The current status of Higher Twist, JET and JETSCAPE

Abhijit Majumder

Berkeley Symposium on hard probes and beyond August 2022.

We are going to talk about this paper

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Multiple parton scattering in nuclei: parton energy loss

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Abstract

Multiple parton scattering and induced parton energy loss are studied in deeply inelastic scattering (DIS) off nuclei. The effect of multiple scattering of a highly off-shell quark and the induced parton energy loss is expressed in terms of the modification to the quark fragmentation functions. We derive such modified quark fragmentation functions and their QCD evolution equations in DIS using the generalized factorization of higher twist parton distributions. We consider double-hard and hard-soft parton scattering as well as their interferences in the same framework. The final result, which depends on both the diagonal and off-diagonal twist-four parton distributions in nuclei, demonstrates clearly the Landau–Pomeranchuk–Migdal interference features and predicts a unique nuclear modification of the quark fragmentation functions.

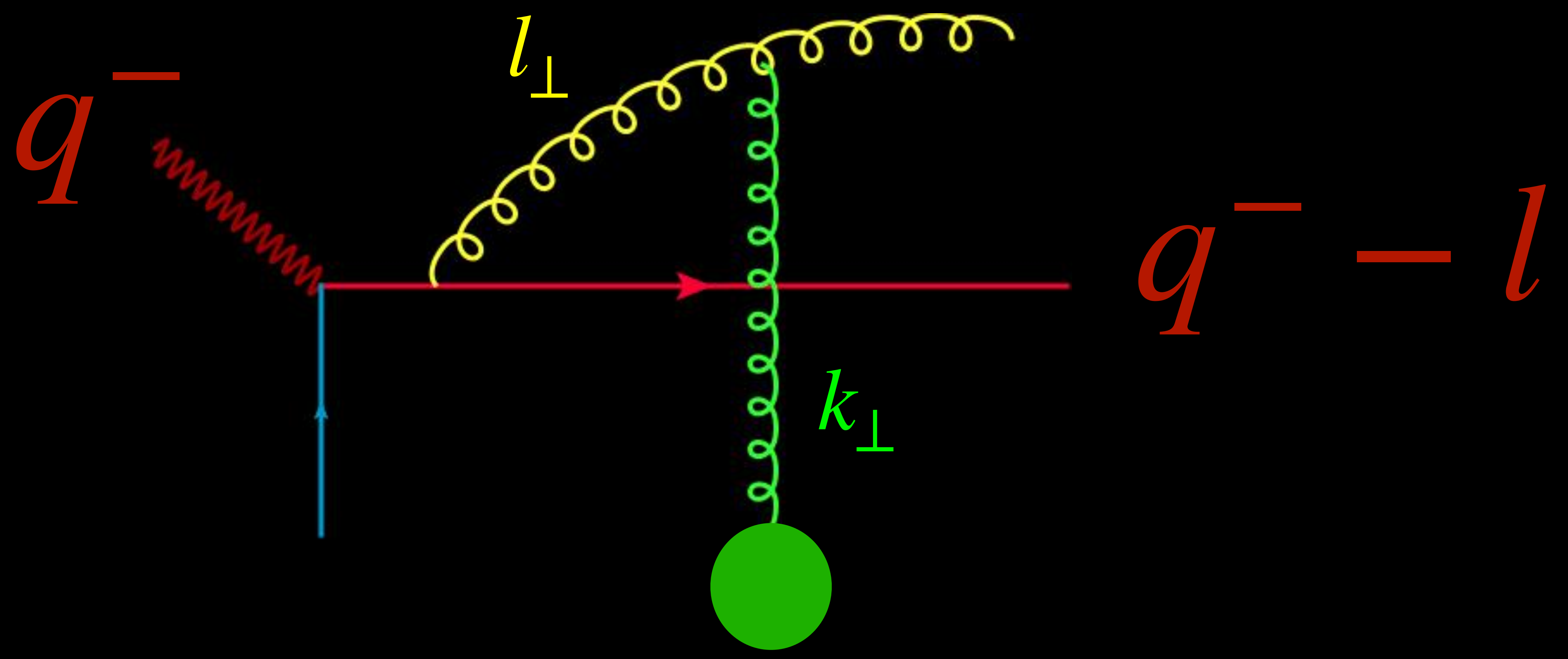
PACS: 24.85.+p; 12.38.Bx; 13.87.Ce; 13.60.-r

$$\frac{W_{\mu\nu}^{D,q}}{dz_h} = \sum_q \int dx\, H_{\mu\nu}^{(0)}(xp,q) \int_{z_h}^{1} \frac{dz}{z} D_{q\to h}(z_h/z) \frac{\alpha_s}{2\pi} C_A \frac{1+z^2}{1-z}$$
$$\times \int \frac{d\ell_T^2}{\ell_T^4} \frac{2\pi\alpha_s}{N_c} T_{qg}^A(x,x_L) + (\text{virtual correction}),$$

$$T_{qg}^A(x,x_L) = \int \frac{dy^-}{2\pi} dy_1^- dy_2^- e^{i(x+x_L)p^+y^-}\left(1-e^{-ix_Lp^+y_2^-}\right)\left(1-e^{-ix_Lp^+(y^- - y_1^-)}\right)$$
$$\times \frac{1}{2}\langle A|\bar{\psi}_q(0)\gamma^+ F_\sigma{}^+(y_2^-)F^{+\sigma}(y_1^-)\psi_q(y^-)|A\rangle \theta(-y_2^-)\theta(y_2^- - y_1^-)$$

- Seminal Higher Twist paper
- Question: where is $\hat{q}$?
- More on that in a bit…

What is meant by the higher twist formalism: strict interpretation



$$q^- = \frac{q^0 - q_z}{\sqrt{2}}, \quad q_z < 0$$

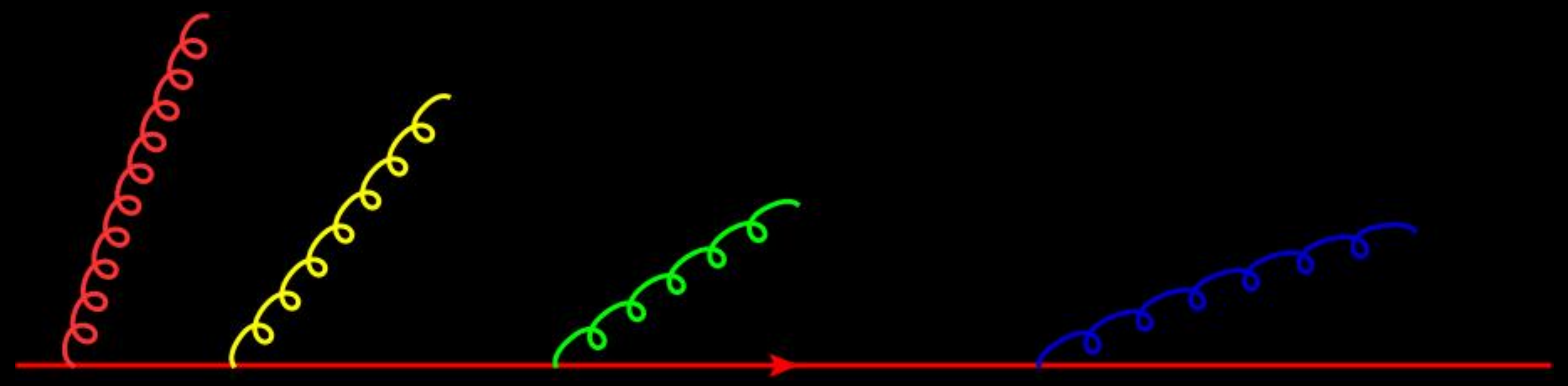
- Power Expansion in $\dfrac{\hat{q}\tau}{l_\perp^2} = \dfrac{\langle k_\perp^2 \rangle}{l_\perp^2} = \dfrac{\int_{l_\perp^2 - \Delta}^{l_\perp^2 + \Delta} dk_\perp^2 \; k_\perp^2 \; \frac{dP(k_\perp^2)}{dk_\perp^2}}{l_\perp^2}$

- Retain first correction

- Small correction to vacuum shower,
  allows the use of vacuum like ordering in multiple emissions.

Multiple Vacuum Emissions in a MC

- Strong ordering in gluon transverse momentum $l_\perp$



- Leads to strong ordering in formation times

$$\tau_f^- = \frac{2y(1-y)q^-}{l_\perp^2}$$

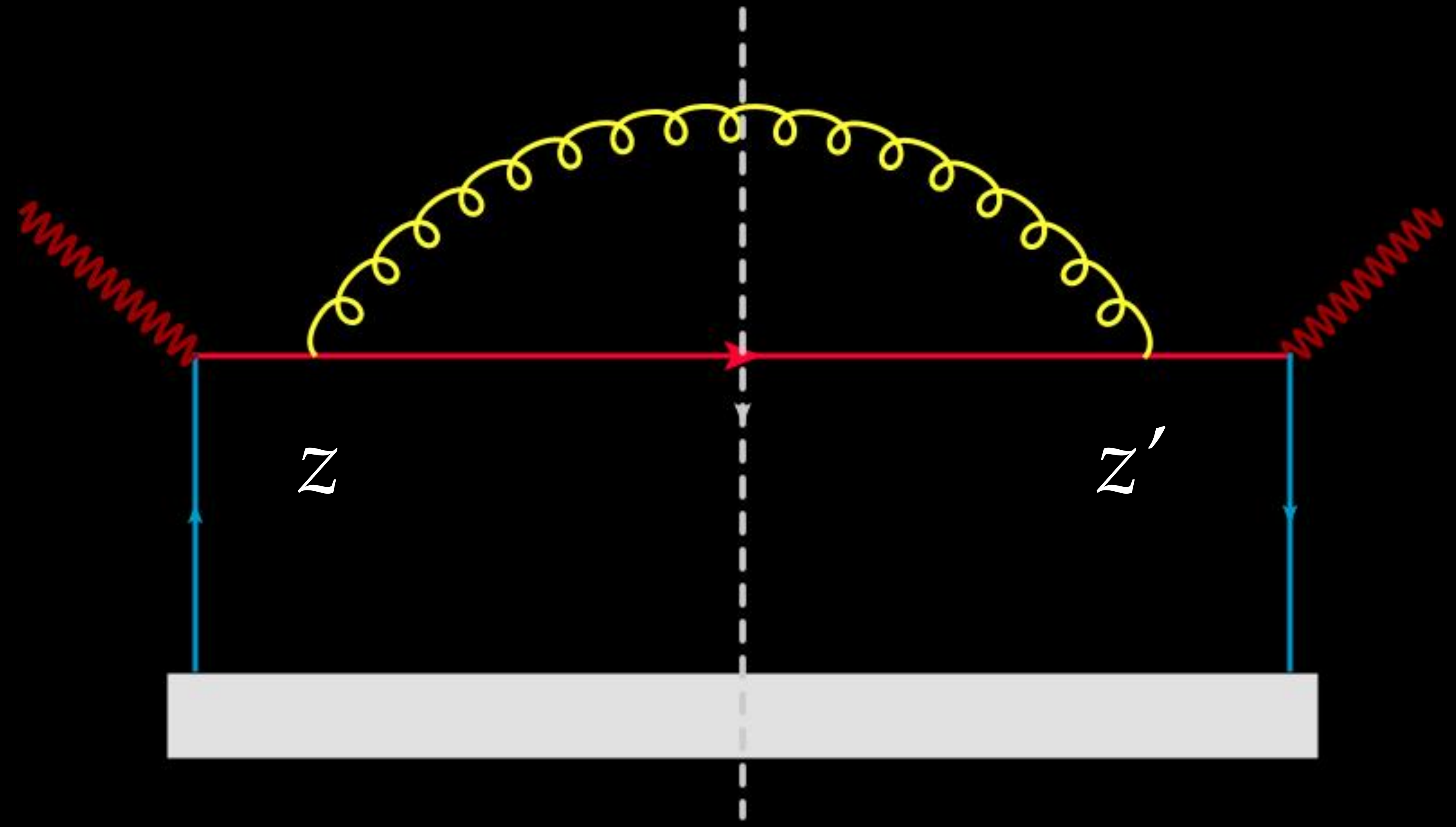
- Simulate as each emission happens strictly within τ_f^-

- Or introduce an uncertainty in $q^+ = \delta q^+$ conjugate to z^- (such that $\langle z^- \rangle \simeq \tau^-$), leads to fluctuations in formation time.

Wave-function for amplitude $\psi(q) e^{iq^- z^+} e^{iq^+ z^-} e^{-iq_\perp z_\perp}$

Phase factors for $\mathcal{M}^* \mathcal{M}$

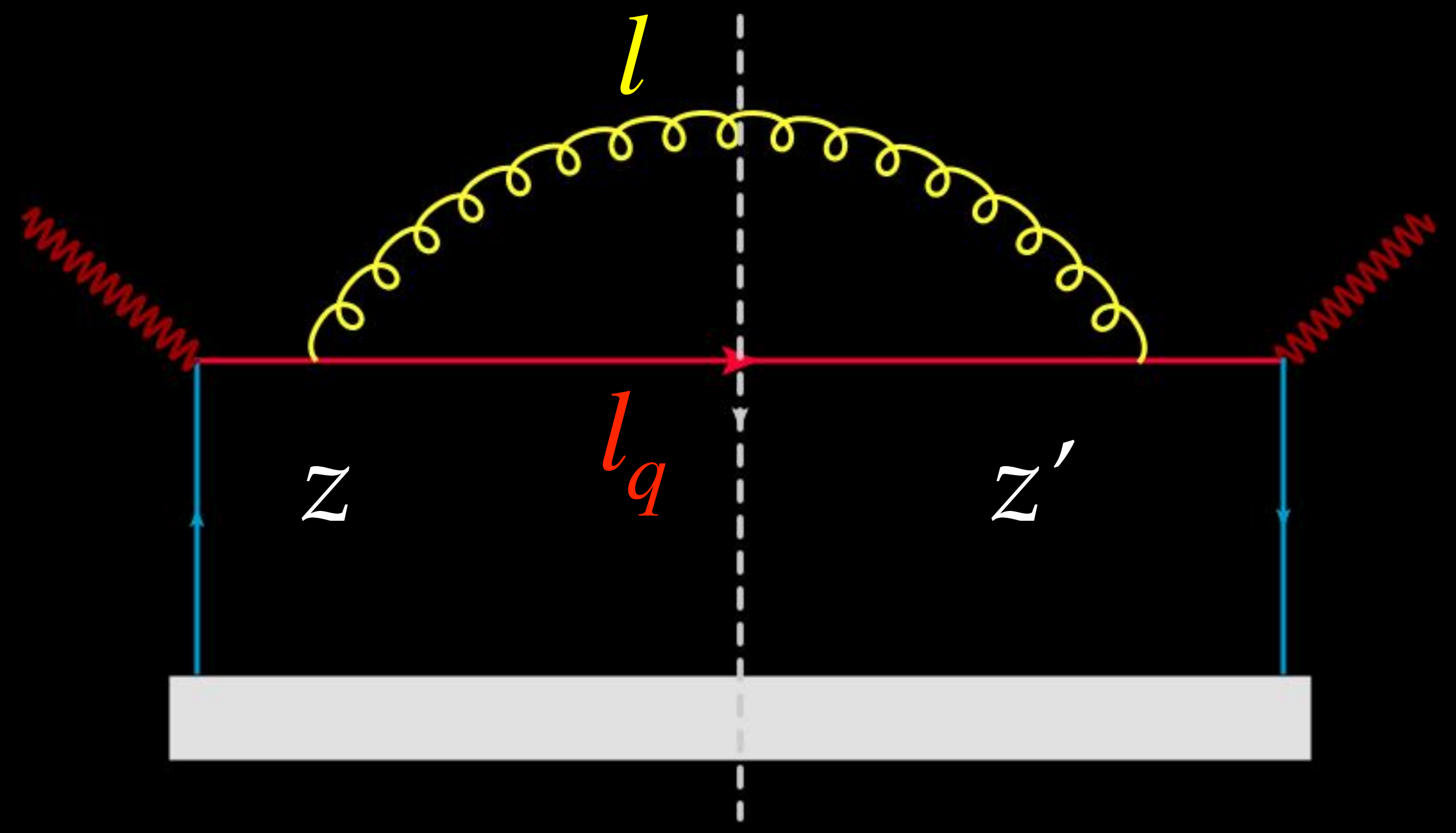
$$[e^{iq^- z^+} e^{iq^+ z^-} e^{-iq_\perp z_\perp}][e^{-iq'^- z'^+} e^{-iq'^+ z'^-} e^{ik'_\perp z'_\perp}]$$



Picking a distribution function for q^+

$$\int_0^\infty d^4\bar{z} \exp\left[i(\delta q)\bar{z}\right] \qquad \bar{z} = \frac{z+z'}{2}$$

$$\int d^4\delta z \exp\left[i\delta z(l + l_q - q)\right] \qquad \delta z = z - z'$$



$$\rho(\delta q^+) = \psi^*\psi = Ae^{-(\delta q^+)^2/\sigma^2}$$

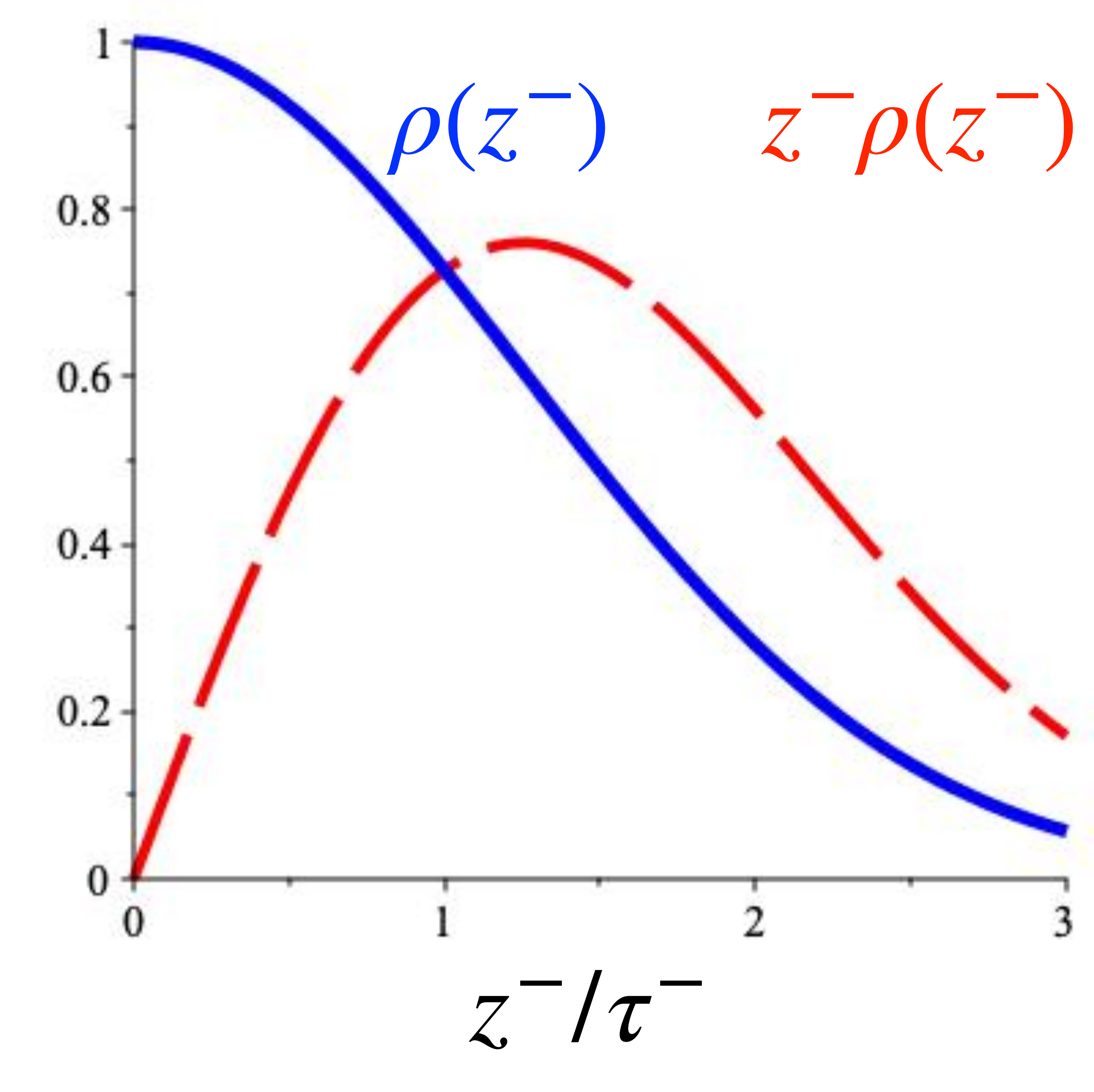
On F.T. gives

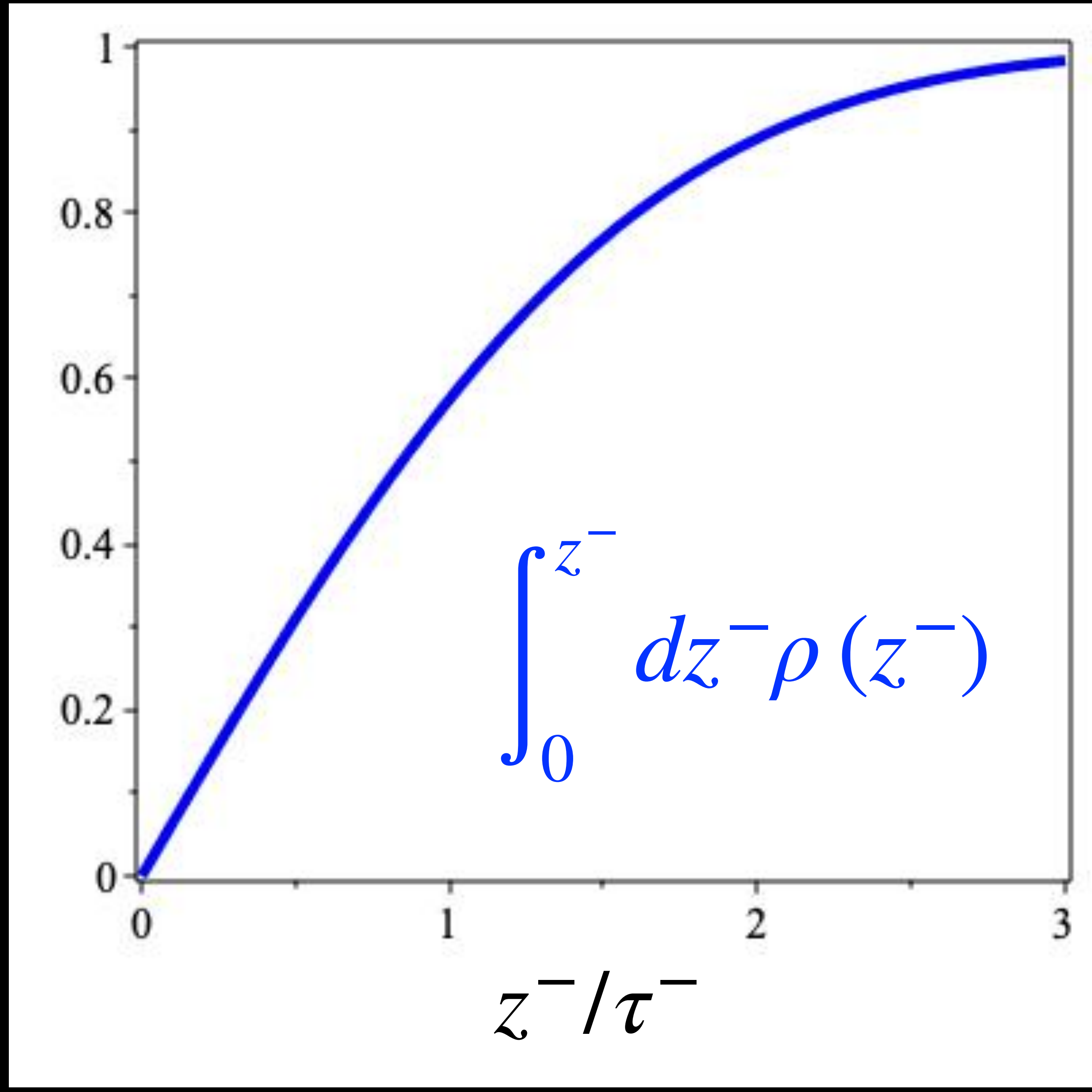
$\rho(z^-) = Be^{-\sigma^2(z^-)^2}$

Adjust σ such that

$$\frac{\int dz^- z^- \rho(z^-)}{\int dz^- \rho(z^-)} = \tau^-$$

require $\langle \delta q^+ \rangle \lesssim \langle q^+ \rangle$





By $z \simeq 3\tau$ 99% of all the radiation has taken place.

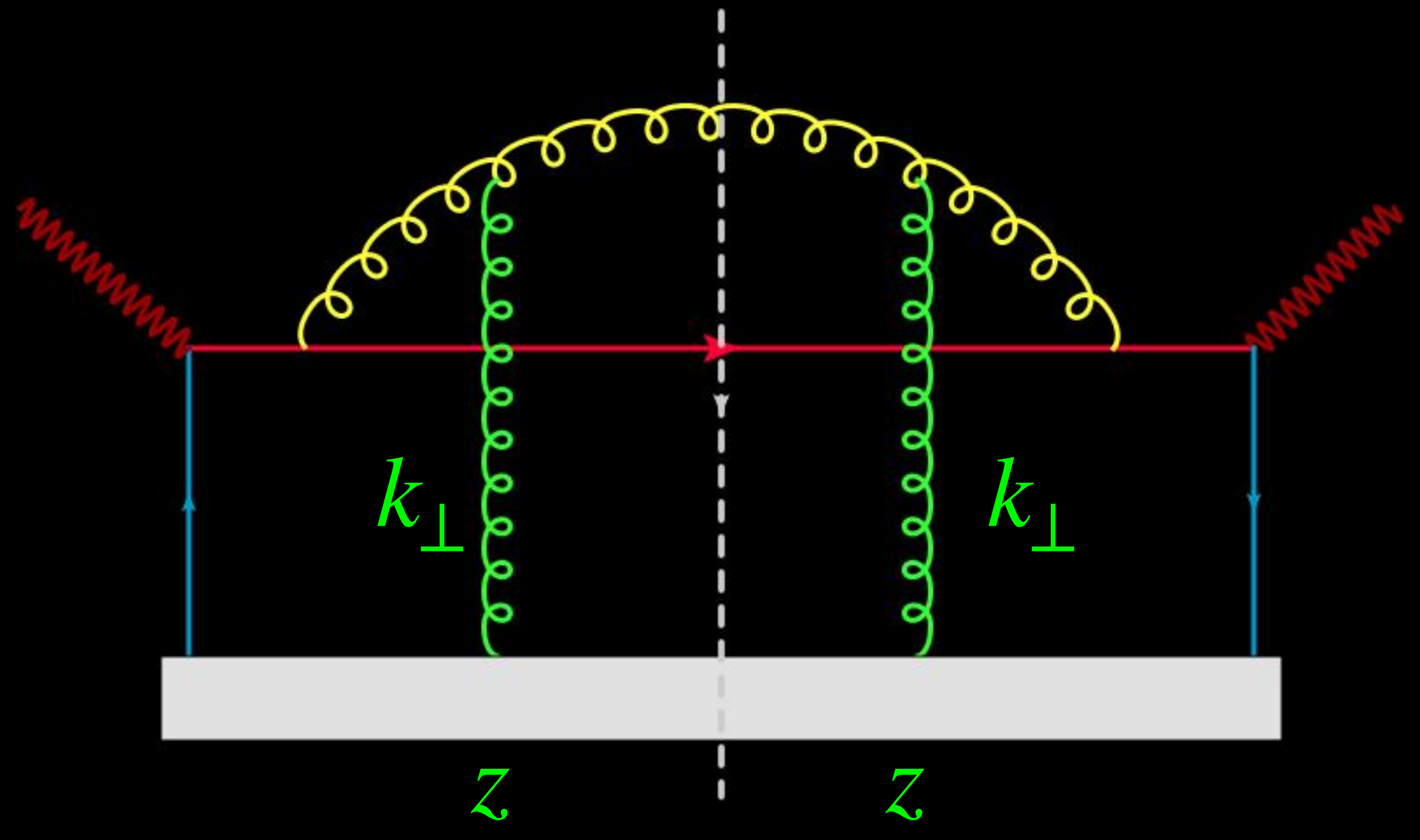
All plots will be made with $z \leq 3\tau$

A.M. Phys.Rev. C88 (2013) 014909

General form for all terms with 1 rescattering

$$d\sigma = \sigma_0 \times \int dz dk_\perp^2 \,|\, \mathcal{M}_i^*(z, k_\perp) \mathcal{M}_j(z, k_\perp) \,|\, e^{-i\Gamma_i(z,k_\perp)+i\Gamma_j(z,k_\perp)}$$

- Guo-Wang prescription: drop $k_\perp$ dependence in $\Gamma(z, k_\perp)$, expand $|\mathcal{M}^* \mathcal{M}|$ as a series in $k_\perp$



$$\frac{dN_g}{dy} = \frac{\alpha_S}{2\pi} P(y) \int \frac{dl_\perp^2}{l_\perp^2} \int dz^- \frac{\hat{q}(z^-)}{l_\perp^2} \left[2 - 2\cos\left(\frac{l_\perp^2 z^-}{2q^- y(1-y)} \right) \right]$$

z is mean location of scattering, $k_\perp$ is the transverse momentum from the medium

