrality: Fluctuation driven
- Higher centrality: Geometry driven

- v_3 : Too correlated at the moment –
Need more statistics



Non-exhaustive list of 3D models

- Phys. Rev. D 74, 045011 (2006), Romatschke and Venugopalan: 2D initial condition plus η_s dependent factorized random noise.
- Phys. Rev. Lett. 111, 232301 (2013), Epelbaum and Gelis: 2D initial condition plus random initial field for the quantum fluctuations.
- Phys. Rev. C 89, 034902 (2014), Ozonder and Fries: Based on Lam and Mahlon: 2D-like initial condition with boosted Coulomb field for the η_s dependence.
- Phys. Rev. D 94, 014020 (2016), Gelfand, Ipp and Müller: 2D MV model performed in (t, z) . The sources move with $v = \pm c$. Spatial geometry provides the η_s dependence.
- Phys. Rev. C 94, 044907 (2016), Schenke and Schlichting: Uses JIMWLK for the 3D structure. 2D initial conditions & 2D evolution for each η_s slice.
- Nucl. Phys. A **1005**, 121771 (2021), McDonald, Jeon and Gale: Uses JIMWLK for the 3D structure. 2D initial conditions & Full 3D evolution.

Summary & Perspectives

- Saturation physics provides good picture of initial interactions
- Going 3D is non-trivial but doable
- Good description of 3D physics possible
- A lot of physics to learn: Saturation physics, JIMWLK evolution, ...
- Update coming soon

Blast from the past



Happy Birthday, Xin-Nian!