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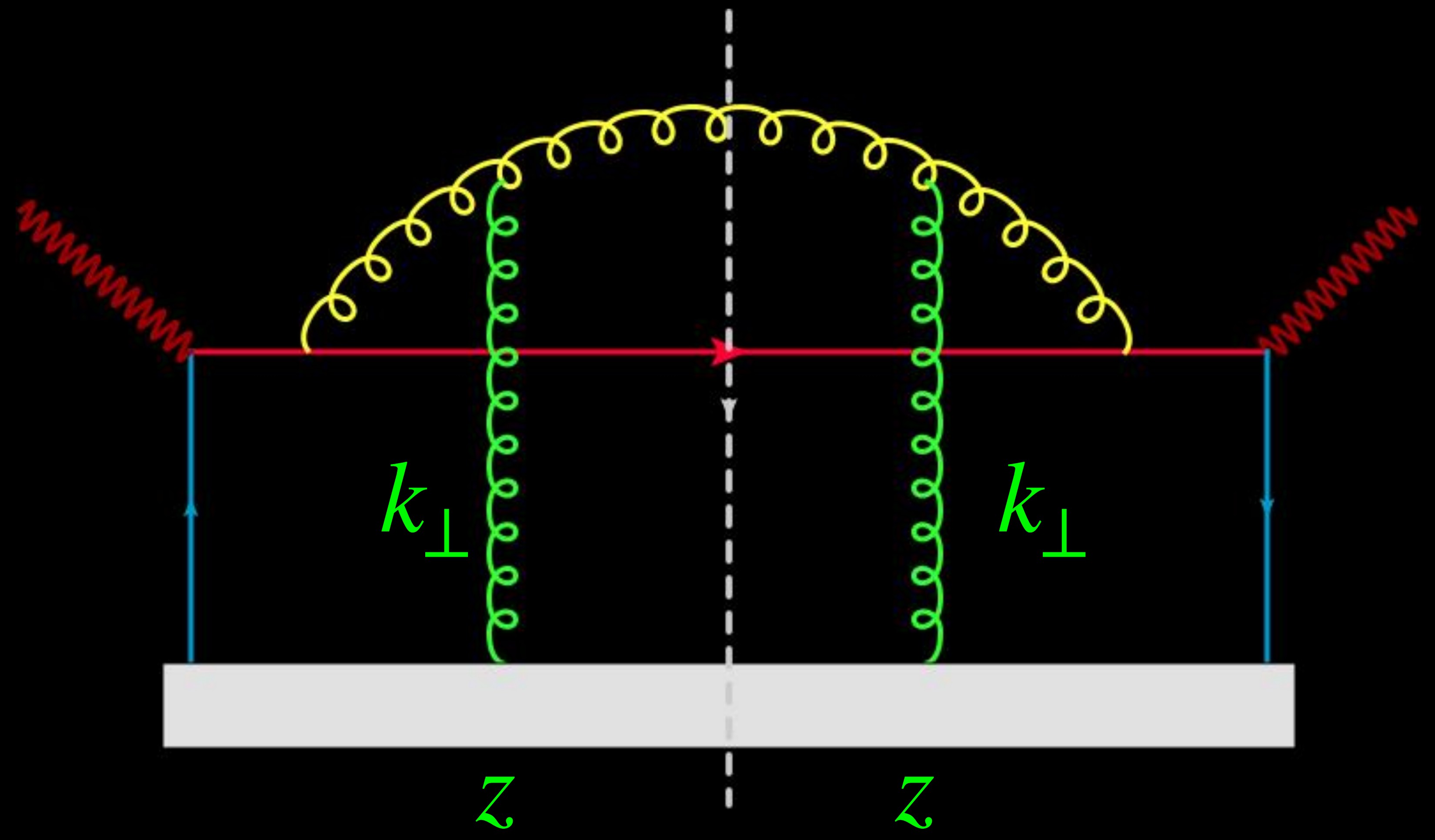

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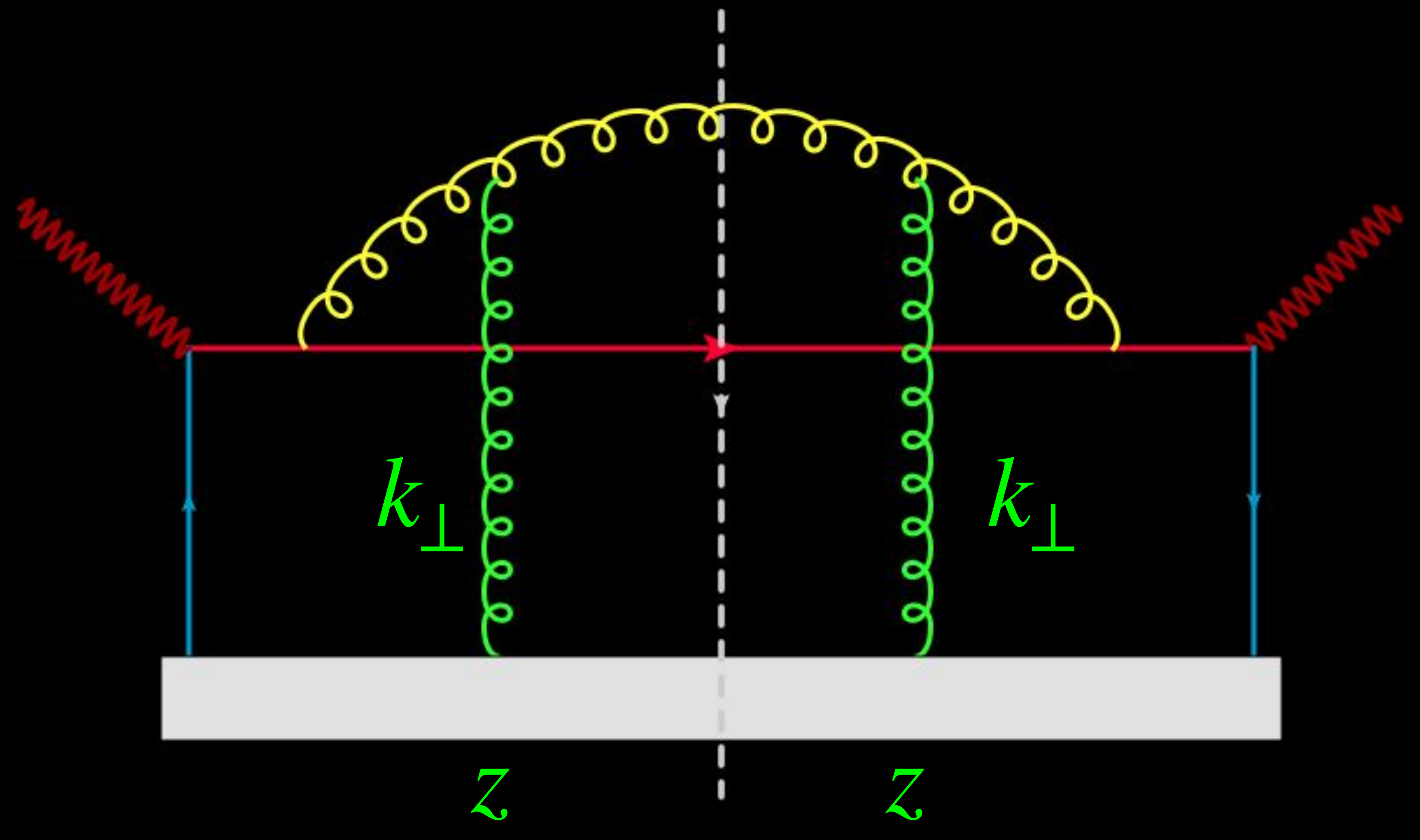

Derived in 15 min, at HP2006

$$\frac{dN_g}{dy} = \frac{\alpha_S}{2\pi} P(y) \int \frac{dl_\perp^2}{l_\perp^2} \int dz^- \frac{\hat{q}(z^-)}{l_\perp^2} \left[2 - 2\cos\left(\frac{l_\perp^2 z^-}{2q^- y(1-y)} \right) \right]$$

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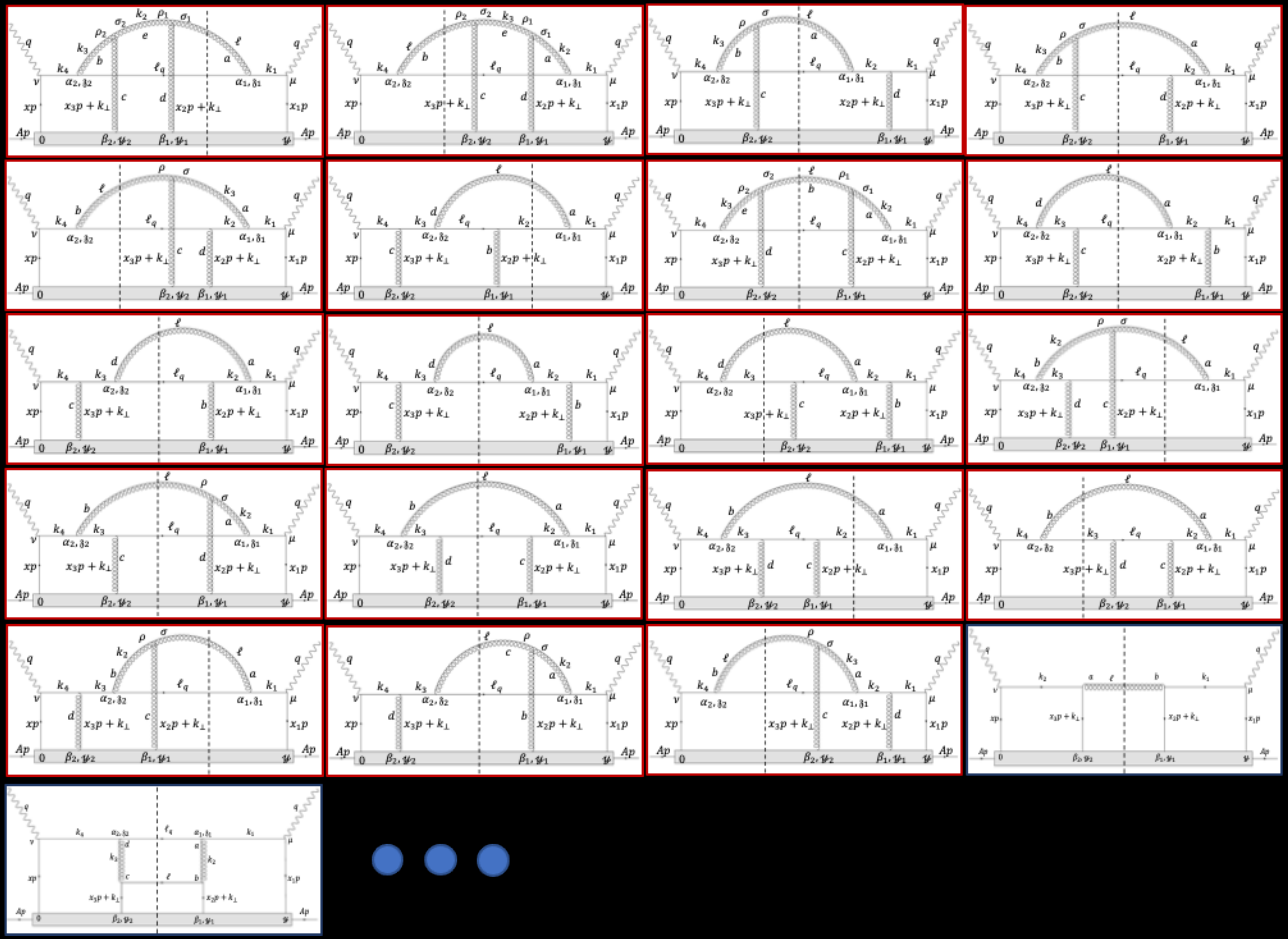
Guo and Wang's original approximation



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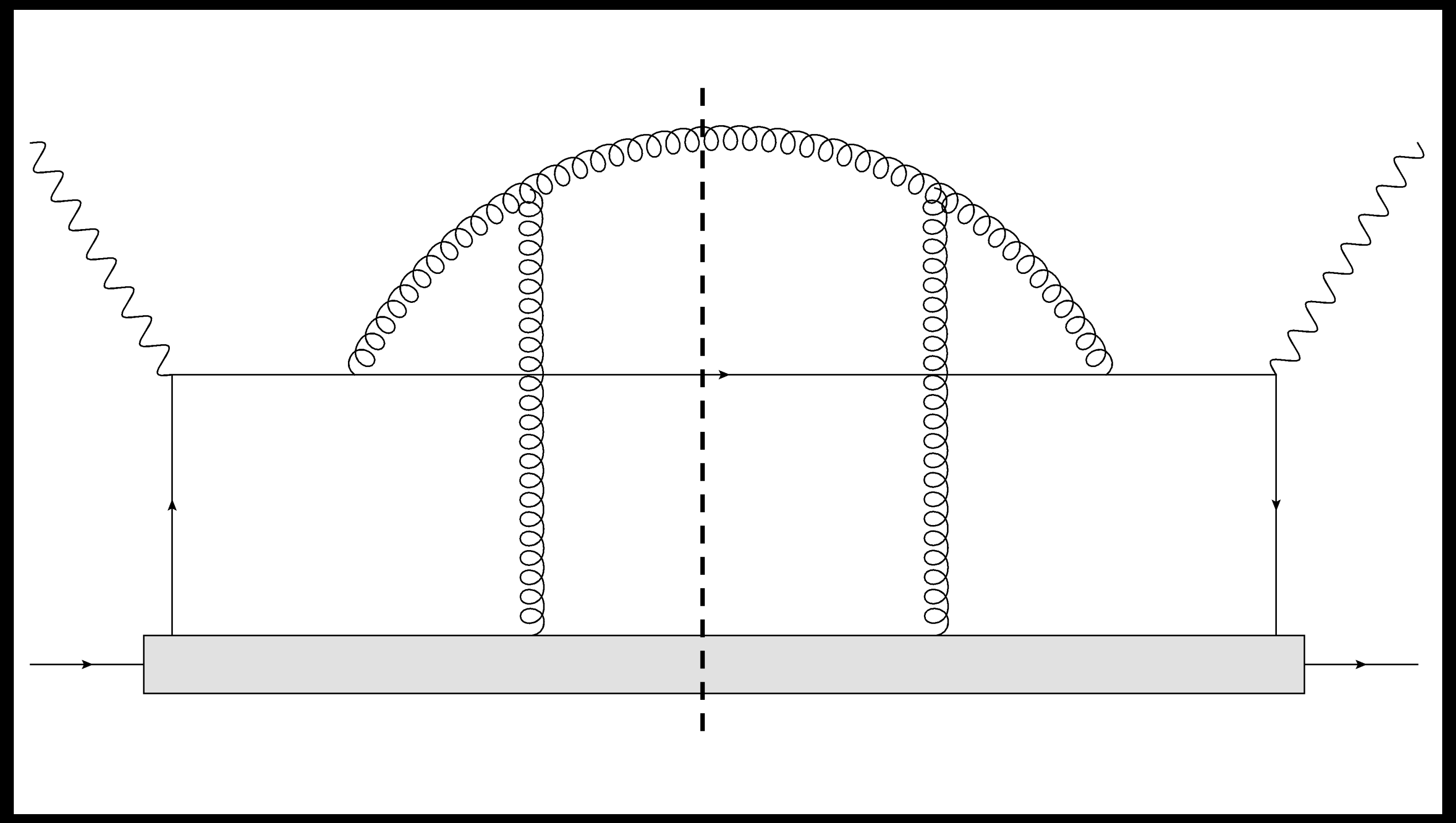


Assuming a static medium with a constant $\hat{q}$

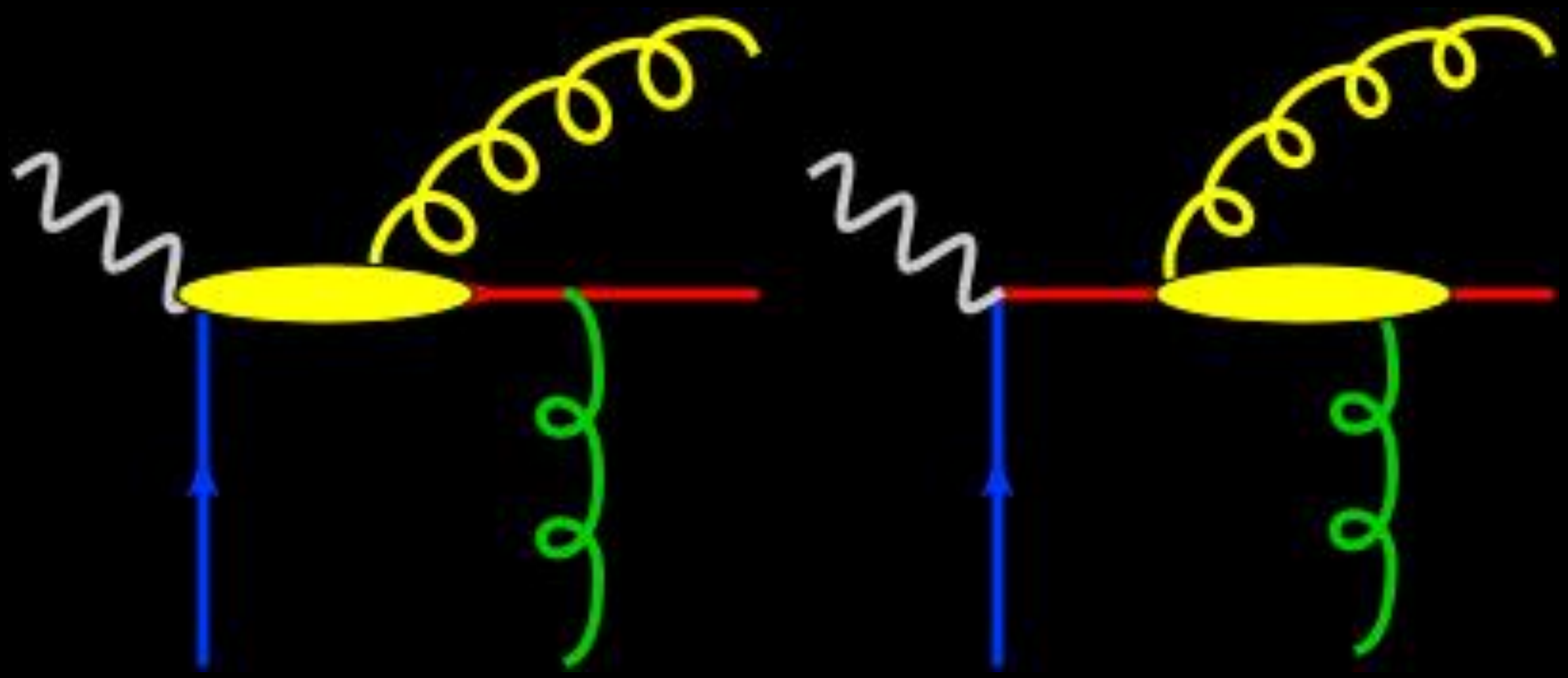
Meaning of this factor:

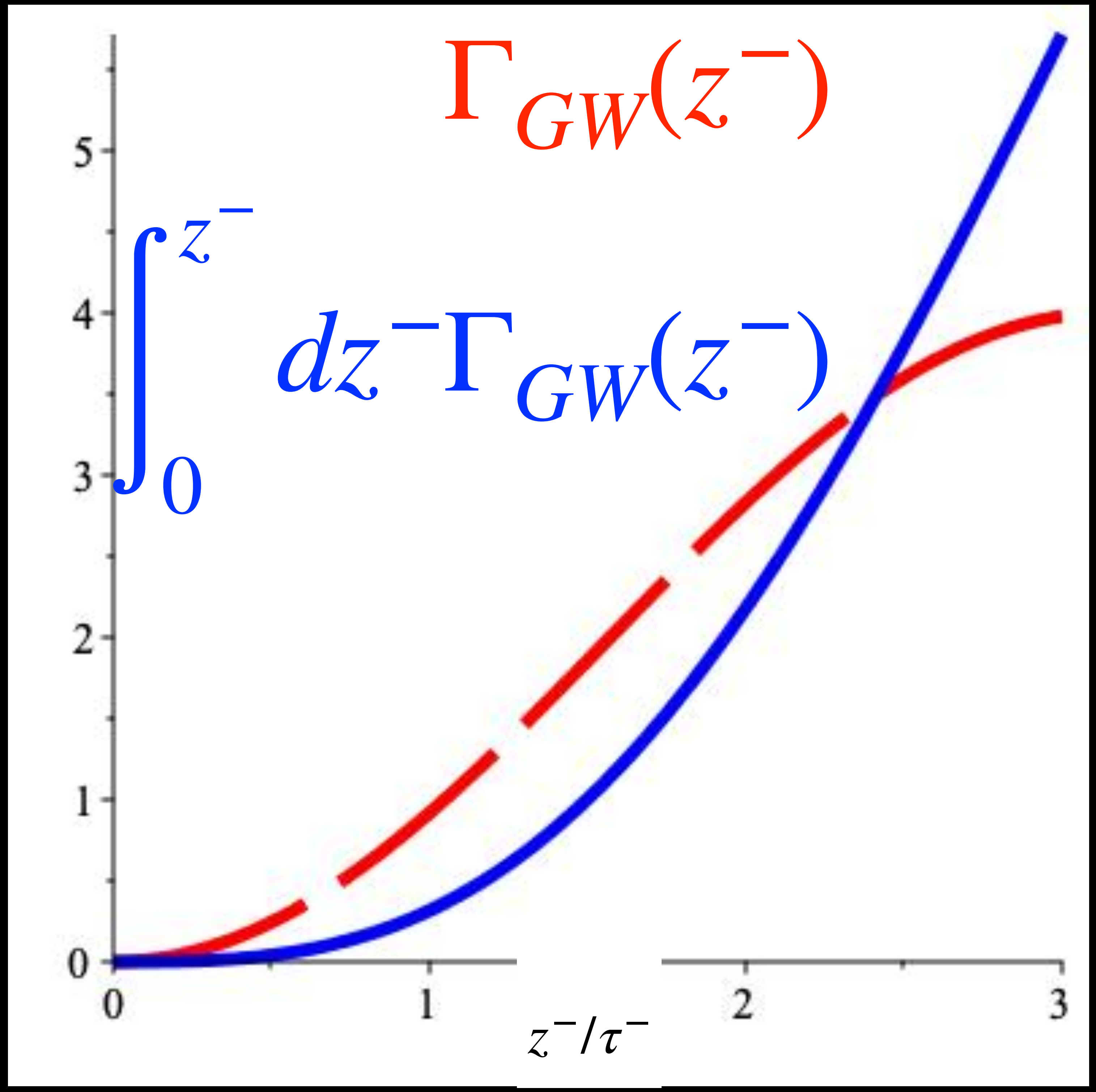
$$\tau^- = \frac{2q^- y(1-y)}{l_\perp^2} = \frac{2q^-}{\mu^2}$$

$$\Gamma(z^-) = 2 - 2\cos(z^-/\tau^-)$$

Interference means the two yellow blobs have to overlap

Given vacuum analysis $t^- \lesssim 3\tau^-$

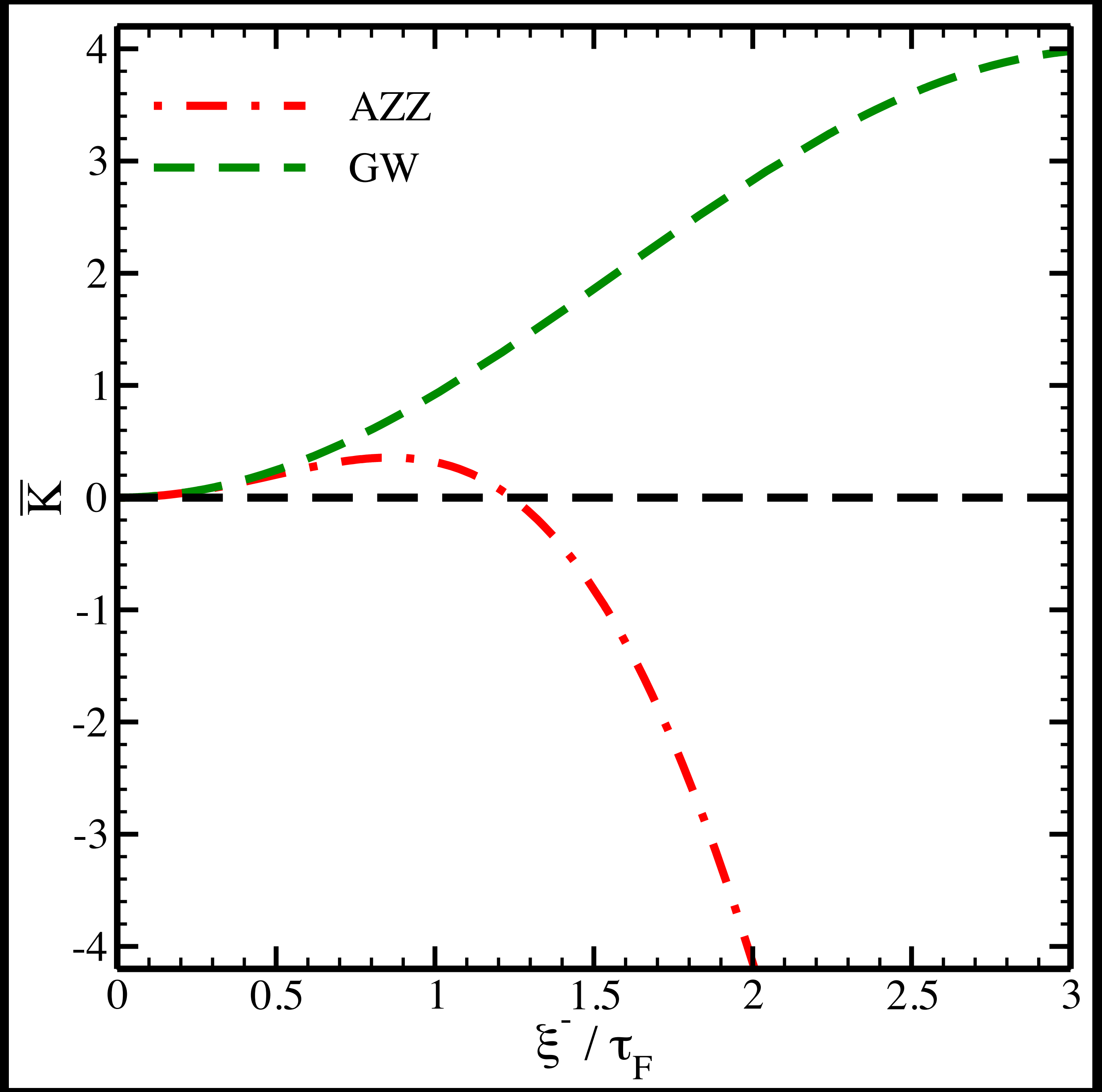


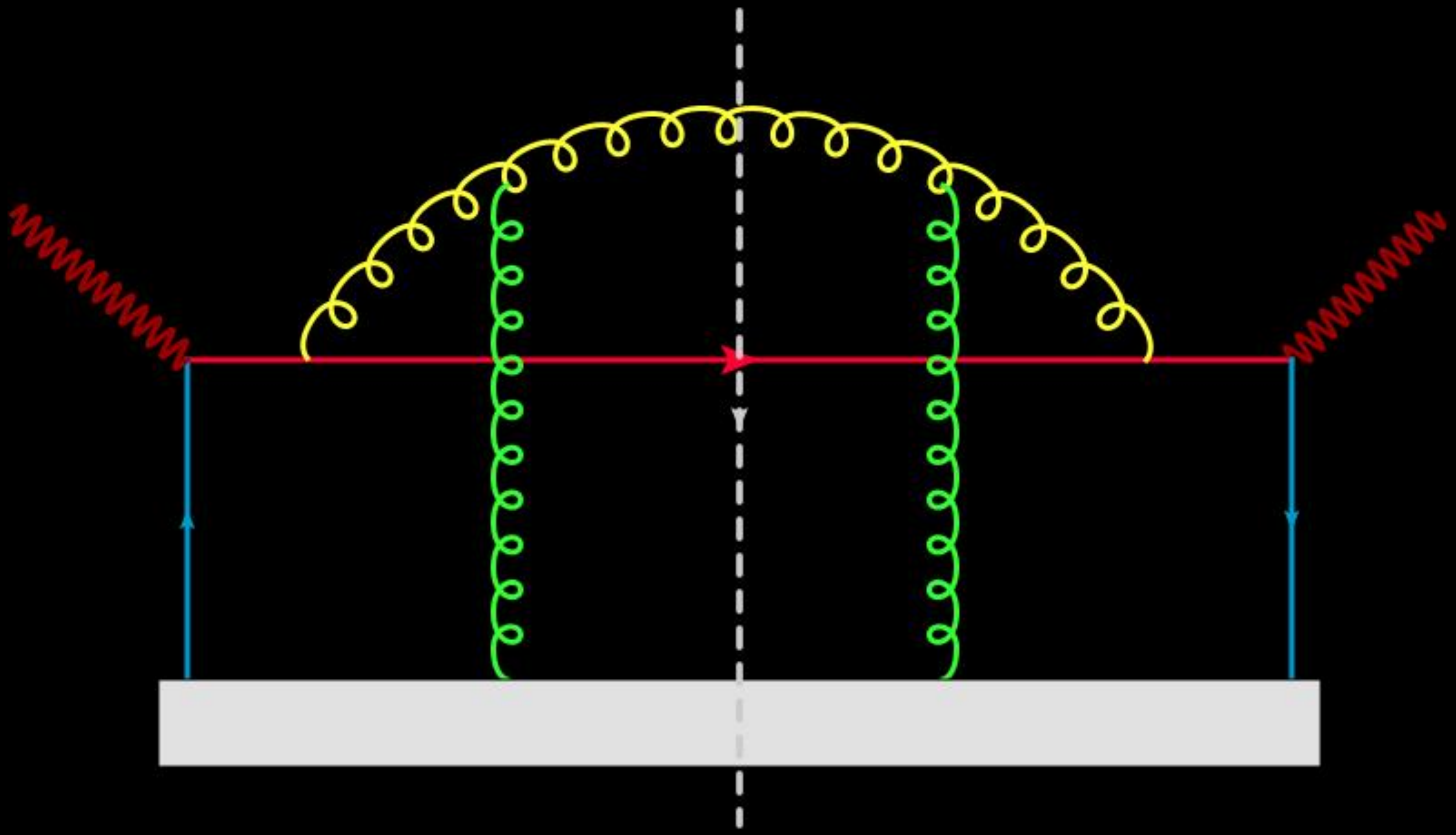


The Aurenche, Zakharov, Zaraket (AZZ) objection!

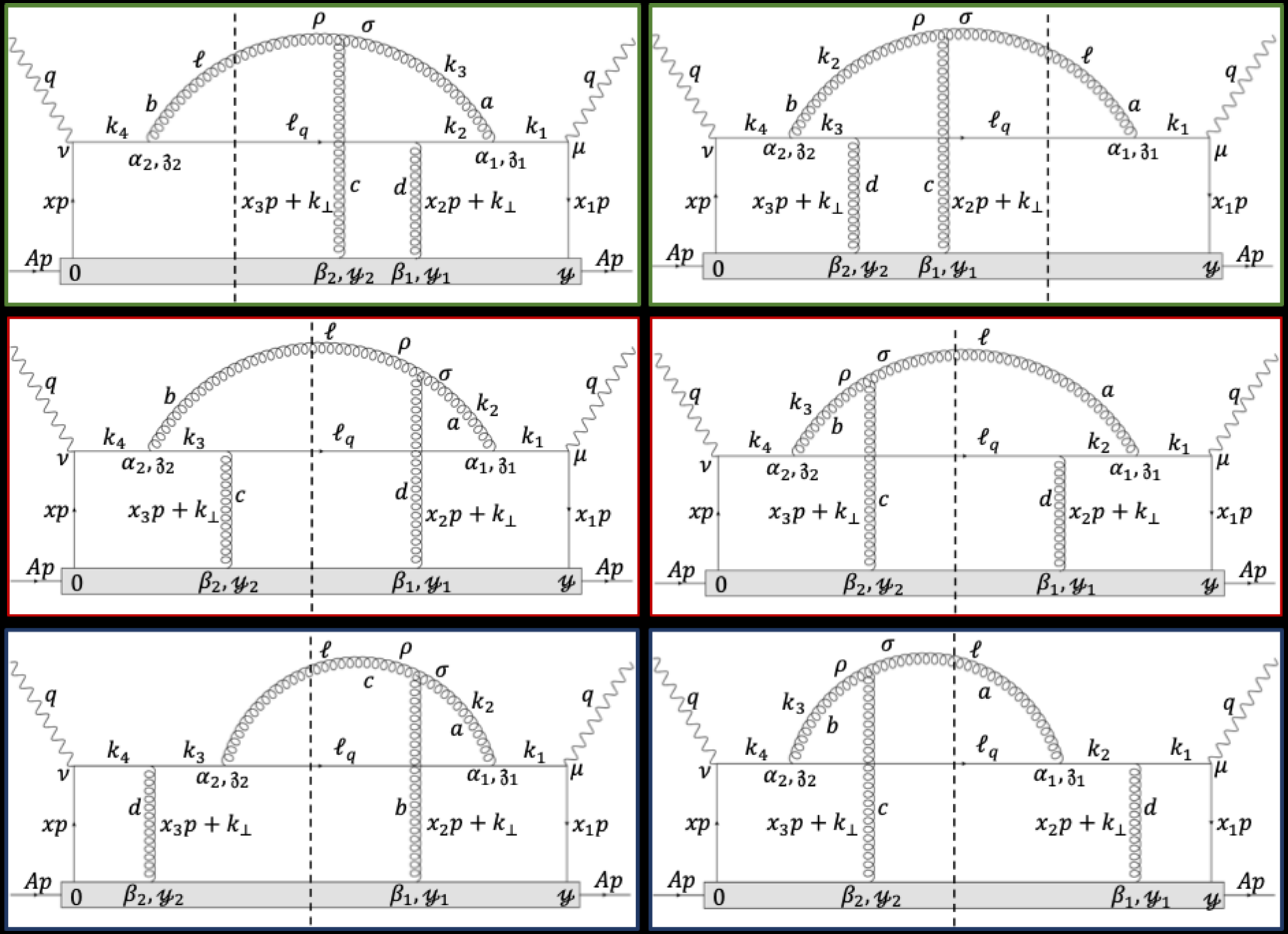
- Why drop the $k_\perp$ dependence in the phase factors?
  they retained all these factors but only for one diagram: JETP Lett. 87 (2008) 605-610, e-Print: 0806.0160 [hep-ph]

- $$\frac{dN}{dy} = \frac{\alpha_S}{2\pi} P(y) \int \frac{dl_\perp^2}{l_\perp^2} \int dz^- \frac{\hat{q}(z^-)}{l_\perp^2} \left[2 - 2\cos\left(\frac{z^- l_\perp^2}{2q^- y\,(1-y)} \right) \right.$$
$$\left. -2 \left(\frac{z^- l_\perp^2}{2q^- y\,(1-y)} \right) \sin\left(\frac{z^- l_\perp^2}{2q^- y\,(1-y)} \right) + 2 \left(\frac{z^- l_\perp^2}{2q^- y\,(1-y)} \right)^2 \cos\left(\frac{z^- l_\perp^2}{2q^- y\,(1-y)} \right) \right]$$





Keeping the power correction in phase of all diagrams



C. Sirimanna, S. Cao, AM, Phys.Rev.C 105 (2022) 2, 024908

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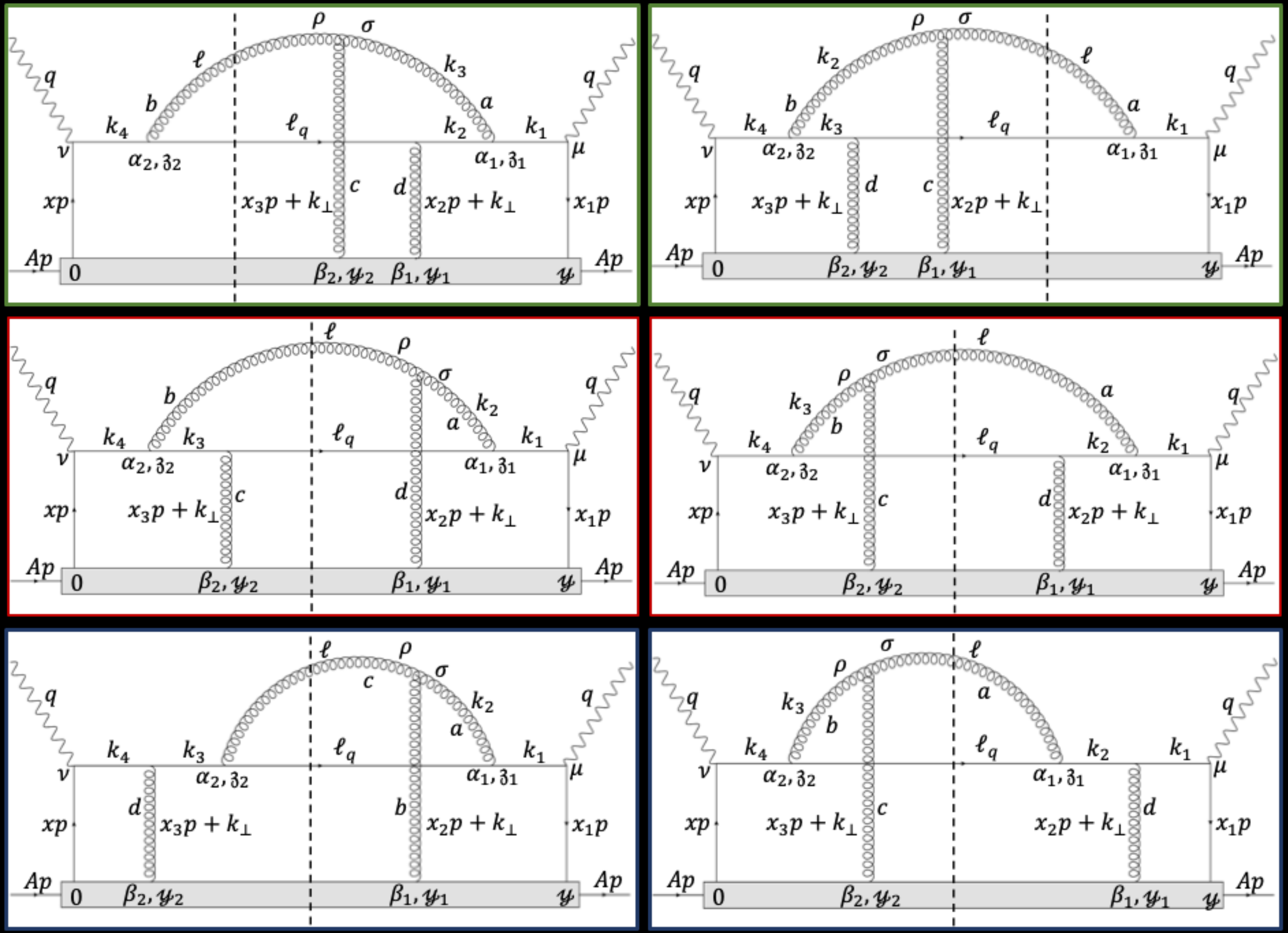

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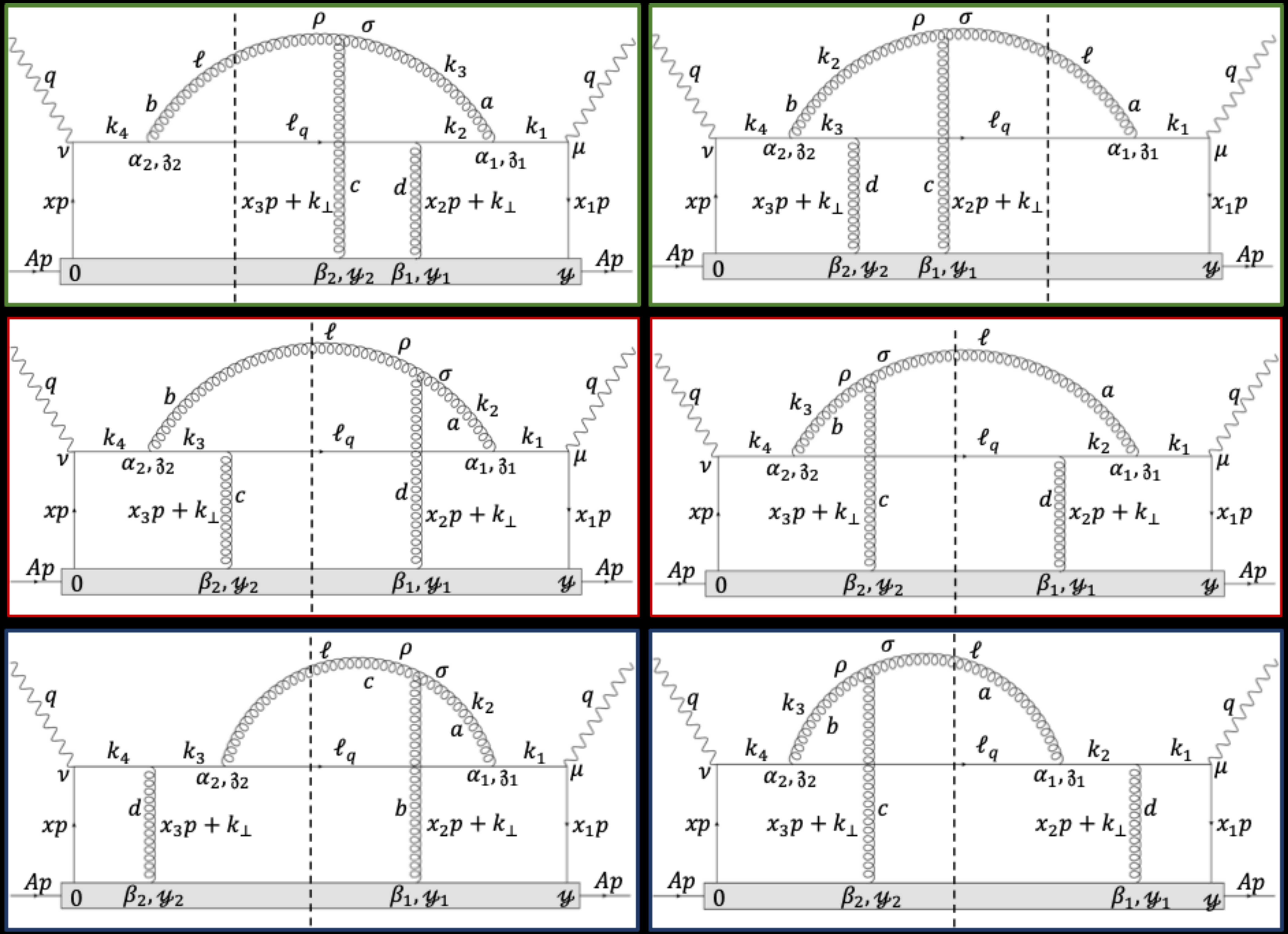

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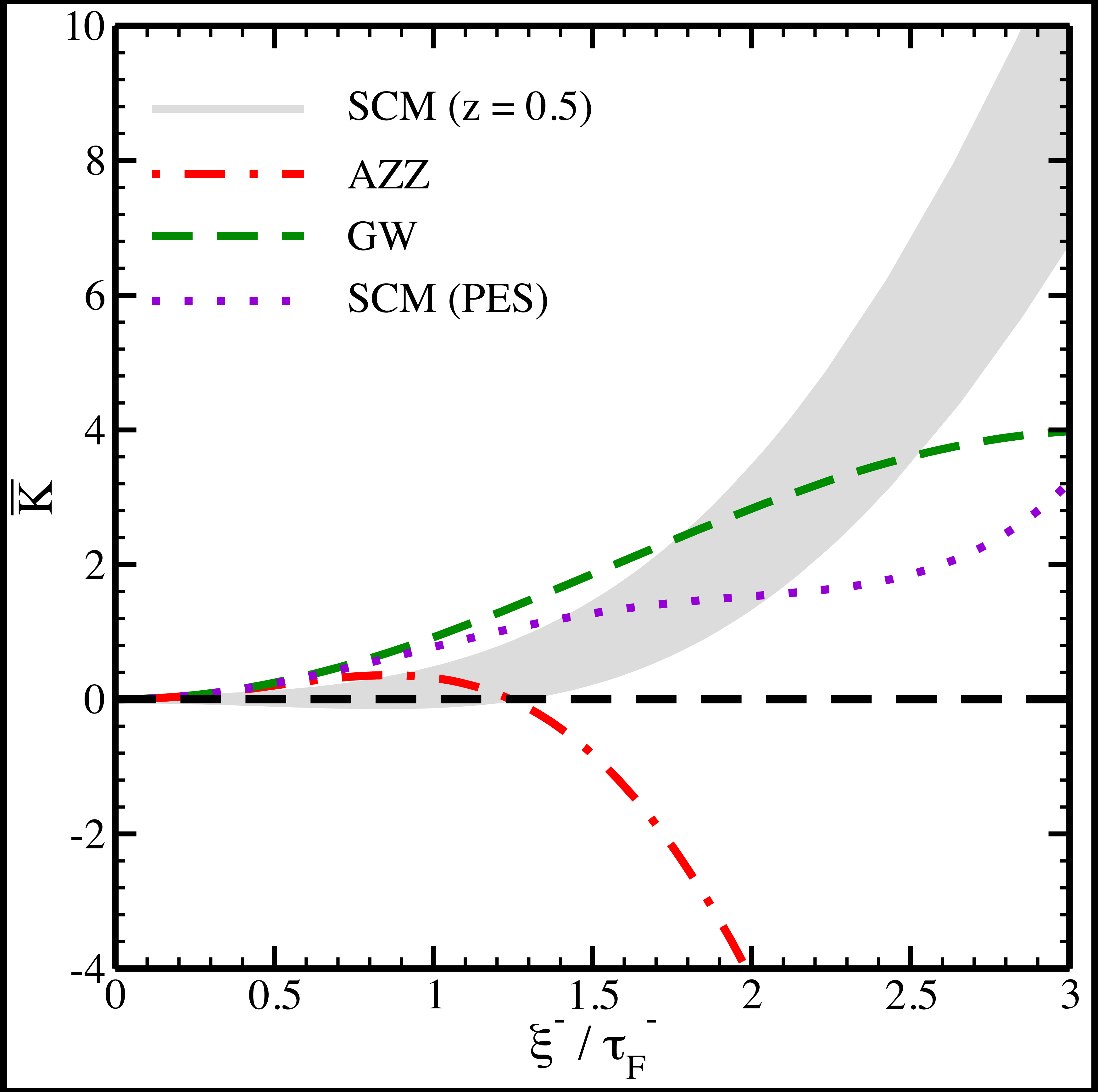
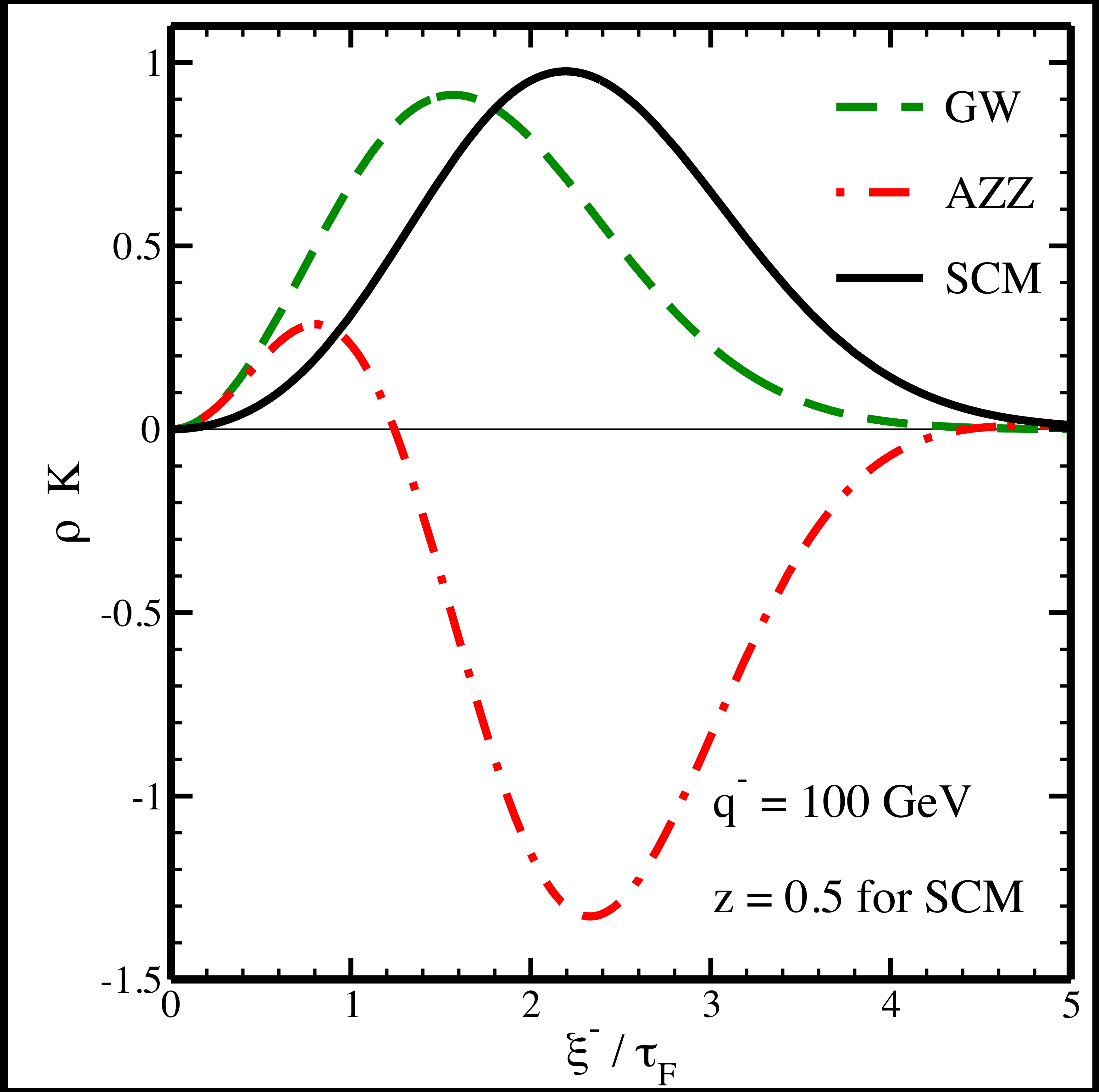
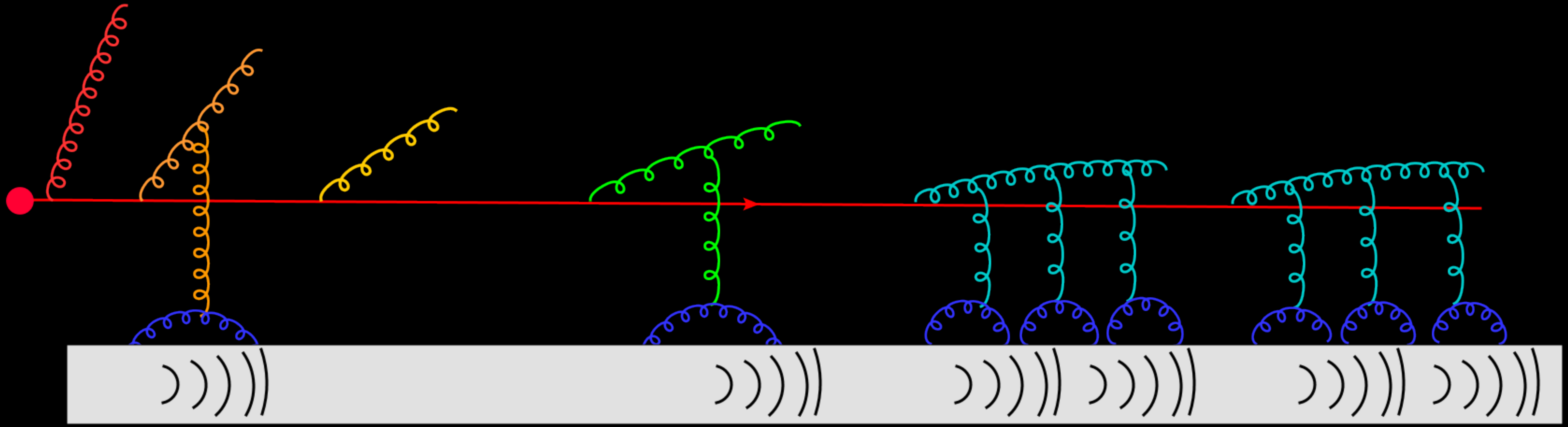




- Final result is positive definite, and very close to GW
- Allows for MC simulation.

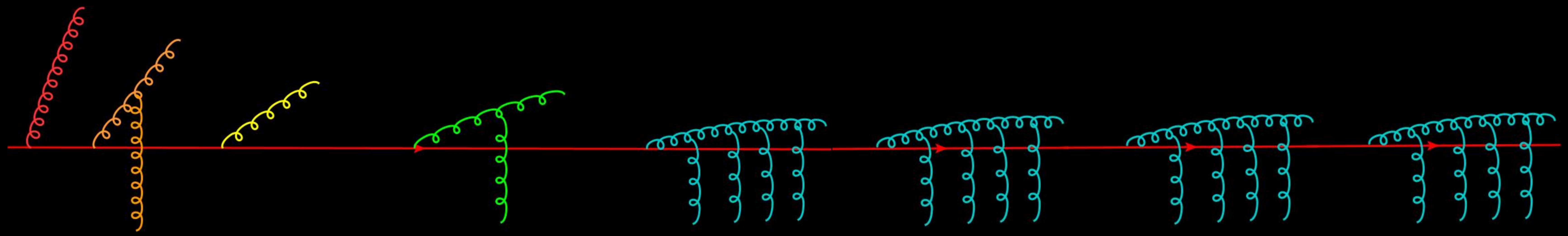
Basic Picture: extra scales in energy loss

- Jet starts in a hard scattering with a virtuality $Q^2 \lesssim E^2$
- First few emissions are vacuum like with rare scattering/emission
- Virtuality comes down to $Q^2_{med} \simeq \sqrt{2E\hat{q}}$ transition to many scattering/emission



- Exchanges with medium lead to excitations/medium response

Jet radiation structure: when does it transition?



- Estimate from formation time arguments: $\tau = \dfrac{2E}{Q^2} \implies Q^2(t) = \dfrac{2E}{t}$

  (Modulo energy loss effects from emitting gluons).

- The maximum virtuality built up from scattering

  at time t is $Q^2_{\rm med} = \hat{q}t \simeq \dfrac{2E}{t} \implies t \simeq \sqrt{\dfrac{2E}{\hat{q}}}$

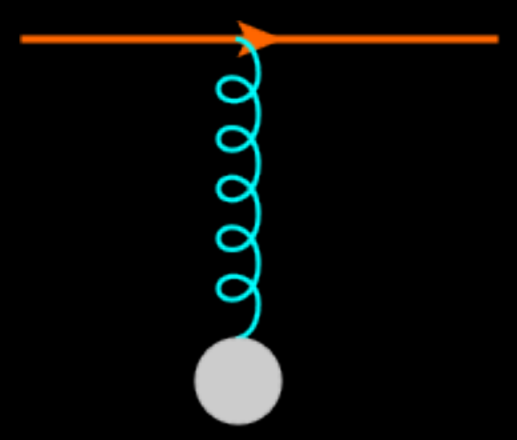
- Highest energy partons (jet core) reach the BDMPS/AMY stage last,

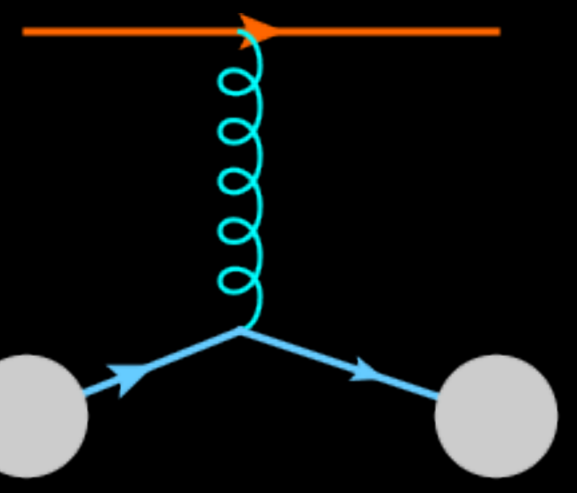
- Smaller the $\hat{q}$, longer it takes to reach the BDMPS/AMY stage: longer DGLAP stage

Multi-scale structure in the medium

- Hard exchanges $k_\perp \gg \Lambda_{QCD}$ will resolve partons in the QGP

$$\int_0^\infty d^3k = \int_0^\mu d^3k + \int_\mu^\infty d^3k$$



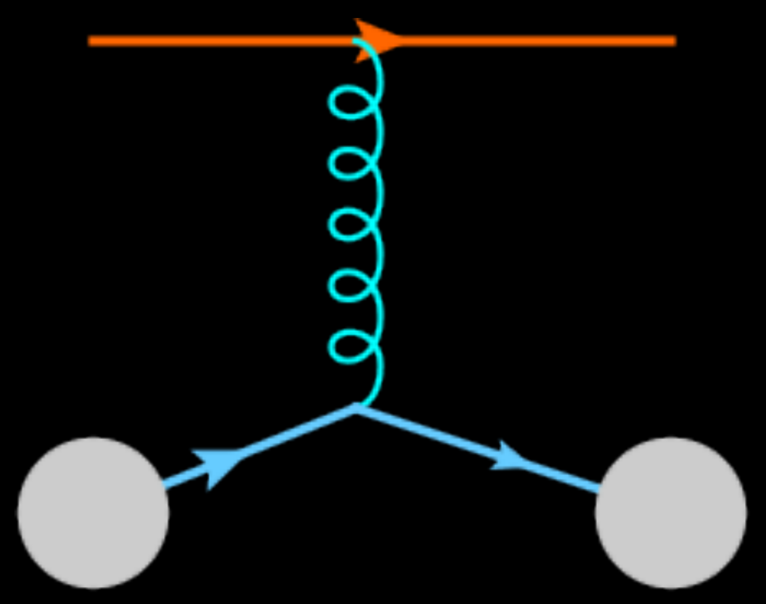


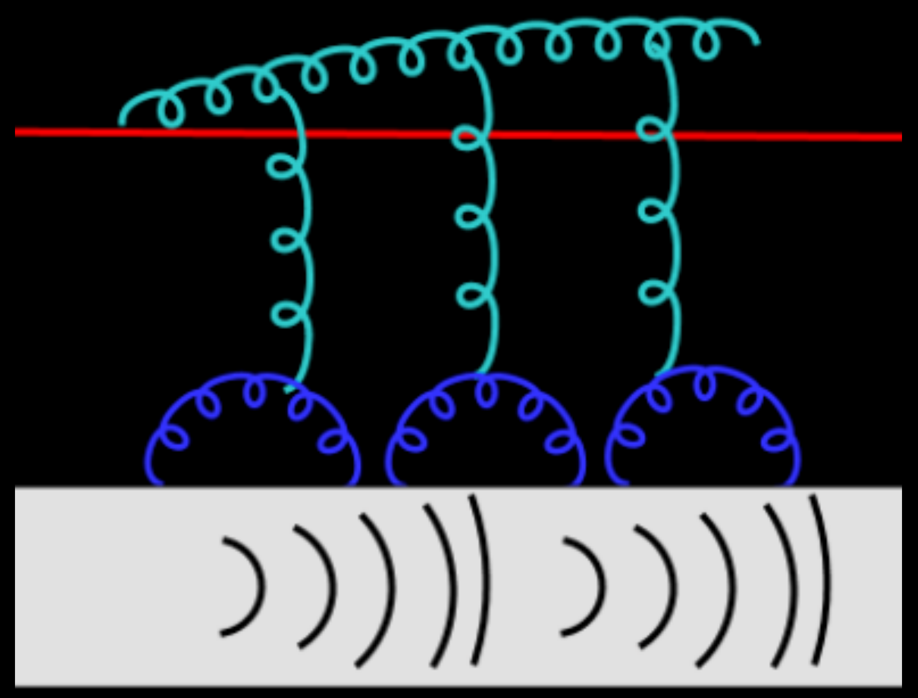
- Incoming "resolved partons" can be modeled with
  - HTL perturbation theory
  - or using QGP PDF (A. Kumar et al., PRC 101 (2020) 034908)
  - Or Both (MATTER + LBT)

- Soft exchanges by generic **broadening** (Lido, Tequila, also do hard exchanges with HTL)

- Outgoing "resolved partons" can be modeled with
  - HTL perturbation theory
  - Or turned into energy momentum source term (liquify)

Structure of the interaction





- Start with low virtuality part: $\mu^2 = \sqrt{2\hat{q}E}$

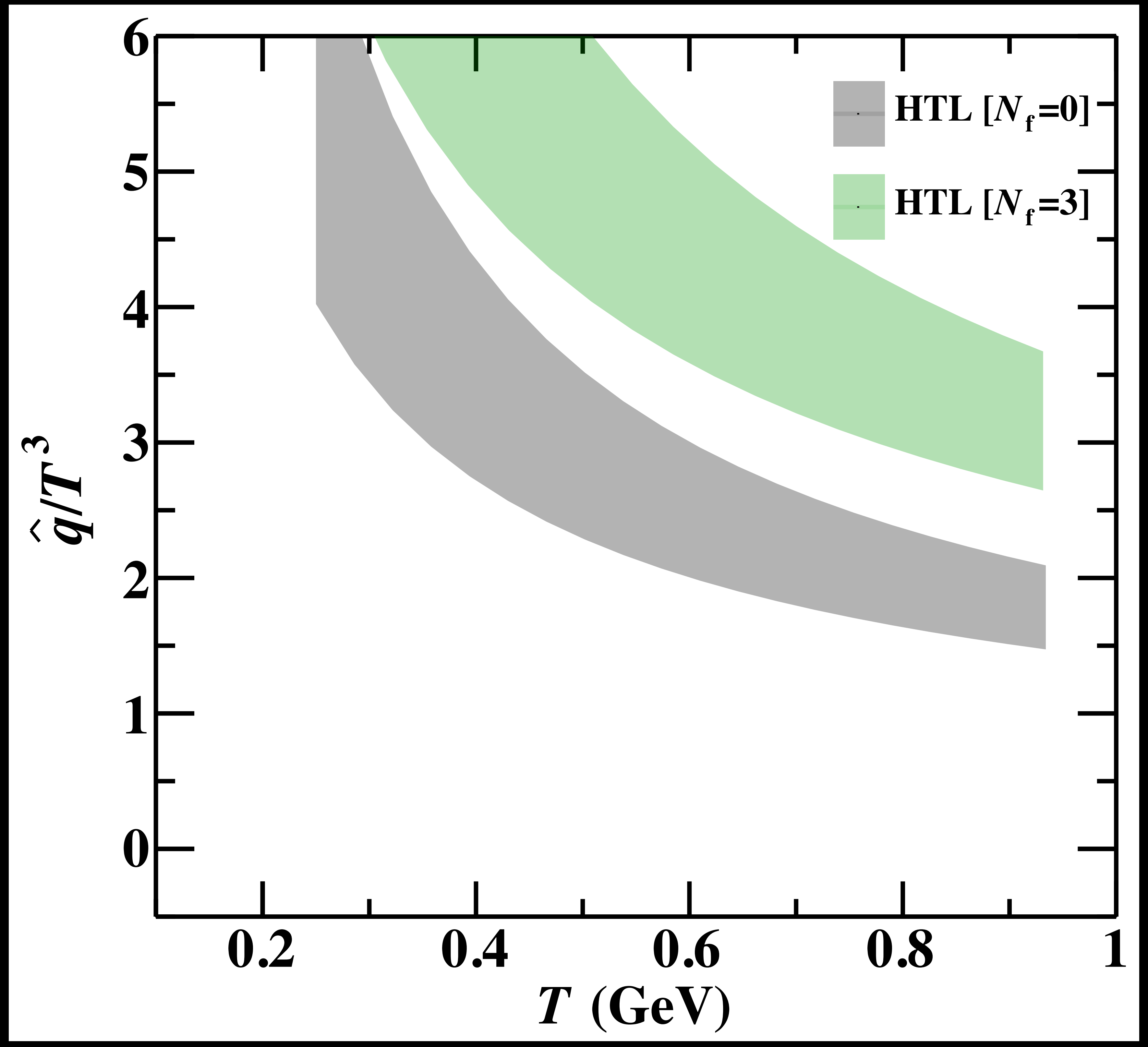
- Use Debye screened potential

$$C(k_\perp) = \frac{C_R}{(2\pi)^2} \frac{g^2 T m_D^2}{k_\perp^2 (k_\perp^2 + m_D^2)}$$

- Running coupling gives,

$$\hat{q} = C\alpha_s(2ET)\alpha_s(m_D)T^3 \log\left(\frac{2ET}{m_D^2}\right)$$

- Struck partons go into medium, and excite medium. Some get clustered into jets, need to keep track of deposited energy

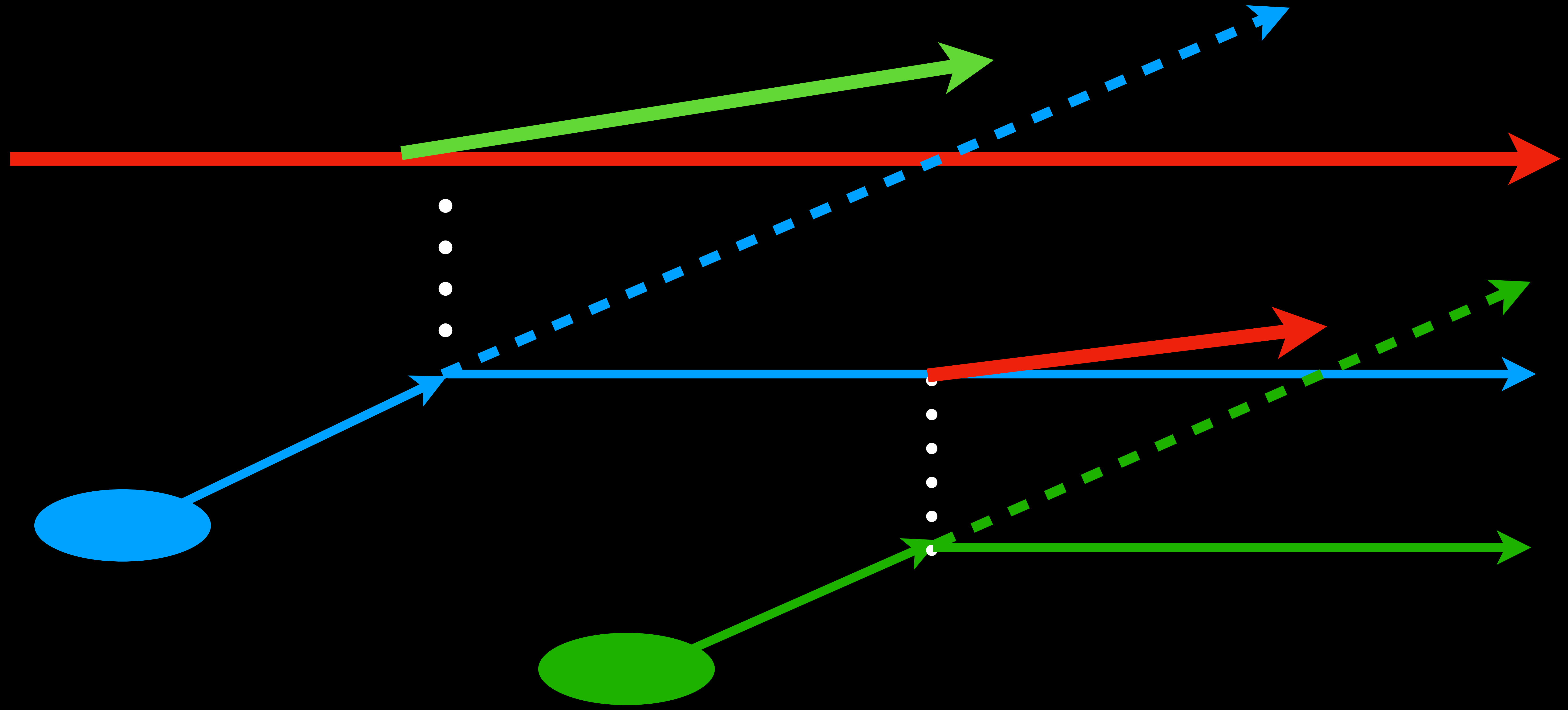


Arnold and Xiao: arXiv: 0810.1026 [hep-ph]

How this is done currently

In LBT, MARTINI, JEWEL, MATTER

Full jet carries recoil particles sampled from a Boltzmann distribution. as regular jet partons, and negative partons or holes



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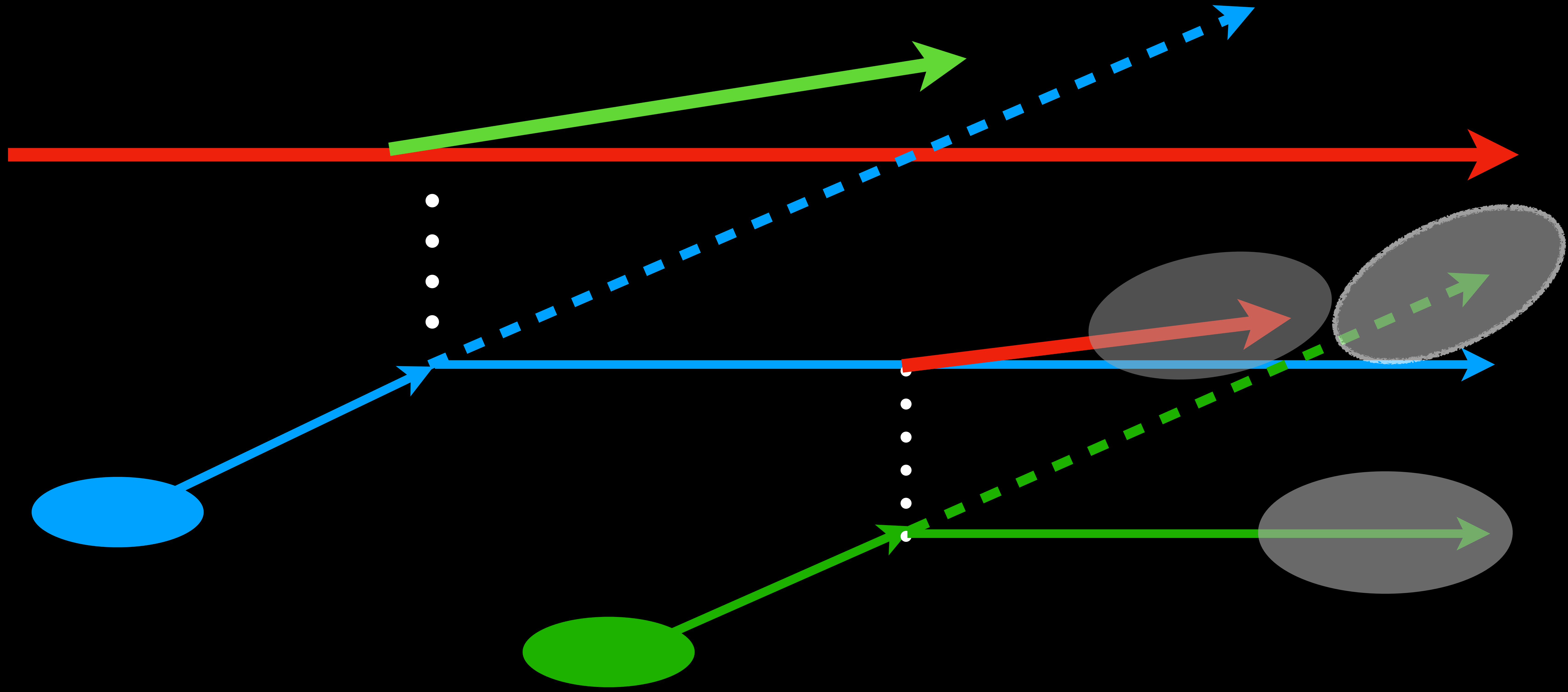
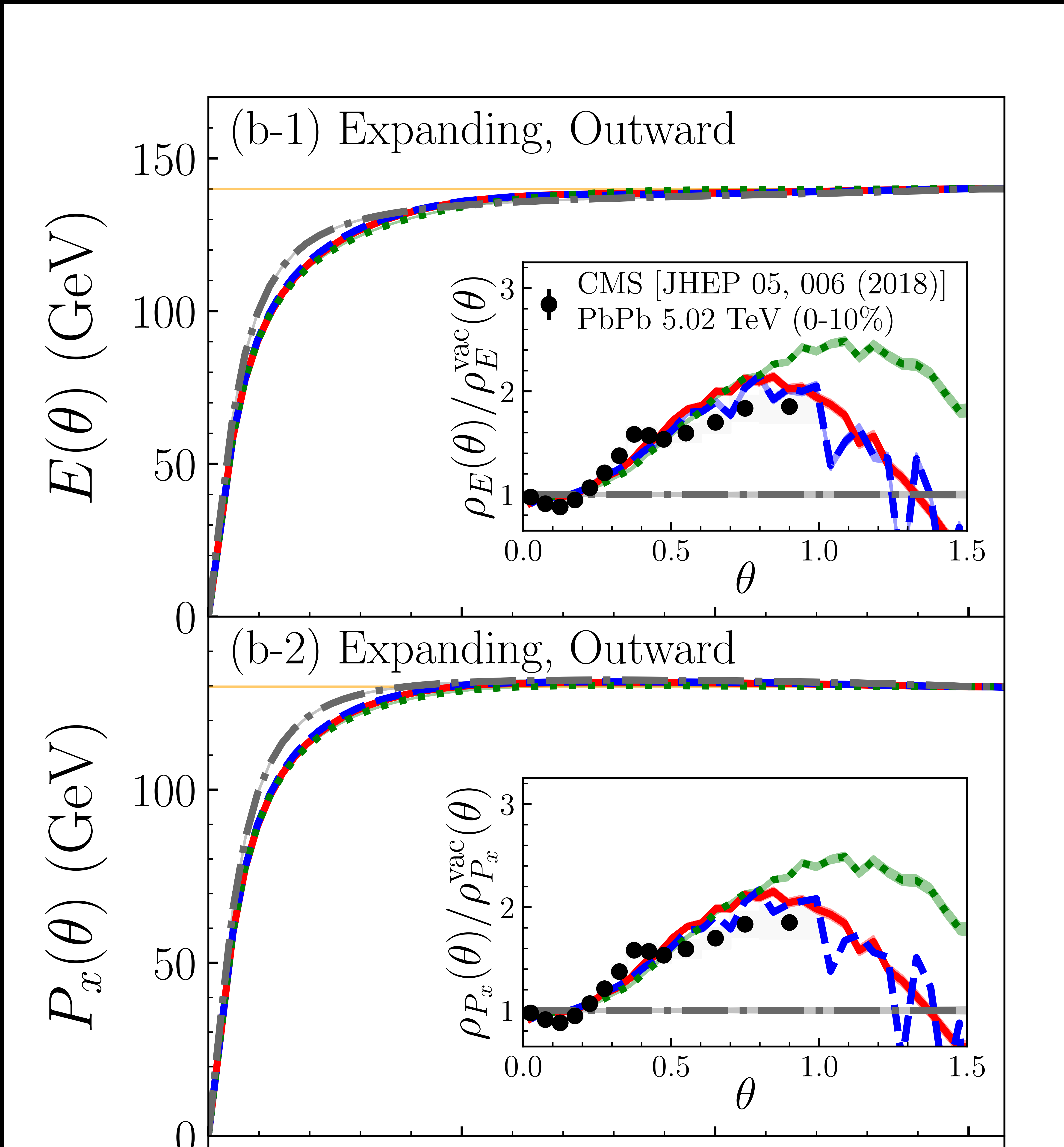


Additionally: Soft partons can be "liquified" into source terms for a subsequent hydro simulation

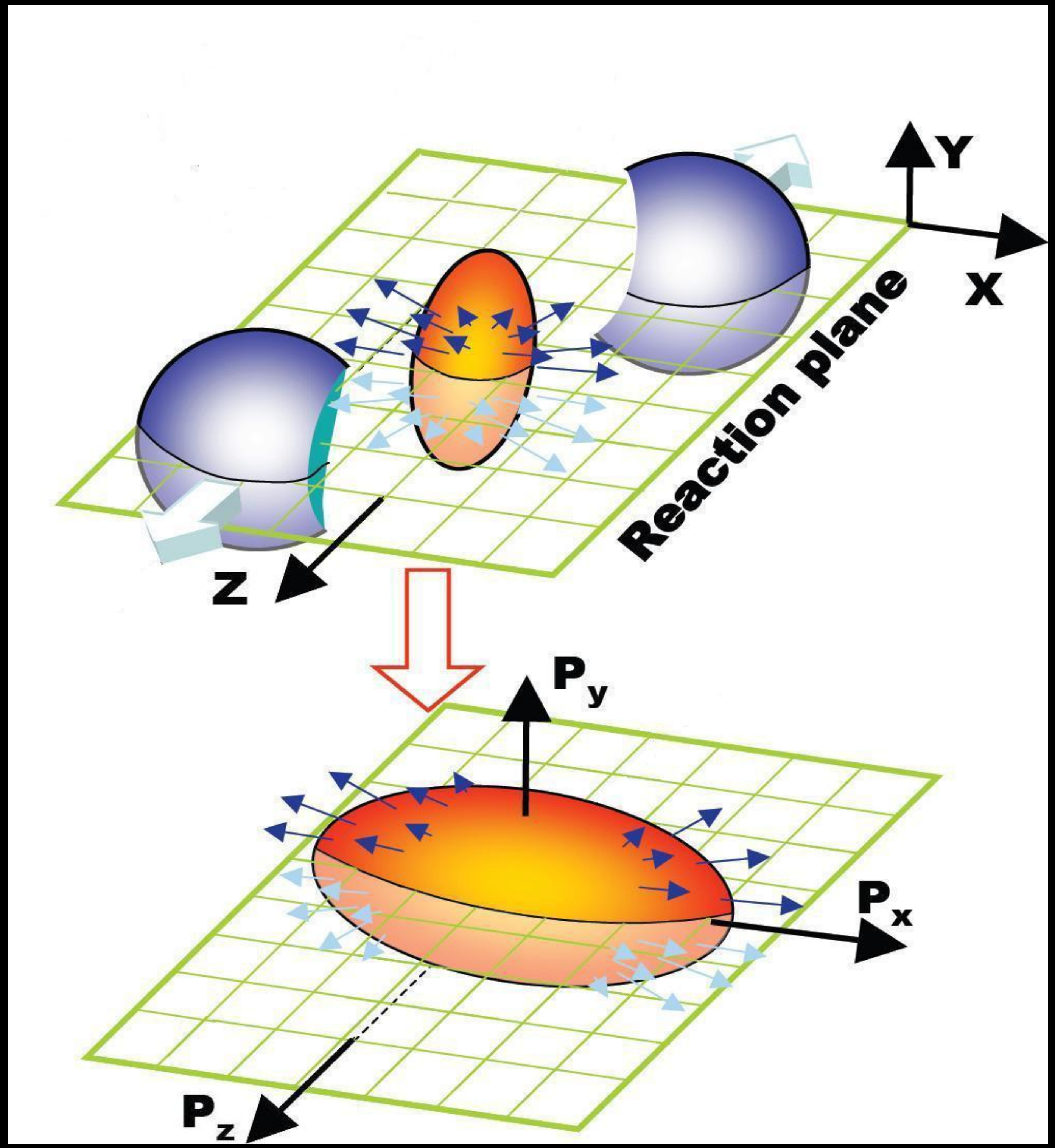
Does not seem to make much difference inside jet cone

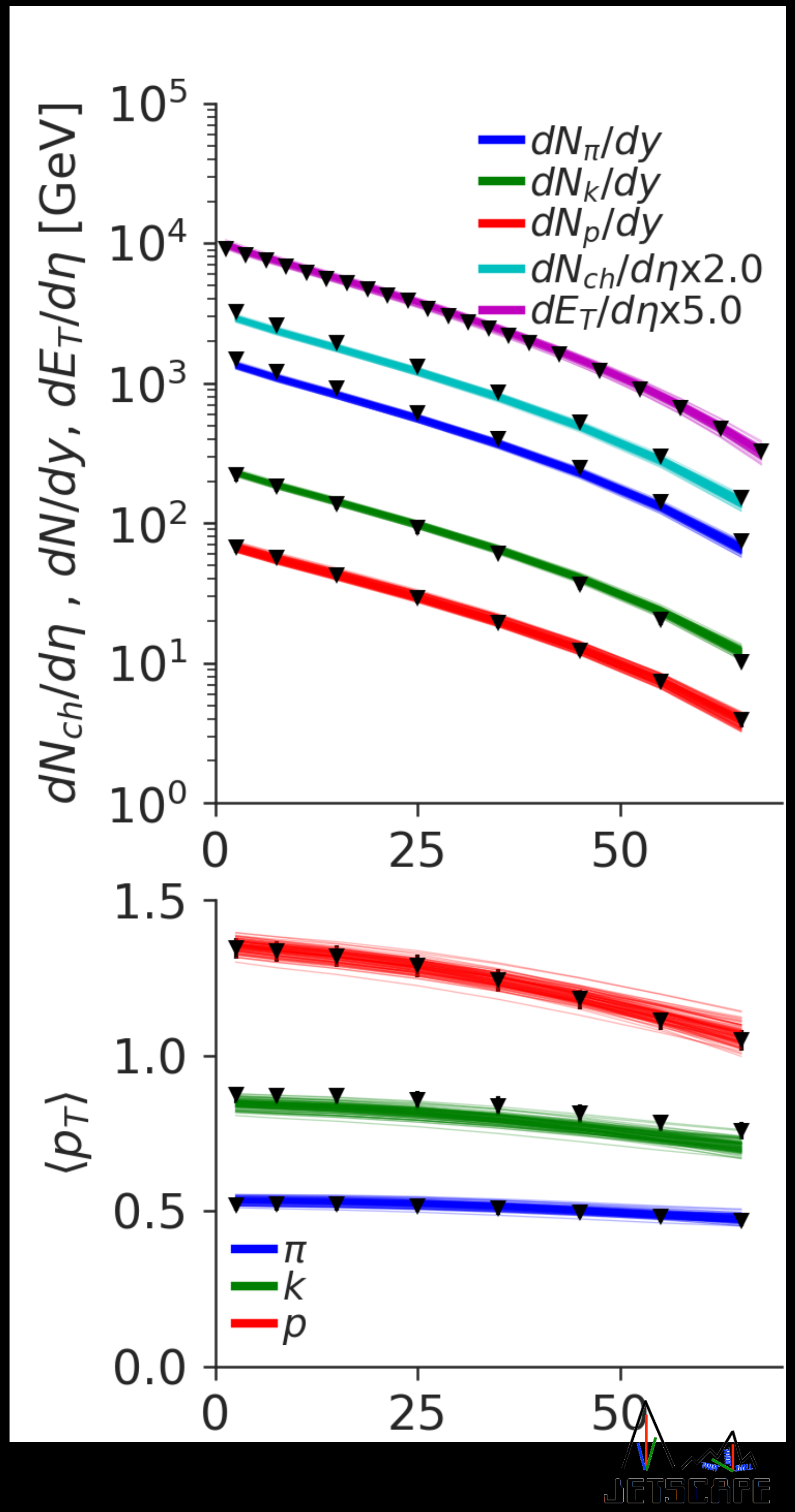
- Simulation (JETSCAPE 0.x) includes:
  - One run of smooth hydro
  - One jet from center outward (left)
  - One jet from out inward (right)
  - Jet simulated for ~10fm/c: MATTER+LBT
  - Jet constructed with partons (weak)
  - Soft partons liquified
  - Source terms developed
  - Hydro re-run
  - Jet reconstructed with hard partons and unit cell momenta (strong)
  - Unit cell particlized (Cooper-Frye), jet reclustered (Strong particlized)

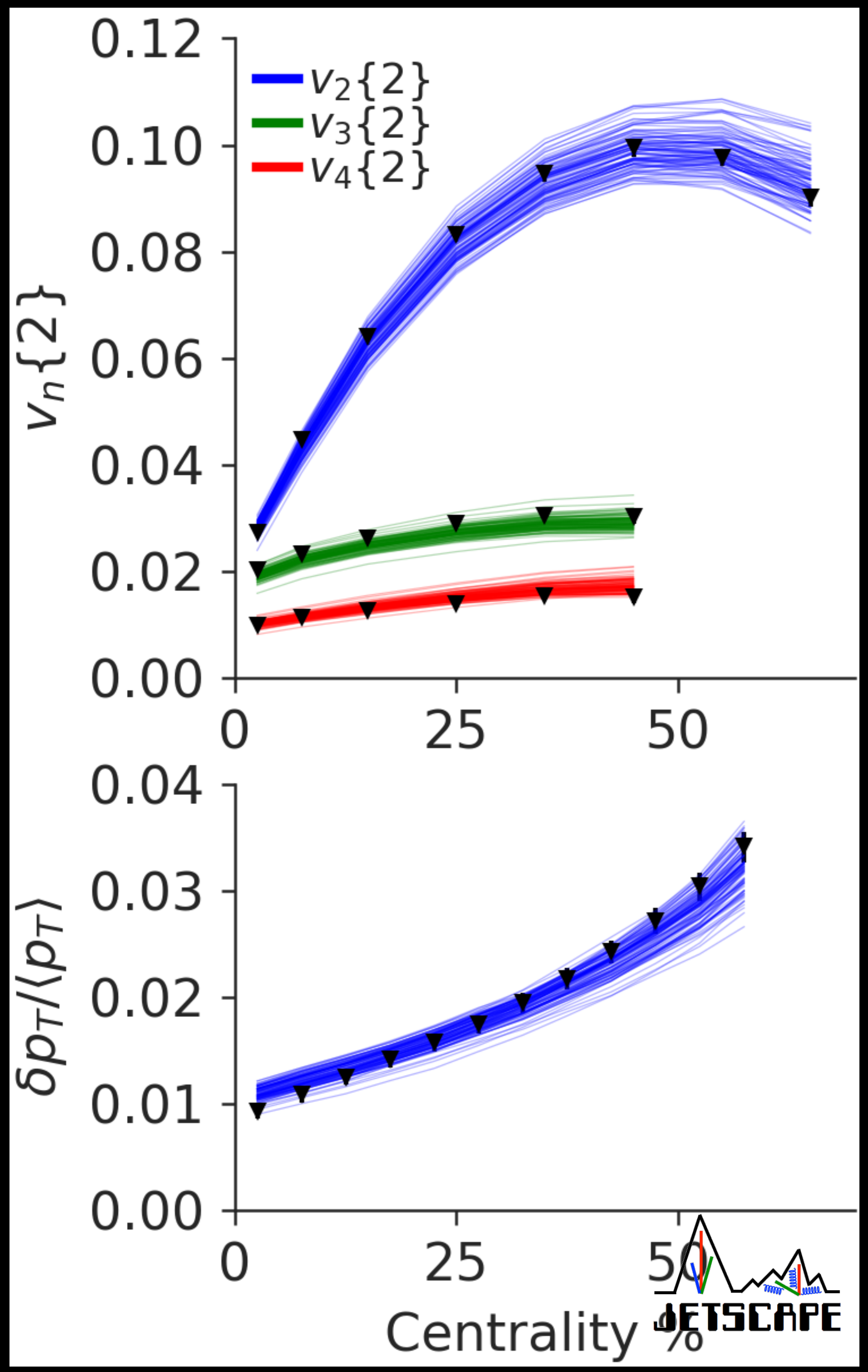
Y. Tachibana, A. M., C. Shen arXiv: 2001.08321 [nucl-th]

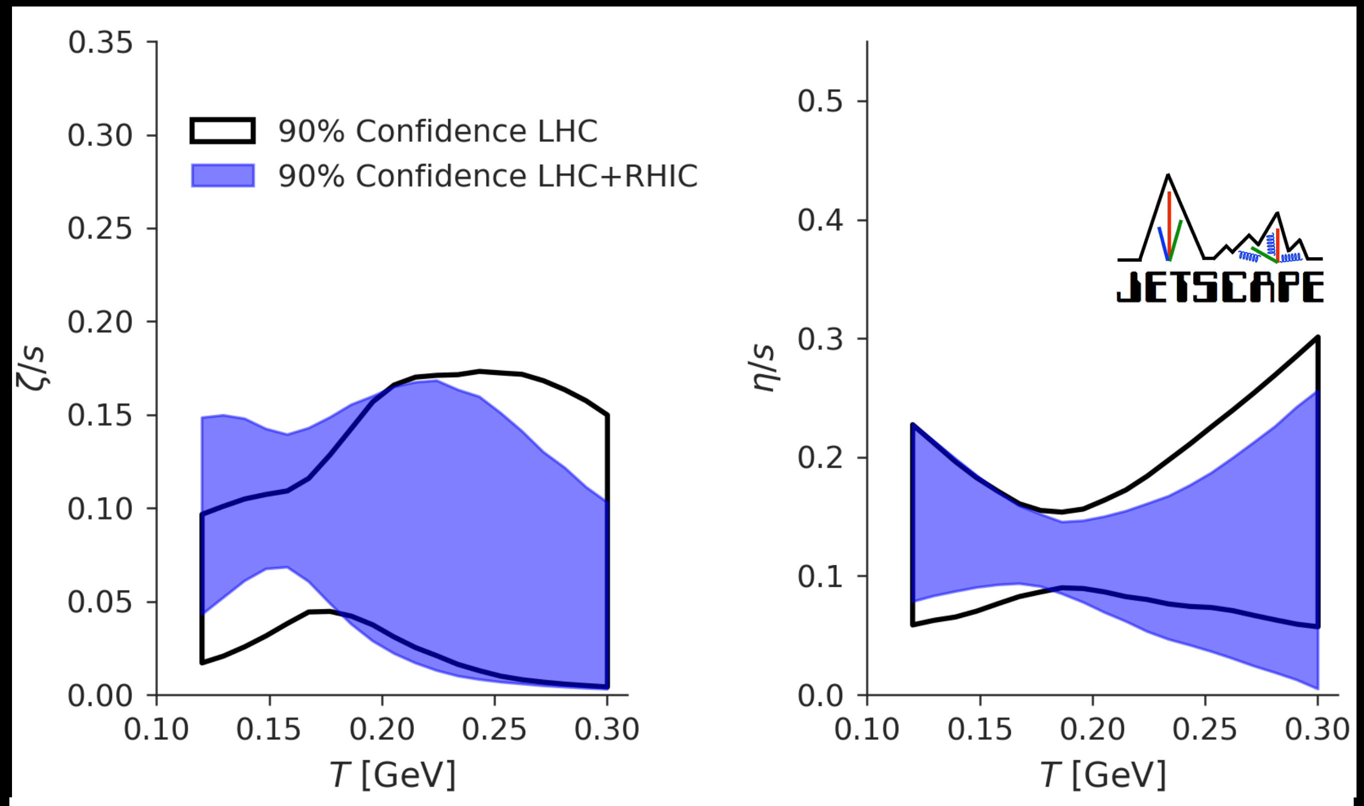


The bulk medium is now extremely well simulated at RHIC & LHC



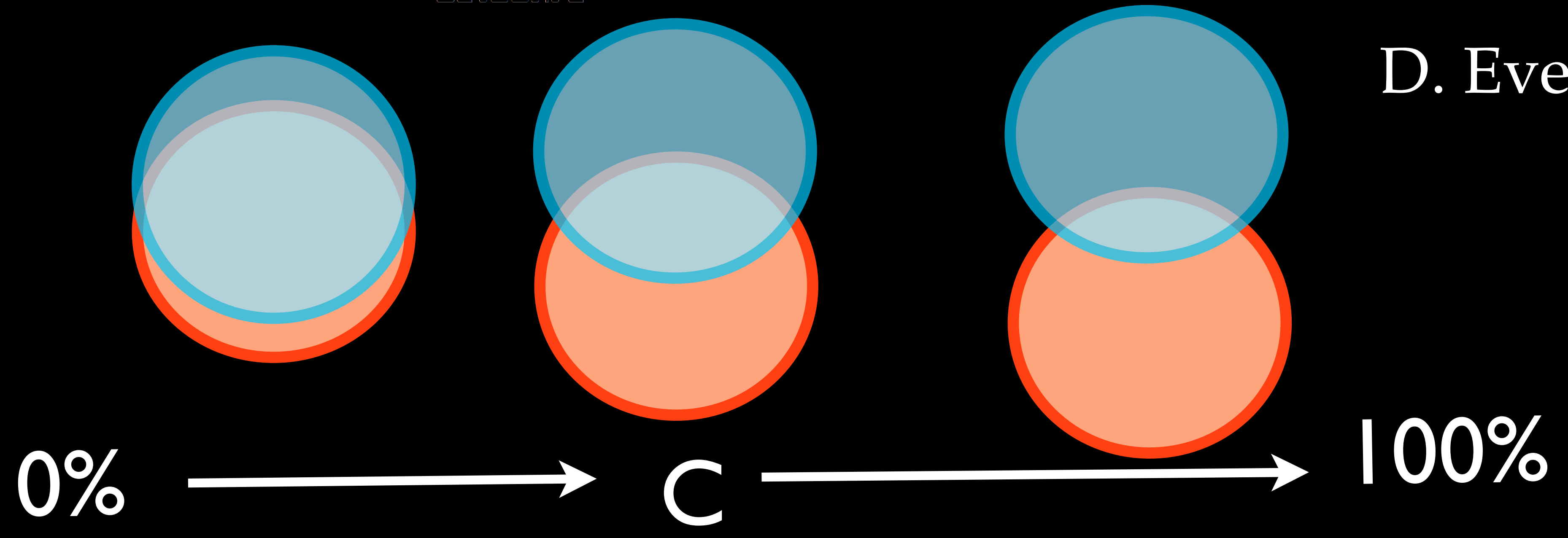






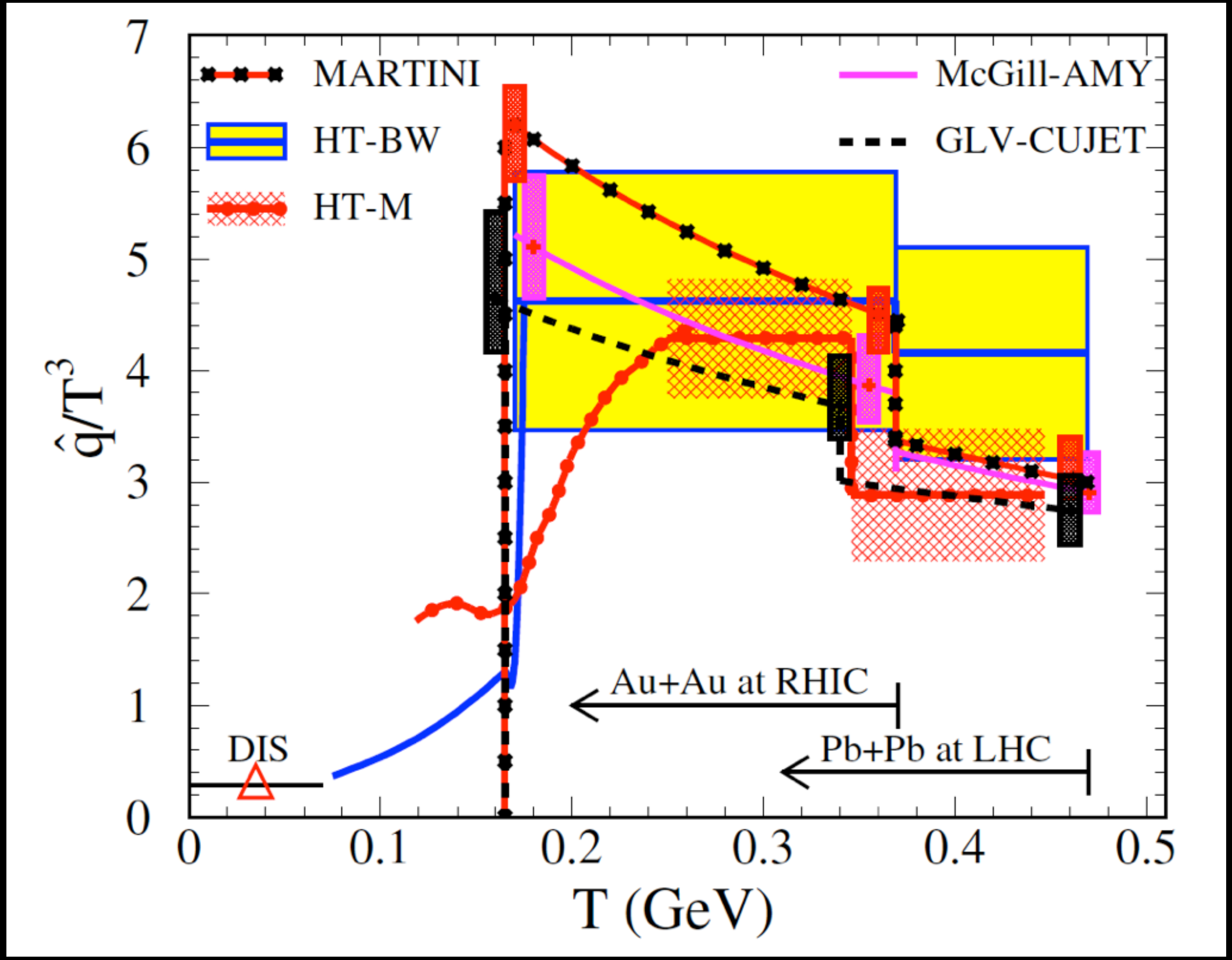
$$\eta \sim \frac{\langle p \rangle}{\sigma}$$

D. Everett et al., Phys. Rev. C 103 (2021) 5, 054904

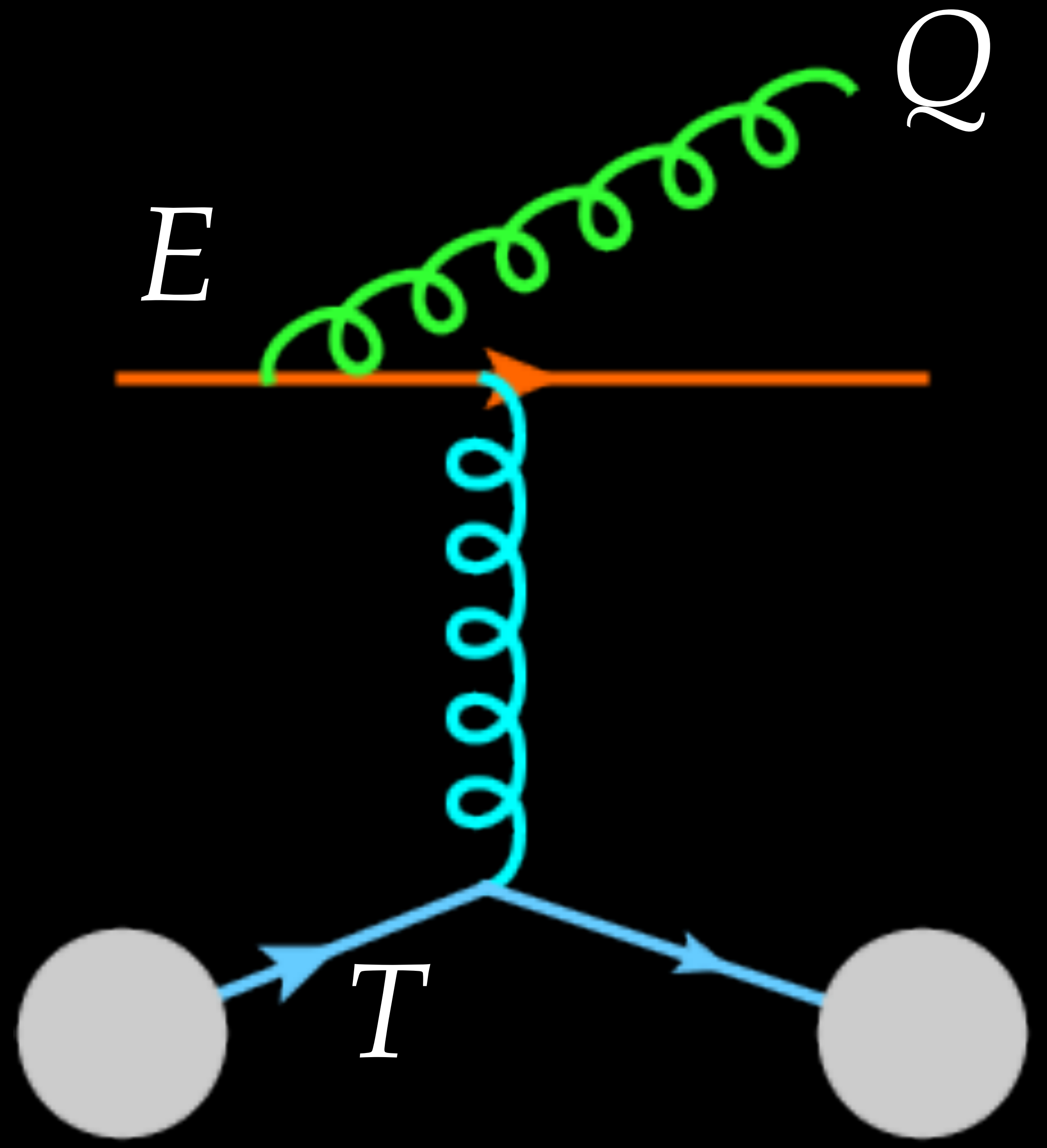


Fluid dynamical simulations and jets

- Fluid simulations are now extremely accurate in determining bulk properties
- Yield well calibrated medium
- Hydrodynamics assumes local thermal equilibrium
- $\hat{q}$ should be constrained by local properties like $T, s, \epsilon, u, \ldots \eta, \zeta \ldots$
- Once the functional form of $\hat{q}$ as a function of T is given, it should not be recalibrated.



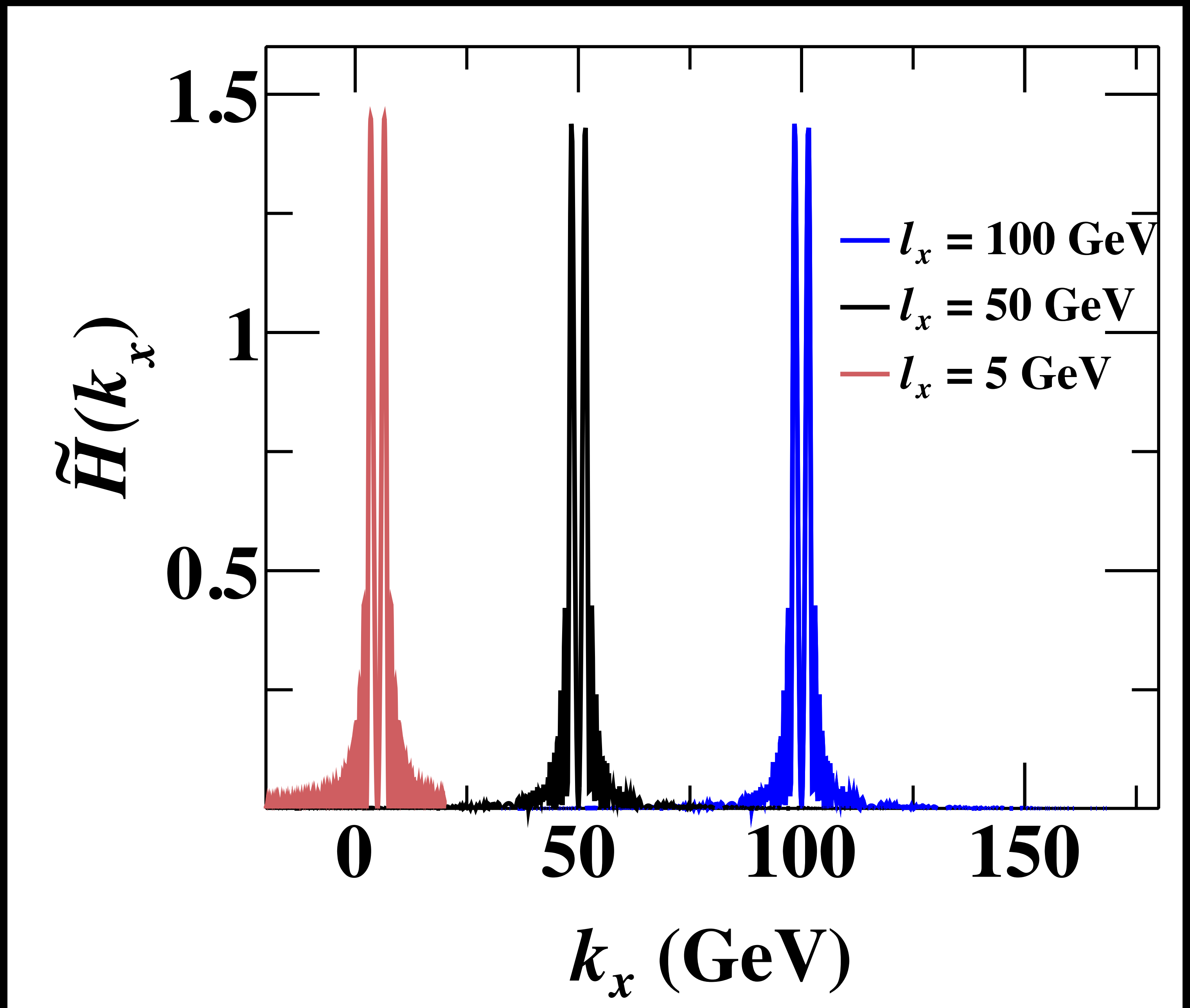
What else can $\hat{q}$ or $\Gamma = \int d^3k C(k)$ depend upon?



- 2 - 2 scattering depends on *s, t, u*
- In general, will depend on *T, E, Q*
- Thermal recoil requires: $\hat{q} = C\alpha_s(2ET)\alpha_s(m_D)T^3 \log\left(\frac{2ET}{m_D^2}\right)$
- $T_{LHC} \sim 1.25\, T_{RHIC}$
- $E_{LHC} \gtrsim 10 E_{RHIC}$
- $Q_{LHC} \gtrsim 10 Q_{RHIC}$

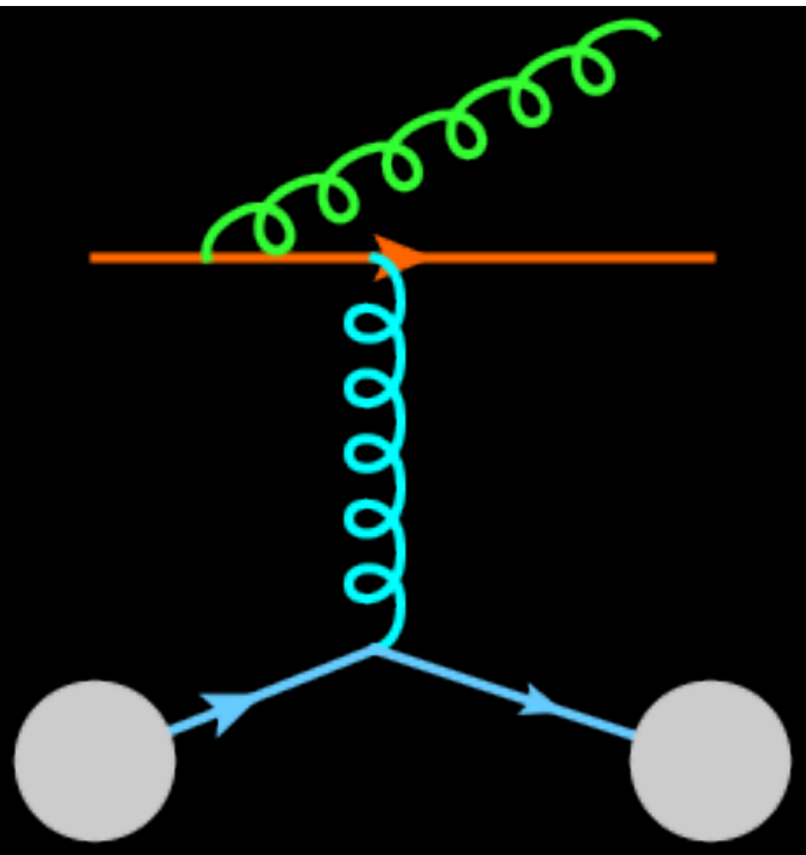
Virtuality dependence/Coherence

- Coherence arguments: $\hat{q}(Q^2 > \sqrt{2\hat{q}E}) \to 0$
- Can be calculated directly in the Higher Twist formalism.



$$\frac{dN_g}{dy d^2 l_\perp} = \frac{\alpha_s}{2\pi} P(y) \int \frac{d^2 k_\perp}{(2\pi)^2} \int d\zeta^- \left[\frac{2 - 2\cos\left(\frac{(l_\perp - k_\perp)^2 \zeta^-}{2q^- y(1-y)}\right)}{(l_\perp - k_\perp)^2} \right]$$

- $$\times \int d(\delta\zeta^-) d^2\zeta_\perp e^{-i\frac{\vec{k}_\perp^2}{2q^-}\delta\zeta^- + i\vec{k}_\perp.\vec{\zeta}_\perp}$$
$$\times \langle P | A^{a+}\left(\zeta^- + \frac{\delta\zeta^-}{2}\right) A^{a+}\left(\zeta^- - \frac{\delta\zeta^-}{2}\right) | P \rangle$$

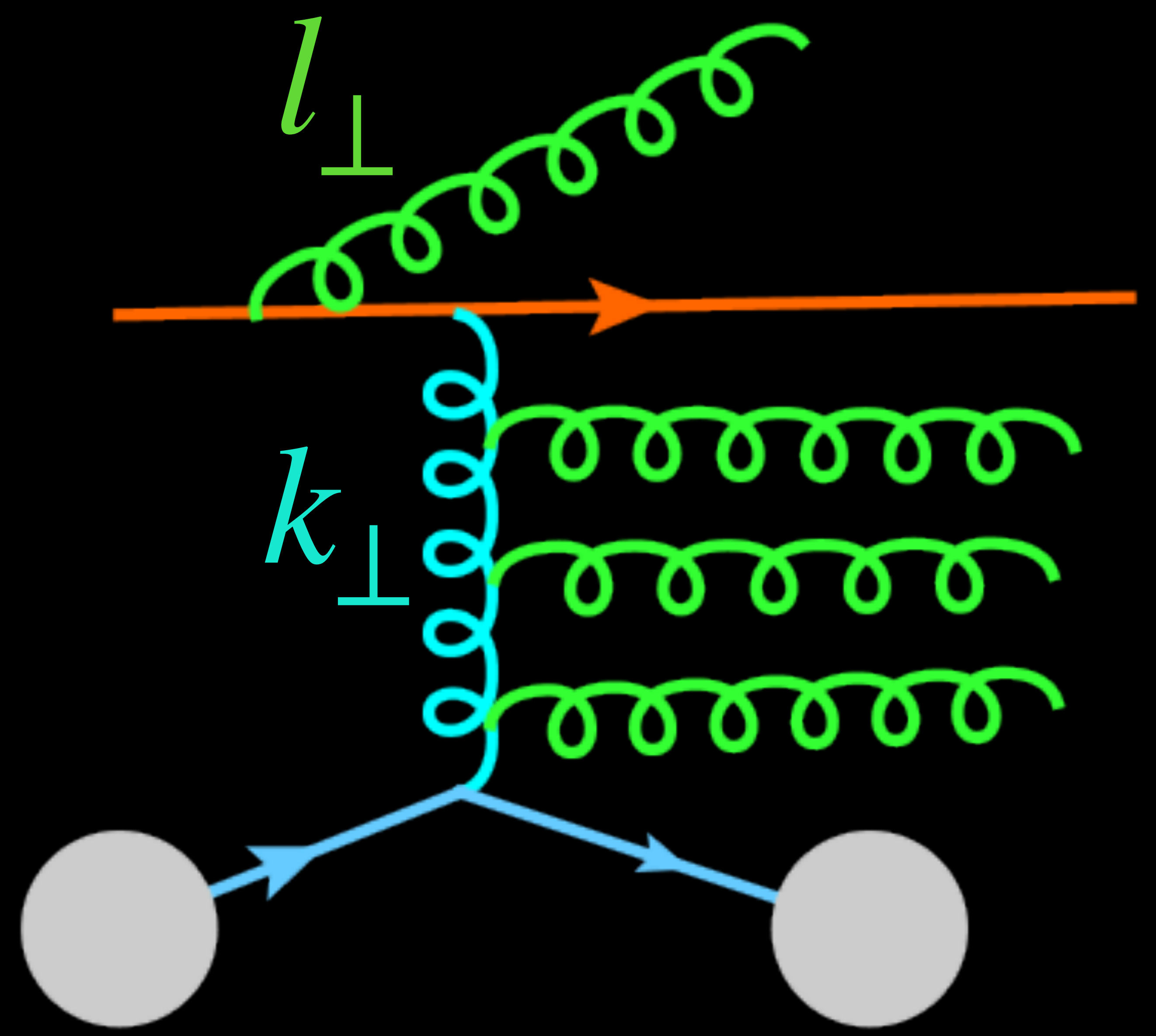


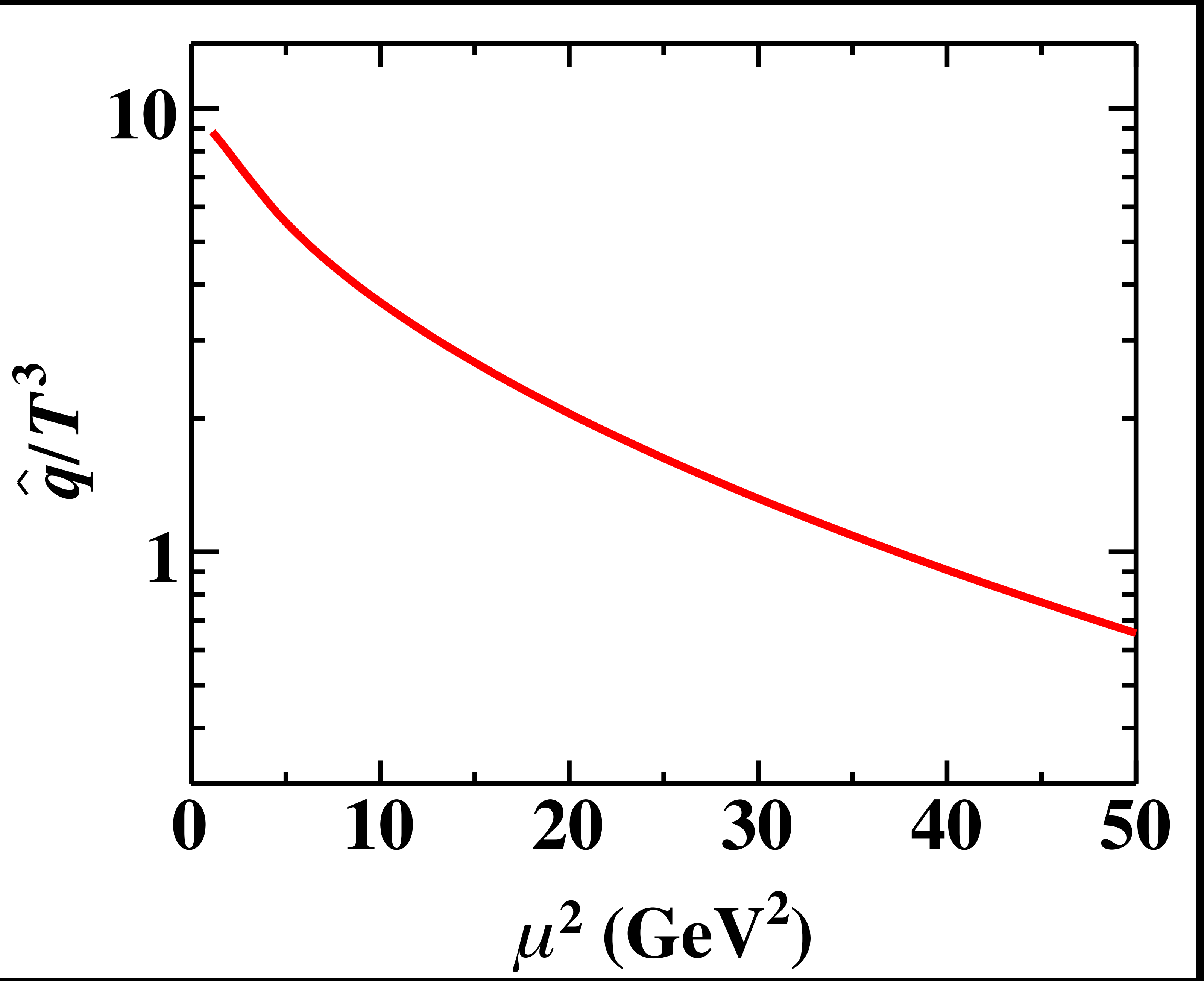
- **The matrix element prefers $k_\perp \sim T$, there is tension between 1st and 3rd line.**

A. Kumar, A.M., C. Shen, PRC 101 (2020) 034908

Virtuality dependence/Coherence

- How does the thermal distribution produce a hard gluon with $k_\perp \gg T$,
- By fluctuation (evolution)
- Reduces the effective $\hat{q}$, as only sensitive to $k_\perp \sim l_\perp$





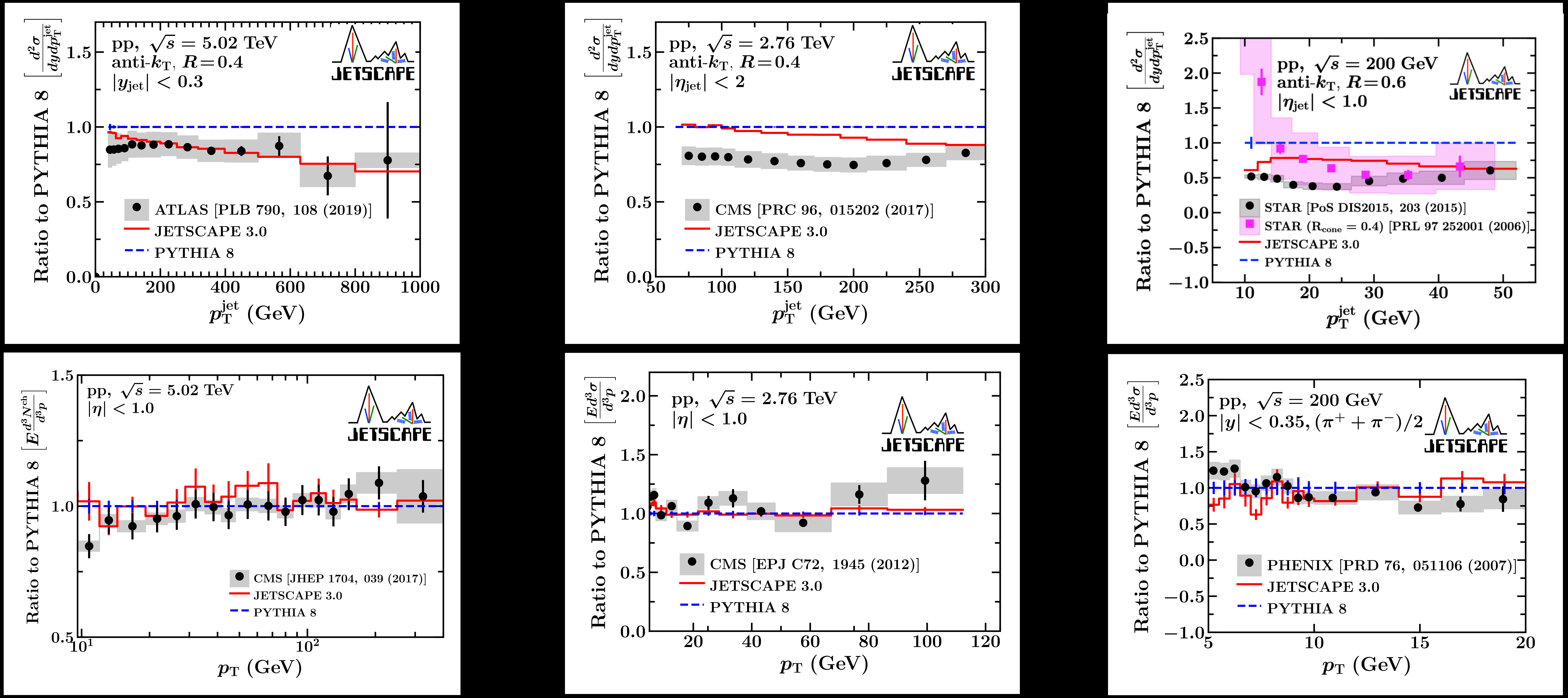
A. Kumar, A.M., C. Shen, PRC 101 (2020) 034908

Transition from MATTER to LBT at $Q_0 = Q_{sw}$

- TRENTO initial state
- Pre Calibrated 2+1D MUSIC gives background
- PYTHIA hard scattering
- High virtuality phase using MATTER
- Lower virtuality phase using LBT
- Both have the same recoil setup
- Evolution starts at Q ~ E and goes down to 1 GeV
- Hadronization applied in vacuum
- Holes subtracted

One more constraint before we start

Any decent event generator should reproduce p-p collisions

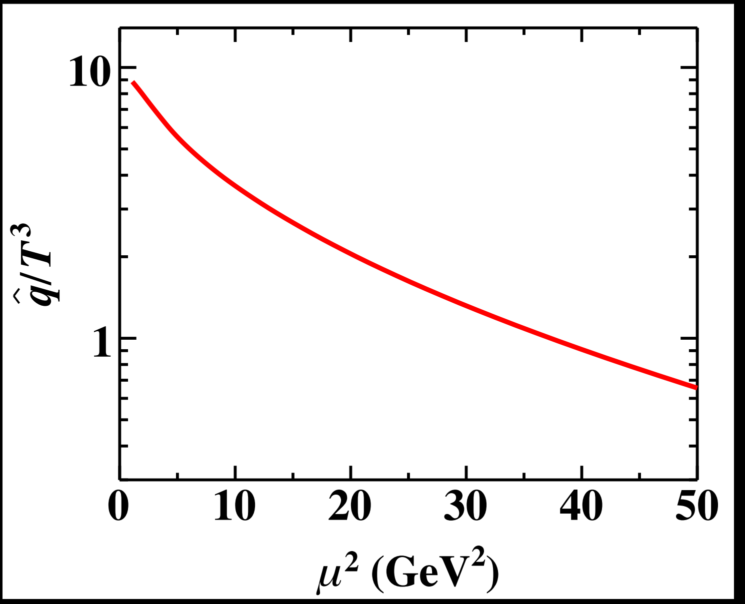


A. Kumar et al., 2204.01163 [hep-ph]

Leading hadrons and jets

At all energies and centralities

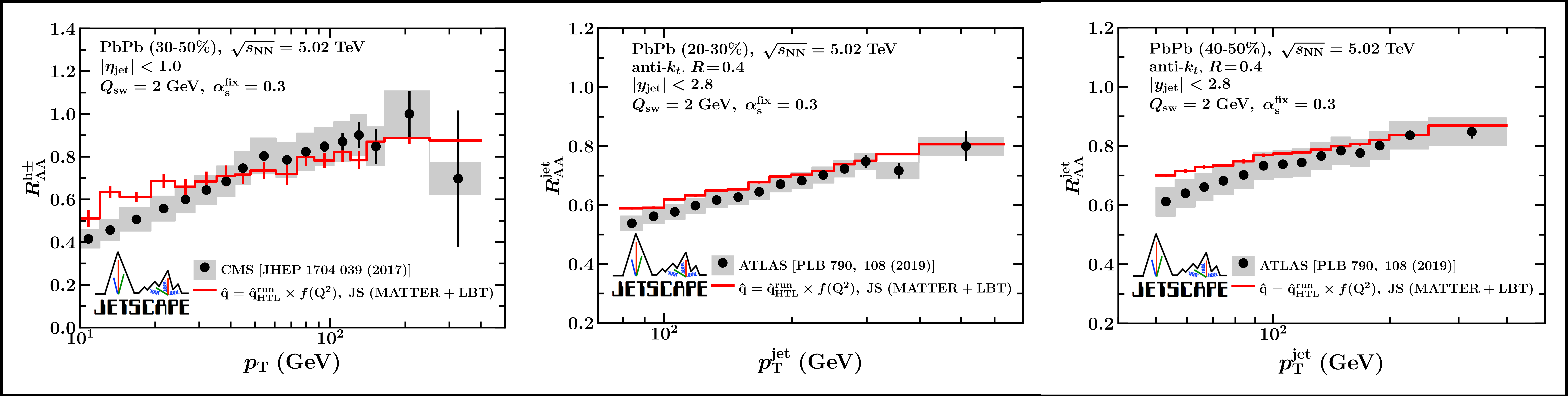
- $\hat{q} = C\alpha_s(2ET)\alpha_s(m_D)T^3 \log\left(\frac{2ET}{m_D^2}\right) \times f(Q^2)$



A. Kumar et al., 2204.01163 [hep-ph]

Centrality

Parameters set in central Pb-Pb at 5 TeV



Note: Quenching stops at 160MeV, no quenching in the hadronic phase,
Expect: low p_T to be less quenched in both jets and leading hadrons

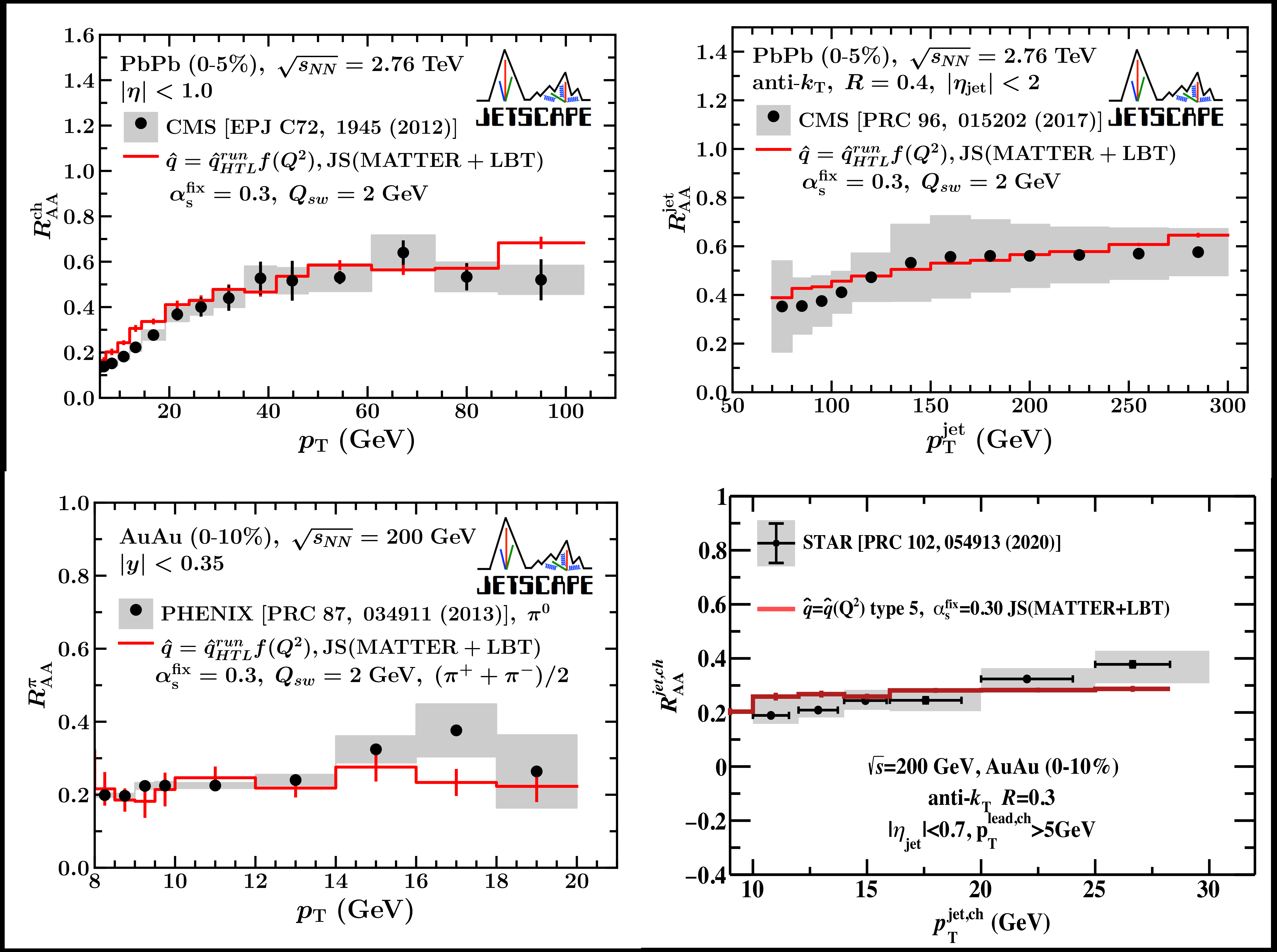
A. Kumar et al., 2204.01163 [hep-ph]

Energy dependence at LHC 2.76 and RHIC 0.2

- Jet and leading hadron RAA show remarkable agreement with experimental data

- Across most centralities and all energies

- No re-tuning or refitting of $\hat{q}$, $C(k)$ or recoil systematics



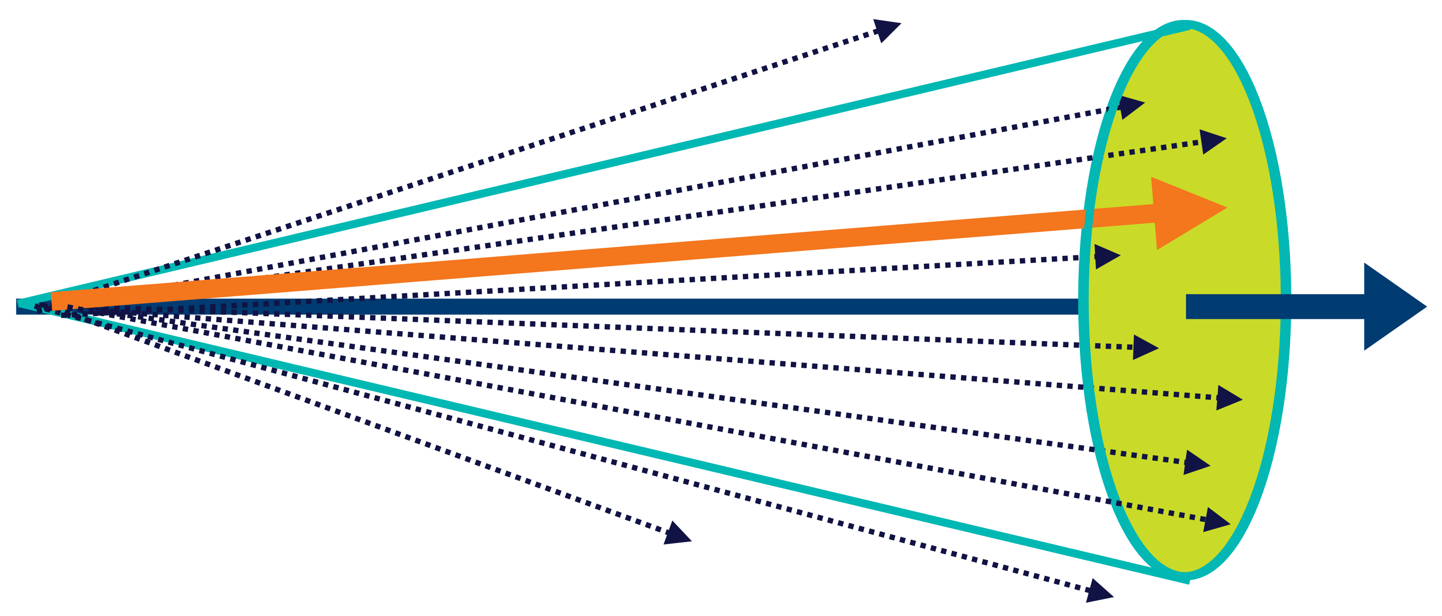
A. Kumar et al., 2204.01163 [hep-ph]

Intrajet

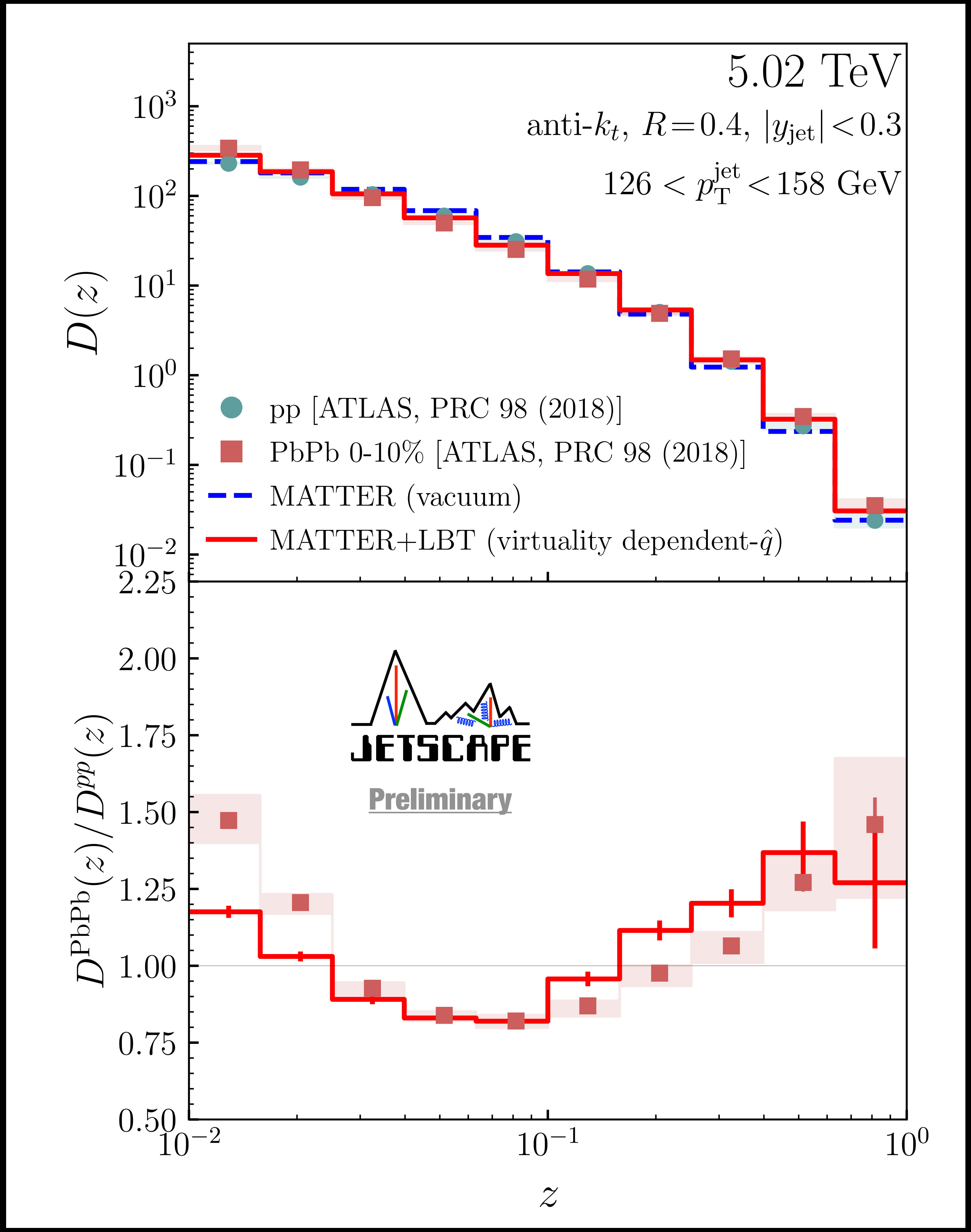
The dependence on E and μ not completely settled

This will probably get done in an upcoming Bayesian analysis

$$D(z) = \frac{1}{N_{\text{jet}}} \sum_{\text{jet}} \frac{dN_{\text{trk}}}{dz}\bigg|_{\text{in jet}}$$

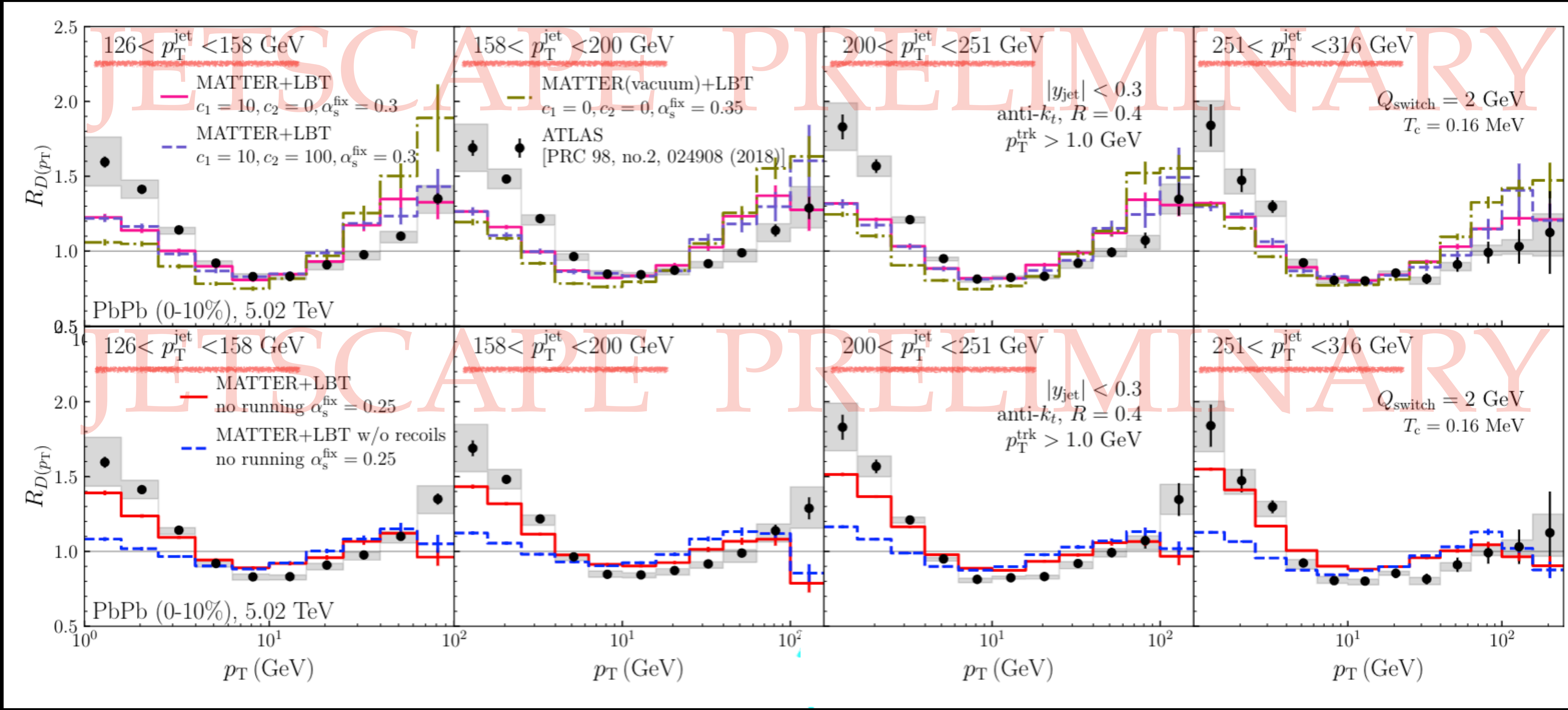


$$z = \frac{p_T^{\text{trk}}}{p_T^{\text{jet}}}$$

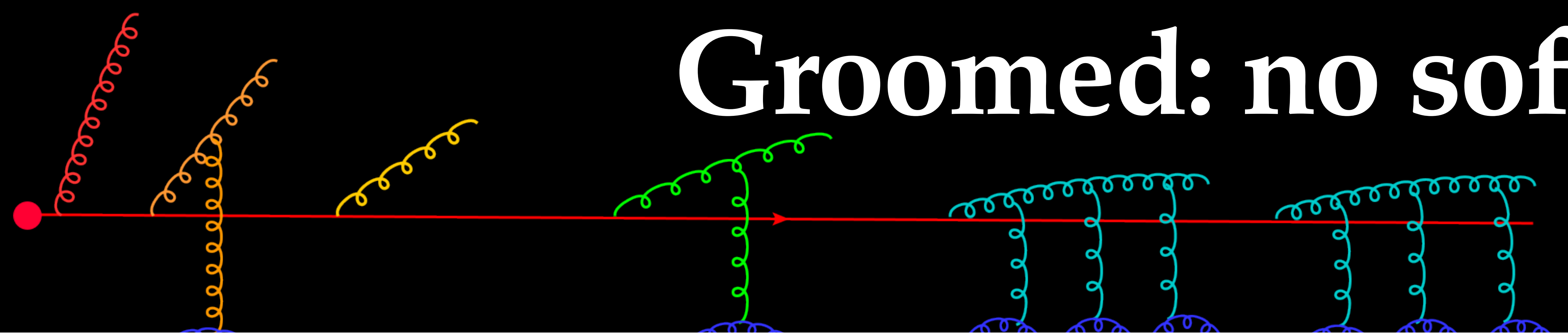


Y. Tachibana et al., *to appear*

Need for quenching in high Q stage



Y. Tachibana et al., *to appear*

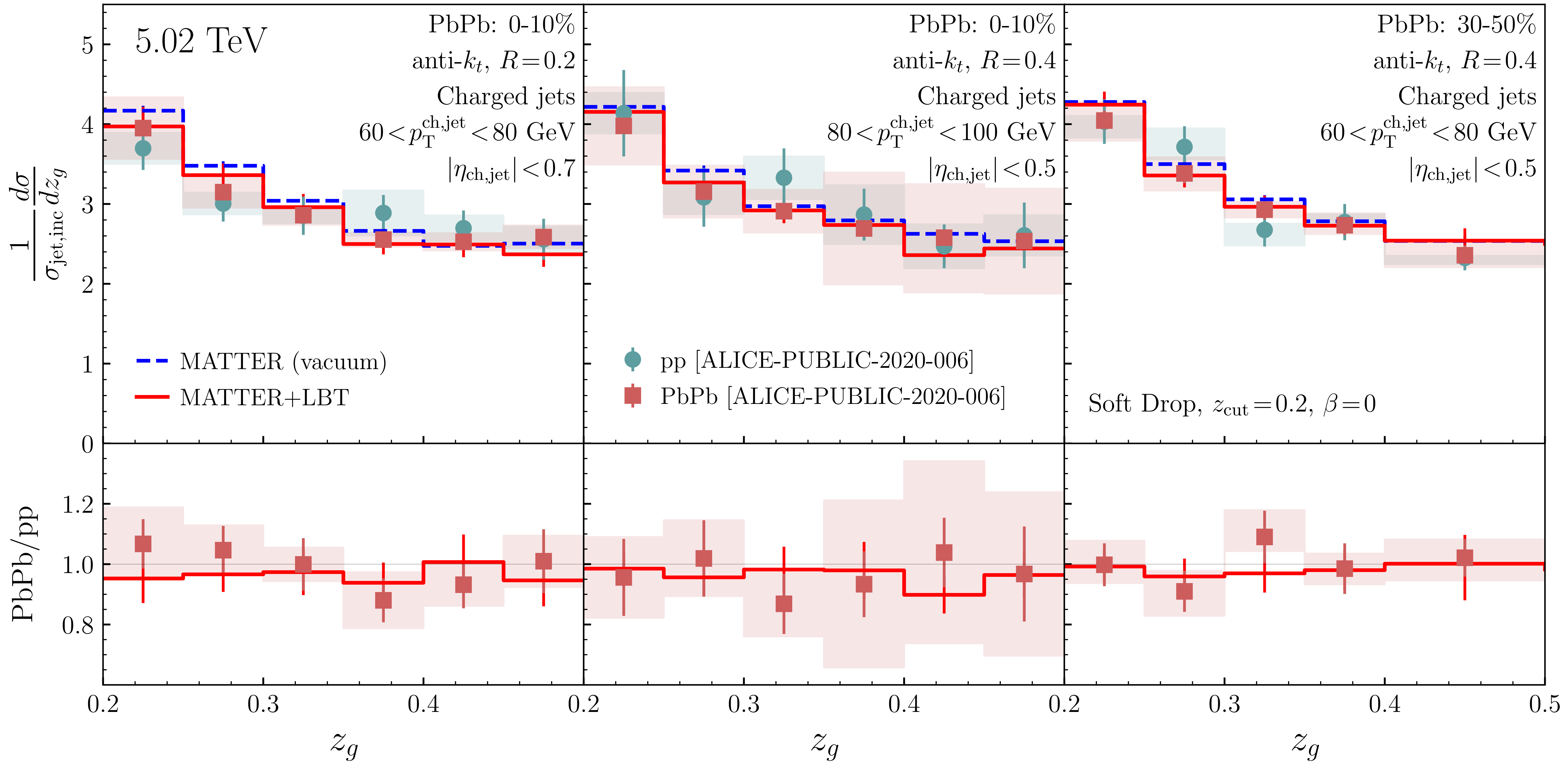


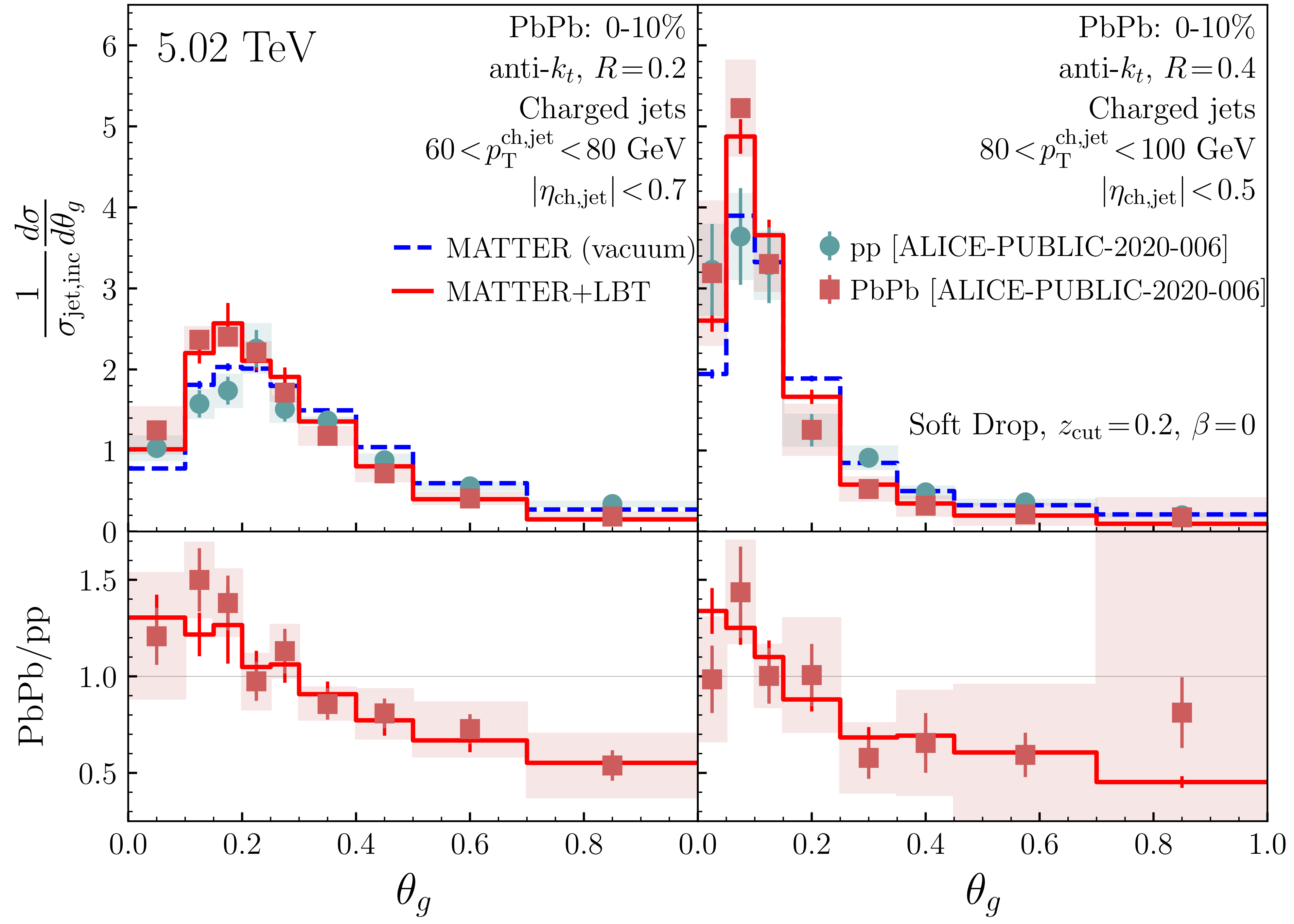
Groomed: no soft modes!

pp: MATTER (vacuum)

PbPb: MATTER+LBT running-α_S, Q^2dependent

$\alpha_S^{\text{fix}} = 0.3, Q_0 = 2\,\text{GeV}, \hat{q}$-paramerization: 5

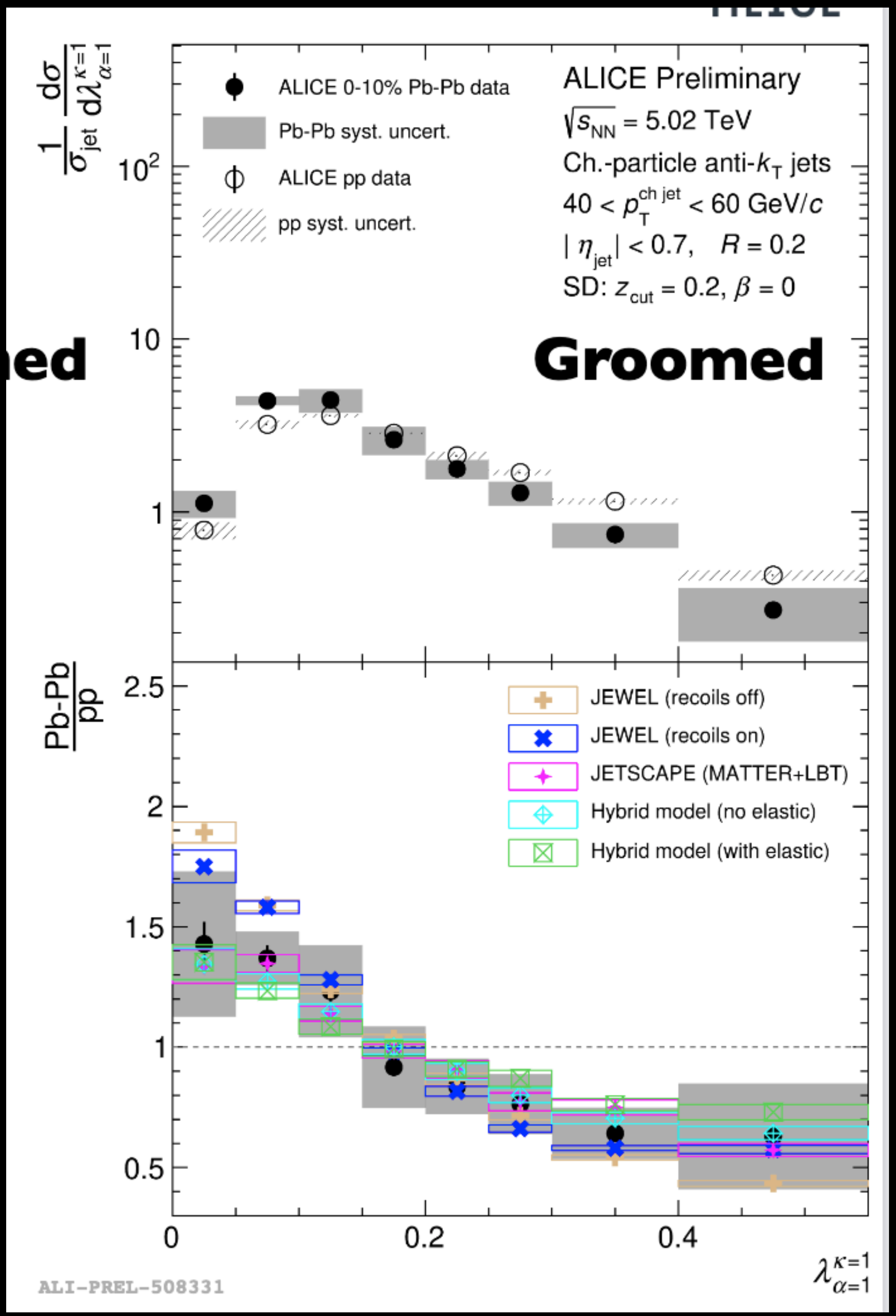




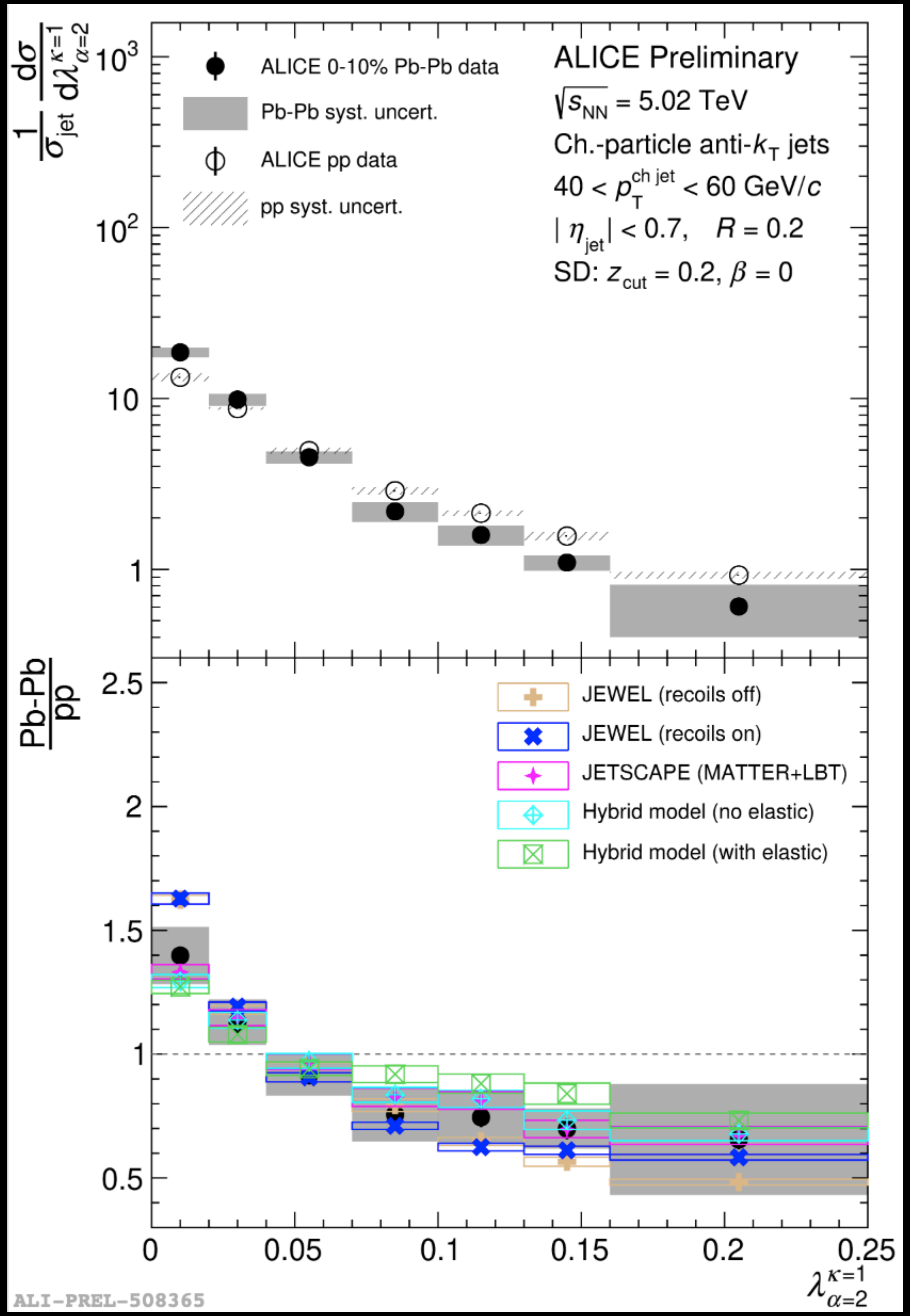
- Soft drop: getting rid of the soft response and looking at the prong structure

Y. Tachibana et al., *to appear*

Groomed Jet angularities

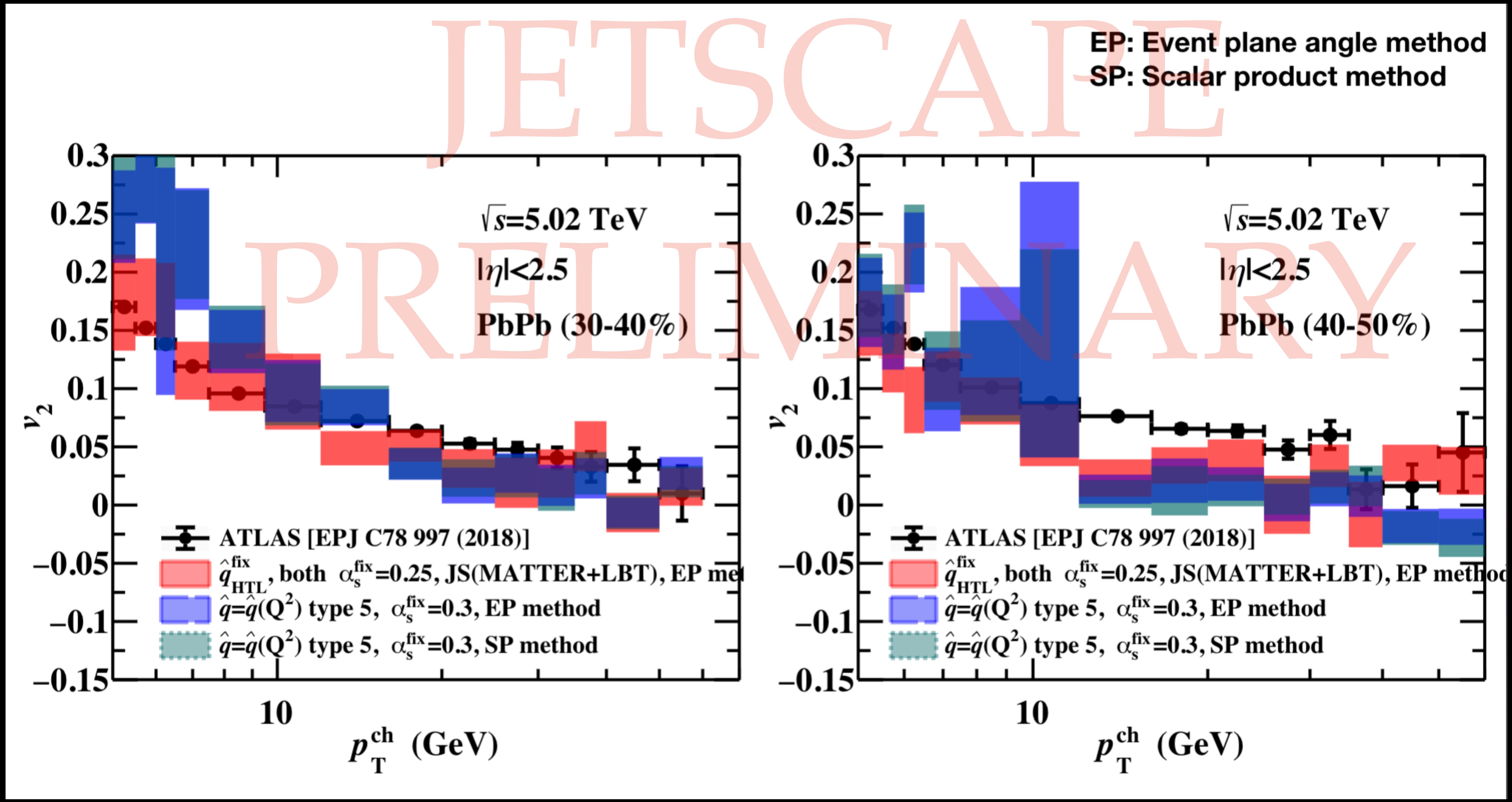


- $\lambda = \sum_{i \in Groomed} z_i \theta_i^{\alpha}$
- Strong constraints on the perturbative part of jet
- Several other similar groomed observables
- JETSCAPE (MATTER +LBT) does very well.

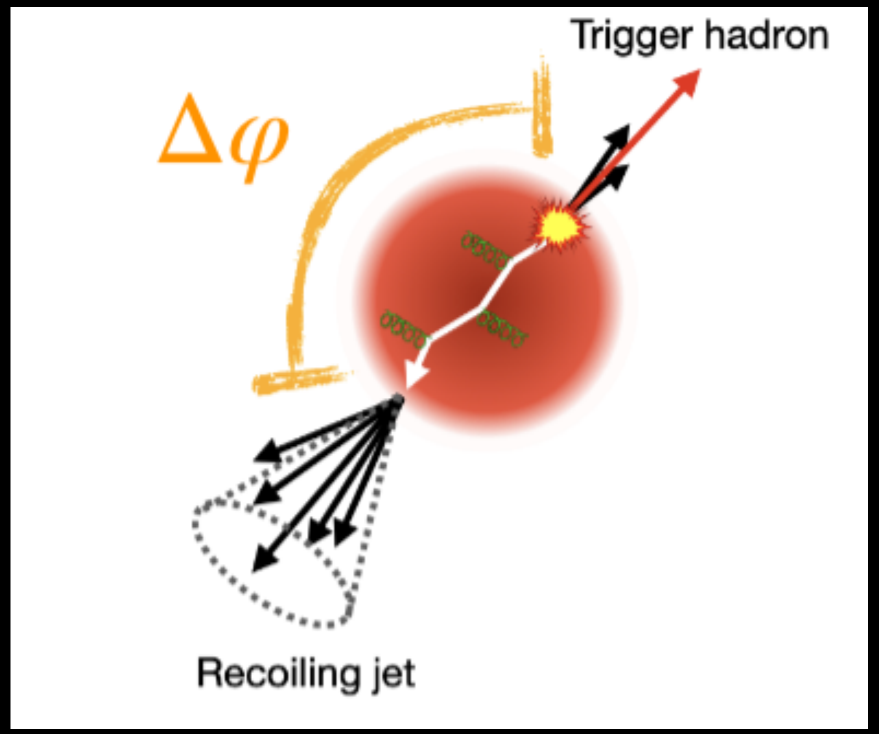


Azimuthal anisotropy

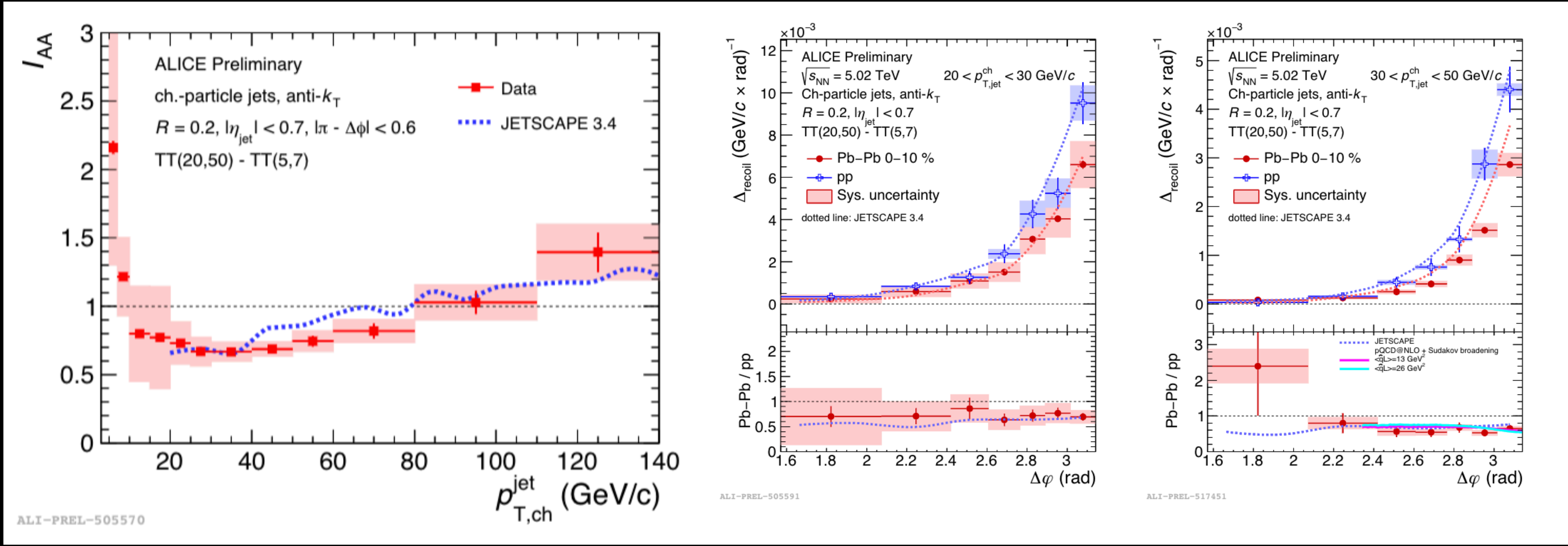
- Note: we haven't played with start and stop times (observation by C. Andres et al, start time important for v_2)
- In the JETSCAPE simulations, hydrodynamics starts around 1fm/c. (Free streaming prior)
- Also with new IP-Glasma, medium has primordial v_2
- Jet modification in the hadronic medium still not known



Coincidence with hadrons

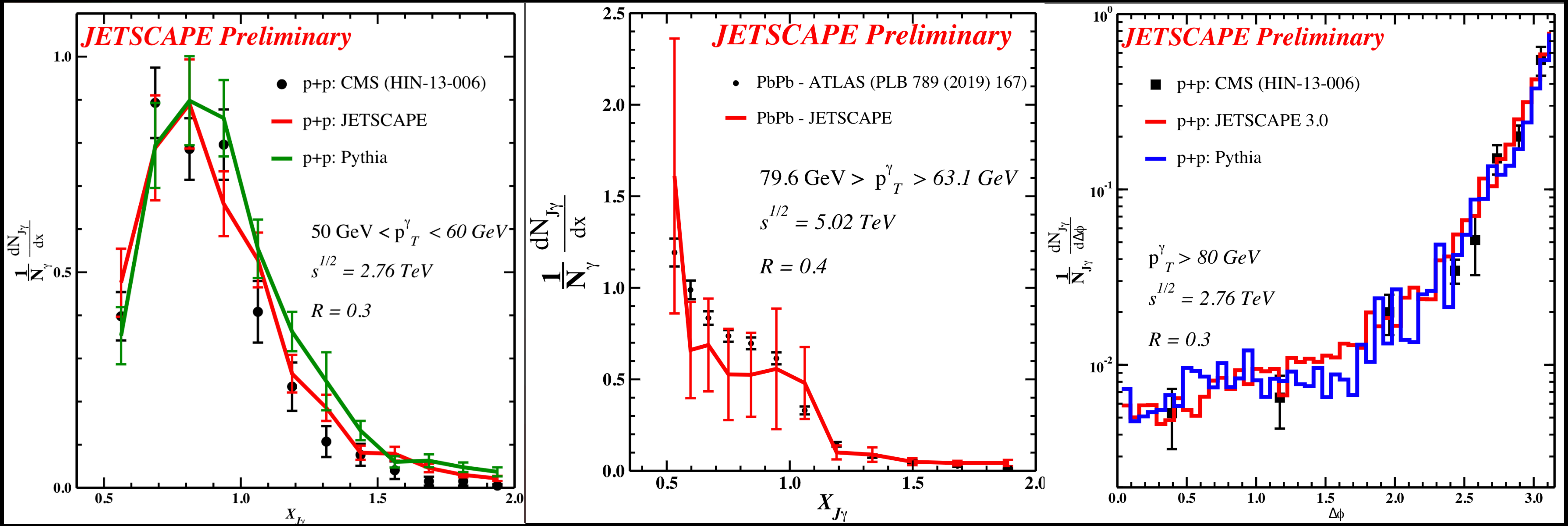


- Results from MATTER+LBT runs use for ratio of difference of triggered jet distribution per trigger.



Photon Trigger

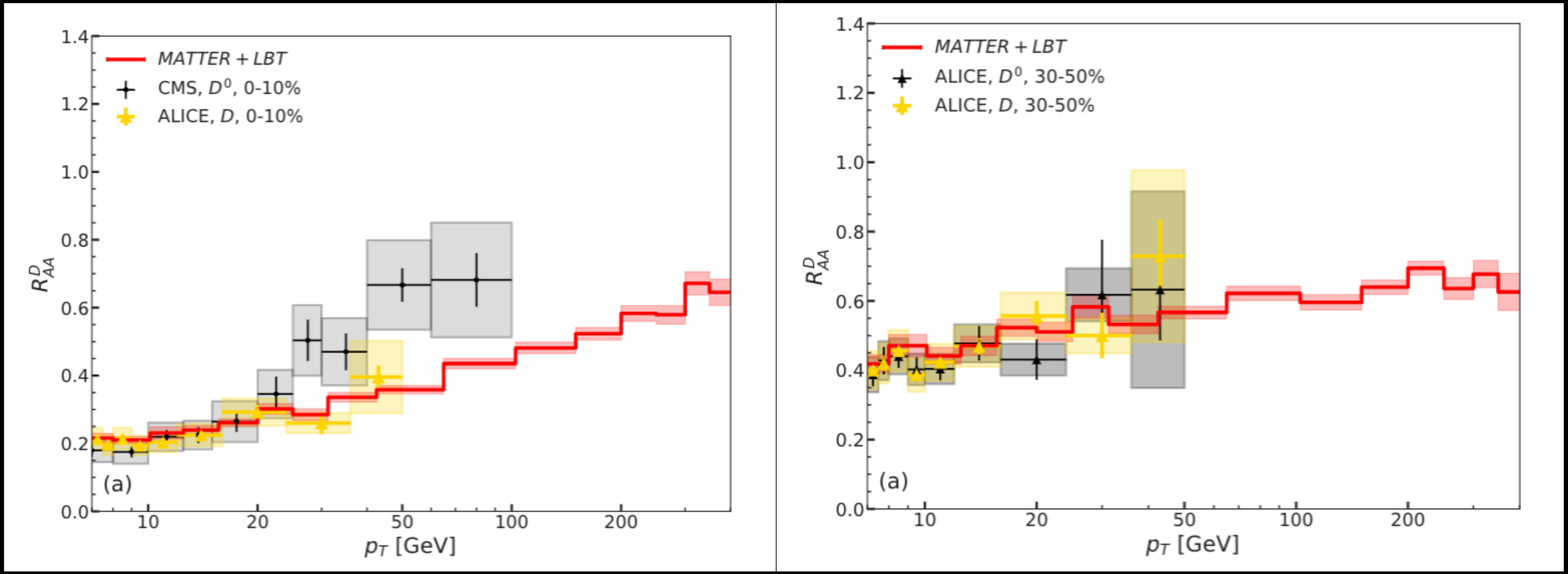
- Higher statistics runs with the exact same parameters as for jets.



C. Sirimanna, *to appear*.

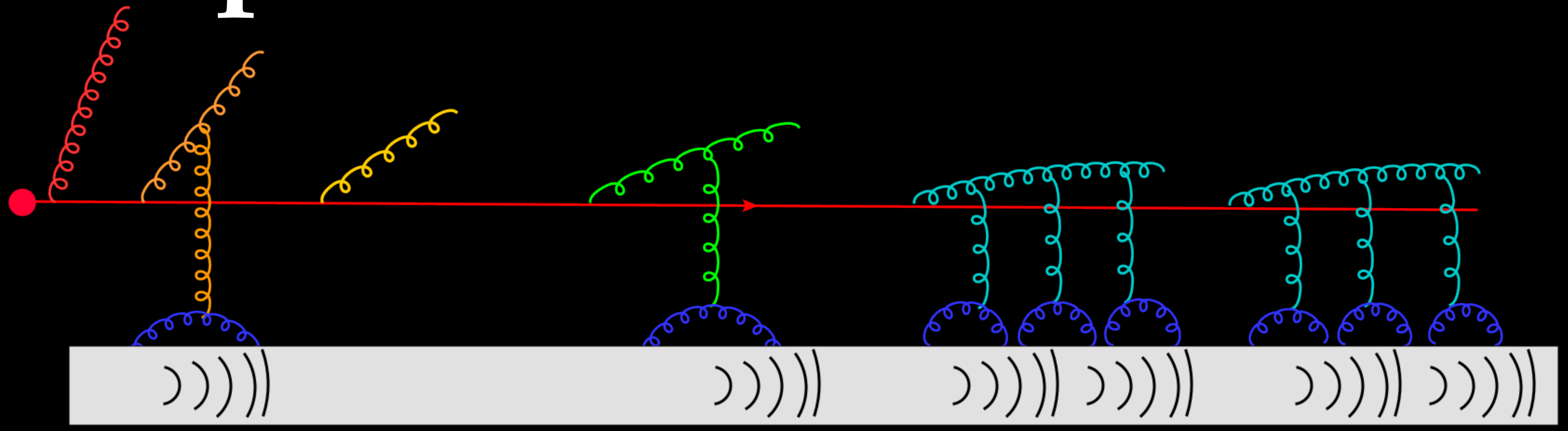
Heavy-quarks

- D meson R_{AA} with identical parameters

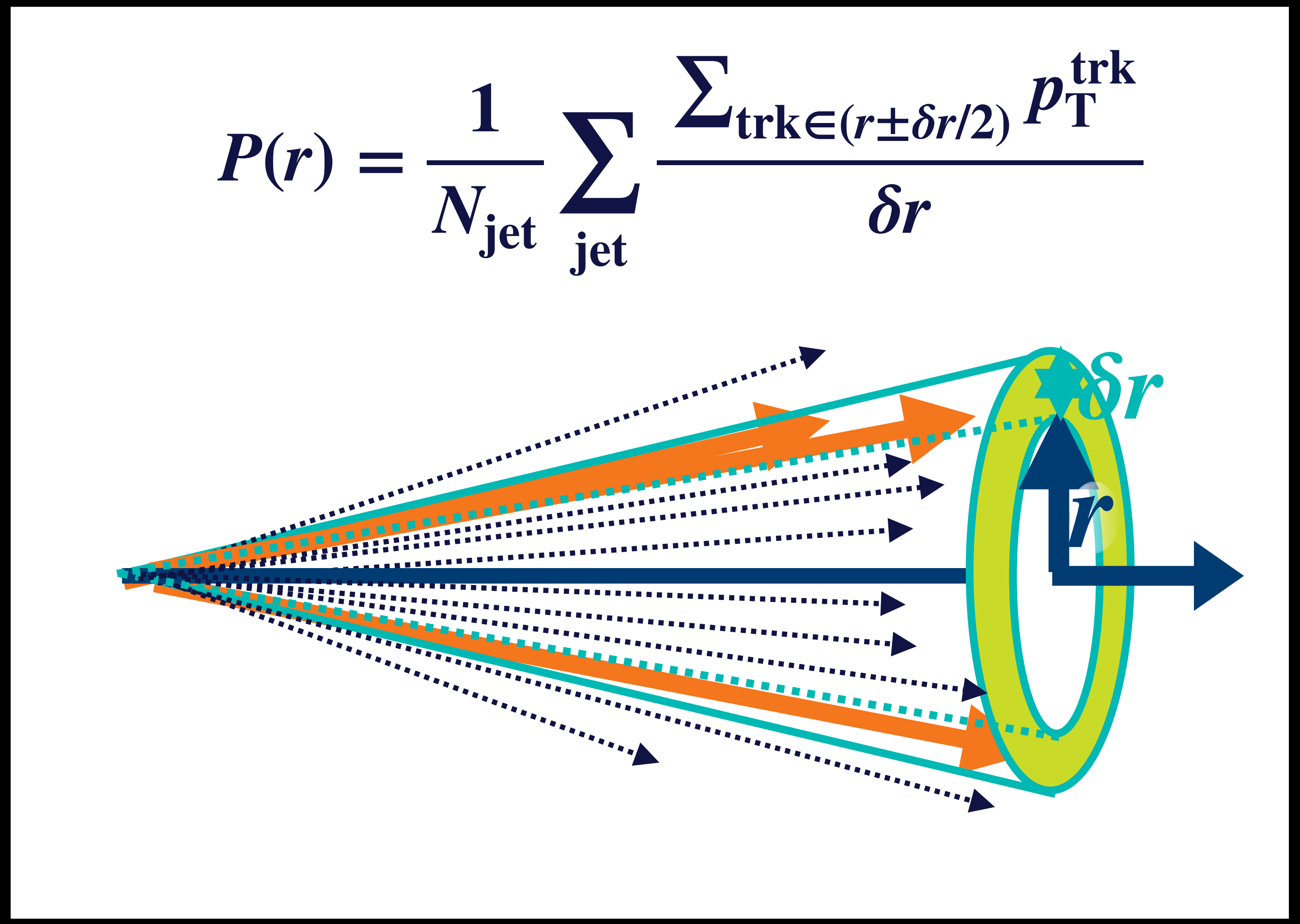


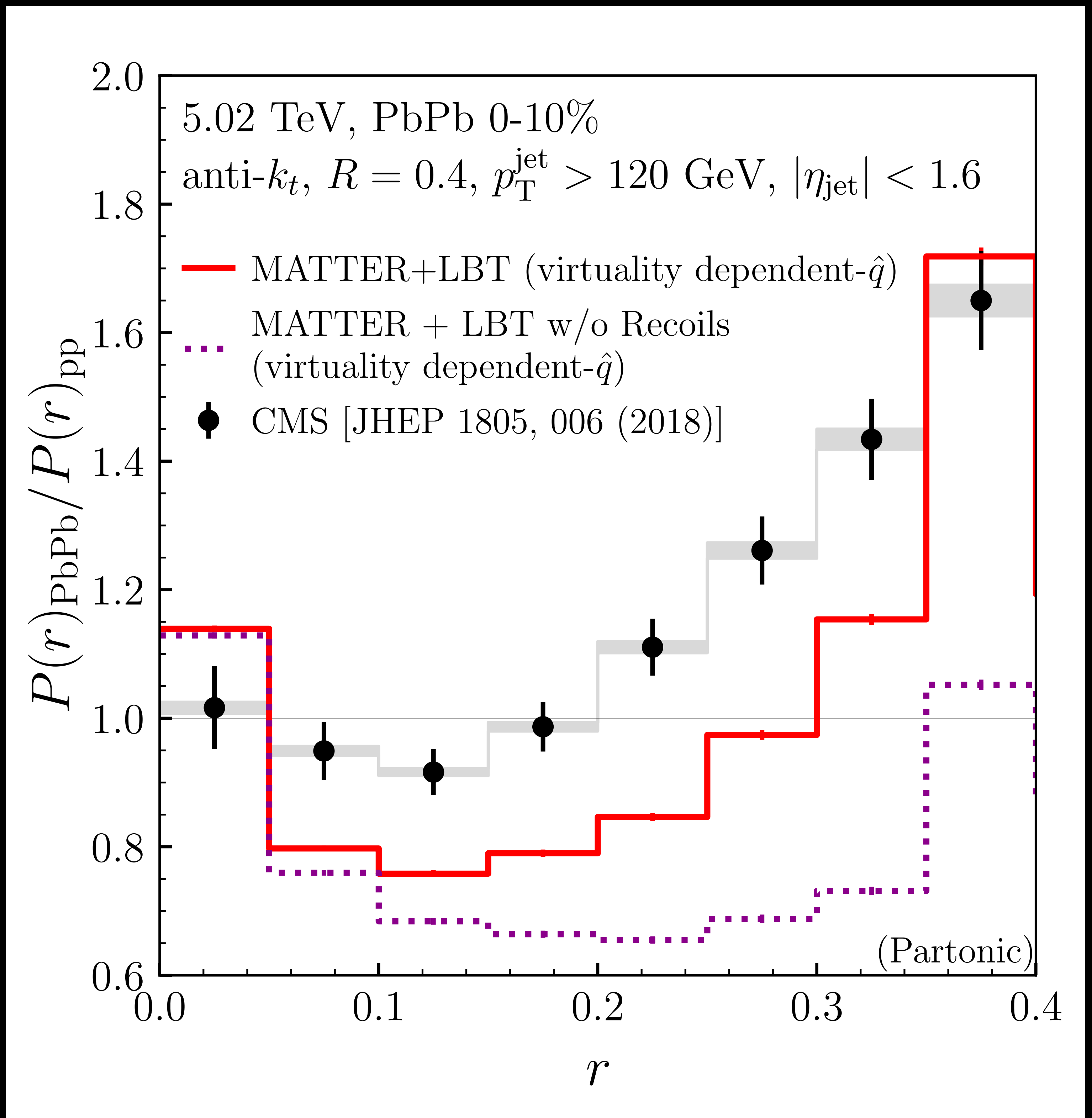
W. Fan, *et al*. e-Print: 2208.00983 [nucl-th]

Jet Shape: more dependence on soft modes



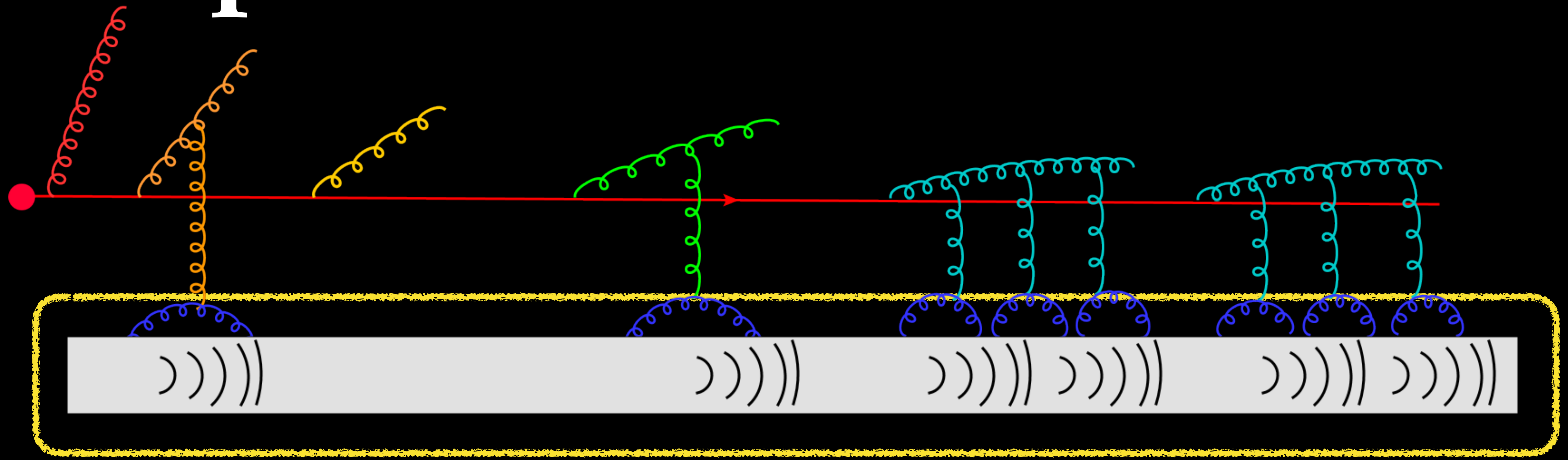
- Jet shape function:
- This depends more on soft non-perturbative modes, especially at larger angles
- Requires 2-stage hydro simulations (hydro+jet+hydro) for response outside jet.





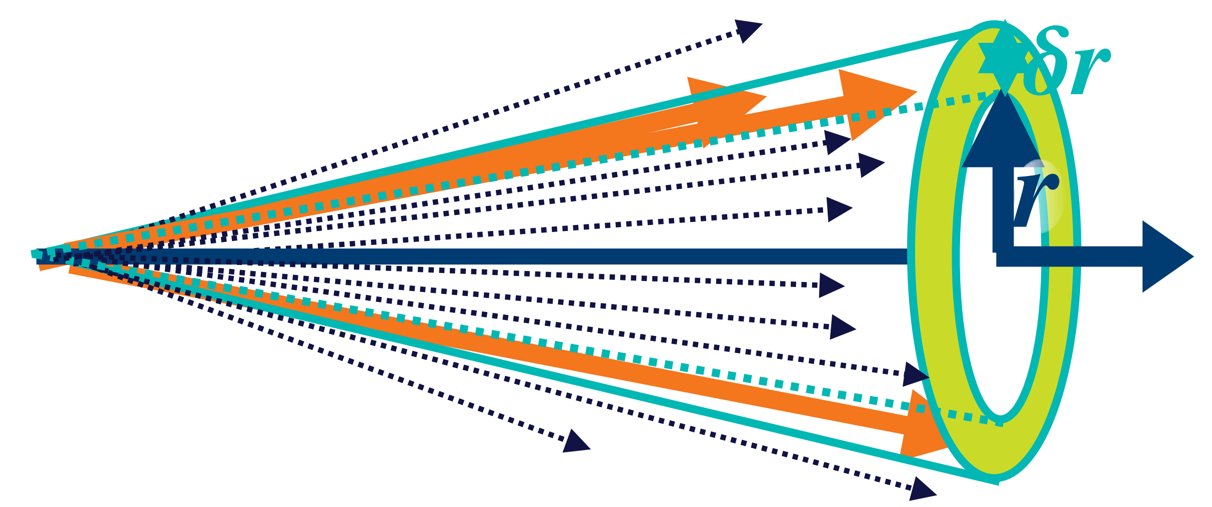
Y. Tachibana et al., *to appear*

Jet Shape: more dependence on soft modes

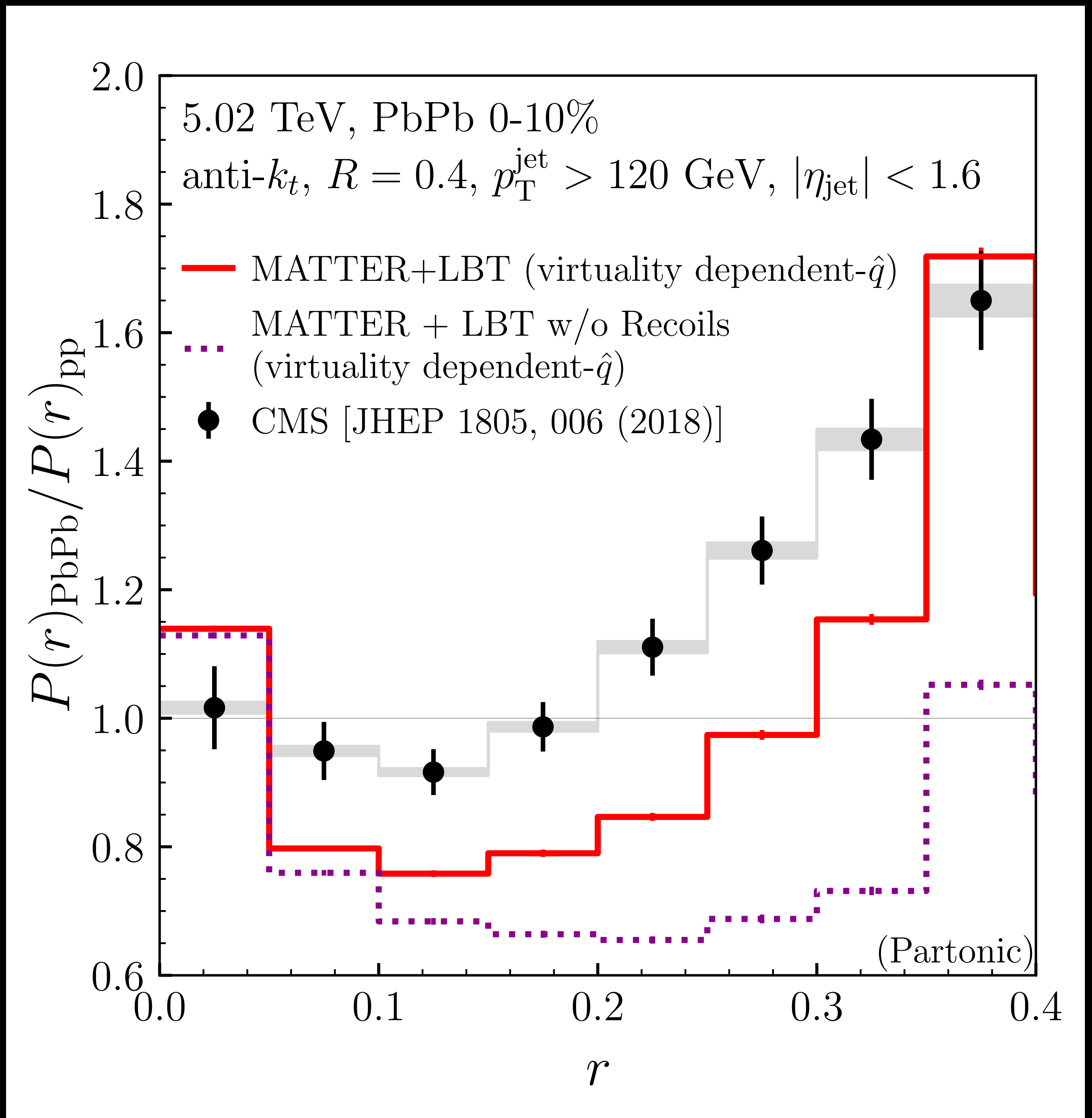


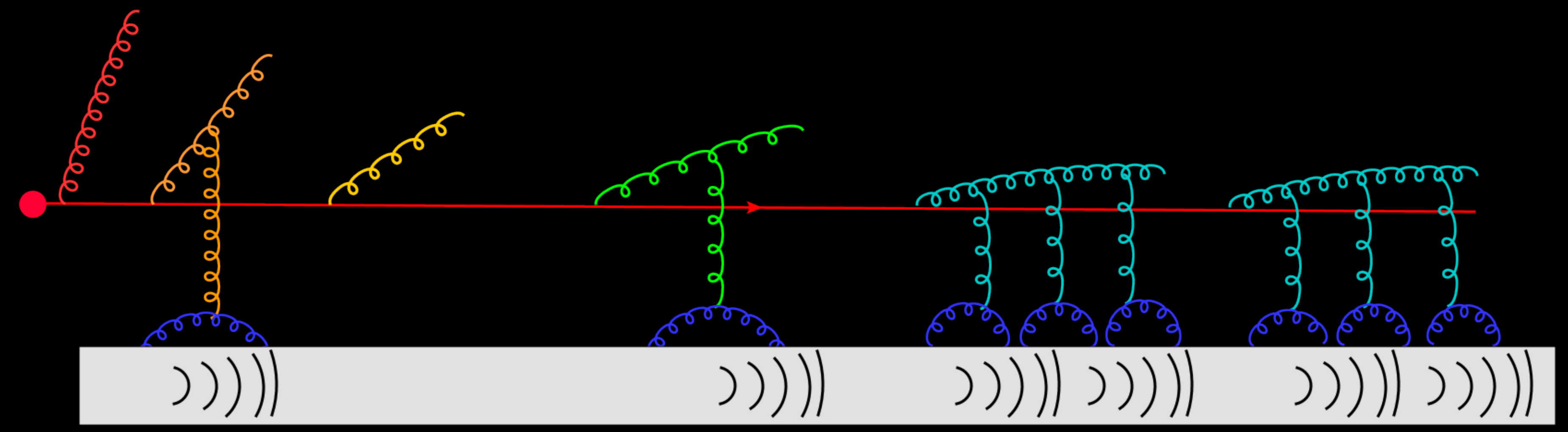
- Jet shape function:
- This depends more on soft non-perturbative modes, especially at larger angles
- Requires 2-stage hydro simulations (hydro+jet+hydro) for response outside jet.

$$P(r) = \frac{1}{N_{\text{jet}}} \sum_{\text{jet}} \frac{\sum_{\text{trk}\in(r\pm\delta r/2)} p_{\mathrm{T}}^{\text{trk}}}{\delta r}$$

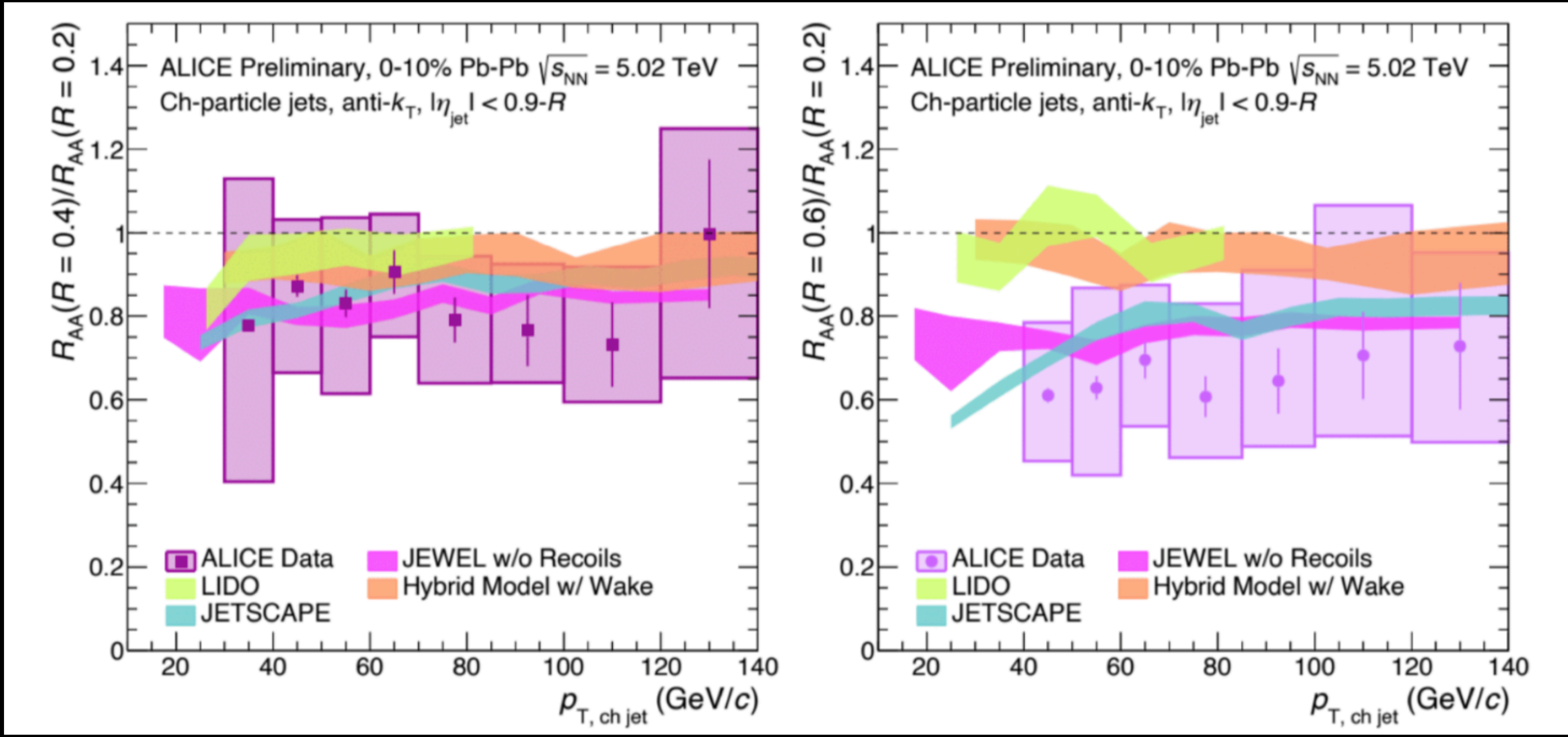


Y. Tachibana et al., *to appear*

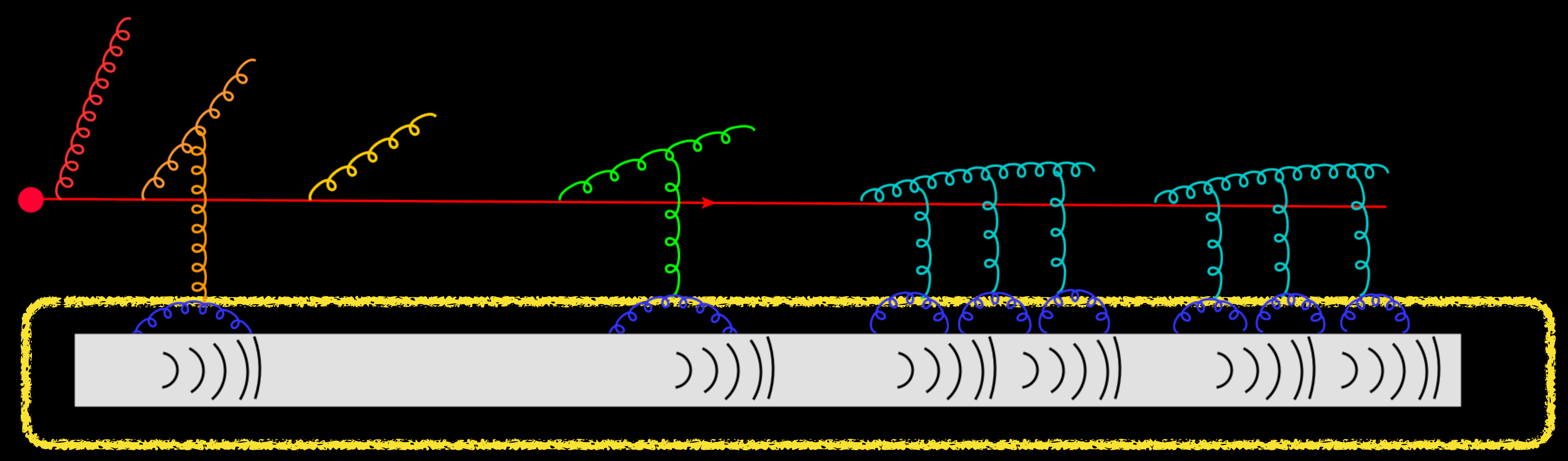


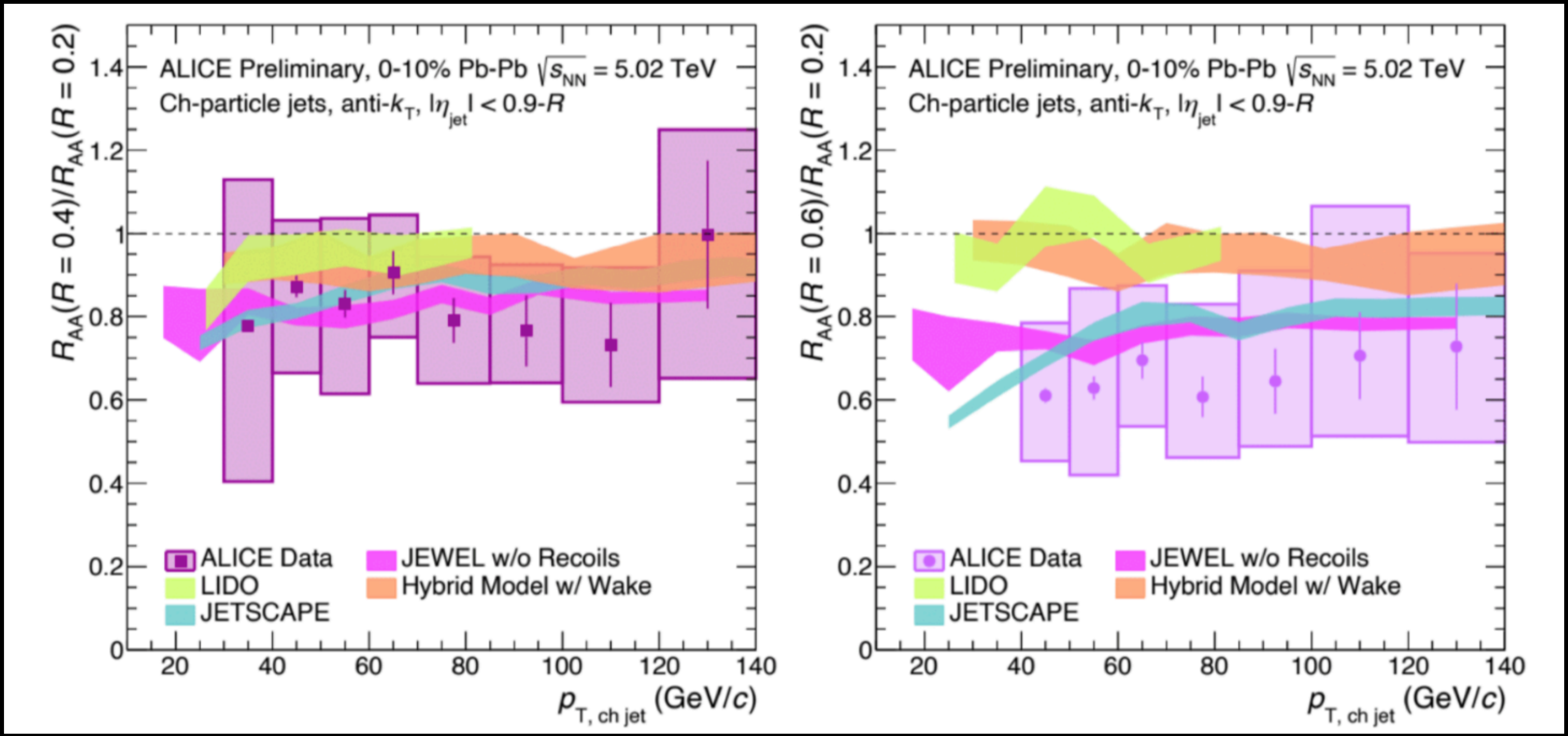


R dependence of R_{AA}



R dependence of R_{AA}





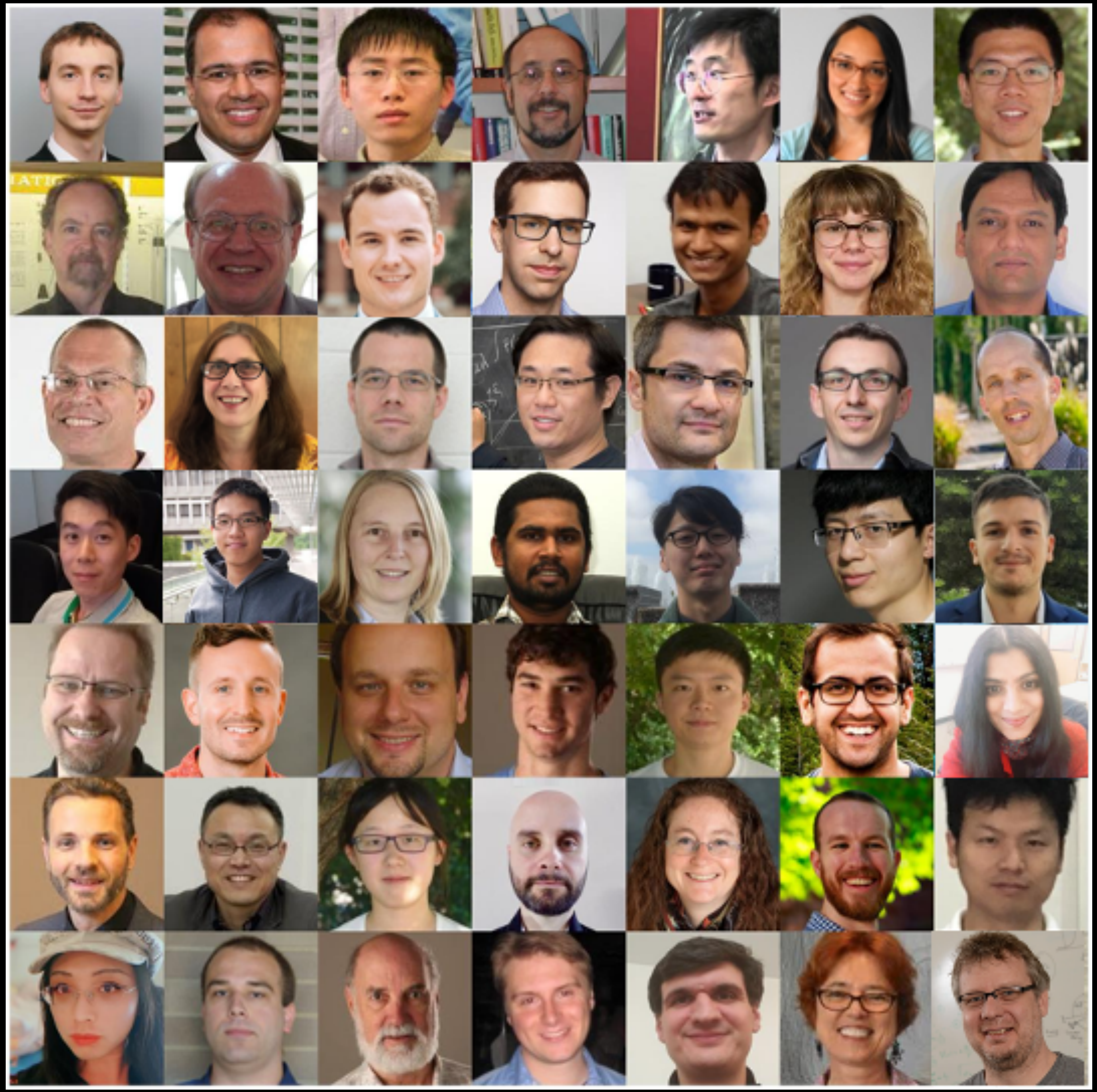
This is where we are now

- We added one more parameter Q_0, transition between high and low virtuality.
- Multi-stage set up seems to able to explain almost all the data
- The Bayesian calibration is being conducted as we speak
- Will rigorously test picture of 2-stage energy loss, with HTL based kernel at $\mu < Q_0$, and gradual weakening for $\mu > Q_0$
- A portion of the quenching will always be non-perturbative and subject to modeling!

Summary

- All simulations carried out on a calibrated fluid profile
- All simulations reproduce p-p on removal of medium
- All simulations have a consistent recoil and $\hat{q}$ incorporation
- The multi-stage (or scale dependent jet modification) is able to describe
    - Jet and leading hadrons simultaneously
    - Centrality dependence
    - Collision energy dependence
    - Intra jet observables
    - Coincidence with hadrons and photons
    - Heavy quarks
    - Azimuthal anisotropy
    - R dependence of R_{AA} (sort of)
- Minor effects still being studied in jet anisotropy, jet shapes etc.
- Is the medium made of quasi-particles or not? We are getting closer to answering this.

Thanks to my collaborators



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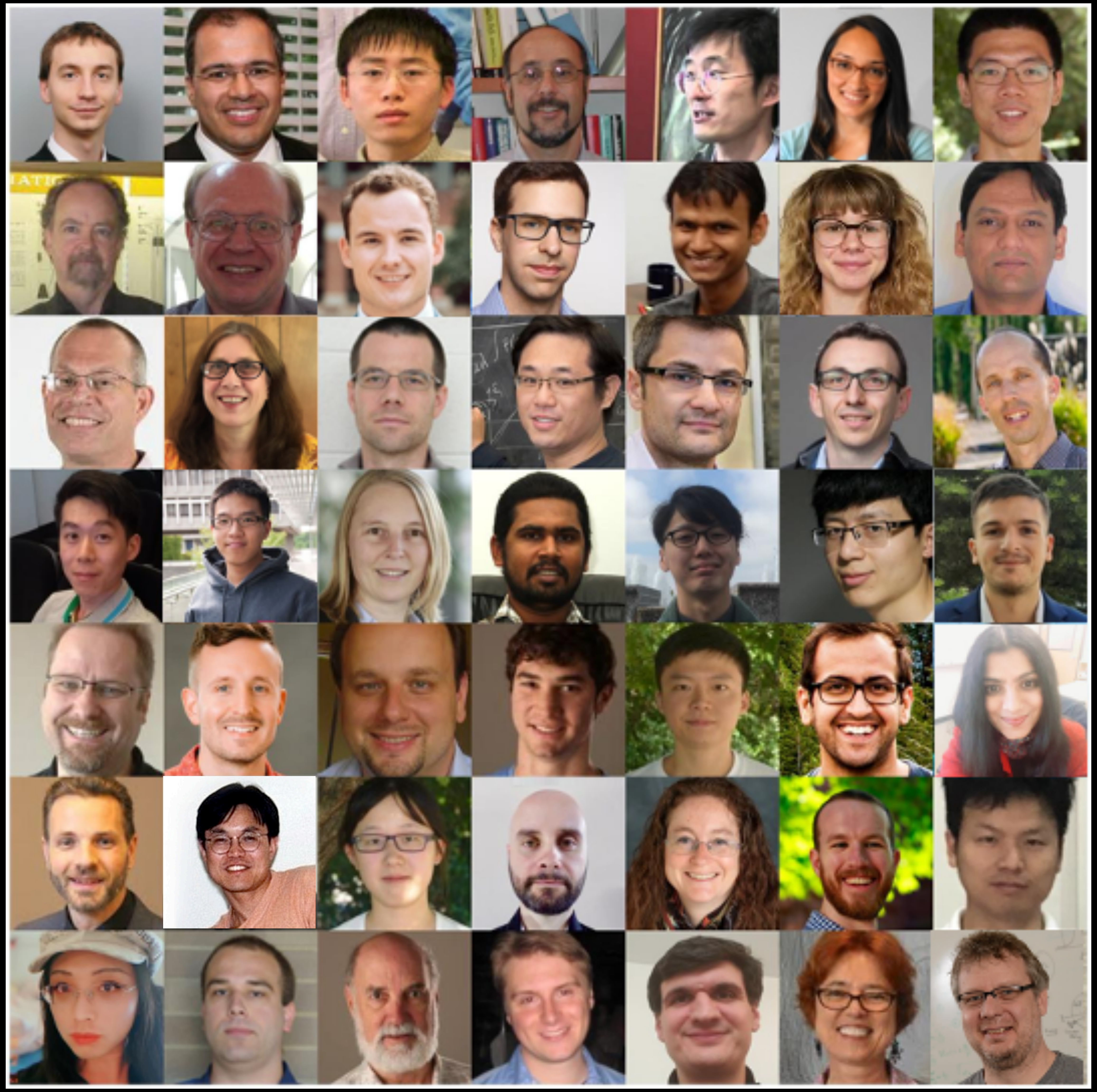
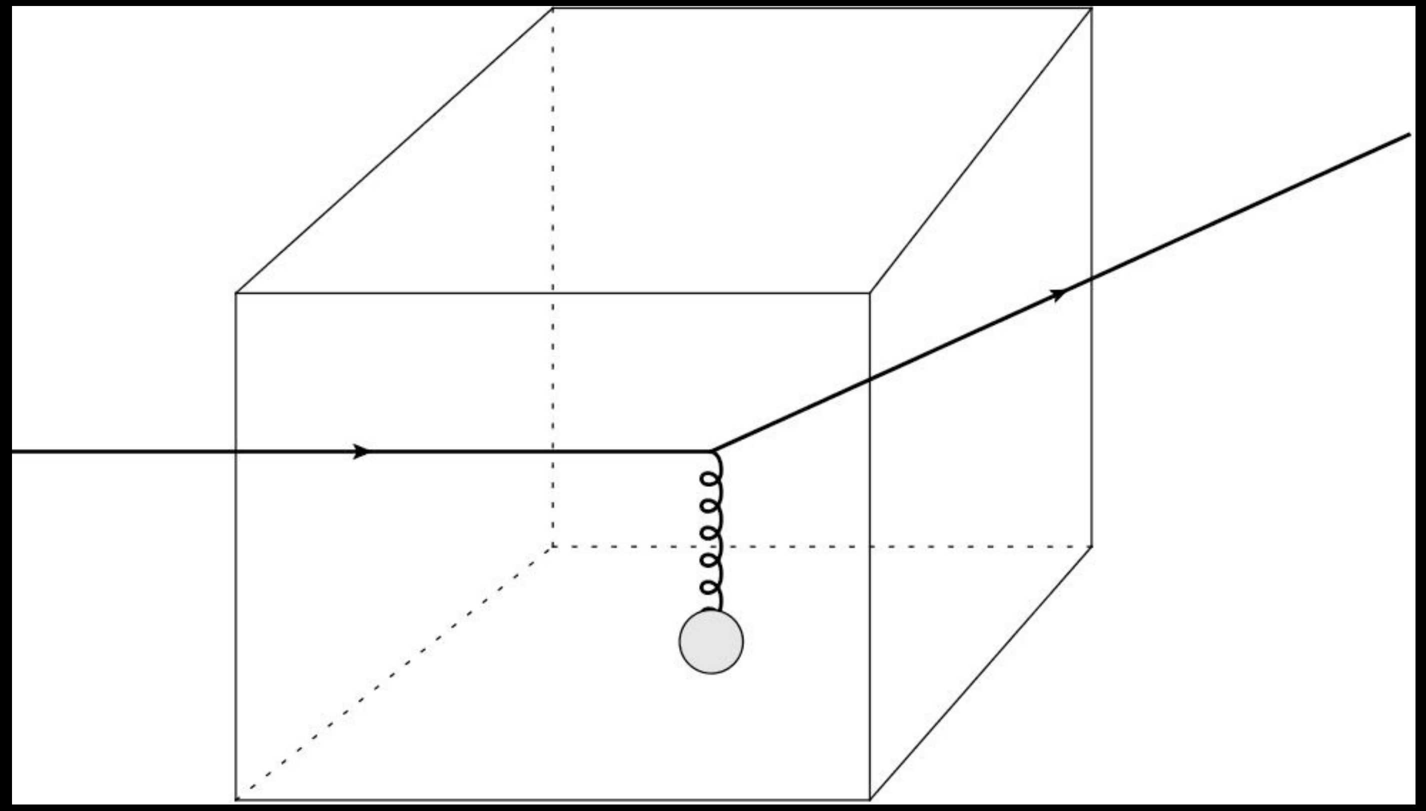


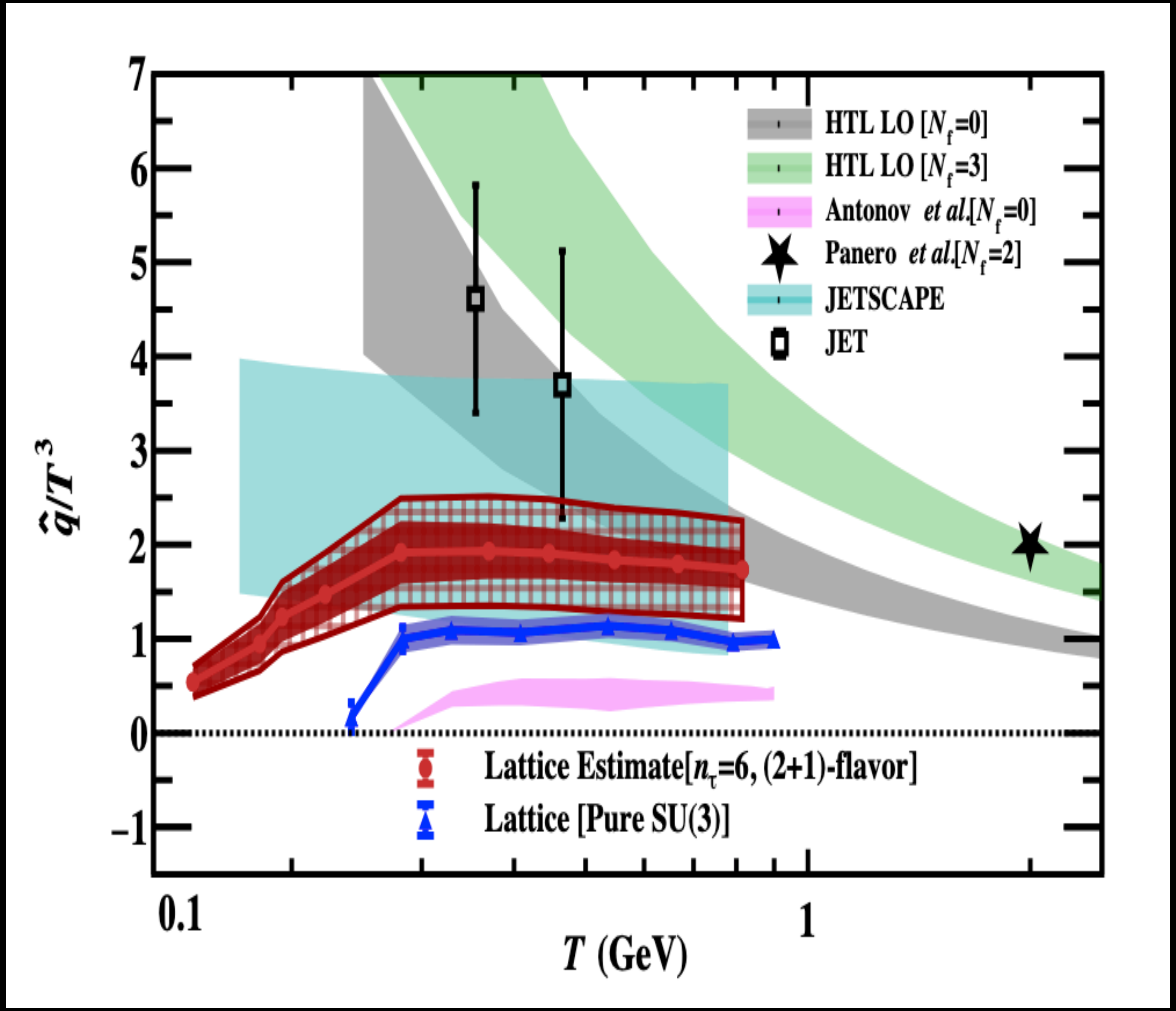
Calculating $\hat{q}$ in Lattice QCD

A. Kumar, A.M., J. Weber, arXiv:2010.14463



$$q^- = \frac{q^0 - q^3}{\sqrt{2}} \to \infty$$

$$q^+ = \frac{q^0 + q^3}{\sqrt{2}} \to 0$$



$$\hat{q} = \frac{4\pi^2\alpha_S}{N_c}\int\frac{dy^- d^2y_\perp}{(2\pi)^3} d^2k_\perp e^{-i\frac{k_\perp^2}{2q^-}y^- + i\vec{k}_\perp\cdot\vec{y}_\perp}$$

$$\times\sum_n \frac{e^{-\beta E_n}}{Z}\langle n|F_\perp^+(y^-,\vec{y}_\perp)F_\perp^+(0)|n\rangle$$

Fully non-perturbative calculation of $\hat{q}$

All calculations for a 100 GeV quark,

Lattice Calculations show weak dependence on E